

# Input Orthogonalization Methods for Third-Order MIMO Volterra Channel Identification

Carlos Alexandre R. FERNANDES<sup>1,2</sup>, Gérard FAVIER<sup>1</sup>, João Cesar M. MOTA<sup>2</sup>

<sup>1</sup>I3S Laboratory, CNRS - UNSA  
2000 route des Lucioles, BP 121, 06903 Sophia-Antipolis Cedex.

<sup>2</sup>GTEL Laboratory - Universidade Federal do Ceará  
Campus do Pici, 60.755-640, 6007 Fortaleza, Brazil.

{acarlos,favier}@i3s.unice.fr, {carlosalexandre,mota}@deti.ufc.br

**Résumé** – Deux méthodes d’orthogonalisation des signaux d’entrée pour l’identification supervisée et aveugle de canaux de communication MIMO de type Volterra sont considérées dans cet article. Dans le cas supervisé, la méthode proposée consiste en une extension de l’utilisation des polynômes orthogonaux aux systèmes de Volterra MIMO. Dans le cas aveugle, on présente une solution originale pour l’identification des canaux à l’aide de codes de modulation avec des signaux PSK. Des algorithmes d’estimation de canal basés sur l’utilisation de ces méthodes d’orthogonalisation sont présentés et leurs performances sont évaluées à l’aide de simulations numériques.

**Abstract** – Input orthogonalization methods for supervised and blind identification of third-order MIMO Volterra communication channels are considered. In the supervised case, we extend the use of orthonormal polynomials to MIMO Volterra systems. In the blind case, we propose an original solution for channel identification based on the use of state-dependent modulation code schemes with PSK signals. Several channel estimation methods using input orthogonalization are presented and their performances are evaluated by means of computational simulations.

## 1 Introduction

In this paper, two input orthogonalization methods are considered for identifying third-order Multiple-Input-Multiple-Output (MIMO) Volterra channels. This kind of nonlinear models has important applications in the field of telecommunications to model wireless communication links with nonlinear power amplifiers [1] and uplink channels in Radio Over Fiber (ROF) multiuser communication systems [2].

The first method assumes the knowledge of the transmitted signals, i.e. a supervised scenario. In this case, the Least Mean Square (LMS) algorithm exhibits a slow convergence speed and the Minimum Mean Squared Error (MMSE) estimate in a block processing (off-line) scheme suffers from an ill-conditioning that can be the source of numerical problems. The diagonalization of the zero delay covariance matrix of the nonlinear regression vector is required in order to overcome these effects and improve the channel estimation. The method presented in this paper is based on the development of orthonormal polynomials for the transmitted signals. Several works have used similar approaches for Single-Input-Single-Output (SISO) Volterra systems, exploiting the advantages of an input orthonormalization in an adaptive context [3], or in a block processing scheme [4]. This paper extends the procedure of construction and the use of orthonormal polynomials to the case of MIMO Volterra systems, allowing different probability density functions (pdf) for the input signals.

The second orthogonalization method considers a blind channel estimation scenario. The identifiability conditions for blind estimation of linear MIMO mixtures from second order statistics (SOS) are not sufficient to ensure the identifiability of a MIMO Volterra system. The proposed method performs the diagonalization of covariance matrices of the nonlinear regression vector for various delays, with the goal of ensuring some identifiability conditions. In this case, the communication channel is assumed to be memoryless. State-dependent modulation codes (constrained codes) [5] are used to ensure the orthogonality of products of the delayed transmitted signals for several time delays, leading to a Parallel Factor (PARAFAC) decomposition [6] of a tensor composed of estimated spatio-temporal covariance matrices.

## 2 Channel Model

The proposed orthogonalization methods require multiple observations at the receiver, which can be obtained through oversampling or an antenna array. The sampled baseband equivalent model of the nonlinear communication channel is assumed to be expressed as:

$$x^{(i)}(n) = \sum_{m_1=1}^{MR} h_1^{(i)}(m_1) \bar{s}_{m_1}(n) + \sum_{m_1=1}^{MR} \sum_{m_2=m_1}^{MR} \sum_{m_3=1}^{MR} h_3^{(i)}(m_1, m_2, m_3) \bar{s}_{m_1}(n) \bar{s}_{m_2}(n) \bar{s}_{m_3}^*(n) + v^{(i)}(n), \quad (1)$$

where  $x^{(i)}(n)$  is the  $i^{th}$  signal measured at the time instant  $n$  with  $i = 1, 2, \dots, I$ ,  $I$  is the number of observations (antennae or oversampling rate),  $M$  is the number of users,  $R$  is the system memory,  $\bar{s}_{m_j}(n)$  corresponds to the  $m_j^{th}$  component of the vector  $\bar{\mathbf{s}}(n) = [s_1(n) \dots s_1(n-R+1) \dots s_M(n) \dots s_M(n-R+1)]^T$ ,  $s_m(n)$ , for  $1 \leq m \leq M$ , are the stationary, complex-valued and mutually independent signals transmitted by the users,  $h_{2k+1}^{(i)}(m_1, \dots, m_{2k+1})$ , for  $k = 0, 1$ , are the kernel coefficients and  $v^{(i)}(n)$  is an Additive White Gaussian Noise (AWGN). The noise components  $v^{(i)}(n)$ ,  $1 \leq i \leq I$ , are assumed to be zero mean, independent from each other and from the transmitted signals  $s_m(n)$ .

The received signals can be expressed in a vector form:  $\mathbf{x}(n) = \mathbf{H}\mathbf{s}(n) + \mathbf{v}(n)$ , where  $\mathbf{x}(n) \in \mathbb{C}^{I \times 1}$ ,  $\mathbf{H} \in \mathbb{C}^{I \times M_V}$  and  $\mathbf{v}(n) \in \mathbb{C}^{I \times 1}$  contains respectively, the received signals, the channel coefficients and the noise components, and the nonlinear regression vector  $\mathbf{s}(n) \in \mathbb{C}^{M_V \times 1}$  contains the linear  $\{\bar{s}_{m_1}(n)\}$  and cubic terms  $\{\bar{s}_{m_1}(n)\bar{s}_{m_2}(n)\bar{s}_{m_3}^*(n)\}$ ,  $M_V$  being the number of coefficients of each sub-channel in (1), given by  $M_V = \sum_{k=0}^1 C_{MR,k} C_{MR,k+1}$ , where  $C_{MR,k} = \frac{(MR+k-1)!}{(MR-1)!k!}$ .

### 3 Input Orthonormalization for Supervised Identification

In this section, a set of orthonormal polynomials is developed in order to improve the performance of the LMS and block MMSE estimates of the channel given by (1). This orthonormalization process extends the method of construction of orthonormal polynomials to the case of MIMO Volterra systems, allowing different probability density functions (pdf) for the input signals, which are assumed to be white in the supervised case. Similar approaches have been considered by some authors, in the case of Single-Input-Single-Output systems [3, 4] and in the case of real-valued MIMO Wiener systems [7].

#### 3.1 Orthonormal Polynomials

The orthonormalization problem consists in finding a lower triangular matrix  $\mathbf{W} \in \mathbb{C}^{M_V \times M_V}$  so that  $\check{\mathbf{s}}(n) = \mathbf{W}\mathbf{s}(n)$ , satisfying  $\mathbf{R}_{\check{\mathbf{s}}\check{\mathbf{s}}} = \mathbf{W}\mathbf{R}_{\mathbf{s}\mathbf{s}}\mathbf{W}^H = \mathbf{I}_{M_V}$ , where  $\mathbf{R}_{\check{\mathbf{s}}\check{\mathbf{s}}} = \mathbb{E}[\check{\mathbf{s}}(n)\check{\mathbf{s}}^H(n)]$ ,  $\mathbf{R}_{\mathbf{s}\mathbf{s}} = \mathbb{E}[\mathbf{s}(n)\mathbf{s}^H(n)]$  and  $\mathbf{I}_{M_V}$  is the  $M_V \times M_V$  identity matrix. So, we may write  $\mathbf{x}(n) = \mathbf{F}\check{\mathbf{s}}(n) + \mathbf{v}(n)$ , with  $\mathbf{F} = \mathbf{H}\mathbf{W}^{-1}$  being the channel matrix associated with the orthonormal polynomials. The components of  $\check{\mathbf{s}}(n)$  are multivariable functions of the delayed inputs that can be expressed as products of monomials, denoted  $\{T_{\alpha_l, \beta_l}(\bar{s}_l)\}$ , where  $T_{\alpha_l, \beta_l}(\bar{s}_l) = (\bar{s}_l)^{\alpha_l} (\bar{s}_l^*)^{\beta_l}$ ,  $0 \leq \alpha_l \leq k+1$ ,  $0 \leq \beta_l \leq k$ ,  $l = 1, \dots, MR$  and  $k = 0, 1$ . Exploiting the hypothesis of independency between the inputs and their white characteristic, an orthonormalization can be carried out by applying the Gram-Schmidt procedure to this set of monomials, considering the following scalar product:  $\langle A(S), B(S) \rangle = \mathbb{E}[A(S)B^*(S)]$ , where  $A(S)$  and  $B(S)$  are polynomials. The orthonormal polynomials are then obtained as products of orthonormal

monomials  $\{P_{\alpha_l, \beta_l}(\bar{s}_l)\}$ , given by:

$$Q_{m_1, \dots, m_{2k+1}}^{(2k+1)}(S) = \prod_{l=1}^{MR} P_{\alpha_l, \beta_l}(\bar{s}_l), \quad (2)$$

where  $\alpha_l$  ( resp.  $\beta_l$ ) is the cardinality of  $\bar{s}_l$  ( resp.  $\bar{s}_l^*$ ) in the set  $\{\bar{s}_{m_1}, \dots, \bar{s}_{m_{k+1}}\}$  ( resp.  $\{\bar{s}_{m_{k+2}}, \dots, \bar{s}_{m_{2k+1}}\}$ ) and the set of orthonormal monomials results from the orthonormalization of the set of monovariate polynomials  $\{T_{\alpha_l, \beta_l}(\bar{s}_l)\}$ , using the Gram-Schmidt procedure:

$$P_{0,0}(\bar{s}) = 1, \quad P_{1,0}(\bar{s}) = \frac{\bar{s}}{\sqrt{\rho_{\bar{s},1,1}}}, \quad (3)$$

$$P_{0,1}(\bar{s}) = \frac{\bar{s}^*}{\sqrt{\rho_{\bar{s},1,1}}}, \quad P_{2,0}(\bar{s}) = \frac{\bar{s}^2}{\sqrt{\rho_{\bar{s},2,2}}}, \quad (4)$$

$$P_{1,1}(\bar{s}) = \frac{\bar{s}\bar{s}^* - \rho_{\bar{s},1,1}}{\sqrt{\rho_{\bar{s},2,2} - \rho_{\bar{s},1,1}^2}}, \quad (5)$$

$$P_{2,1}(\bar{s}) = \frac{\rho_{\bar{s},1,1}\bar{s}^2\bar{s}^* - \rho_{\bar{s},2,2}\bar{s}}{\sqrt{\rho_{\bar{s},1,1}^2\rho_{\bar{s},3,3} - \rho_{\bar{s},1,1}\rho_{\bar{s},2,2}^2}}. \quad (6)$$

where  $\rho_{\bar{s},p,q} = \mathbb{E}[\bar{s}^p \bar{s}^{*q}]$ .

The main advantage of this method is that the Gram-Schmidt orthogonalization is applied to calculate only few monomials, even if the system has a high number of non-linear input terms.

#### 3.2 Estimation Algorithms

The LMS equation for updating the estimated channel matrix  $\hat{\mathbf{F}}$  associated with the orthonormal polynomials is given by:  $\hat{\mathbf{F}}(n+1) = \hat{\mathbf{F}}(n) + \mu (\mathbf{x}(n) - \hat{\mathbf{F}}(n)\check{\mathbf{s}}(n)) \check{\mathbf{s}}^H(n)$ , where  $\mu$  is the step-size parameter, the matrix  $\hat{\mathbf{F}}(n)$  represents the estimation of  $\mathbf{F}$  at the  $n^{th}$  iteration and  $\mathbf{x}(n) = [x^{(1)}(n) \dots x^{(I)}(n)]^T \in \mathbb{C}^{I \times 1}$  is the vector of received signals (output vector). On the other hand, the MMSE block estimate of  $\mathbf{F}$  is given by:  $\hat{\mathbf{F}} = \mathbf{R}_{\mathbf{x}\check{\mathbf{s}}}\mathbf{R}_{\check{\mathbf{s}}\check{\mathbf{s}}}^{-1} = \mathbf{R}_{\mathbf{x}\check{\mathbf{s}}}$ , where  $\mathbf{R}_{\mathbf{x}\check{\mathbf{s}}} = \mathbb{E}[\mathbf{x}(n)\check{\mathbf{s}}^H(n)]$ .

### 4 Input Orthogonalization for blind Identification

In this section, state-dependent modulation code schemes using Phase-Shift Keying (PSK) modulated signals are developed to ensure some identifiability conditions from SOS, by means of a PARAFAC analysis. The redundancy provided by the codes introduces temporal correlation in a controlled way, which means that the transmitted signals are not white. In this case, the channel is assumed to be memoryless, i.e.  $R = 1$  in (1). In addition, for PSK signals, the cubic terms corresponding to  $m_3 = m_1$  and  $m_3 = m_2$  can be excluded from (1), as they have the form:  $s_{m_j}(n) |s_{m_3}(n)|^2$ ,  $j \in \{1, 2, 3\}$ , where  $|s_{m_3}(n)|^2$  is a multiplicative constant absorbed by the channel coefficients. Thus, we have  $M_V = \frac{M}{2}(M^2 - M + 2)$ .

## 4.1 PARAFAC Decomposition of a Tensor of Covariance Matrices

The spatio-temporal covariance matrices of the received signals are given by  $\mathbf{R}(\tau) = \mathbb{E}[\mathbf{x}(n+\tau)\mathbf{x}^H(n)] = \mathbf{H}\mathbf{C}(\tau)\mathbf{H}^H$ , with  $\mathbf{C}(\tau) = \mathbb{E}[\mathbf{s}(n+\tau)\mathbf{s}^H(n)]$ , where  $\tau \in \Upsilon = \{\tau_1, \tau_2, \dots, \tau_T\}$ . The noise term is not considered in  $\mathbf{R}(0)$  since it can be estimated and then subtracted [8]. A third-order tensor  $\mathbf{R} \in \mathbb{C}^{T \times N \times N}$  can be defined from the matrices  $\mathbf{R}(\tau)$ , for  $\tau \in \Upsilon$ . The PARAFAC decomposition of the tensor  $\mathbf{R}$  can be obtained if the  $\mathbf{C}(\tau)$  matrices are diagonal for  $\tau \in \Upsilon$ , leading to the following scalar notation:

$$r_{t,i_1,i_2} = \sum_{q=1}^{M_V} h_{i_1,q} h_{i_2,q}^* c_{q,q}(\tau_t), \quad (7)$$

where  $r_{t,i_1,i_2} = [\mathbf{R}]_{t,i_1,i_2}$ ,  $h_{i,q} = [\mathbf{H}]_{i,q}$  and  $c_{q,q}(\tau_t) = [\mathbf{C}(\tau_t)]_{q,q}$ . The following theorem states sufficient conditions to ensure the diagonality of  $\mathbf{C}(\tau)$  for  $\tau \in \Upsilon$  [9].

**Theorem 1:** Suppose that all the signals transmitted by the users are mutually independent and have constant moduli. Then, the following conditions are sufficient to ensure the diagonality of the covariance matrices  $\mathbf{C}(\tau)$ ,  $\forall \tau \in \Upsilon$ : (i)  $\mathbb{E}[s_m(n)] = 0$ , for all the users; (ii)  $\mathbb{E}[s_m^2(n)] = 0$ , for  $(M-1)$  users; (iii)  $\mathbb{E}[s_m^2(n+\tau)s_m(n)] = 0$  and  $\mathbb{E}[s_m^2(n)s_m(n+\tau)] = 0$ , for  $(M-1)$  users; (iv)  $\mathbb{E}[s_m(n+\tau)s_m(n)] = 0$ , for  $(M-1)$  users (see [9]).

## 4.2 Design of Coding Schemes

State-dependent modulation codes can be designed to ensure that the transmitted signals satisfy the constraints of Theorem 1. In these modulation code schemes, the modulation makes part of the encoding process and it introduces redundancy by expanding the signal constellation.

The modulated signals are characterized by Discrete Time Markov Chains (DTMC) with  $R_m$  states, given by the PSK symbols  $a_r = \{A_m \cdot e^{j2\pi(r-1)/R_m}\}$ , for  $r = 1, 2, \dots, R_m$ , where  $A_m$  is the amplitude of the signal of the  $m^{\text{th}}$  user. The symbol transitions are state-dependent and defined by a block of  $k_m$  bits, denoted by  $B_n = \{b_n^{(1)}, b_n^{(2)}, \dots, b_n^{(k)}\}$ , where  $b_n^{(k)}$ , for  $k=1, \dots, k_m$ , is uniformly distributed over the set  $\{0, 1\}$  and  $2^{k_m} < R_m$ . In addition, it is assumed that  $b_n^{(k)}$  ( $k=1, \dots, k_m$ ) are mutually independent. For each of the  $R_m$  states, the block of bits  $B_n$  defines  $2^{k_m}$  equiprobable possible transitions. Therefore, the coding imposes some restrictions on the symbol transitions. For each state, there is  $(R_m - 2^{k_m})$  not assigned transitions. The code rate of the  $m^{\text{th}}$  user is then given by  $(k_m/l_m)$ , where  $l_m = \log_2 R_m$ .

Let us denote by  $\mathbf{T} = \{T_{r_1,r_2}\}$ , with  $r_1, r_2 \in \{1, 2, \dots, R_m\}$  the Transition Probability Matrix. Note that  $\sum_{r_2=1}^{R_m} T_{r_1,r_2} = 1$  and  $T_{r_1,r_2} \in \{0, 1/2^{k_m}\}$ . The matrix  $\mathbf{T}$  defines the possible state transitions for each state. The following theorem proposes some constraints on the transition probability matrix  $\mathbf{T}$  in such a way that the conditions of Theorem 1 are verified for  $\tau = 0$ .

**Theorem 2:** Let us assume that for all the users: (i) the DTMC associated with the modulation code is ir-

reducible and aperiodic and (ii)  $\sum_{r_1=1}^{R_m} T_{r_1,r_2} = 1$ , for  $1 \leq r_2 \leq R_m$ . Then conditions of Theorem 1 are verified for  $\tau = 0$  (see [9]).

Let  $T_{r_1,r_2}^n$  be the  $(r_1, r_2)^{\text{th}}$  entry of  $\mathbf{T}^n$ . By definition,  $T_{r_1,r_2}^n$  represents the probability of being in the state  $a_{r_2}$  after  $n$  transitions, supposing that the current state is  $a_{r_1}$ . Concerning the conditions of Theorem 1 in the case  $\tau \neq 0$ , conditions (iii) and (iv) can be rewritten as

$$\mathbf{a}^T \mathbf{T}^\tau \mathbf{a}_2 = 0, \quad \mathbf{a}_2^T \mathbf{T}^\tau \mathbf{a} = 0 \quad \text{and} \quad \mathbf{a}^T \mathbf{T}^\tau \mathbf{a} = 0, \quad (8)$$

where  $\mathbf{a} = [a_1, a_2, \dots, a_{R_m}]^T$  and  $\mathbf{a}_k = [a_1^k, a_2^k, \dots, a_{R_m}^k]^T$ . Equations (8) only depend on the constellation order, which means that the transition probability matrices can be a priori designed to verify these constraints. For example, for 4-PSK signals ( $\mathbf{a} = [1 \ j \ -1 \ -j]^T$ ) with 1/2-rate code, the following matrices:

$$\mathbf{T}_{2,A} = 0.5 \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}, \quad \mathbf{T}_{2,B} = 0.5 \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{pmatrix}, \quad (9)$$

verify the conditions of Theorem 2. Moreover,

$$\mathbf{T}_{1,A} = 0.5 \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{T}_{1,B} = 0.5 \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix}, \quad (10)$$

verify the conditions of Theorem 2 and the equations (8).

## 4.3 Estimation Algorithms

Based on the orthogonalization provided by the modulation codes, two estimation methods are proposed for estimating the channel: (i) a two-step Alternate Least Squares (ALS) algorithm [9] and (ii) a joint diagonalization algorithm [10], corresponding to an extension of the Second Order Blind Identification (SOBI) algorithm [8] to nonlinear channels. The covariance matrices of the sources  $\mathbf{C}(\tau)$  are assumed to be known, which explains why the ALS algorithm needs only two steps.

Once the matrices  $\mathbf{C}(\tau)$  are diagonal, the identifiability conditions of the methods (i) and (ii) can be obtained respectively from the Kruskal Theorem [11] and the Theorem of Essential Uniqueness of Joint Diagonalization [8]. We must remark that  $I \geq M_V$  is only required for the method (ii). Moreover, if the channel matrix is full column rank, then both identifiability conditions become equivalent.

## 5 Simulation results

A MIMO Wiener filter corresponding to the model of an uplink channel of a Radio Over Fiber (ROF) multiuser communication system [12] is considered for the simulations. Fig. 1 shows the Normalized Mean Square Error (NMSE) of the estimated output vector  $\hat{\mathbf{x}}(n)$  using the LMS algorithm with the canonical and orthonormal polynomials, for  $I = 4$ ,  $M = 4$ ,  $R = 2$  and a fixed Signal-to-Noise-Ratio (SNR) of 30dB. The four users transmit uniformly distributed q-QAM (Quadrature Amplitude Modulation) signals, for  $q = 16, 16, 32$  and 64. The adaptation using orthonormal polynomials converges approximately

after 5000 iterations and using canonical polynomials after 22000 iterations. Moreover, Fig. 2 shows the NMSE of the estimated output vector  $\hat{\mathbf{x}}(n)$  using the block estimation with the orthonormal and canonical polynomials, for  $I = 4$ ,  $M = 4$ ,  $R = 2$ , 16-QAM signals and various values of  $L$ , the length of the data block used for the moment estimation. The gain provided by the orthonormal approach is evident, specially for high SNR's. Moreover, the better performance is obtained with a smaller complexity, due to the absence of matrix inversion.

Fig. 3 shows the NMSE of the estimated output vector  $\hat{\mathbf{x}}(n)$  using the blind estimation techniques of Section 4, for  $I = 4$ ,  $M = 2$ ,  $R = 1$ ,  $T = 5$ ,  $L = 1000$  and  $L = 3000$ . The used modulation is 4-PSK and the code rate is 1/2. We can conclude that the joint diagonalization estimator performs significantly better than the ALS for high SNR's.

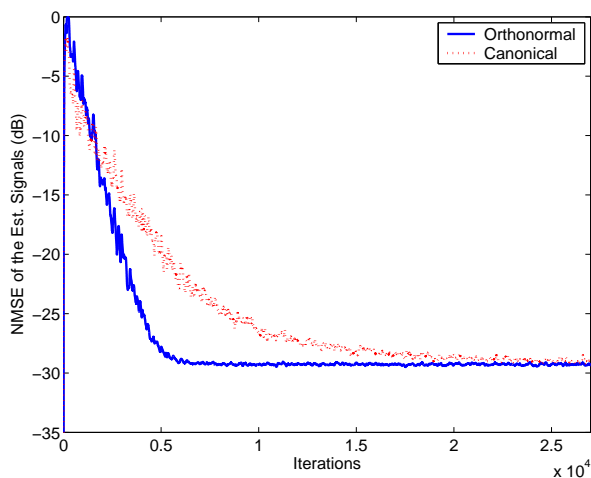


FIG. 1: NMSE of the est. output vector - LMS Estimation.

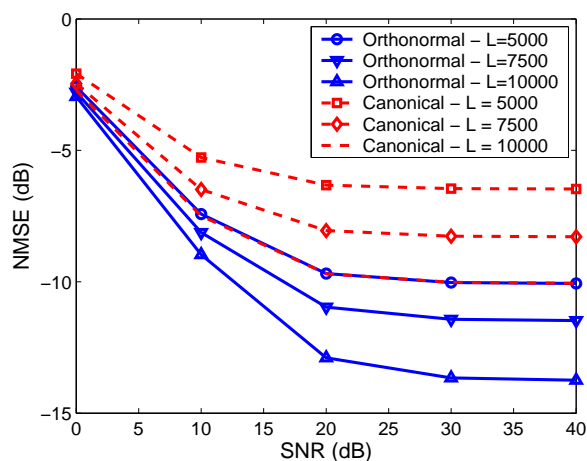


FIG. 2: NMSE of the est. output vector - Block MMSE Estimation.

## 6 Conclusion

In this paper, two input orthogonalization methods for identification of third-order MIMO Volterra communica-

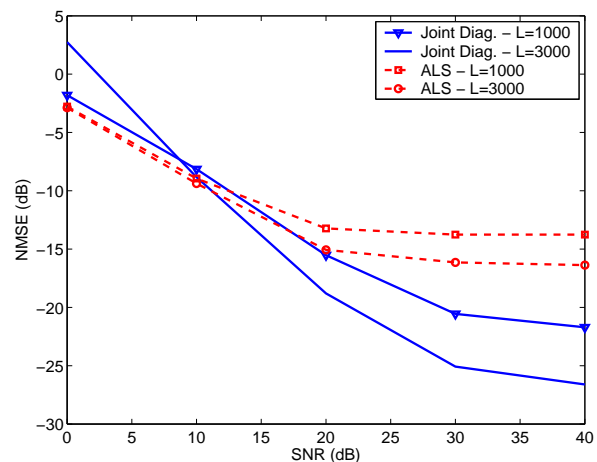


FIG. 3: NMSE of the est. output vector - Blind Techniques.

tion channels have been presented. In the supervised case, the proposed method is designed for white input signals and it is based on the utilization of orthonormal polynomials. The use of this orthonormalization method accelerates the convergence of the LMS algorithm and improves the MMSE estimate in a block processing scheme while needing a smaller computational cost. In the blind case, the proposed orthogonalization method is based on the use of modulation codes to ensure the orthogonality of non-linear interfering terms for different time delays, which constitutes a new application of modulation codes. Two channel estimation techniques have been considered for this case. Some simulation results illustrate the performance of the proposed identification methods.

## References

- [1] A. Ziehe, M. Kawanabe, S. Harmeling and K.-R. Müller, *Journal of Machine Learning Research*, vol. 4, no. 7-8, pp. 1319-1338, 2003.
- [2] S. Z. Pinter and X. N. Fernando, *Canadian Conf. on Electrical and Computer Engineering*, pp. 1-4, May. 1-4, 2005.
- [3] E. Biglieri, A. Gersho, R. Gitlin and T. Lim, *IEEE J. Select. Areas Commun.*, vol. 2, pp. 765-777, 1984.
- [4] S. Serfaty, J. L. LoCicero and G. E. Atkin, *IEEE Trans. on Communications*, vol. 38, no. 10, pp. 1835-1843, 1990.
- [5] J. G. Proakis, *Digital Communications*, McGraw-Hill, 4th ed., ISBN 0-07-232111-3, 2001.
- [6] R. A. Harshman, *UCLA Working Papers in Phonetics*, vol. 16, pp. 1-84, Dec. 1970.
- [7] C. A. R. Fernandes, A. Kibangou, G. Favier and J. C. M. Mota, *International Telecommunications Symposium (ITS)*, Fortaleza, Brazil, Sept. 3-6, 2006.
- [8] A. Belouchrani, K. Abed-Meraim, J.-F. Cardoso and E. Moulines, *IEEE Trans. Sig. Proc.*, vol. 45, no. 2, pp. 434-444, Feb. 1997.
- [9] C. A. R. Fernandes, G. Favier and J. C. M. Mota, *SPAWC*, Helsinki, Finland, Jun. 17-20, 2007.
- [10] C. A. R. Fernandes, G. Favier and J. C. M. Mota, *EUSIPCO*, Poznan, Poland, Sept. 3-7, 2007.
- [11] J. Kruskal, *Linear Algebra Appl.*, vol. 18, pp. 95-138, 1977.
- [12] X. N. Fernando and A. B. Sesay, *IEEE Trans. Vehic. Tech.*, vol. 54, no. 6, pp. 1980-1991, 2005.