Ageostrophic instabilities of a front in a stratified rotating fluid

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Abstract :

It is known that at large Rossby numbers geostrophically balanced flows develop specific ageostrophic instabilities. We undertake a detailed study of the so-called Rossby -Kelvin instability and of its nonlinear evolution.

The collocation method is used to study linear stability of balanced baroclinic flow in the framework of the 2-layer rotating shallow water model. We reproduce Sakai's results [4], and obtain similar results for the cases of layers of different depth and of outcropping of the density interface.

In order to investigate the RK instability in the more realistic framework of continuously stratified flow, and to study its nonlinear stage, we then use an atmospheric mesoscale model WRF. We confirm the appearance of the RK-instability with characteristics which are qualitatively and quantitatively close to the 2-layer case. We observe the saturation of the instability and formation of secondary vortices at the nonlinear stage of evolution.

Résumé :

Les instabilités d'une région de front dans un fluide stratifié en rotation sont étudiées. On s'intéresse plus particulièrement à l'instabilité dite de Rossby-Kelvin, qui existe grâce au couplage d'une onde de Rossby et d'une onde Kelvin.

La stabilité linéaire d'un front dans un fluide à deux couches est étudiée numériquement par la méthode de collocation. Les résultats de Sakai [4] pour le cas de deux couches d'égale hauteur sont confirmés et ceci valide notre approche. Celle-ci permet d'étendre ces résultats aux cas non-symétriques et au cas où le front intersecte le fond ou la surface.

Ensuite, la stabilité d'un front dans un fluide continuement stratifié est analysée par des simulations numériques à l'aide du modèle méso-échelle WRF (Weather Research and Forecast). L'existence de l'instabilité de Rossby-Kelvin dans un fluide stratifié est ainsi confirmée, avec des taux de croissance comparables au cas du fluide à deux couches.

Key-words :

fronts; baroclinic instability ; Rossby-Kelvin instability

1 Introduction

Some fundamental aspects of meso-scale dynamics of atmospheric fronts remain poorly understood, especially the generation of gravity waves and front destabilisation leading to secondary cyclogenesis. In order to understand better these aspects of frontal dynamics we investigate the instabilities of a frontal region in a rotating stratified fluid, without traditional restraining to geostrophic (or balanced) motions.

In addition to the classical baroclinic instability, other instabilities of the front such as symmetric, Kelvin-Helmholtz and Rossby-Kelvin instabilities may appear. The latter exists due to the coupling of a Rossby wave and of a Kelvin wave, and has attracted only little attention until recently.

The stability analysis and identification of unstable modes are first carried out for a 2-layer rotating shallow water model, using the collocation method (section 2). The relevance of these modes for continuously stratified flows is then demonstrated using idealized simulations with a mesoscale meteorological model (section 3).

2 Linear stability analysis of the frontal configurationfor in the two-layer fluid

2.1 Overview of the model and method

We consider the 2-layer rotating shallow water model on the f-plane with a vertical shear flow as shown in figure 1. The domain is a vertically bounded channel of width $2\,Y_{max}$ and height H_0. The linearized equations for the perturbation in non-dimensional form are:

$$\begin{cases} \partial_t u_j + Ro\partial_x u_j - v_j = -\partial_x \pi_j \ , \\ \partial_t v_j + Ro\partial_x v_j + u_j = -\gamma\partial_y \pi_j \ , \\ \partial_t h + Ro\partial_x h = \pm(Ro(H_j\partial_x u_j + \gamma\partial_y(H_j v_j))) \ , \end{cases} \tag{1}$$

where $Ro = U_1/(fR_d)$ is the Rossby number, $R_d = (g'H_1/2)^{\frac{1}{2}}/f$ is the Rossby deformation radius, $\gamma = R_d/Y_{max}$ and $j = 1, 2$ indicates the layer. The lateral boundary conditions are $v_j(\pm Y_{max}) = 0$.

Assuming a sinusoidal form of the solution in the x-direction, we obtain an eigenvalue problem of order 6 which can be solved by the collocation method as described in [2] by discretizing the y-interval. We fixed $\Delta H = Ro\,Y_{max} = 0.5$ so that parameter space can be explored by varying two parameters, Ro and k.

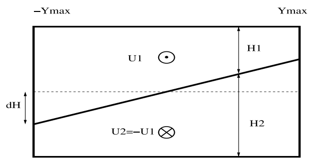

Figure 1: Schematic diagram of the flow considered in the two-layer shallow water model discussed in section 2.

2.2 Instabilities and growth rates

In the pioneering paper [4], Sakai studied a symmetric configuration, with $H_2 = H_1$. His results are well reproduced by our method, and this calculation served as a benchmark. The symmetric configuration is a degenerate case for which some features disappear. Below we present the results of the stability analysis for a non-symmetric configuration, for which the depths of the two layer are no longer equal: $H_1 = H_2/0.7$. Growth rates and phase speeds of these different modes are shown in figures 2 and 3.

Figure 2 shows the growth rates in the (Ro, k) plane for the different types of instabilities. At the lower-left, for small Ro and k, one finds baroclinic instability, which can be understood

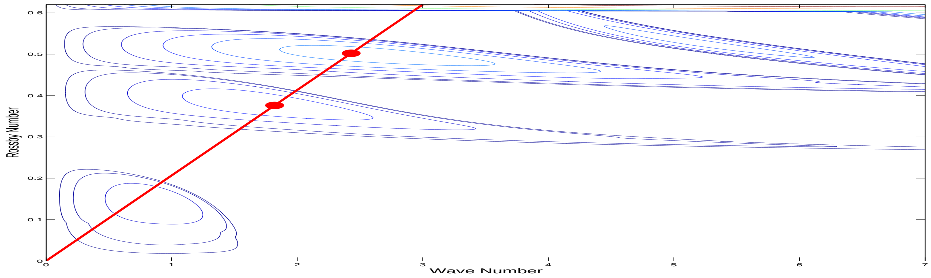

Figure 2: Growth rate of most unstables modes in (Ro, k)-space for $H_1 = H_2/0.7$. Contours displazed are 0.01, 0.02 and further interval 0.02

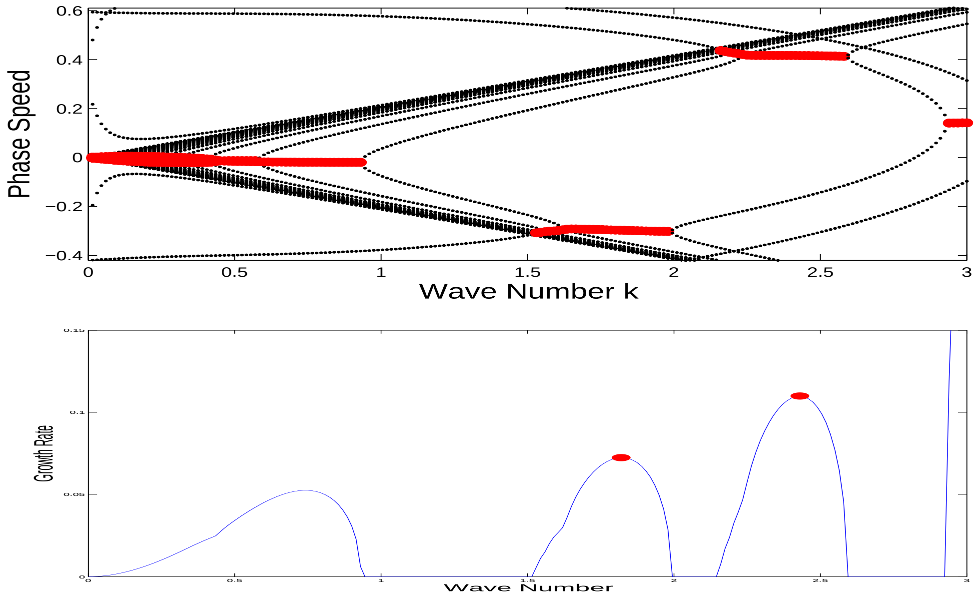

Figure 3: Dispersion diagram (upper panel) and growth rate (lower panel) of the modes along the section $Ro = k/5$ (as shown in figure 2). Red arguments on the upper panel correspond to the unstables modes

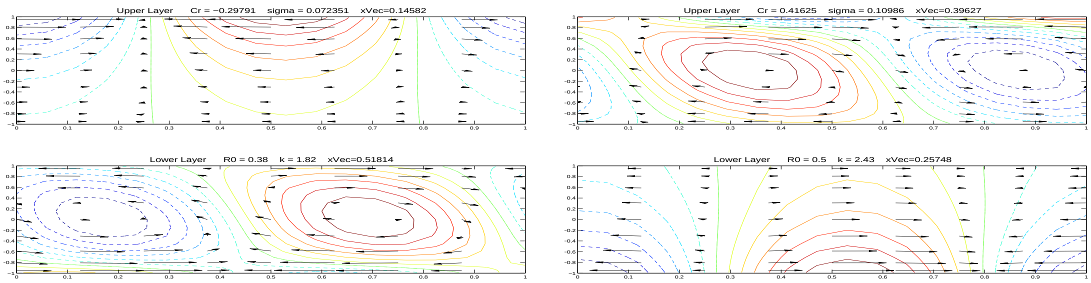

Figure 4: Pressure and velocity fields of the Rossby-Kelvin instability in the upper layer and the lower layer at the maximum growth rate for $H_2 = H_1/0.7$. Left: $k = 1.8$ et $Ro = 0.4$ and right: $k = 2.4$ et $Ro = 0.5$, as shown by the red points in figure 3

as the interaction between two Rossby waves, cf. [4]. For stronger shear, $Ro \approx 0.6$, Kelvin-Helmoltz instability occurs, with larger growth rates ($\omega_i \sim 1$). For intermediate values of the Rossby number, one finds two regions of instability ($Ro \approx 0.4$ and $Ro \approx 0.5$) which correspond to Rossby-Kelvin instability as described in [4]. These modes exist due to the interaction of a Rossby wave in one layer and a Kelvin wave in the other. However, contrary to the symmetric case analysed by Sakai (see his figure 6), two different Rossby-Kelvin instabilities occur here, instead of one.

Figure 3 shows of instability in the phase-space and growth rates along the section $Ro = k/5$. Baroclinic instabilty occurs for smaller k and Kelvin-Helmholtz instability for k greater than 2.9. The two other peaks, for intermediate values of k are the two types of Rossby-Kelvin instability, with different growth rates.

Following Ripa [3] and Sakai [4] the flow is unstable if there is a pair of wave which match several conditions. They must propagate in the opposite direction with respect to the basic flow ($\omega_1\omega_2 < 0$), and they must have almost the same Doppler-shifted frequency ($\omega_1 + kU_0 \sim \omega_2 - kU_0$).

The dispersion curves, shown in figure 3, are different with respect to the symmetric case with a higher phase speed for the positively propagating Kelvin waves (upper part of the diagram), but the Rossby waves phase speed barely change. Hence, the intersection between the two modes moves toward higher k for a positive phase speed. We thus get two distinct instability areas. One corresponds to a Kelvin wave in the upper layer and a Rossby wave in the lower layer as shown on the left panel of the figure 4. The other corresponds to a Kelvin wave in the lower layer and a Rossby wave in the upper layer as shown on the right panel of the figure 4.

These differences are consistent with estimations of the phase speeds of the two types of waves: the Kelvin wave's phase speed is about $\sqrt{g'H_j}$, i.e. strongly linked to the depth, whereas the Rossby waves have a phase speed like $-2kU_j/(2R^2|k^2|+1)$ which does not depend on the depth of the layer.

3 Stability analysis in a continuously stratified fluid

3.1 The model and the experimental setup

The Weather Research and Forecast Model (WRF, [5]) was developped to allow both operational and idealized simulations. We use an idealized configuration of the model to investigate stability of a frontal region in a more realistic configuration (continuous stratification), and then to investigate the non-linear evolution of these instabilities.

The domain is a channel ($L_x.L_y.L_z$) on the f-plane, periodic in x and bounded by lateral walls in y, with a flat bottom without a boundary layer, as in the previous 2-layer model. Two main differences with the 2-layer model are the background stratification (in addition to the front), and the top boundary condition (free surface and not rigid lid). The basic state is defined by including a stantard stratification and a potential temperature jump along the front:

$$z_{fr}(y) = z_0 + S\ y, \quad \theta = \theta_0 + \Theta_z z + \frac{\theta_{ju}}{2}\left(1 + \tanh(\frac{z - z_{fr}}{z_{ju}})\right), \tag{2}$$

where z_0 is the average height of the front, S the slope, θ_{ju} the potential temperature jump, z_{ju}

the thickness of the frontal zone, and Θ_z describes the basic stratification.

An example of an initial state is shown on figure 5. Note that here we return to a configuration with the two layers having equal mean depths.

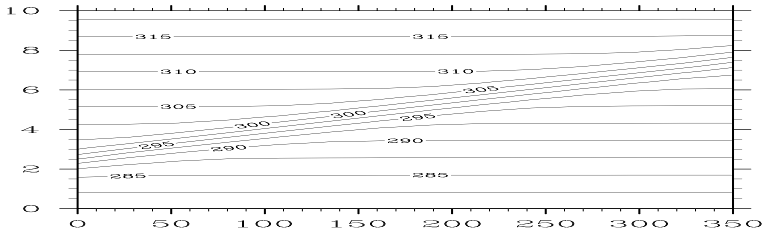

Figure 5: Initial distribution of potential temperature for (z,y) in km.

3.2 Rossby-Kelvin instability in a continuously stratified fluid

Simulations started with a slightly perturbed front were carried out for various values of Ro and k in order to identify unstable modes along the line $Ro = k/3$. A breeding procedure was carried out to isolate the most unstable normal mode, if any, for each set of parameters used. The different modes of instability described in section 2 are reproduced, with growth rates that are very close to those obtained with the 2-layer model. In particular, the Rossby-Kelvin instability is also present in the continously stratified case.

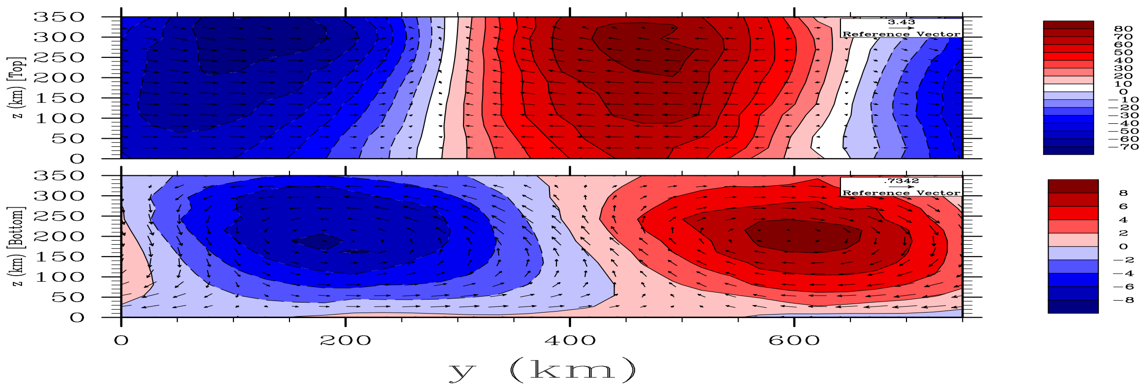

Figure 6: Pressure and velocity field for a Rossby-Kelvin instability for non-dimensionnal parameters $Ro = 0.5$ and $k = 2.4$

Figure 6 shows the pressure and velocity fields vertically averaged below and above the front. The structure of a Rossby wave (geostrophic wind turning around pressure extrema) can be identified clearly below the front, and the structure of a Kelvin wave (wind parallel to the boundaries, pressure extrema at the lateral boundaries) can be identified above the front. This instability is growing in about 5 days. The growth rate was evaluated from plots of the growth of kinetic energy of the perturbation over the whole domain. Similar simulations were run for different parameters along the line $Ro = k/5$. Figure 7 shows the corresponding non-dimensional growth rates, remarkably close to those found for the two-layer model (see [4], figure 7).

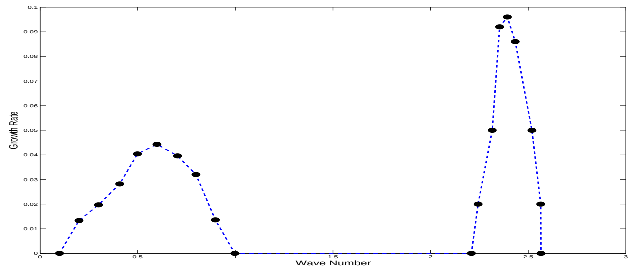

Figure 7: Growth rates of the different instabilities of a front in the continuously stratified fluid. The peak at low k corresponds to classical baroclinic instability. The peak for larger k corresponds to Rossby-Kelvin instability.

4 Conclusions

The linear stability of a front in a rotating stratified fluid has been investigated both for a 2-layer fluid and a continuously stratified fluid. In the first case we benchmarked the collocation method by reproducing the results obtained by Sakai [4] for the symmetric case (2 layers with equal depth). The method allows us to investigate the instabilities in a more general case (2 layers with different depth, outcropping of the interface, relevant for oceanic fronts), and to show the presence of new unstable modes in these configurations.

We then demonstrated the relevance of these unstable modes for a more realistic configuration of continuously stratified front by using idealized simulations carried out with the atmospheric mesoscale model WRF. Rossby-Kelvin modes occur again, with a structure quite similar to the one deduced from the 2-layer model. Growth rates are also quite comparable to the growth rates of the 2-layer model, in spite of important differences between the two models (background stratification, compressibility, upper boundary condition).

This work confirms existence of the unstables modes in a frontal region of continuously stratified fluid. Such modes provide a mechanism of coupling between balanced and unbalanced motions (e.g. [1]). The study of the non-linear saturation of these modes and their manifestations in more realistic configurations (no side boundaries) is in progress.

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