Influence of an elliptic instability on the merging of a co-rotating vortex pair

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Abstract :

We study the nonlinear evolution of the elliptic instability and its influence on the merging process of two corotating Batchelor vortices using a spectral DNS approach.
First, we analyse the nonlinear saturation of the elliptic instability for a single strained vortex, with and without axial jet, for moderate Reynolds numbers ($Re = \Gamma/\nu \approx 12500$, where Γ is the circulation and ν the kinematic viscosity). We show that the vortex deformation induced by the instability remains limited to the vortex core region.
The second part of our work focuses on the influence of the elliptic instability on the merging process. We compare three cases : no instability (2D), elliptic instability without axial jet, and elliptic instability with axial jet, the latter case being relevant to aircraft wakes. Qualitative and quantitative differences between the three different cases are pointed out and discussed in the context of aircraft vortices.

Résumé :

Nous étudions l'évolution non-linéaire de l'instabilité elliptique et son influence lors de la fusion de deux vortex de Batchelor corotatifs, à l'aide de simulations numériques.
Dans un premier temps, nous analyson la saturation non-linéaire de l'instabilité elliptique pour un tourbillon dans un champ d'étirement, avec et sans jet axial, à des nombres de Reynolds modérés ($Re = \Gamma/\nu \approx 12500$, où Γ est la circulation et ν la viscosité cinématique). Nous montrons que la déformation du vortex induite par l'instabilité reste limitée au cœur.
La seconde partie de ce travail examine l'influence de l'instabilité elliptique lors du processus de fusion. Nous comparons trois cas de fusion : sans instabilité (2D), avec l'instabilité elliptique sans jet axial, et avec l'instabilité elliptique avec jet axial, ce dernier cas ayant des applications aux sillages des avions. Les différences qualitatives et quantitatives entre ces trois cas sont examinées et discutées dans le contexte aéronautique.

Key-words :

vortex merging; elliptic instability

1 Introduction

A vortex elliptically deformed by an external strain field is subjected to the so-called elliptic instability, resulting in the deformation of the vortex core, along the vortex axis. The linear properties of this instability have been studied extensively, in various cases : co-rotating and counter-rotating vortex pair, with or without axial jet. This work focuses on the non-linear evolution of this instability and its effect on the merging of a co-rotating vortex pair.

The elliptic instability has been thoroughly reviewed by Kerswell (2002). It can be viewed as a resonance between two Kelvin waves (or eigen modes, see Fabre *et al.*, 2006) of the vortex and the external shear. Theoretical predictions have been done by Le Dizès & Laporte (2002), and a study of the instability of counter-rotating vortices with axial jet has been carried out recently by Lacaze *et al.* (2007). Only little is known about the non-linear regime : a weakly non-linear analysis has been done by Sipp (2000) and Meunier & Leweke (2005) performed some experiments on counter-rotating vortices without axial flow.

Using numerical simulations on Batchelor vortices, we first study the saturation of the elliptic instability on a single vortex, showing that the sinuous displacement induced by the elliptic instability remains bounded. The second part of this work shows that a merging with elliptic instability happens earlier and makes the resulting vortex wider.

2 Method

The initial, z-invariant, vorticity field ω is composed of two vortices of size a and circulation Γ, separated by a distance b :

$$\omega = \frac{\Gamma}{\pi a^2}\left(\exp\left(-(x-b/2)^2/a^2 - x^2/a^2\right) + \exp\left(-(x+b/2)^2/a^2 - x^2/a^2\right)\right)$$

The axial velocity v_z in a Batchelor vortex is proportional to the vorticity and its intensity is measured by the parameter $W_0 = 2v_z/\omega a$.

The vortex core size a increases due to viscous diffusion, and when a/b reaches a threshold value close to 0.23 in 2D, the merging process begins. This process is well-documented when the system remains two-dimensional. When the Reynolds number is large enough, each vortex develops an elliptic instability, deforming the vortex core and eventually modifying the merging process.

In order to tackle this problem, we use a full spectral method. The computation domain is a square of size L with the origin at its center. In the z direction, a simple Fourier decomposition is performed.

3 Non-linear evolution of the elliptic instability

For this part, we freeze the two-dimensional flow, and let the elliptic instability develop on top of the z-invariant stationary flow. From previous studies it is known that without axial jet, the most unstable mode is a combination of $m = 1$ and $m = -1$, leading to a sinuous deformation.

The structure of the vortex perturbed by the elliptic instability is illustrated in figure 1. We have observed that during the nonlinear evolution, the vorticity field gets more and more localised until viscous effects stop this process. The case with axial jet, is similar but the most unstable mode is now a combination of $m = 2$ and $m = 0$, leading to a helical deformation.

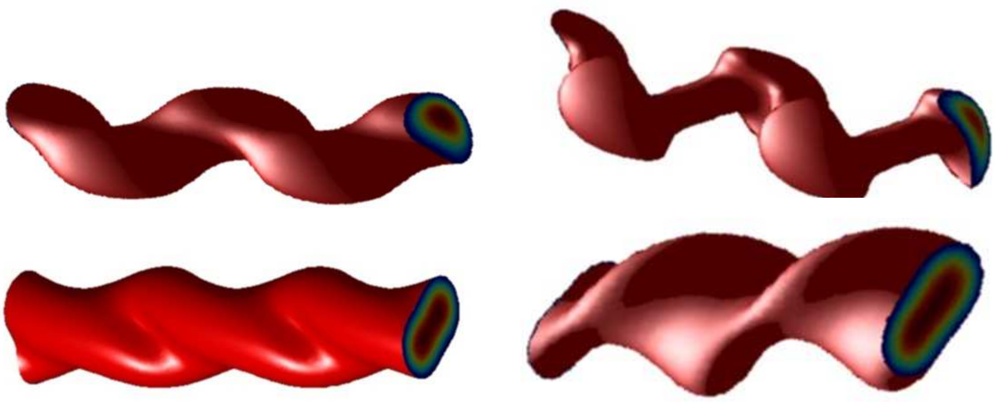

Figure 1: Vortex deformed by the elliptic instability materialized by a total vorticity isosurface, without axial jet (top line) and with $W_0 = 0.6$ (bottom line). A weak, almost linear deformation is shown on the left, whereas a strongly non-linear case is shown on the right.

4 Merging of a vortex pair subjected to elliptic instability

In order to compare how three different configurations of a vortex pair will merge, we start with the two-dimensional flow described above, and let it evolve to reach a quasi-steady state. At a given time, we add the most unstable mode at a given amplitude, as well as the optional axial jet, and let the resulting three different system evolve further. The evolution of the distance between the two vortices is shown in figure 2. The most unstable mode is added et $t = 100$ (time is based on the single vortex turn-over time) and we can see that the merging occurs earlier when an elliptic instability is allowed to develop, with or without jet.

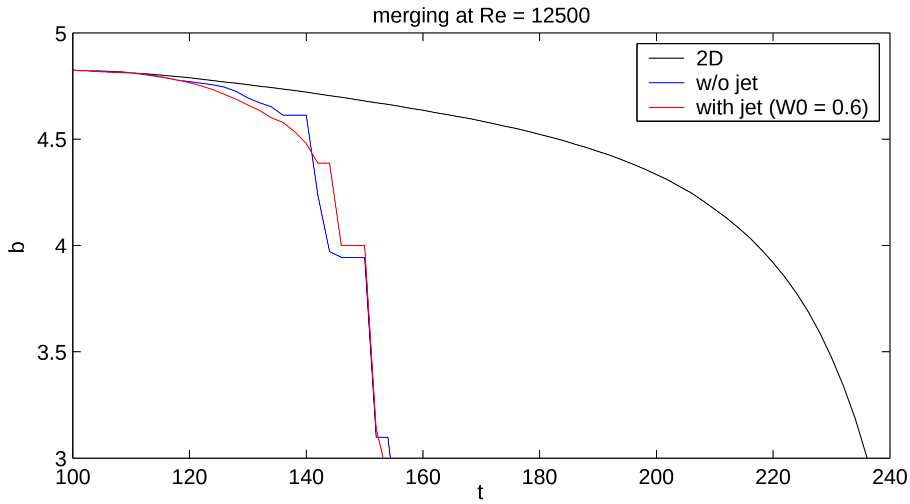

Figure 2: Distance b between the two vortices as a function of time, at $Re = 12500$

When the merging process is completed and the three systems have reached a nearly axisymmetric state, we compare the vorticity profiles (figure 3). When compared to the two-dimensional case, the merging processes with elliptic instability evolve toward wider vortices, with lower peak vorticity. The vortex widening is even stronger with axial jet. We found also that the peak angular velocity is reduced by about 20%.

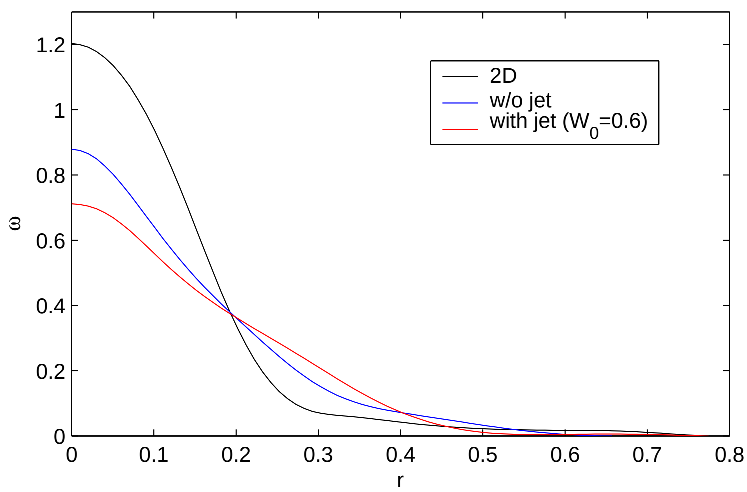

Figure 3: Vorticity profiles after merging, taken at the same time, at $Re = 12500$

5 Conclusions

The non-linear evolution of the elliptic instability has been investigated. For a single strained vortex, the displacement of the vortex core remains bounded, even though the amplitude of the instability can grow quite large, resulting in important deformations. This can be explained by the presence of one or more zeros in the radial profile of the most unstable mode.

The merging of a co-rotating vortex pair is strongly affected by the elliptic instability : the process happens earlier, and the resulting vortex is significantly wider and weaker. For aircraft wakes, this means that in tuning the jet and the circulation, it is possible to achieve a faster decay of the strong trailing vortices.

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