Analysis of Experimental Homogeneous Turbulence Time Series by Hilbert−Huang Transform

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Abstract :

In this paper the Empirical Mode Decomposition (EMD) method and Hilbert-Huang transform are used to analyse experimental homogeneous turbulence time series. With this method, one can decompose nonlinear time series into a sum of different modes, each narrow-banded. Here we consider experimental turbulent velocity time series with a large Reynolds number ($Re_\lambda = 720$). The Fourier power spectrum reveals a wide inertial range with a classical $-5/3$ Kolmogorov power-law spectrum. We show that the EMD method applies very nicely to the turbulent velocity time series, with a dyadic filter bank in the inertial range. We estimate the Fourier power spectra of each mode, showing that adding more and more modes corresponds to including lower and lower frequencies. This filtering property can have interesting applications in the field of turbulence modelling. We estimate the Hilbert-Huang power spectrum of the turbulent time series and show its scaling properties, with an exponent different from $-5/3$.

Résumé :

Il s'agit d'une mise en application de la méthode d'analyse de séries temporelles non-linéaires EMD (décomposition modale empirique), et de la transformation de Hilbert-Huang, à des données expérimentales de turbulence, possédant des fluctuations invariantes d'échelle dans la zone inertielle de cascade d'énergie. Nous montrons que la méthode EMD permet de décomposer une série temporelle turbulente en une somme de modes intrinsèques appartenant aux échelles inertielles. Nous estimons le spectre de Fourier de chaque mode, et montrons qu'ajouter des modes correspond à remonter en échelles, incluant les basses fréquences dans la zone inertielle. Cette propriété de filtre peut avoir d'intéressantes applications en modélisation de la turbulence. Nous montrons aussi que le spectre de Hilbert-Huang est invariant d'échelle, avec une pente différente de la pente classique turbulente de $-5/3$.

Key-words :

Fully developed turbulence ; Hilbert−Huang Transform, Empirical Mode Decomposition

1 Introduction

In this paper the Empirical Mode Decomposition (EMD) method and the Hilbert-Huang transform are used to analyse experimental homogeneous turbulence time series. With this method, one can decompose nonlinear time series into a sum of different modes, each one having characteristic frequencies Huang *et al*. (1998, 1999). Due to the simplicity of its algorithm, the EMD method has met a large success; this technique has already been applied to several fields, including acoustics Loutridis (2005), climate Salisbury and Wimbush (2002); Coughlin and Tung (2004) and nonlinear waves in oceanography Hwang *et al*. (2003); Veltcheva and Guedes Soares (2004). It has also been applied to numerically simulated fractional Gaussian noise (fGn) time series, and shown to act as a dyadic filter bank Flandrin *et al*. (2004). In the same paper, it was shown how to use the hierarchy of modes to estimate the fGn scaling exponent H.

However, to our knowledge, it has seldom been applied to fully developed turbulent time series, characterized by a high Reynolds number, a large scaling range for the fluctuations, and strong intermittency Frisch (1995). Here we consider experimental turbulent velocity time series with a large Reynolds number ($Re_\lambda = 720$). We show that the EMD method applies very nicely to the turbulent velocity time series, with a dyadic filter bank in the inertial range. Section 2 presents the data; section 3 the EMD method and Hilbert-Huang transform. Section 4 presents the results obtained on the velocity time series.

2 Presentation of the experimental database

We consider here a database obtained from measurements of nearly isotropic turbulence downstream an active-grid. The experiment is characterized by the Taylor-based Reynolds number $Re_\lambda = 720$. The sampling frequency is $f_s = 40kHz$, and a low-pass filtered at a frequency of $20kHz$ is applied on the experimental data. The sampling time is $30\,s$, and the total number of data points per channel for each measurement is 1.2×10^6. We used data in the streamwise direction at position $x_1/M = 20$, where M is the grid size (the mean velocity at this location is $12\,m/s$ and the turbulence intensity is about 15.4%). For details about the experiment and the data see Kang *et al.* (2003); the data can be found at http://www.me.jhu.edu/˜meneveau/datasets.html.

3 Empirical Mode Decomposition and Hilbert−Huang Transform

Empirical Mode Decomposition is a recently developed method Huang *et al.* (1998, 1999) that can be applied to study the nonlinear and non-stationary properties of a time series. This method contains the following two steps: Empirical Mode Decomposition (EMD) and Hilbert Spectra Analysis (HSA). The main idea of EMD is to locally estimate a signal as a sum of a local trend and a local detail: the local trend is a low frequency part, and the local detail a high frequency. When this is done for all the oscillations composing a signal, the high frequency part is called an Intrinsic Mode Function (IMF) and the low frequency part is called the residual. The procedure is then applied again to the residual, considered as a new times series, extracting a new IMF and a new residual. At the end of the decomposition process, the EMD method expresses a time series $x(t)$ as the sum of a finite number of IMFs $C_i(t)$ and a final residual $r_n(t)$ Huang *et al.* (1998); Flandrin *et al.* (2004). The procedure is precisely described below.

An IMF is a function that satisfies two conditions: (i) the difference between the number of local extrema and the number of zero-crossings must be zero or one; (ii) the running mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero. The procedure to decompose a signal into IMFs is the following Huang *et al.* (1998, 1999):

1 The local extrema of the signal $x(t)$ are identified;

2 The local maxima are connected together forming an upper envelope $e_{\max}(t)$, which is obtained by a cubic spline interpolation. The same is done for local minima, providing a lower envelope $e_{\min}(t)$;

3 The mean is defined as $m_1(t) = (e_{\max}(t) + e_{\min}(t))/2$;

4 The mean is subtracted from the signal, providing the local detail $h_1(t) = x(t) - m_1(t)$;

5 The component $h_1(t)$ is then examined to check if it satisfies the conditions to be an IMF. If yes, it is considered as the first IMF and denoted $C_1(t) = h_1(t)$. It is subtracted from the original signal and the first residual, $r_1(t) = x(t) - C_1(t)$ is taken as the new series in step

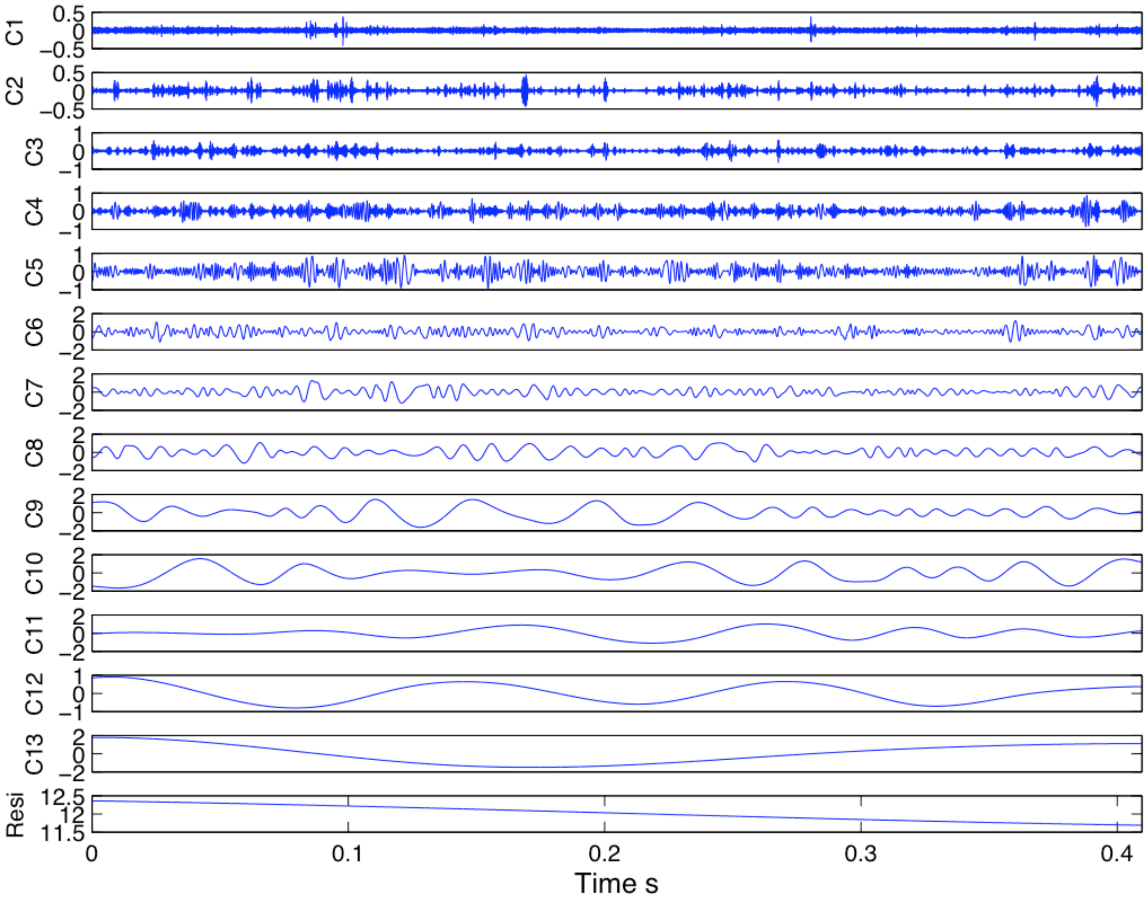

Figure 1: IMFs estimated from one 2^{14} points segment of the velocity. The time scale is increasing with the mode.

1. If $h_1(t)$ is not an IMF, a procedure called "sifting process" is applied as many times as needed to obtain an IMF. In the sifting process, $h_1(t)$ is considered as the new data; the local extrema are estimated, lower and upper envelopes are formed and their mean is denoted $m_{11}(t)$. This mean is subtracted from $h_1(t)$, providing $h_{11}(t) = h_1(t) - m_{11}(t)$. Then it is checked if $h_{11}(t)$ is an IMF. If not, the sifting process is repeated, until the component $h_{1k}(t)$ satisfies the IMF conditions. Then the first IMF is $C_1(t) = h_{1k}(t)$ and the residual $r_1(t) = x(t) - C_1(t)$ is taken as the new series in step 1.

By construction, the number of extrema decreases when going from one residual to the next; the above algorithm ends when the residual has only one extrema, or is constant, and in this case no more IMF can be extracted. The complete decomposition is then achieved in a finite number of steps, of the order $n \leq \log_2 N$, for N data points. The signal $x(t)$ is finally written as:

$$x(t) = \sum_{i=1}^{N} C_i(t) + r_n(t) \tag{1}$$

The IMFs are orthogonal, or almost orthogonal functions (mutually uncorrelated). This method does not require stationarity of the data and is especially suitable for nonstationary and nonlinear time series analysis Huang *et al.* (1998, 1999). Each mode is localized in frequency space Flandrin and Gonçalvès (2004); Wu and Huang (2004). EMD is a time-frequency analysis Flandrin *et al.* (2004) since it can represent the original signal in a energy-frequency-time form

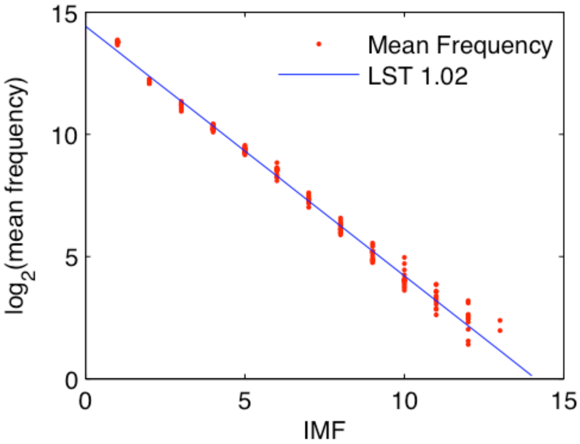

Figure 2: Mean frequency versus mode number for the turbulent velocity time series. There is an exponential decrease with a slope very close to 1. This indicates that EMD acts as a dyadic filter bank.

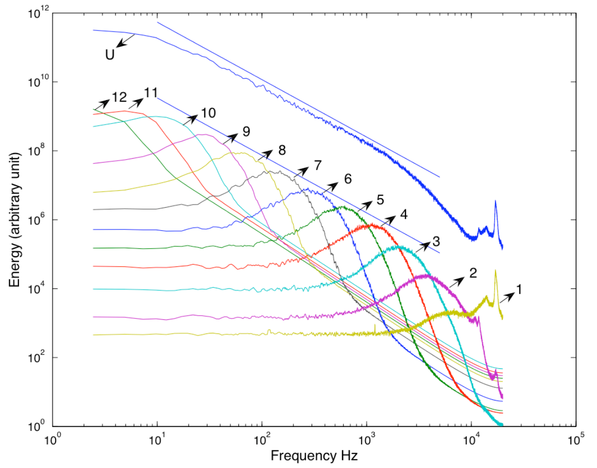

Figure 3: Fourier spectrum of each mode (from 1 to 12) showing that they are narrow-banded. The slope of the reference line is $-5/3$.

at local level, using a complementary method called Hilbert-Huang spectrum Huang *et al.* (1998). This decomposition can be used to express the original time series as the sum of a trend (sum of modes from p to N) and small-scale fluctuations (sum of modes from 1 to $p-1$), where p is an index whose value depends on the trend decomposition which is desired.

In the second step of this method, Hilbert Spectra Analysis, Hilbert transform is applied to each IMF. Then we can design the Hilbert spectrum $H(\omega, t)$, which represent the energy as the function of instantaneous frequency and time. Here the Hilbert transform is a singular integration, it can be taken as the best local fit of an amplitude and phase varying trigonometric function to $x(t)$ (Huang *et al.* (1998)). Therefore the Hilbert spectrum can provide sufficient locality information in both physics and frequency space. In global sense we also can define the Hilbert marginal spectrum $h(\omega)$ which, in some sense, is an equivalence of power spectrum in Fourier analysis. In fact, here the definition of instantaneous frequency is different with the one in Fourier frame. The interpretation and the detailed physical meaning of Hilbert marginal spectrum should be paid more attention in future research. The locality and adaptivity abilities make this method unique and suitable for nonlinear and nonstationary time series analysis. Since it was proposed, HHT has been applied successfully to many fields. However, to our knowledge, it has seldom been applied to fully developed turbulent time series, characterized by a high Reynolds number, a large scaling range for the fluctuations, and strong intermittency

4 Results

The original velocity time series is divided into 73 segments (without overlapping) of 2^{14} points each. After decomposition, the original velocity series is decomposed into several IMFs (see Fig.1), from 11 to 13 modes with one residual. It is clear that the time scale is increasing with the mode; each mode has a different mean frequency, which is estimated by considering the (energy weighted) mean frequency in the Fourier power spectrum. The relation between mode number k and mean frequency Huang *et al.* (1998) is displayed in Fig. 2. The straight line in log-linear plot which is obtained suggests the following relation:

$$\overline{f}(k) = f_0 \rho^{-k} \qquad (2)$$

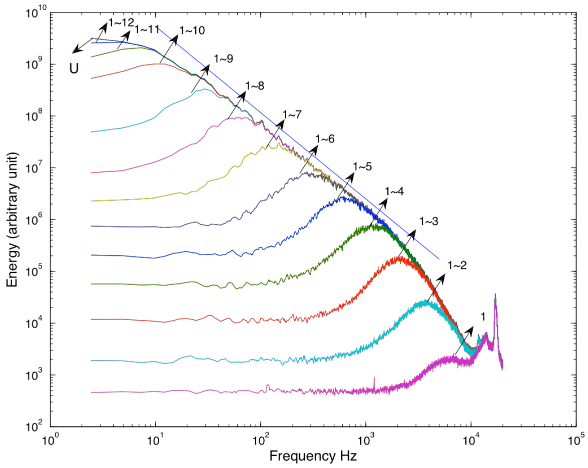

Figure 4: Fourier spectrum of the sum of modes from 1 to p, with $p = 2, 3...12$. It shows a clear asymptotic behavior.

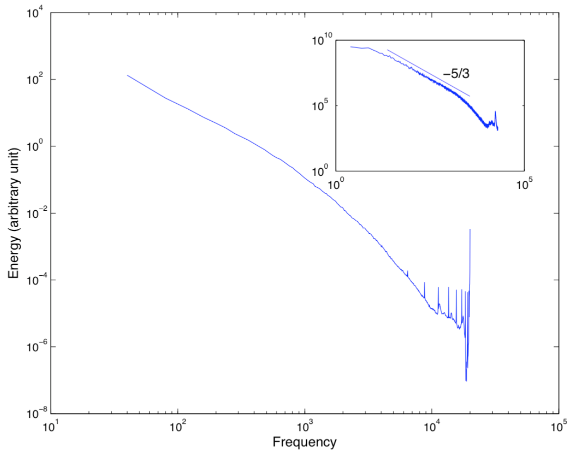

Figure 5: Hilbert marginal spectrum of the velocity signal. For comparison Fourier spectrum is displayed in the up-right pannel.

where $\overline{f}$ is the mean frequency, f_0 is a constant and ρ is very close to 2. This indicates that EMD acts as a dyadic filter bank in the frequency domain; it was shown previously using stochastic simulations of Gaussian nose and fBm Flandrin *et al.* (2004); Wu and Huang (2004), and it is interesting to note here that the same result holds for fully developed turbulence time series.

When compared with the original Fourier spectrum of the turbulent time series (see Fig.3 and 4), these modes can be termed as follows: the first mode, which has smallest time scale, corresponds to the measurement noise; modes 2 and 3 are associated to the dissipation range of turbulence; mode 4 corresponds to the Kolmogorov scale; modes 5 to 11 all belong to the inertial range; larger modes belong to the large turbulent forcing scales. Fig. 3 and 4 represent the Fourier power spectra of each mode and of the sum of the modes, respectively. They show (i) that each mode in the inertial range is narrow-banded; (ii) that adding more and more modes corresponds to going farther and farther towards large scales in the inertial range, reconstituting the $-5/3$ Kolmogorov spectrum. This property can be very interesting to decompose a turbulent signal into a mean and small-scale fluctuations, as is often done for turbulence modelling purposes.

The Hilbert marginal spectrum $h(\omega)$ (defined in Huang *et al.* (1998)) of the velocity is displayed in Fig. 5 together with the Fourier spectrum. It is clear that the following relation

$$h(\omega) \sim \omega^{-\beta_H} \tag{3}$$

holds in some range, with an exponent β_H different from the $-5/3$ Fourier exponent. We recall here that the frequency ω defined in EMD is different from the Fourier frequency, and the precise physical meaning of Hilbert marginal spectrum is still to be explored.

Let us finally note here that, due to the limitation of this paper, we just present here the results of velocity U at location $x/M = 20$. For other points and velocity V we get the same results, which does not present here.

5 Conclusion

In present paper, we applied Hilbert−Huang transform to analyze a high Reynolds number, $Re_\lambda = 720$, turbulent experimental time series. After decomposition, the original velocity time series is separated into several intrinsic modes. This method acts as a dyadic filter bank in the frequency domain (in Fourier frame). Comparing the Fourier spectrum of each mode, we can draw that the first mode contains the smallest scale and the most noise of the measurement, and that many modes are associated to the inertial subrange. Finally, when the Fourier spectrum of each mode is compared with the original one, these modes can be divided into three terms: the smallest scales corresponding to the dissipation range, the moderate scales corresponding to the inertial subrange and the large scales corresponding to the coherent structures (energy-contain structure). However, if all these modes are added back step by step, it illustrates a clearly asymptotic approximation behavior. This will be very useful for turbulence modeling: some model parameters can be adjusted based on these interesting results. And also this provides a possible way to establish a low dimensional dynamical system. Otherwise, the Hilbert marginal spectrum demonstrates a generalized power-law, which is different with the Fourier spectrum. Detailed interpretation should be given in future investigations.

In Hilbert spectra analysis, instantaneous frequency is used to represent the relation between energy, time and frequency, and Hilbert spectrum reveals a direct relation between frequency and energy. For Hilbert marginal spectrum, an approximate power-law has been obtained, whose slope, different from $-5/3$, is still to be interpreted.

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