A discrete model for fracture of rigid solids based on a damaging interface

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Abtract:

We describe the progressive and delayed fracture of rigid solids by a discrete modelling. Each rigid solid is considered as an assembly of particles with initial cohesive bonds, the latter decreasing progressively during the loading. A damaging interface model is proposed to describe this progressive pehenomenon. This model has been implemented in a numerical code based on a discrete element method. The illustrative example is related to the crushing of an assembly of rigid solids - i.e. a granular medium - due to an œdometric compression.

Résumé :

Nous décrivons la rupture progressive et différée de solides rigides par une approche discrète. Chaque solide rigide est représenté par une collection de particules, initialement liées par une cohésion qui peut progressivement diminuer au cours du chargement. Un modèle d'endommagement interfacial est proposé pour décrire cette décroissance progressive. Implémenté dans un code de calcul par éléments discrets, ce modèle permet de simuler la rupture de collections de solides rigides. L'exemple illustratif traite de la rupture et de l'attrition d'une collection de grains sous compression œdométrique.

Key-words :

Granular medium; rigid solids; interface; damage; fracture

1 Introduction

The general frame of this study is this of the progressive (finite cracking velocity) and delayed (with respect to the loading) fracture of rigid solids interacting by contact and friction. An illustrative example of such a structural problem is this of a rockfill dam, which can globally settle due to the local fracture of rock blocks in the time, see *e. g.* Deluzarche and Cambou (2006), Oldecop and Alonso (2002) or Tran (2006).

Choice is here made to get numerically approximated solutions of the contact-friction part of the problem by using the discrete element method proposed by M. Jean (1999) and J.J. Moreau (1988). However, due to the fact that the rigid solids (or grains) - which will be all assumed of the same characteristic size D^S - can break, each of them is considered as an assembly of rigid particles - which will be also all assumed of the same characteristic size $D^p \ll D^S$. These particles are assumed to be initially "glued". From a numerical point of view, a grain, i. e. an assembly of rigid particles, must thus be seen as a mesh of the rigid solid, in which a crack can initiate (resp. propagate) only on (resp. through) the contact zones between rigid grains. Consequently, from a physical point of view, these contact zones have to be considered as rigid but breakable interfaces.

Strong cohesive forces are supposed to exist initially on the interfaces (see e. g. Delenne *et al.* (2004)), giving to them their initial tensile strength. It is then assumed that, when a given interface I - characteristic area $S \propto (D^p)^2$ in 3D and $S \propto D^p$ in 2D - is submitted to a sufficiently strong tensile force, microcracks and/or microcavities, i.e. damage initiate, growth and, eventually, coalesce, that leads to the fracture of the interface (and so, to the irreversible vanishing of the cohesive forces).

Following this introduction, Section 2 is devoted to the presentation of the damaging interface model where, in agreement with the general frame of this study, the evolution of the damage is at the same times progressive and delayed. An illustrative examples is presented in Section 3, where the numerical results have been obtained using a numerical code in which the damaging interface model has been implemented. This example is related to the crushing of an assembly of two-dimensional rigid solids - i. e. a two-dimensional granular medium - due to an œdometric compression.

2 A damaging interface modelling

The (thermo)dynamic system considered in this section is an interface I between two grains. Like the grains, I is assumed to be rigid : the area of the surface S occupied by I is then constant, whatever the forces acting on are. Furthermore, the displacement jump $[\![\boldsymbol{u}]\!]$ through S is assumed to be zero whenever I is not destroyed (i. e. whenever S is clearly defined). Actually, only one evoluting state variable will be considered, denoted by d (scalar) and characterizing the damage by microcracking and/or microcavitation of the constitutive material of I. It is assumed that $0 \leq d \leq 1$. It is emphasized that, as soon as $d = 1$, I is destroyed and the contact-friction interactions between the both grains have to be considered on the basis of the Signorini-Coulomb equations (see e.g. Moreau (1998)). These equations will not be detailed in the present paper.

The damaging interface model is actually based on previous work on continuum damage mechanics by Marigo (1981), where the necessary and sufficient condition for the intrinsic dissipation to be non negative is simply given by $\dot{d} \geq 0$. We take $\boldsymbol{\sigma}$ to denote the stress tensor, taken here to be homogeneous in S. It is assumed that, due to the damage, the effective tough surface of I is not S but the undamaged part $(1-d)S$. The force vector acting on S is then $\boldsymbol{F} = S(1-d)\boldsymbol{\sigma} \cdot \boldsymbol{N}$. We take $F_N = \boldsymbol{F} \cdot \boldsymbol{N}$ to denote the normal force (such that $F_N > 0$ when I is submitted to a tensile force), $\boldsymbol{F}_T = \boldsymbol{F} - F_N \boldsymbol{N}$ the tangential force, $u_N = [\![\boldsymbol{u}]\!] \cdot \boldsymbol{N}$ the normal relative displacement and $\dot{\boldsymbol{u}}_T = [\![\dot{\boldsymbol{u}}]\!] - \dot{u}_N \boldsymbol{N}$ the tangential relative velocity.

A damage yield surface is introduced. Once more, it is clearly inspired by the works by Marigo (1981). However, for a sake of consistency between the present interfacial damage model and the Mohr-Coulomb Signorini one, which must merge in the latter one as soon as $d = 1$, the damage yield surface is

$$g^d(\boldsymbol{F}_T, F_N, d) = \mu^{-1} \left\| \boldsymbol{F}_T \right\| + F_N - (1-d)F^0 .$$

where μ is the friction coefficient between grains when I is destroyed, and $F^0 > 0$ the undamaged yield. As for the fracture of I, which can occur suddenly when I is sufficiely damaged, it is controlled by a fracture yield surface, which reads:

$$g^f(\boldsymbol{F}_T, F_N, d) = \mu^{-1}\left\|\boldsymbol{F}_T\right\| + F_N - (1-d)F^f \,.$$

where $F^f \geq F^0$ is the maximal tensile force I can undergo. It is emphasized that $g^d \geq g^f$, whatever the reachable mechanical state is: damage takes place before fracture, apart from the limit case of a perfectly brittle interface ($F^f = F^0$), where damage and fracture are concomitant. We distinguish the history of the contacting bodies: the left status denotes the status before the current time, while the right status denotes the status before the current time.

The state laws are

$$\begin{array}{l} \text{if the left status is cohesive } (\textit{i.e. } d < 1) \\ u_N \geq 0,\ F_N - (1-d)F^f \leq 0,\ \left[F_N - (1-d)F^f\right]u_N = 0 \\ \text{else} \\ u_N \geq 0,\ F_N \leq 0,\ F_N u_N = 0 \end{array} .$$

The complementary laws include:

- the evolution laws:

$$\begin{cases} g^f(\boldsymbol{F}_T, F_N, d) \leq 0,\ \dot{\lambda} \geq 0,\ \dot{\lambda} g^f(\boldsymbol{F}_T, F_N, d) = 0 \\ \dot{\boldsymbol{u}}_T = \dot{\lambda}\dfrac{\boldsymbol{F}_T}{\left\|\boldsymbol{F}_T\right\|} \end{cases} ,\ \dot{d} = \frac{< g^0(\boldsymbol{F}_T, F_N, d) >}{\eta F^0} ,$$

where $< . >$ denotes the MacCauley brackets, and η is a characteristic time;

- the status change rules:

$$\begin{array}{l} \text{the right status is the same as left status,} \\ \text{except if the left status is cohesive } (d < 1), \\ \text{and if the solution satisfies } [\![\dot{\boldsymbol{u}}]\!] \neq 0 \\ \text{then the right status is set to ``non-cohesive'' } (d = 1) \end{array} .$$

This description is inspired by the work of Jean *et al.* (2001). This is a non-smooth model: d may jump from its failure value given by $g^a(\boldsymbol{F}_T, F_N, d) = 0$ to its maximum value. This rule does not allow the contacting bodies to be glued again, one the interface is destroyed. This damaging interface model has been implemented in a non-smooth contact dynamics code (LMGC90).

3 Application to œdometric compression of a granular medium

We consider an assembly of two-dimensionnal rigid solids (grains) - i.e. a two-dimensional granular medium - submitted to a compressive force $|\ T\ |$ in œdometric conditions (i.e no lateral displacements). In the initial state, see Fig. 1, the sample (initial heigth 45 cm, width L = 48cm) is composed by 75 grains (diameters between 5 and 6 cm), each of them being constituted by 60 to 70 particles (diameters D^p between 5 and 6 mm). The numerical simulations involve 4980 particles.

Each grain is an assembly of initially glued particles of a same diameter D^p. From a numerical point of view, D^p is the characteristic length of a mesh. As a consequence, F^0 and F^f appear to be numerical parameters depending on D^p: the smaller D^p, the smaller F^0 and F^f. We consider therefore as material parameters $\sigma_0 = F^0 / (D^p e)$ and $\sigma_u = F^f / (D^p e)$ (in the three-dimensionnal case, $(D^p)^2$ should be considered instead of $(D^p e)$). The dimensionless number $r = F^0 / F^f = \sigma_0 / \sigma_u$ is a material parameter. All the simulations were performed with $\mu = 1$.

The axial strain is defined by $\varepsilon =| U | /L$ where U is the global displacement induced by T; the axial stress is denoted by $\sigma =| T | /(eL)$ where e is the (unit) thickness of the sample. We take ν to denote the ratio between the current number of non-cohesive contacts and the initial number of cohesive contacts.

The loading T is first defined by a Heaviside step, in order to highlight the creep like response of the granular medium. Figs. 2 and 3. show the influence of the characteristic time η : this parameter simply delays the response (Fig. 2 left). The use of η leads to an efficient dimensionless scaling in time (Fig. 2 right). Parameter η slightly modifies the response quantitatively, as shown by the evolution of ν (Fig. 3 left). Notice eventually that ν and ε evolves in the same way during the creep phase: the kinetics is mainly governed by the fracture of the interfaces (Fig. 3 right). For a given value of σ_u, Fig. 4 shows that r influences the amplitude of the strain during the creep phase. The lower bound of the final amplitude corresponds to a perfectly brittle interface (r=1), while the upper bound corresponds to a damaging interface with no damage yield (r=0).

Fig. 5 shows the œdometric response of the sample during a constant stress rate. This sample behaves like most granular media: decreased compressibility along with increased load yields a convex stress/strain curve. However, this curve presents several breaks: they correspond mainly to the abrupt breakage of one or more grains. Stopping the loading and maintaining constant stress leads to a creep response, while maintaining constant strain leads to a stress relaxation.

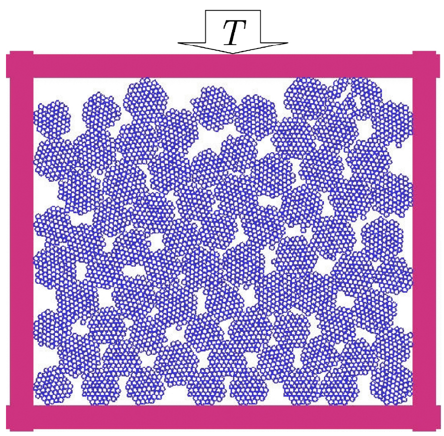

FIG. 1 – Sample composed by an assembly of 75 non "glued" grains (initial heigth 45 cm, width 48cm) and submitted to an œdometric loading; each grains is composed of ≈65 particles, initially "glued".

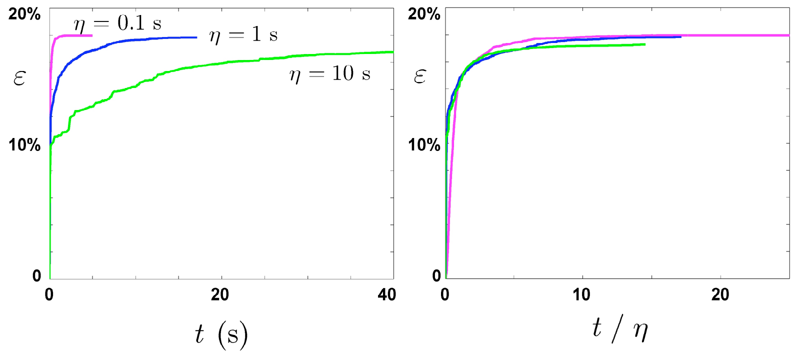

FIG. 2 – Œdometric creep test. Axial strain vs. time (left) and axial strain vs. dimensionless time (right). Influence of parameter η (σ_u=200 kPa, r=0.1, loading is σ=16 kPa).

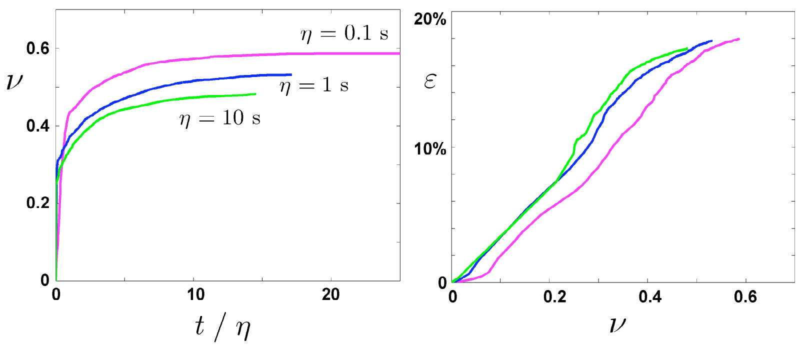

FIG. 3 – Œdometric creep test. Relative number of non-cohesive contact vs. dimensionless time (left), and axial strain vs. relative number of non-cohesive contact (right). Influence of parameter η (σ_u=200 kPa, r=0.1, loading is σ=16 kPa).

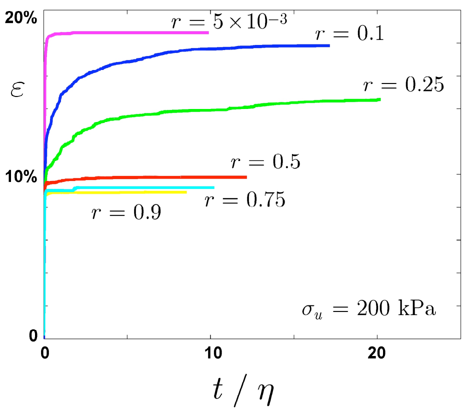

FIG. 4 – Œdometric creep test. Axial stress vs. dimensionless time. Influence of parameter r (σ_u=200 kPa, η=1 s, loading is σ=16 kPa).

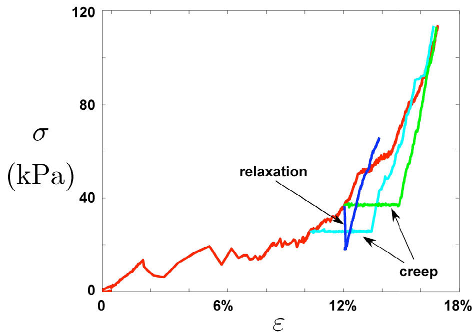

FIG. 5 – Œdometric compression test (axial stress vs. axial strain) showing creep under constant stress and stress relaxation under constant strain (r =0.5, σ_u =200 kPa, η =1 s, $\dot{\sigma}$ =11 kPa/s).

6 Conclusions

Most of the structural failures are due to the pre-existence of various kinds of micro-defects (microcracks and/or microvoids) in the materials, which propagate and eventually coalesce in a macro-crack. The modelling of these propagation and coalescence is an important issue. The discrete approach presented here is intended as a step toward this issue. The proposed damaging interface model is based on a reduced set of five parameters. The illustrative example clearly deals with dam engineering: rockfill material is characterized by delayed grain breakage under constant load. This is the main cause of the majority of post-constructive displacements observed in high rockfill dams, which can induce the failure of the impervious element or piping erosion.

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References

J.-Y. Delenne, M.S. El Youssoufi, F. Cherblanc, J.-C. Benet, Mechanical behaviour and failure of cohesive granular materials, *Int. J. Numer. Anal. Geomech.*, 28(2004), 1577-1594.

R. Deluzarche and B. Cambou, Discrete numerical modelling of rockfill dams, *Int. J. Numer. Anal. Geomech.*, Vol. 30, pp 1075-1096, 2006.

M. Jean, The non-smooth contact dynamics method, *Computer Methods in Applied Mechanic and Engineering*, Vol.177, pp 235-257, 1999.

M. Jean, Acary V., Monerie Y., Non-smooth contact dynamics approach of cohesive materials, *Philosophical Transactions: Mathematical, Physical & Engineering*, The Royal Society, London A, A359 (1789), pp 2497-2518, 2001.

J.J. Marigo, Formulation d'une loi d'endommagement d'un matériau élastique, *C. R. Acad. Sci. Paris*, série II, t292 (1981) 1309-1312.

J.-J. Moreau, Unilateral contact and dry friction in finite freedom analysis. In J.J Moreau et P.D Panagiotopoulos, editors, *Non Smooth Mechanics and Application*, chapitre CISM Courses and Lectures, Vol.302, Springer-Verlag, pp 1-82, 1988.

L.A. Oldecop and E.E. Alonso, Fundamentals of rockfill time-dependent behaviour. In *Unsaturated Soils*, Juca, de Campos & Marinho (eds), pp 793-798, 2002.

H. Tran, Analyse et modélisation du vieillissement des barrages en enrochements par une approche micromécanique, PhD Thesis, Ecole Centrale de Lyon, 2006.