The Dynamics of Vortex Breakdown

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Abtract :

Swirling flows with jet-like axial velocities are studied by means of direct numerical simulations for various control parameters. The numerical code solves the low-Mach number approximation of the Navier-Stokes equations in cylindrical coordinates which allows the incorporation of compressible and variable-density effects without the adverse effect of acoustic waves on the numerical time-step. The governing parameters, such as swirl and coflow, are varied, and their effect on the breakdown type and breakdown location is studied. The nonlinear direct numerical simulation traces the steady states of the swirling flow, and the structure of the bifurcation is described as a function of the swirl parameter, the Reynolds number and the coflow parameter. In particular, unstable branches are computed and their physical relevance is discussed. The analysis gives new insight into the prevalence of coherent states and their controllability.

Résumé :

Les écoulements tournants avec une vitesse axiale, semblable à un jet sont étudiés par simulation numerique directe pour différents ensembles de paramètres de contrôle. La simulation résout numériquement l'approximation à faible nombre de Mach des équations de Navier-Stokes en coordonnées cylindriques. Ceci permet l'incorporation des effets de compressibilité et de densité variable sans devoir prendre en compte les ondes acoustiques qui requiéreraient l'utilisation de pas de temps trop petit. La variation des paramètres de contrôle, tels que le paramètre de la rotation et de coflow, permet l'étude de l'éclatement et sa localisation. La simulation numérique directe non linéaire conduit à l'observation des états stables de l'écoulement tournant. La structure de la bifurcation est décrite en fonction du paramètre de rotation, du nombre de Reynolds et du paramètre de coflow. Nous nous attacherons plus particulièrement à calculer et discuter la physique des branches instables.

Key-words :

swirling flow; vortex breakdown; bifurcation

1 Introduction

Vortex breakdown is a feature of rotating flows involving a concentrated core of vorticity embedded in a largely irrotational flow that is moving in a direction approximately parallel to the vortex. It arises in a number of natural settings, such as tornadoes, dust devils, or water spouts. In reactive flows, swirl is frequently used to achieve a larger spreading angle of the jet which in turn stabilizes the flame (Beer and Chigier, 1972). The prediction and control of vortex breakdown in swirling jets is also of interest in aeronautical applications, such as the vortex core over delta wings at high angles of attack, trailing vortices shedding off the wing tips, etc.

As reported by Lopez (1994) changes in the topology of the stream surfaces due to the swirl parameter yielded a classification criterion which states that a swirling flow has undergone vortex breakdown when specific critical points appear in the velocity field. These critical points are only well defined if the flow is steady and axisymmetric. In this case, they appear as stagnation points on the axis of symmetry and the separatrix, connecting them, forms the boundary of what is commonly referred to as a vortex breakdown bubble.

As emphasized by Faler & Leibovich (1977), the term axisymmetric breakdown is a misnomer as the breakdown form is not truly axisymmetric. Nevertheless, numerical simulations of internal, axisymmetric swirling flow have been rather successful in reproducing the breakdown structure with high accuracy (Beran & Culick, 1992).

The direct numerical simulation of nominally axisymmetric, swirling flows revealing vortex breakdown can be achieved solving the full Navier-Stokes equations in cylindrical coordinates for unsteady flow. Special care has to be taken in treating the singular behaviour of certain expressions near the axis. As far as the conditions at the open boundaries are concerned, swirling flows pose a great challenge due to their ability to support upstream travelling waves, which render the flow particularly sensitive to small disturbances near the outflow boundary. Furthermore, it has been found that prescribing an axial pressure gradient far away from the vortex can greatly influence the onset of vortex breakdown and govern its mode selection; see the review by Spall & Snyder (1999) for details.

2 Governing equations

The numerical simulations are based on the low Mach-number approximation of the time-dependent axisymmetric Navier-Stokes equations in cylindrical coordinates (r, θ, x). To render the governing equations dimensionless, a characteristic length (L) and velocity (U) are introduced. The convective time scale is $T = L/U$, and the characteristic pressure is $P = \rho U^2$, where ρ denotes the constant density. The Reynolds number is thus defined as

$$\mathrm{Re} = \frac{UL}{\nu}.$$

The computational domain has the dimensions $R_d = 10$ and $Z_d = 20$, it is numerically resolved by $n_r = 127$ and $n_x = 257$ grid points in the radial and axial directions.

A Grabowski profile (see Grabowski & Berger, 1976) is used for the radial velocity and the axial and azimuthal velocity components are defined piecewise for the region inside and outside a characteristic radius R. The axial velocity component can exhibit a jet-like or wake-like character inside R and approaches a constant free-stream velocity $\tilde{v}_{x,\infty}$ outside R. The non-dimensional form of the velocity profile is obtained by scaling the radius with the characteristic core radius $L = R$. The velocity profile at the inflow boundary is forced to be axisymmetric and constant over time; moreover, no perturbations are imposed

$$\begin{aligned}
&v_\theta(0 \le r \le 1) = Sr(2 - r^2),\\
&v_\theta(1 \le r) = S/r,\\
&v_r(r) = 0,\\
&v_x(0 \le r \le 1) = \alpha + (1-\alpha)r^2(6 - 8r + 3r^2),\\
&v_x(1 \le r) = 1.
\end{aligned}$$

Here the swirl parameter S represents the ratio of azimuthal velocity at the edge of the core and the axial free-stream velocity, i.e., $S = \tilde{v}_\theta(R)/\tilde{v}_{x,\infty}$. The coflow parameter α denotes the ratio of the axial velocity at the axis and the axial free-stream velocity, i.e., $\alpha = \tilde{v}_{x,c}(R=0)/\tilde{v}_{x,\infty}$. Setting α greater or less than one yields a jet-like or wake-like behavior, respectively.

The outflow boundary condition takes on the form

$$\frac{\partial u_i}{\partial t} + C\frac{\partial u_i}{\partial x} = 0,$$

where the exact value of C is not critical to the solutions, since we only consider steady-state solutions. In order to reach the steady state the simulations were run for a physical time of up to $t = 1000$.

3 Reference case

As a representative reference case, a swirling jet is selected with the governing dimensionless parameters of $\mathrm{Re} = 200$, $\alpha = 1$, and $S = 1.095$. This choice is identical to the reference cases obtained by Grabowski & Berger (1976) and by Ruith, Chen, Meiburg & Maxworthy (2003).

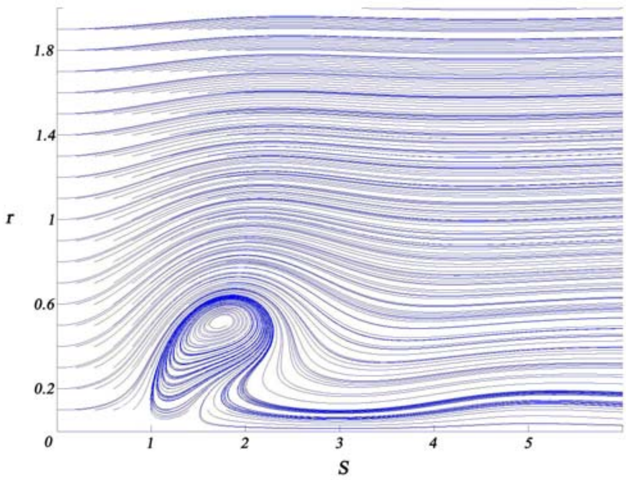

FIG. 1 – Reference case: projected streamlines in the meridional plane

In FIG. 1 the projected streamlines in the meridional plane are shown. These results match closely the streamlines patterns presented in figures 3 frame *a* and frame *b* of Ruith, Chen, Meiburg & Maxworthy (2003).

In the above case, a steady state is defined when the velocity components change by less then 10^{-7} over a time interval of $\Delta t = 10$.

4 Bifurcation diagram

In order to demonstrate the bifurcation structure of the flow, some quantitative measure of the flow has to be introduced and monitored as the governing parameters are varied. An appropriate diagnostic quantity is

$$u_{x,\min} = \min(\min(u_x)),$$

that is, the minimum of the axial velocity at any point in the computational domain. Here, as in Beran & Culick (1992) and Lopez (1994), only two of the governing parameters will be varied. These are the swirl parameter S, which represents the ratio of the azimuthal velocity at the edge of the core to the axial free-stream velocity, and the Reynolds number Re. For each choice of these parameters we compute the steady-state branch of the solutions, where each new steady-state computation uses the previously calculated steady state as an initial condition. In the case of two governing parameters we obtain a surface parameterized by (S, Re), which shows a characteristic fold representing the existence of multiple solutions as well as hysteresis and limit point behavior.

The main results of the present investigation are summarized in FIG. 2, where the steady-state solution branch is shown as a function of the swirl parameter S. As described by the theoretical study of Wang & Rusak (1997), we confirm that the minimum streamwise velocity (the chosen bifurcation parameter) increases as the Reynolds number increases. The green curve corresponds to $\mathrm{Re} = 200$, the blue one to $\mathrm{Re} = 500$, and the red one to $\mathrm{Re} = 1000$. At some critical point an intersection of the lines occurs; this critical point signifies the point on the (S, Re) surface where multiple solutions start to exist.

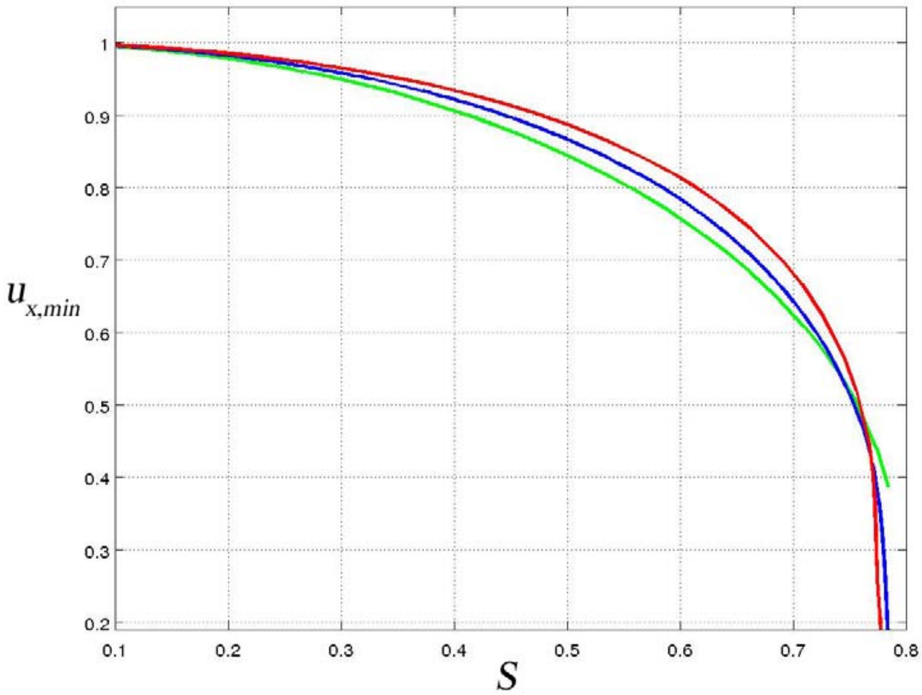

FIG. 2 – Bifurcation steady state diagram: $u_{x,\min}$ against S at $\mathrm{Re} = 200$ (green line), $\mathrm{Re} = 500$ (blue line), and $\mathrm{Re} = 1000$ (red line).

Using the Recursive Projection Method (RPM), introduced by Shroff & Keller (1993), we are able to determine the unstable solution branches yielding the s-shape bifurcation curve for higher Reynolds numbers. In particular, we apply the extended technique described in Janovsky & Liberda (2003) to stabilize the unstable eigenvalues of the linearized right-hand side of the Navier-Stokes equations; furthermore, a Cayley transform (see Garratt, Moore & Spence 1993) is employed to determine unstable solutions.

4 Conclusions

We study the dynamics of axisymmetric swirling flows in a finite domain with a special emphasis of vortex breakdown; benchmark numerical simulations agree well with both theoretical efforts and numerical simulations in the literature. Long-time simulations to obtain steady-state solutions have determined the bifurcation diagram based on the minimum axial velocity as the swirl parameter and the Reynolds number are varied. The diagram identifies the onset of multiple steady-state solutions, the occurrence of hysteresis effects and the existence of unstable steady branches. The exact structure of the bifurcation diagram, including stable as well as unstable steady branches, is determined by embedding the direct numerical simulations into a Recursive Projection Method (RPM) algorithm. In this manner, the abrupt nature of vortex breakdown and the prevalence of observed states over multiple theoretical solutions can be assessed by numerical means.

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