Ageostrophic Instabilities in a Baroclinic Flow over Sloping Topography

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Abstract :

The ageostrophic normal modes of a spatially uniform, vertically sheared flow along a sloping bottom are considered in two layers underneath a deep motionless third layer (two-and-half layer model). The variations of the layer thickness are assumed to be small to derive the six-order dispersion relation with constant coefficients valid for finite Froude number typical for oceanic currents. The dispersion curves for the Rossby waves and inertia-gravity waves (IGW) are investigated to identify different types of instabilities occurs if there is a pair of wave components which have almost the same Doppler-shifted frequency related to crossover of the branches when the Froude number increases. Ageostrophic instabilities due to a resonance between the IGW modes and the Rossby wave in either lower, or middle layer, are described. In both cases the growth rate and the width of the unstable wavenumber window are shown to be proportional to the square root of the corresponding gradient of the layer thickness. These powerful types of ageostrophic instability can coexist together (and with Kelvin-Helmgoltz instability) and may play an important role in mixing processes in geophysical fluids.

Résumé :

On considère les modes normaux agéostrophiques d'un écoulement spatialement uniforme et cisaillé verticalement le long d'une pente, dans un modèle deux couche et demi. La relation de dispersion du 6ème ordre à coefficients constants valide pour un nombre de Froude fini (représentatif des courants océaniques) est obtenue en considérant des faibles variations des épaisseurs de couche. Une étude des relations de dispersion des ondes d'inertie gravité et des ondes de Rossby permet d'identifier différent types d'instabilités. Celles-ci apparaissent s'il existe une paire de composantes ayant quasiment la même fréquence relativeincluant le déphasage doppler (elle est liée à un croisement des branches quand le nombre de Froude augmente). On décrit par ailleurs des instabilités agéostrophiques dues a une résonnance entre les modes des ondes d'inertie gravité et les ondes de Rossby dans la deux couches actives. Dans les deux cas le taux de croisance et la largeur de la fenêtre d'instabilité du nombre d'onde sont proportionnels à la racine carré du gradient d'épaisseur de couche. Ces diverses instabilités agéostrophiques peuvent coexister (ainsi qu'avec l'instabilité de Kelvin-Helmoltz) et elle sont susceptibles de jouer un rôle important dans les processus de mélange des fluides géophysiques.

Key-words :

baroclinic flow ; rotating fluid ; inertia-gravity waves

1   Introduction

Stratified rotating flows support various types of instabilities which can be interpreted in terms of resonances between different wave modes (Hayashi and Young 1987). Three major types of resonances between inertia-gravity waves (IGW) and the Rossby wave modes are known for horizontally uniform, vertically sheared flows (Sakai 1989). Kinetic energy of mean flow is the most important for Kelvin-Helmholtz instability due to high-frequency resonances between IGW modes at the order one Froude number. It provides mixing at small scales and has been studied

mainly in a non-rotating frame. Available part of potential energy is a source for baroclinic instability which can be interpreted as vertical coupling between Rossy waves related to gradients of the basic potential vorticity (e.g., Pichevin 1998). Because the Rossby waves have typically low frequencies, most of these studies were done in a framework of quasi-geostrophic dynamics at small Froude and Rossby numbers when IGW are excluded a priori. The mechanism of this instability is most simply illustrated by geostrophic two-layer Phillips' model where the variations of layer thickness are small.

An ageostrophic version of Phillips' model (two-layer channel model on an f-plane with large variations of the layer thickness) was used to reveal a third type of instability (Orlanski 1968) which has been recognized as an instability caused by resonance between IGW and Rosby waves (Sakai 1989). It was found at finite Froude number and called the Rossby-Kelvin (R-K) instability to indicate the different types of waves that resonate in the lowest mode. The instability occurs if there is a pair of IGW and Rossby wave components which propagate in the opposite direction to the basic flow and these wave components have almost the same Doppler-shifted frequency. In the R-K instability the Rossby waves are almost completely in geostrophic balance while the ageostrophic IGW is the same as in a one-layer system. Doppler shifting matches frequencies which would otherwise be very different.

R-K-type instability is also found in a continuously stratified model (ageostrophic version of the Eady model). Stone (1966, 1970) found some unstable modes with phase speed different from that of the average basic flow (note that the conventional baroclinic instability has the same phase speed as the average basic flow). It is identified by Nakamura (1988) as an instability due to the inertial critical layer. He showed that this unstable mode is caused by an interaction between a vorticity mode trapped at the boundary and an IGW mode which has intrinsic frequency of order the Coriolis parameter and is trapped in the inertial critical layer. Recently Molemaker, McWillias and Yavneh (2005) have investigated R-K-type instability in the Eady model with an emphasis on how it relates to the breakdown of balance in the neighborhood of loss of balanced integrability and on how its properties compare with examples of ageostrophic anticyclonic instability of rotating, stratified, horizontally sheared currents.

Here we consider a multi-layer ageostrophic version of Phillips' model with sloping topography in a configuration with small variations of layer thickness which allows to consider analytically all mentioned types of instability (Kelvin-Helmholtz, Rossby-Kelvin, baroclinic) together in the system of ODE with constant coefficients. Therefore, dispersion curves and wave resonances can be analyzed explicitly that helps to distinguish ageostrophic R-K instabilities related to a gradient of potential vorticity in each layer.

2 Model Formulation

We consider a three-layer, rotating fluid in the Boussinesq, hydrostatic approximation at the *f*-plane. The layer densities are ρ_j, the depths are H_j, the pressure field is P_j, and the velocity vector is (U_j, V_j), where $j = 1$, 2 and 3 represent variables in the lower, middle and upper layer, respectively. The right-hand coordinate system corresponds to the depth topography $H(X)$, with the *X*-axis directed onshore, and the *Y*-axis parallel to the isobath. Further we assume the upper layer to be infinitely deep and motionless ($P_3 = 0$). The basic state has horizontally uniform flows in each layer: $U_j = 0$, $V_j = dP_j / dX$, $H_j = \overline{H}_j + S_j X$;

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where $S_1 = V - S$, $S_2 = \gamma V_2 - V$, $\gamma = (\rho_1 - \rho_2)/(\rho_2 - \rho_3)$, S is the topographic slope and $V = V_1 - V_2$ characterizes the Froude number.

The linear stability of this flow is addressed by adding infinitesimal disturbances of the form $(iU_j(X), v_j(X), p_j(X))\exp(ikY - i\omega t)$ and linearizing. Here k is the along-flow wavenumber and ω is the disturbance frequency (a positive imaginary part implying instability). Assuming the changes in the layer thickness are small, we obtain the dispersion relation which has six roots corresponding to four branches of IGW and two branches of the Rossby waves depending on parameters $\overline{H}_j$, γ, S_j, V and the cross-flow wavenumber α.

3   Transformation of dispersion curves

Examples of dispersion curves calculated for increasing Froude number $0 \leq V \leq 1$ are shown in figure 1 for $\overline{H}_1 = \overline{H}_2 = 0.5$, $\gamma = 1$, $S_1 = -S_2 = 0.2$, $\alpha = 1$. The phase speed is shown for the second IGW modes (downstream propagating, red curve, and upstream propagating, brown curve) as well as for upstream propagating Rossby wave, blue curve, related to $S_1 > 0$ in the lower layer, and downstream propagating Rossby wave, green curve, related to $S_2 < 0$ in the middle layer.

When $V = 0$ (figure 1a), the phase speed of neutral topographic Rossby waves is much smaller than ones of IGW modes. When $V = S_1$ (figure 1b), the conventional baroclinic instability has the maximum growth rate at $k = 1$ due to resonance between Doppler-shifted Rossby waves within the unstable wavenumber window $0 \leq k \leq 1.8$. When $V = 0.4$ (figure 1c), the upstream propagating Rossby wave has larger Doppler shift, so that the baroclinic instability growth rate and the unstable wavenumber window are much smaller.

When $V = 0.6 > \sqrt{\overline{H}_2/(1+\gamma)} = 0.5$ (figure 1d), ageostrophic R-K instability becomes possible due to a resonance between the lower layer Rossby wave and downstream propagating IGW with the maximum growth rate proportional to $\sqrt{S_1}$, while the Doppler shift becomes too large for a resonance between the Rossby waves. When $V = 0.8 > \sqrt{\overline{H}_1} = 0.7$ (figure 1e), another R-K instability becomes possible due to a resonance between the middle layer Rossby wave and Doppler-shifted upstream propagating IGW with the maximum growth rate proportional to $\sqrt{-S_2}$; it coexists here with the first R-K instability in more narrow unstable wavenumber window. Finally, when $V = 1$ (figure 1f), the Kelvin-Helmgoltz instability becomes possible for $k > 6$; it coexists here with both types of the R-K instabilities which unstable wavenumber windows become to overlap.

4   Conclusions

A multi-layer ageostrophic version of Phillips' model over sloping topography with small variations of the layer thickness allows to consider analytically different types of instability (Kelvin-Helmholtz, Rossby-Kelvin, baroclinic) in horizontally uniform flows. The dispersion curves for the Rossby waves and IGW are investigated for two-and-half layer configuration to identify different R-K instabilities related to crossover of the branches when the Froude number

increases. Simple criteria for these ageostrophic instabilities are derived: either $(1+\gamma)V^2 > \overline{\overline{H}}_2$ due to a resonance between the IGW modes and the Rossby wave in lower layer, or $V^2 > \overline{H}_1$ due to a resonance with the Rossby wave in the middle layer. In both cases the growth rate (and the width of the unstable wavenumber window in the vicinity of the resonant wavenumber) are shown to be proportional to the square root of the corresponding gradient of the layer thickness.

Examples of dispersion curves demonstrate that these types of ageostrophic instability can coexist together (and with Kelvin-Helmgoltz instability) and their growth rates may exceed the growth rates of conventional baroclinic instability (figure 1). Such instabilities may play an important role in submesoscale mixing processes in the ocean. Further investigations are planned to clarify their finite-amplitude forms and the relation to the breakdown of balance for finite Froude number in agradient velocity model (Sutyrin 2004) and to the loss of balanced integrability for finite Rossby number in anticyclonically sheared flows (Molemaker, McWilliams \& Yavneh 2005).
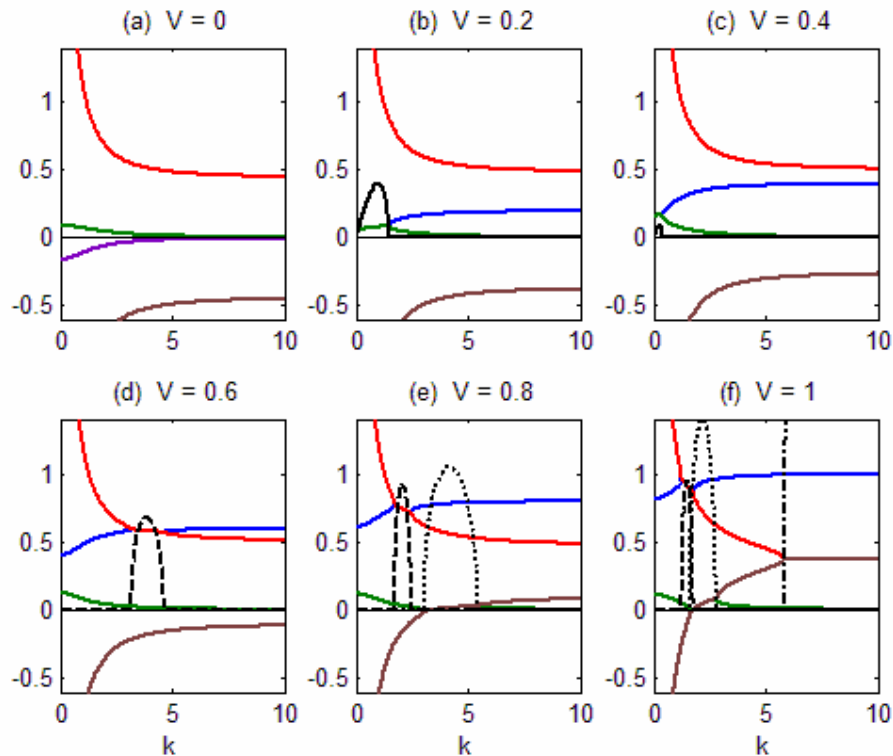
Figure 1. The real part of the phase velocity for the second IGW modes (red and brown curves) and the Rossby wave modes (blue and green curves) depending on the wavenumber *k* for six values of *V*. The growth rates (multiplied by 10) corresponding to crossover of the branches are shown for the baroclinic instability (solid line at (b) and (c)); R-K instability (dashed and dotted lines at (d), (e), (f)) and Kelvin-Helmgoltz instability (dashed-dotted line at (f)).

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