

Dynamics of turbulent jet with positive buoyancy in stratified fluid

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Abstract :

*The modeling of flow induced by submerged sewer system was carried out within laboratory physical experiment. Results of experiments performed at two experimental facilities (Large Termostratified Tank LTSB (20*4*2 m) and small ltank (1.2*0.5*0.5 m) with saline stratification) are presented. The theoretical model of Fan & Brooks (1969) was verified within the series of experiments in saline pycnocline-type stratification in wide range of parameters. The conditions of scaling modeling were determined on the basis of this model, and laboratory scale modeling with geometrical scale 1:27 was performed in LTSB. Excitation of intensive temperature oscillations in the pycnocline was observed. They were interpreted as self-sustained oscillations of the buoyant plume.*

Key-words :

jet ; buoyancy ; stratification

1 Introduction

Disposal of sewage of coastal cities to the ocean is a usual world practice producing sensible stress on coastal water areas including effects on hydrodynamics of coastal zone and coastal ecosystems. In the sewer systems almost fresh wastewater is discharged from submerged diffusers to form buoyant plumes in salt ambient seawater. Density stratification strongly affects the dynamics of the buoyant plumes, for example, the pycnocline-type stratification can trap the wastewater and prevent rising them to the sea surface.

Scale laboratory modeling of complex phenomena, which occur within buoyant jet-stratified fluid interaction is of interest both for hydraulic engineering and environmental applications. The main aim of the present work is investigation of fluid dynamics of wastewater discharge from submerged sewer systems basing on laboratory scale modeling. The theoretical model of Fan & Brooks (1969) was verified within the series of experiments in saline pycnocline-type stratification in wide range of parameters. Non-stationary effects of buoyant jet interaction with ambient stratified fluid are of special attention. In particular, the effect of excitation of internal waves by buoyant plumes is studied in laboratory tank with temperature stratification.

2 Theoretical model of the flow induced by submerged sewer system.

Disposal of wastewaters of coastal cities to the ocean is a usual world practice. A typical outfall consists of a submarine pipeline with a diffuser section at the far offshore end – a manifold with many small holes of the diameter d (see Koh & Brooks (1975)). Fresh water is discharged to ambient salty stratified ocean water at rates 1-5 m/s to form buoyant plumes. The parameters of the laboratory experiment were chosen to model the Sand Island Honolulu wastewater outfall in Mamala bay (Hawaii). The characteristics of ambient pycnocline stratification in this region have been investigated sufficiently (see Koh & Brooks (1975), Keeler et al (2005)).

Initially the diffuser flow could be considered as n parallel round horizontal turbulent jets, with the velocity U_0 and initial buoyancy equals to $\beta_0 = g((\rho_1 - \rho_0)/\rho_0)$. This flow was described within the theoretical model of inclined round turbulent buoyant jet evolution developed by Fan & Brooks (1969).

We used the dimensionless variables: $\tilde{s} = s/b_0$ - a coordinate of jet centerline, $\tilde{x} = x/b_0$ - horizontal coordinate, $\tilde{z} = z/b_0$ - a vertical coordinate, $B(\tilde{s}) = b(s)/b_0$ - a diagonal jet scale, $V(\tilde{s}) = u(s)/U_0$ - centerline velocity, $\gamma = \beta/\beta_0$ - a jet buoyancy ($b_0 = d/\sqrt{2}$ - an initial diagonal jet size). In these variables, the system of equations (Fan and Brooks (1969)) for the averaged jet parameters is as follows:

$$\frac{d}{ds}(VB^2) = 2\alpha VB \quad (1)$$

$$\frac{d}{ds}(V^2 B^2 \cos \Theta) = 0 \quad (2)$$

$$\frac{d}{ds}(V^2 B^2 \sin \Theta) = 2\lambda^2 B^2 \gamma Ri \quad (3)$$

$$\frac{d}{ds}(VB^2 \gamma) = \left((1 + \lambda^2) / \lambda^2 \right) VB^2 n^2 (\tilde{z}/\tilde{h})(Str), \quad (4)$$

$$d\tilde{x}/d\tilde{s} = \cos \Theta, \quad d\tilde{z}/d\tilde{s} = \sin \Theta, \quad (5)$$

where $\alpha = 0.06$ - is an empirical coefficient of entrainment for a round buoyant jet and $\lambda=1,16$ - the Schmidt number. $N^2(z/h)$ - is a profile of buoyancy frequency, which could be described as $N^2(z/h) = N_0^2 n^2(z/h)$, where h - is a pycnocline thickness and N_0 - is the maximum buoyancy frequency, Θ - an angle of inclination of a jet.

The initial conditions are taken as follows:

$$B = 1, V = 1, \gamma = 1, \tilde{x} = 0, \tilde{z} = z_0/b_0 \quad (6)$$

The system (1-5) and initial condition (6) are defined by four dimensionless parameters: the parameter of ambient stratification - $Str = N_0^2 b_0 \rho_0 / (g \Delta \rho_0)$, the global Richardson number of the jet flow - $Ri = g \Delta \rho_0 b_0 / (\rho_0 U_0^2)$, the dimensionless thickness of pycnocline $\tilde{h} = h/b_0$, the dimensionless depth of diffuser location $\tilde{z} = z_0/b_0$.

A verification of the model applicability in laboratory experiments was made for the case without jet junction, therefore the dynamics of flat jet was not considered.

3. Laboratory modeling. Results of experiments.

Principal scheme of all experiments is shown in fig.1.

To verify the proposed model a series of experiments was carried out in a small plexiglass tank (a=50 cm, b=120 cm, c=25 - 40 cm) with a pycnocline-type saline stratification. The flow was visualized by dye. The intervals of used parameters of the experiment are presented in the second column of the Table 1.

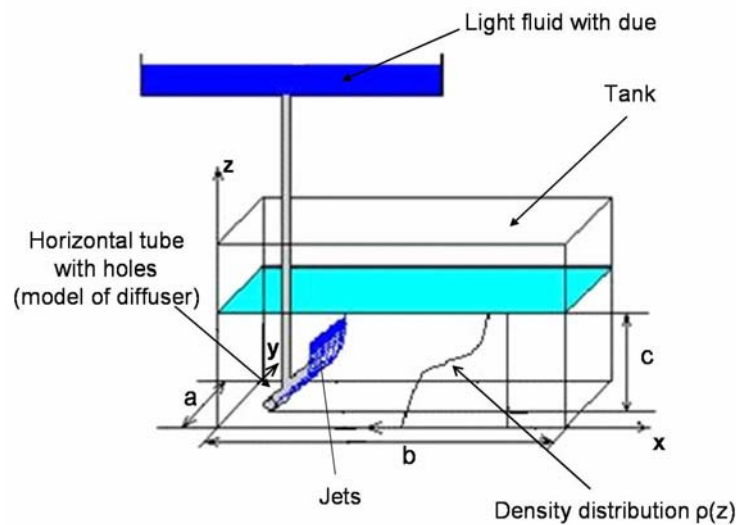


Fig. 1. – Principal scheme of experiments.

Two regimes of a flow were observed, depending on the parameters of laboratory experiment with saline stratification. The first one corresponds to a jet rising to the surface and the second one describes jet trapping by the stratification (fig.2). In each experiment the jet trajectory $z(x)$ is found and a terminal height of rise of the jet is defined within the system of equations (1-5). The results of such forecast were in good agreement with the results of experiments for certain parameters.

Parameter	Diffuser of Honolulu	Exp in saline str.	Exp. in LTSB
Diameter of diffuser holes d	8 cm	0.13 - 0.3 cm	0.3 cm
Distance from diffuser to pycnocline z_p	30 m	0.06 – 0.18 m	1.1 m
Thickness of pycnocline h	5.5 m	4 – 8 cm	0.2 m
Distance between the holes l	7 m	4.5 – 6 cm	0.3 m
Maximum of buoyancy frequency N_0	$5 \cdot 10^{-2} - 3 \cdot 10^{-2} \text{ rad/s}^{-1}$	0.5 - 1 rad/c	0.45 rad/s
Initial density defect in jet $\rho_1 - \rho_0$	0.0235 g/cm^3	$0.04 - 0.1 \text{ g/cm}^3$	0.07 g/cm^3
Initial velocity of the jet U	3 m/s	0.5 – 1 m/s	1 m/s

Table 1. Parameters of experiments.

After the verification of the used theoretical model, a series of 14 experiments was set up in a Large Thermostratified Tank (LTSB) ($a=4$ m, $b=20$ m, $c=1.8$ m) of IAP RAS. The scheme of the experiments was similar to experiments with saline stratification (fig. 1). In this experiments the initial jet velocity varied from 30 cm/sec to 190 cm/sec in fixed temperature stratification with $N_0=0.45 \text{ rad/c}$. All the parameters of outflow and stratification in LTSB, presented in third column of Table 1, provided a physical scale modeling of the Sand Island Honolulu wastewater outfall in Mamala bay (Hawaii) in Ri number, Str number, \tilde{z}_0, \tilde{h} and geometrical similarity

(1:27). The temperature fluctuations, induced by the buoyant jet, were measured by 14 vertically mounted sensors.

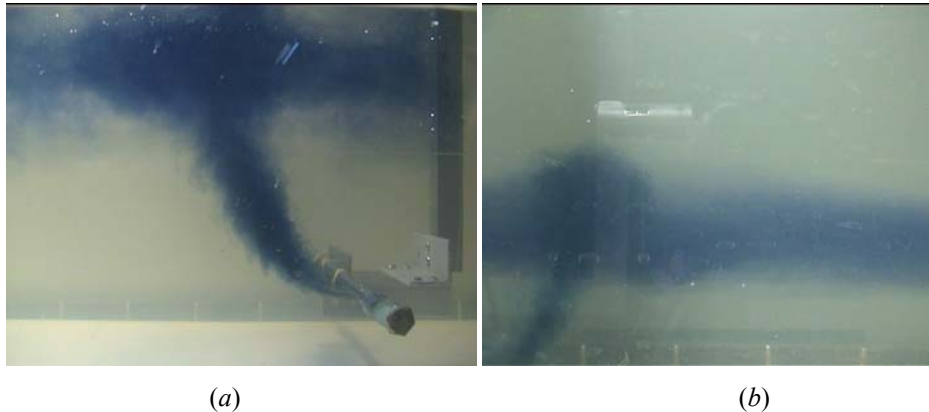


Fig.2 – Rising to surface regime (a) and trapping regime (b) in experiments with saline stratification.

For all the experiments, set in the LTSB, the jet trapping at the lower horizon of the thermocline was observed. It means that at the model conditions the sewage didn't reach the surface. An example of measurements under scale modeling conditions (initial velocity - 100 cm/sec) is presented in fig. 3 - 5. A temporal section of temperature field measured by 14 sensors, is demonstrated in fig.3. Fig. 4 indicates the isotherms of 10°C and 17°C, and fig. 5 represents the temporal spectrums of these isotherms. The isotherm of 10°C was convected to be a lower horizon of the thermocline, and the isotherm of 17°C is the upper horizon of the thermocline. Intensive isotherm oscillations are visible in these figures.

The variances of the amplitude fluctuations for the high frequency oscillations of 10°C and 17 °C isotherms, $\langle \sigma_{\eta}^2 \rangle = \int_{f_0=0.01\Gamma u}^{f_{max}} Sp(f)df$ were studied as functions of Froude number:

$$Fr = Ri^{-1/2} = U / \sqrt{g\Delta\rho_0 / (\rho_0 b_0)}$$

which is the control parameter of the problem.

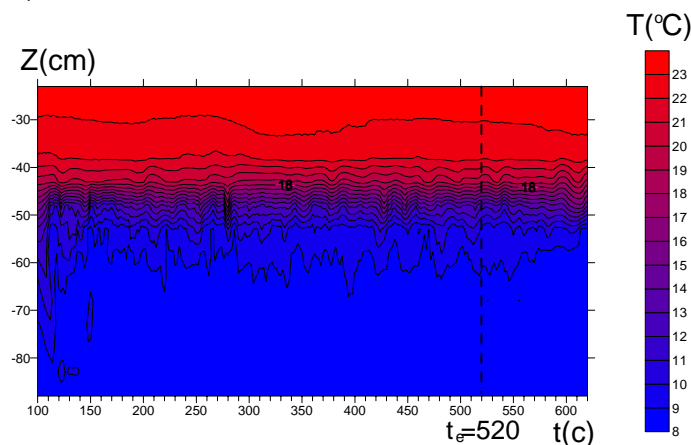


Fig.3 – Temporal oscillations of isotherms.

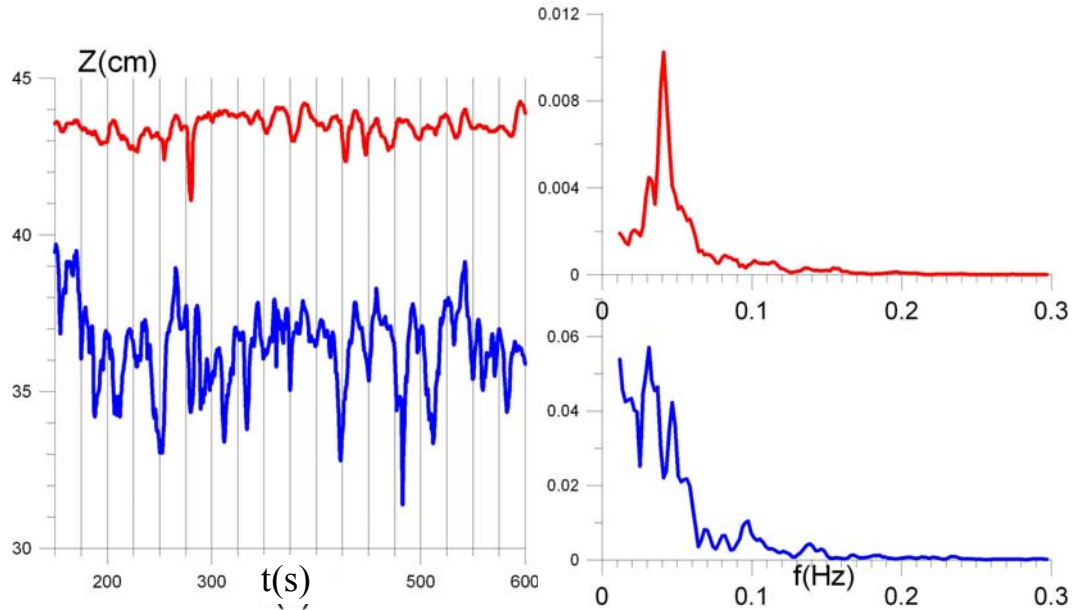


Fig. 4. - Oscillations of the isotherms of 10°C (blue curve) and 17°C (red curve)

Fig 5. - Spectrums of these isotherms

This dependence $\langle \sigma_\eta^2 \rangle (Fr)$ for lower horizon of the thermocline (isotherm 10°C) is demonstrated in fig. 6a. It is clear that the amplitude of oscillations monotonically grows with the increase of Froude number. We supposed that the global mode instability (GMI) exists in this system, leading to excitation self-sustained oscillations (see Chomaz et al (1988), Huerre et al. (1990)). To verify the supposition the observed dependence $\langle \sigma_\eta^2 \rangle (Fr)$ was compared with the solution of stationary Ginzburg-Landau equation:

$$\frac{\gamma}{\mu} + \left((Fr - Fr_c) - \frac{\nu}{\mu} \langle \sigma_\eta^2 \rangle \right) = 0 \quad (7)$$

Here Fr_c - is a critical value of a control parameter, γ - is an external force, ν - is a parameter of nonlinear damping. Figure 6a demonstrates a measured data fitting with (7) ($\gamma/\mu = 7.4$, $Fr_c = 23.6$, $\nu/\mu = 2.3$), i.e. existence of the Hopf bifurcation to the self-sustained oscillations in the system. As it is mentioned in the paper by Huerre & Monkewitz (1990) such typical amplitude dependence on the control parameter is considered as the most rigorous criterion for GMI.

Figure 6b demonstrates the similar dependence $\langle \sigma_\eta^2 \rangle$ for the upper horizon of the thermocline (isotherm 17°C) on Fr . The non-monotonically dependence could be caused by the resonant character of excitation of the thermocline oscillations by the buoyant jet.

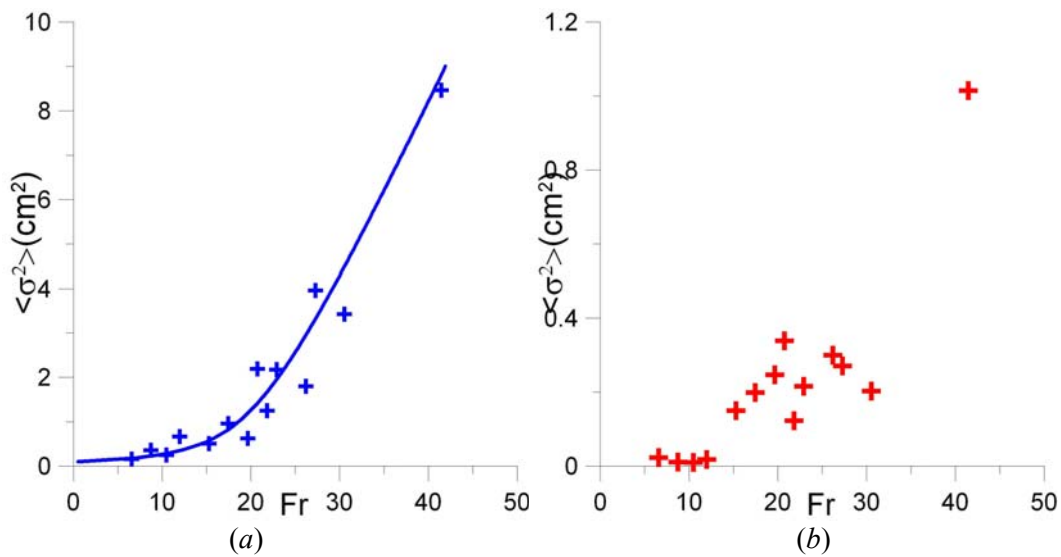


Fig. 7. – (a) Symbols demonstrate dependence $\langle \sigma_\eta^2 \rangle(Fr)$ (isotherm 10°C), and firm line is result of approximation with (7) (b) dependence $\langle \sigma_\eta^2 \rangle(Fr)$ (isotherm 17°C)

4 Conclusions

The scale modeling conditions of the turbulent buoyant jet flow induced by a typical sewer system were determined on the basis of the theoretical model developed by Fan & Brooks (1969). The model was verified in wide range of parameters on the basis of the series of experiments in saline stratification. Scale modeling of turbulent buoyant jet dynamics was performed in LTSB. Phenomenon of generation of short-period internal waves was observed for the certain values of outflow velocity, which correspond to the typical in-situ conditions.

We supposed that global mode instability (GMI) exists in this system, what leads to a self-oscillation of the jet excitation. An existence of the self-sustained oscillation regime is confirmed with the measured dependency of the amplitudes of oscillations on the control parameter typical for the presence of the Hopf bifurcation in the system.

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