Laboratory, numerical and theoretical modeling of a far wake flow in a stratified fluid

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Abtract :

A laboratory study and direct numerical simulation (DNS) of far wake flow in a stratified fluid are performed. The laboratory study employs the PIV technique to measure the velocity field in a wake behind a towed sphere at high Reynolds and Froude numbers. The DNS parameters and initialization are prescribed in accordance with the experimental data, which allows a direct comparison between numerical and experimental results. The results of the DNS and the laboratory experiment are compared with predictions of a theoretical model, which considers the wake as a quasi-two dimensional turbulent jet flow with the main mechanism of evolution associated with transfer of momentum from the mean flow to quasi-two dimensional sinuous disturbances growing due to hydrodynamic instability. The time evolution of the wake axis velocity and its width obtained within the framework of the model is in good agreement with the experimental and numerical data.

Key-words : wake ; stratified fluid; hydrodynamic instability

1 Introduction

Turbulent stratified wakes are studied extensively since late 60-th (cf. review papers by Lin J.–T., Pao Y.–H., 1979; D.L.Boyer, A.Srdic–Mitrovic, 2001 and references therein). Substantial progress in studies of such flows has been achieved in laboratory experiment with the use of the digital particle image velocimetry (DPIV) technique developed and applied by Spedding et al (1996) (see also Spedding (1997); Spedding (2001); Spedding (2002); Bonnier & Eiff (2002), Fincham, Spedding (1997) as well as other visualization methods developed and applied by Lin et al, (1992), Lin et al, (1993), Chomaz et al, (1993). Direct numerical simulation (DNS) of the turbulent stratified wake was performed by Gourlay,et al, (2001) and Druzhinin (2003) and the large eddy simulation was performed by Dommermuth et al (2002). These studies show that there are three distinct stages of the wake evolution, which are defined with respect to the product Nt, where N is the characteristic value of the buoyancy frequency at the level of towing and t is the time elapsed from the moment of the body pass at a given point. The near wake occurs at $Nt < 1$, and its dynamics strongly depends on details of the flow around the obstacle. For the case of towed sphere it is determined by the Froude and Reynolds numbers, $Fr= 2U_t/ND$ and $Re=U_tD/\nu$ (where D is the sphere diameter, U_t the towing speed, and ν the fluid molecular viscosity). This stage was studied in detail experimentally for example by Chomas et al (1993), Lin et al (1992). The results of these experiments show that the fluid motion in the wake remains three-dimensional at large Froude and Reynolds numbers until times $Nt \sim 1$, when the action of the buoyancy forces causes the collapse of the vertical velocity pulsations. At larger times there occur two distinct stages of the wake evolution, namely an intermediate stage at times $2<Nt<50$ (defined also as non-equilibrium (NEQ) stage by Spedding, 1997), and the late wake flow regime at times $Nt>50$ (or quasi-two-dimensional (Q2D) stage, cf. Spedding, 1997). It should be mentioned that, as it was demonstrated by Spedding (2001), at theses two stages (NEQ and Q2D) the wake flow is quite anisotropic in that the horizontal flow scale strongly exceeds the vertical scale.

Recent theoretical analysis of the stratified wake flow performed by Troitskaya (2002) and Balandina, et al (2004), Troitskaya, et al (2006), shows that the temporal development of the wake axis mean velocity, observed in the laboratory experiments by Spedding (1997) and Bonnier & Eiff (2002), can be related to the development of a quasi-2D hydrodynamic instability mode of the wake flow. The theoretical model employs the following assumptions. Firstly, it is assumed that the dynamics of the wake at the quasi 2D-stage is governed by the development of a quasi-2D hydrodynamic instability mode. As another assumption the effect of internal waves on the wake evolution is also neglected and the wake as a quasi-2D jet-like flow subject to quasi-2D disturbances is considered. Thus, the reduction of the wake mean axis velocity is caused by the momentum transfer from the mean flow to the sinuous quasi-2D disturbances. The third model assumption is related to neglecting the viscous dissipation associated with the vertical shear at the considered stage of the wake evolution. Thus, the model is not applicable in the late wake region, where the viscous dissipation is known to be significant (see, Spedding, 1997).

The theoretical model employs a quasi-linear approximation, where the non-linear contribution to the mean wake velocity is the only effect of non-linearity taken into account. The mean flow disturbances are considered under the linear approximation as a superposition of unstable modes and the mode interaction is neglected. Applicability of the quasi-linear approximation was investigated in Troitskaya, et al (2006), on the basis of comparison with the results of DNS. It is revealed that the quasi-linear approximation is valid beyond the conditions of its formal applicability.

In the present paper, we compared the results of a laboratory study and direct numerical simulation (DNS) of far wake flow in a stratified fluid with predictions of the quasi-linear quasi-two-dimensional theoretical model. The laboratory study employs the PIV technique to measure the velocity field in a wake behind a sphere towed under a picnocline at high Reynolds and Froude numbers. The DNS and the theory parameters and initialization are prescribed in accordance with the experimental data, which allows a direct comparison between numerical and experimental results.

2 Description of the laboratory experiment

The experiments were performed in a tank with sizes 3×2×0.5 m. A salty stratification was created where the initial fluid density distribution was characterized by a pycnocline with the total density jump 0.0464 г/см^3. A sphere with diameter 3.8 cm were towed along a string with the use of a tow-wire at a constant speed 70 cm/s and at the vertical level corresponding to the location of 10 cm below the pycnocline center. The buoyancy frequency at the level of the towing was equal to N = 0.5 rad/s.

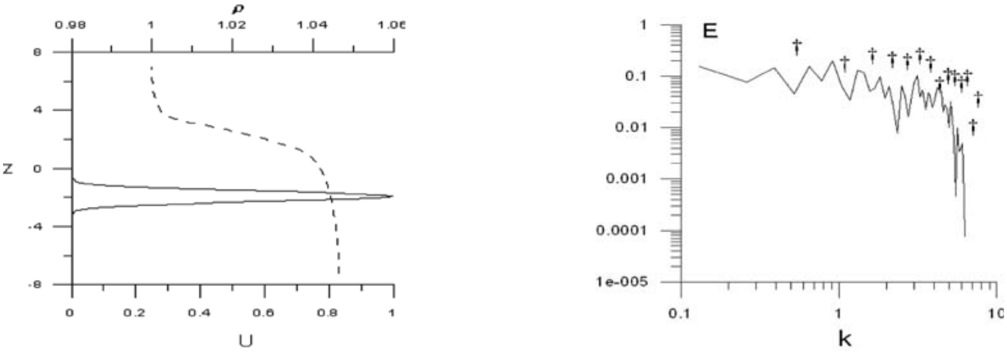
The PIV (Particle Image Velosimetry) technique was employed to measure the fluid velocity where polystirol particles with diameter 0.8 mm were used as tracers. The density of the particles was 1.04 g/cm^3 calibrated with the accuracy ±0.07 so that the deviation of the particles location from the axis of towing was 1.5 cm, i.e. smaller than the diameter of the towed body. The trajectories of the particles were digitally recorded with the use of a video camera Sony DCR-TRV17E positioned above the measurement area. The mean velocity profile at a given time moment t was evaluated by spatial averaging over the longitudinal coordinate x as $U(y) = \frac{1}{L_x}\int_0^{L_x} u(x, y)dx$, where L_x=43.cm. These profiles were approximated by the Gaussian function $U(y) = a + U_0 \exp\left(-(y-y_0)^2/\delta^2\right)$. Parameters of the flow δ =3.25 cm and U_0= 1.3 cm/sec measured at t=0.6 s from the towing moment were used as initialization in DNS.

3 Numerical simulation of a dynamics of a turbulent jet flow below the pycnocline

In order to perform the numerical simulation of the flow in the far wake we introduce dimensionless variables by normalizing the spatial coordinates (x,y,z) and fluid velocity with the use of the length and velocity scales $L_0=2\delta$ and U_0. We consider a fluid jet flow with the initial (reference) profile of the dimensionless average horizontal component of the velocity in the form:

$$U_{ref} = -\exp(-4(y^2 + (z+1.3)^2)), \qquad (1)$$

where y and z are the coordinates in the transverse and vertical directions (solid line in Fig.1).. The initial density profile is also obtained from the experimental data and is shown in



(a) (b)

FIG. 1 – Reference profiles of the fluid velocity (solid line) and density (dashed line) –(a). Power spectra of the fluid velocity in the laboratory experiment at time t=0.6 sec (symbols) and in the numerical simulation at the initialization time t=0 (solid line) – (b).

The fluid velocity at the initial moment is prescribed as a sum of the random component and the reference horizontal velocity U_{ref} (1). Velocity fluctuations at the initial moment are evaluated as a sum of independent Fourier harmonics with random phases. The power spectrum $E(k)$ of the velocity fluctuations is isotropic and prescribed as $E(k)=E_0(k/k_p)\exp(-k/k_p)$, where k is the modulus of the harmonics wave number. Parameters E_0 and k_p are chosen to match the initial velocity spectrum in the numerical simulation to the corresponding velocity spectrum obtained in the experiment. Figure 2 shows a comparison between the numerical spectrum (solid line) and experimental data (symbols) obtained by the Fourier transform of the fluid velocity over the horizontal coordinate x at time t = 0.2 (in dimensionless units, corresponding to the dimensional time L_0t/U_0=0.6 sec). The initial distribution of velocity fluctuations in the transverse and vertical directions is made proportional to the mean flow profile (1), as it is also observed in the experiment.

The Navies-Stokes equations in the Boussinesq approximation, the incompressibility condition for the fluid velocity and the equation for the fluid density were solved by the numerical method described by Druzhinin (2003). In the numerical calculation the Reynolds number is assigned sufficiently large (Re=U_0L_0/ν = 400), so that the effects of molecular viscosity remain negligibly small during the simulation. This value is also quite close to Re=483 observed in the experiment at time t =0.6 sec. The global Richardson number is set equal to $Ri = (L_0 g \Delta\rho_0)/(U_0^2\rho_0)$=15. This value is less than the experimental value (Ri = O(100)). The choice of parameter *Ri* in DNS is restricted by the constraints imposed by numerical resolution of the large density and velocity gradients of the flow field in our numerical simulation presented below. However, as it was observed in the previous numerical studies (see e.g. Gourlay etal, 2001), the choice of *Ri* does not affect significantly the dynamics of the jet flow. Since *Ri* is considered much larger than unity in the present study, we can assume that the current choice (*Ri*=15) does not degrade the comparison with the experimental results.

4. A theoretical model of the evolution of a quasi-two dimensional wake in a stratified fluid.

A theoretical model of the stratified wake flow describing the development of the quasi-2D instability of the wake flow and the dynamics of the wake integral parameters (mean axis velocity and horizontal width) is described in Troitskaya et al (2006). The system of quasi-2D equations was derived by Lilly (1983), Flór & van Heijst (1994) and Embid & Majda, (1998) in the limit of the large Richardson number for the case of a laminar flow. We employ these equations rewritten in terms of the flow stream function ψ and vorticity Ω and take into account the momentum transfer due to small-scale turbulent fluctuations present in the wake.

Then the set of the equations for ψ and Ω is written as follows:

$$\frac{\partial\Omega}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial\Omega}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\Omega}{\partial y}=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)K_T\Omega+2\left(2\frac{\partial^2 K_T}{\partial x\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial^2 K_T}{\partial x^2}\frac{\partial^2\psi}{\partial y^2}-\frac{\partial^2 K_T}{\partial y^2}\frac{\partial^2\psi}{\partial x^2}\right) \quad (2)$$

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=\Omega\,. \quad (3)$$

Here K_T is the horizontal eddy viscosity coefficient.

We seek solution to the set of equations (2,3) as a sum of a mean flow, which does not depend on *x* coordinate, and disturbances which are presented as a superposition of harmonics with random phases as:

$$\left(\psi,\Omega\right)(x,y,t)=\left(\psi_0,\Omega_0\right)(y,t)+\mathrm{Re}\sum_k a(k,t)\left(\psi_1,\Omega_1\right)(y,t,k)exp(ik\,x+i\varphi_k) \quad (4)$$

Equations for the mean fields can be obtained by averaging of eqs.(2,3) over the *x* coordinate, which in our case is equivalent to the statistical averaging under the approximation of the random phases. Under the random-phases approximation, the nonlinear terms in equations for disturbances are diminished after averaging over phases (Galeev & Sagdeev, 1973. If we now assume that the disturbance characteristic time scale T_{dis} is much smaller than the characteristic time scale of the mean flow T_{avr}, so that factor $\mu = T_{dis} / T_{avr} << 1$. , then the linear equation for $\psi_1(y,t,k)$ can be solved with the use of the WKB-approximation, i.e.:

$$\psi_1=\left(\Phi_0+\mu\Phi_1+\ldots\right)e^{-i\frac{\Theta}{\mu}}. \quad (5)$$

Denoting $\omega=\frac{d\Theta}{d\tau}$, we have $\Theta=\mu\int_0^t\omega dt$. In the 0-th order in μ we obtain the eigenvalue problem:

$$ik\left(\left(\frac{\omega}{k}-U\right)\left(\frac{\partial^2\Phi_0}{\partial y^2}-k^2\Phi_0\right)+\frac{\partial^2 U}{\partial y^2}\Phi_0\right)+\left(\frac{\partial^2}{\partial y^2}-k^2\right)K_T\left(\frac{\partial^2\Phi_0}{\partial y^2}-k^2\Phi_0\right)+2\Phi_0k^2K_{Tyy}=0, \quad (6)$$

$$\Phi_0\big|_{y\to\pm\infty}=0\,.$$

Solution to the problem (6) gives the dispersion relation $\omega=\omega(k,t)$ and the eigenmodes. The solvability condition of the equation in the 1-st order in μ gives the equation for the amplitude *a*:

$$\int_{-\infty}^{\infty} \Psi \frac{\partial}{\partial t}\left(a\left(\frac{\partial^2 \Phi_0}{\partial y^2} - k^2 \Phi_0 \right)\right) dy = 0 \qquad (7)$$

Here $\Psi(y,t)$ is the solution of equation conjugated to the eq. (6). Substitution (5) in the averaged over the x eqs.(2,3) gives the following equation for mean velocity $U=\Psi_{0y}$:

$$\frac{\partial U}{\partial t} - \frac{\partial}{\partial y}\frac{1}{2}\sum_k k\left|a(k,t)\right|^2 e^{2\int_0^t \mathrm{Im}\,\omega dt}\ \mathrm{Im}\left(\Phi_0(y,t,k)\frac{\partial \Phi_0}{\partial y}\right) = \frac{\partial}{\partial y}\left(K_T \frac{\partial U}{\partial y}\right) \qquad (8)$$

The theoretical model of the quasi-two-dimensional wake flow consists of: 1) equation (8) for the mean axis velocity $U(y,t)$; 2) eigenvalue problem (6); and 3) the equation (7) for the harmonics amplitude.

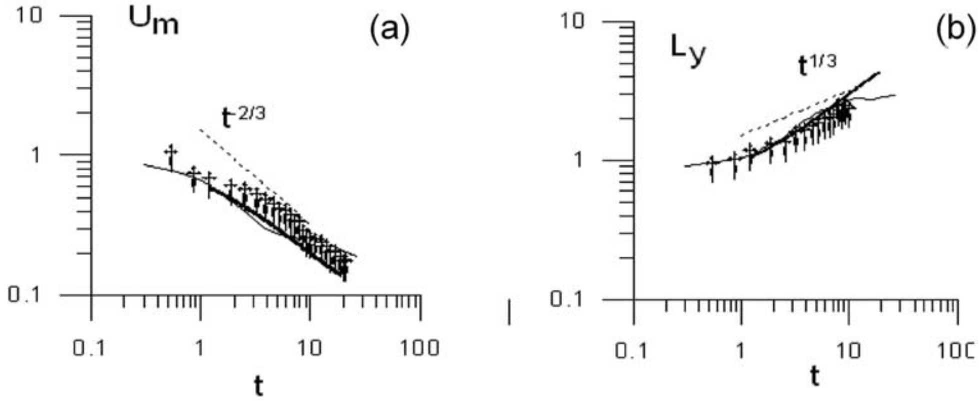


FIG.2 Temporal development of the (a) mean velocity; (b) transverse length scale (thick line –theory, thin lines – DNS) . Symbols show the corresponding experimental data. Temporal asymptotics of the non-stratified jet flow are shown in dotted lines.

These equations were solved numerically for the parameters of the experiment described in section 2. Since the theoretical model is applicable only to the case of the late wake we initialize the solution at Nt=1.8. The initial spectrum of the streamfunction disturbances was taken from the experiment. The initial mean velocity profile was taken in the gaussian form with the parameters measured at Nt=1.8. Figure 2 shows the temporal development of the normalized by the scales U_0 and L_0 average velocity maximum U_m and the length scale L_y obtained in the numerical simulation, within the theoretical model and in the laboratory experiment (in dashed and solid lines and symbols, respectively). The figure shows that there is a qualitative and quantitative agreement between the theoretical and numerical results and experimental data in the decay of the velocity maximum and the transverse length of the stratified wake.

5 Conclusions

In this paper we present results of a laboratory study, direct numerical simulation (DNS) and theoretical prediction of far wake flow in a stratified fluid. We compared the results of all these approaches, where the wave field in the theory and DNS was taken from the experiment was employed as initial conditions. A qualitative and quantitative agreement between the theoretical results and numerical and experimental data in the decay of the velocity maximum and the transverse length of the stratified wake was demonstrated. It enables us to conclude that probably the principal physical process providing evolution of the mean flow in the wake after a towed body in the stratified fluid is associated with demodulation of disturbances growing due to hydrodynamic instability.

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