Experimental study of a turbulent Ekman layer

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Résumé :

Cette communication présente les résultats d'une campagne de mesures en fluide tournant sur des couches limites turbulentes sur fond plat horizontal. Une techique de PIV stéréoscopique est utilisée pour obtenir les trois composantes des champs de vitesse dans un plan. Les profils de vitesse obtenus en régime laminaire sont en très bon accord avec les prédictions théoriques d'Ekman. Les résultats obtenus en régime turbulent valident la méthode de mesure et permettent d'envisager une étude extensive des couches d'Ekman turbulentes.

Abstract :

This paper reports on laboratory experiments concerning frictional rotating turbulent boundary layer in spin-up flow over flat horizontal bottom. Stereoscopic Particle Image Velocimetry technique is used to obtain two-dimensional three components fluctuating velocity fields. Velocity profiles measured in laminar regime show a remarkable agreement with the Ekman theoretical predictions. Results obtained in turbulent regime confirm the measurement method validity and allow to plan an extensive analysis of the turbulent Ekman layers.

Key-Words :

Ekman layer, turbulence, SPIV

1 Introduction

The role of friction in geophysical flows is closely related to the structure of the boundary layer which appears on the bottom. In its pioneering study of boundary layers in presence of background rotation, V.W.Ekman (1905) shown that the rupture of the geostrophic balance due to velocity defect induces a rotation of the velocity vector near the bottom, leading to the well-know Ekman spiral.

Although numerous field and laboratory observations have confirmed the general features of the Ekman model in the atmospheric and oceanic bottom boundary layer, few detailed laboratory experiments have been able to quantitatively analyse the rotating boundary layers, particularly in the turbulent case (Greenspan 1968). Stability studies (Lilly 1966) have shown that the Ekman layer becomes unstable for boundary Reynolds numbers around $Re_\delta = 55$; here $Re_\delta = \left(2u_0^2 / f\nu\right)^{1/2}$, where u_0 is the velocity outside the boundary layer, *f=2Ω* the Coriolis parameter and *ν* the kinematic viscosity. Using laboratory experiments, Caldwell and van Atta (1970) gave support for the theoretical Lilly criterion and found that the transition threshold to fully turbulent regime does not occur until Re_δ = 148. Some measurements in fully turbulent rotating boundary layers have been obtained on smooth and rough surfaces (Howroyd and Slawson 1975, Ferrero et al 2005), and compared with atmospheric boundary layer theory (Garratt 1992) in terms of scaling laws, surface stress and fluxes.

However, our understanding of the physical processes governing the turbulent Ekman layer is still limited by the lack of reliable data on local turbulence properties. The present experimental work aims to employ recent measurement techniques such as Stereoscopic Particle Image Velocimetry (SPIV) to enhance our knowledge of the bottom Ekman layer, particularly focusing on the turbulent regime.

2 Experimental set-up

The experimental campaign took place on the hydrodynamic rotating tank of Coriolis/LEGI (Grenoble). The tank is filled with salted water of constant density 1,02 kg/m^3. The depth at rest is 85cm, while during rotation the free surface is paraboloid. The anticlockwise rotation period T of the tank was impulsively decreased from T=53s to T=40s to generate a relative spin-up flow. Coriolis parameter is f=0,31 and the Rossby number defined by $\Delta\omega/\omega=0{,}33$

Quantitative measurements of the two-dimensional three-components velocity fields are provided by Stereoscopic Particle Image Velocimetry (SPIV). This technique requires two cameras to record simultaneous but distinct off-axis views of the same acquisition area. Two CCD cameras (1024x1024 pixels, B&W 12 bits) are placed outside the tank, viewing the same acquisition area through a Plexiglas window in the vertical tank rim and recording simultaneously two series of images at adjustable time rate. The acquisition area, which size is about 9x9cm^2, is illuminated by a vertical continuous Argon laser sheet. The angle between laser sheet and azimuthal direction is 8° and the cameras are placed at -14° and +30° with respect to the axis view. The homogeneous fluid is uniformly seeded with by 30µm neutrally buoyant particles.

Measurements in a spin-up seeded flow have thus been performed, consisting in the simultaneous acquisition of images series for both camera. In-plane displacements of the particle patterns between two successive images are calculated by direct cross-correlation of the image luminosity using Correlation Image Velocimetry (CIV, Fincham and Delerce 2000). Specific mesh grids are used to get higher points density in the boundary layer. The out-of-plane component of the velocity is finally reconstructed from the two in-plane displacements fields using an original stereoscopic method, based on a linear approximation around the acquisition plane.

3 Results

The three components of the velocity vector are defined as: *u* azimuthal clockwise , *v* radial inward, and *w* vertical upward. Experimental parameters are choosen so as to generate a fulle turbulent boundary layer slowly decaying toward laminar regime.

We present first the results obtained in the laminar regime which can easily be compared with the Ekman theoretical solutions for a homogeneous rotating bottom boundary layer.

$$u = U(1 - e^{-z/\delta_E}\cos(z/\delta_E))$$
$$v = U(e^{-z/\delta_E}\sin(z/\delta_E))$$

where U is the geostrophic velocity, z the distance above the bottom and $\delta_E = (2\nu/f)^{1/2}$ is the Ekman layer depth.

Figure 1 presents the comparison between instantaneous experimental and analytical Ekman profiles for spin-up case for boundary layer Reynolds number Re=60. One notes the very good agreement between measured velocity profiles and theoretical predictions. The azimuthal velocity is constant above the boundary layer, i.e. for z>3 cm, slightly increases in the upper part of the boundary layer and then decreases toward 0. The radial velocity is very weak for z>2cm and then increases to reach its maximal value around z=0,5cm. This reveals clearly the presence of a radial inward Ekman transport in the boundary layer, due to rupture of the geostrophic balance, meaning that the Coriolis force is no more balanced by the pressure gradient.

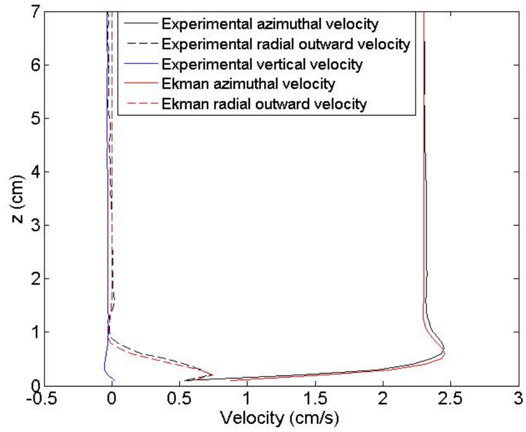

FIG. 1 –Comparison between measured laminar profiles and Ekman theoretical solutions

Figure 2 shows the evolution of the profiles of the three components of velocity during the complete spin-up process. Profiles have been averaged over 50 successive velocity fields to remove the influence of turbulent fluctuations. One notes the influence of turbulence on the profiles, which tends to smooth the velocity gradients.

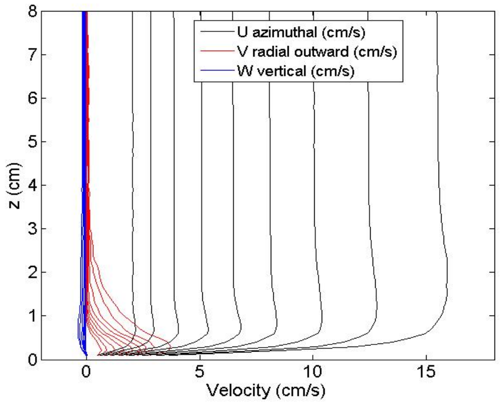

FIG. 2 – Evolution of the velocity profiles during the spin-up process

Turbulent fluctuations are thus calculated as the local deviance to the mean value, and averaged in space and time to obtain the turbulent momentum fluxes. Figure 3 shows the evolution of the profile of vertical flux of azimuthal momentum. One notes that the turbulence in essentially localized in the boundary layer, slowly decreasing during time. The negative sign is consistent with a downward flux of azimuthal momentum. The usual assumption of a constant flux in the turbulent layer is not verified.

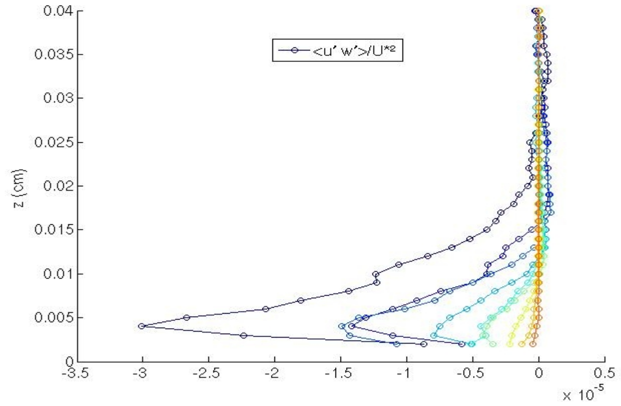

FIG. 3 – Evolution of the profiles of vertical flux of azimuthal momentum

During the spin-up process, the azimuthal velocity adjustement of a vertical fluid column is equal to the total momentum flux, i.e. the sum of viscous flux, turbulent flux and the Coriolis force on the radial component of velocity:

$$H\frac{dU}{dt}=f\int_0^z v-\langle u'w'\rangle+\nu\frac{\partial U}{\partial z}$$

The right hand term is theoretically constant in the fluid column. Figure 4 plots the comparison between the left ahnd and the right hand term of the momentum equation. The results presented are obtained at z=1cm above the bottom, i.e. in the boundary layer, but very close results are obtained for different heights (not shown here).

One notes the very good agreement between the momentum decay and momentum flux in the boundary layer. This confirms the validity of the measurements by relating local turbulent data to mean flow features.

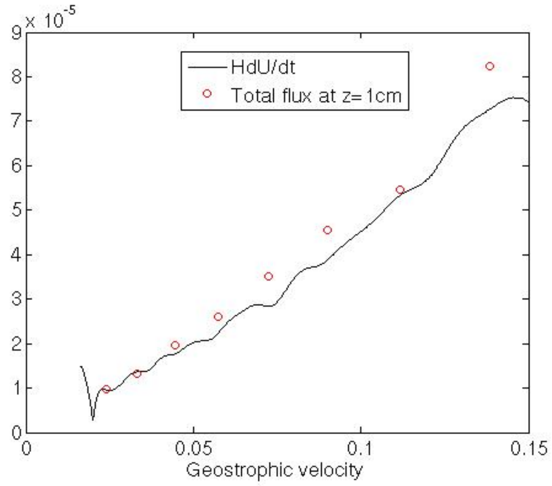

FIG. 4 – Comparison between momentum decay and momentum flux in the boundary layer

4 Conclusions

Spin-up experiments of homogeneous fluid have been performed on the Coriolis turntable to study the development of frictional rotating « Ekman » layer on a flat horizontal solid bottom. The experimental parameters have been chosen so as to generate turbulent boundary layer slowly decaying toward laminar regime. Accurate measurements of turbulence properties are performed using Stereoscopic Particle Image Velocimetry. An original method based on a linear approximation has been used to reconstruct of the out-of-plane velocity component from the two off-axis views.

The measurements have been first successfully validated by comparing the velocity profiles in the laminar regime with the theoretical prediction of V.W.Ekman. Small negative vertical velocity has been measured near the solid bottom, which can be related to the presence of Ekman suction in this anti-cyclonic spin-up flow. Vertical velocity profiles show a smoothing due to turbulent diffusion. Computations of turbulent fluctuations have thus been performed to give access the turbulent properties of the flow. A clear downward flux of streamwise momentum has been observed in the boundary layer, but the assumption of quasi-constant flux is verified only over a small part of the layer. Finally, a relation relating local turbulent measurements with mean flow decay in terms of momentum decay have confirmed the reliability of the measurement method.

These conclusions indicate that the measurement method presented is useful for analysing the turbulent Ekman layer and further experimental campaigns are planned. From an experimental point of view, spatial and temporal resolutions will be improved to allow the tracking of instabilities inside and outside the boundary layer. More physically, it will of great interest to analyse the influence of external parameters (rotation rate, bottom roughness, flow direction, topography or depth) on the turbulence dynamics and friction processes, and to compare experimental results with the similarity laws used by atmospheric boundary layer theory. Due to its ubiquitous influence on geophysical flows, a particular attention will be paid to the fluid stratification.

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