

## Mixing efficiency and entrainment at an atmospheric inversion layer

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### Abstract :

*The context is that of the convectively-driven atmospheric boundary layer capped by an inversion layer (i.e. a stably-stratified interface) and we focus on the regime of equilibrium entrainment, i.e. when the boundary-layer evolution is in a quasi-steady state. The parameterization of the entrainment process across the interfacial layer is usually based on the entrainment ratio, namely the ratio of the negative of the heat flux at the interface to the heat flux at the ground surface. Hence the issue is to relate the entrainment ratio to measurable parameters. In this study, we rely on a formulation of convective entrainment in terms of mixing efficiency, which can be computed directly for instance from high-resolution vertical profiles of virtual potential temperature. We discuss the applicability of this parameterization for an explicit treatment of the entrainment process in classical boundary-layer parameterization schemes implemented in meso-scale models.*

### Résumé :

*Dans cette étude, on s'intéresse à la couche limite atmosphérique convective présentant une inversion à son sommet (i.e. une interface stablement stratifiée) et l'on se limite au cas où son évolution est quasi stationnaire. La paramétrisation du processus d'entraînement au sommet de la couche limite est classiquement basée sur le rapport, en valeur absolue, entre le flux de chaleur à l'interface et le flux de chaleur à la surface du sol. Ainsi l'enjeu est de relier ce rapport quantifiant l'entraînement à des paramètres mesurables. Dans ce travail, on utilise une formulation de l'entraînement exprimée en termes d'efficacité de mélange, laquelle peut être calculée directement par exemple à partir de profils verticaux de température potentielle virtuelle à haute résolution. On discute de l'applicabilité de cette paramétrisation pour un traitement explicite du processus d'entraînement dans les schémas classiques de paramétrisations de la couche limite implémentés dans les modèles à méso échelles.*

### Key-words :

**Atmospheric flows ; Mixing and dispersion ; Stratified flows**

### 1 Introduction

In the convectively-driven atmospheric boundary layer, buoyancy-driven thermals overturn continuously the air, thereby mixing the convective layer (referred to as the mixed layer). Hence, profiles of conserved variables (e.g. the virtual potential temperature) are uniform with height within the mixed layer. Thereafter we consider a horizontally homogeneous boundary layer without large-scale subsidence. Let  $\Theta$  and  $\mathcal{F}$  denote the horizontally-averaged virtual potential temperature and heat flux, respectively. The mixed layer is generally capped by an inversion layer (i.e. a stably-stratified interface) at a height  $z_i$ , which is defined as the level where  $\mathcal{F}$  is minimum (being negative). The interfacial layer is dominated by both local entrainment effects and the properties of the inversion and stably-stratified region above. As a result of entrainment of air from the free atmosphere above, the mixed layer deepens, the evolution of which is governed by the entrainment velocity  $w_e \equiv \dot{z}_i$ . Several entrainment law formulations have been derived out to date to relate  $w_e$  to measurable parameters of the mixed and interfacial layers.

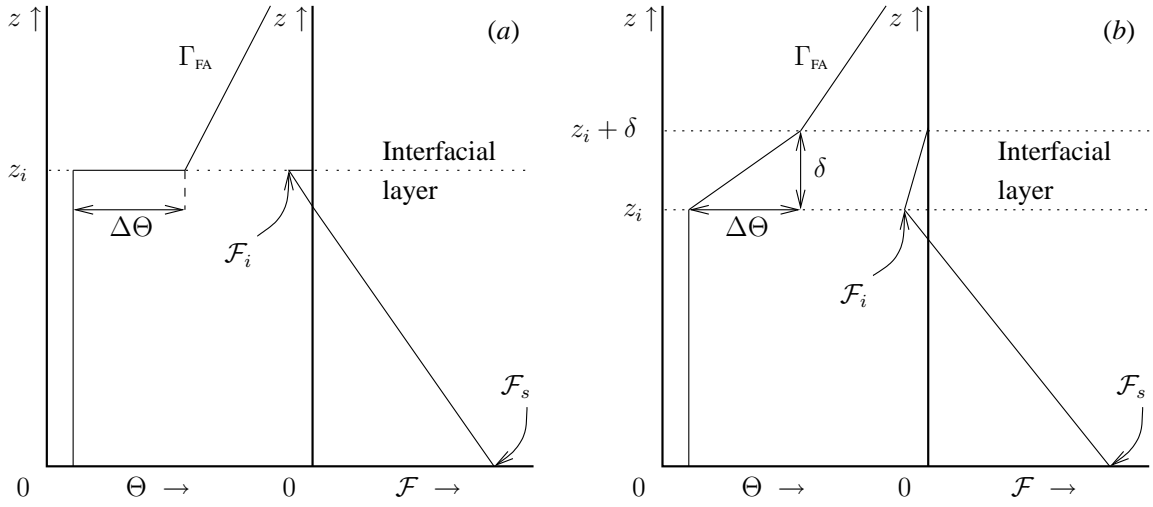


Figure 1: Sketch profiles of typical virtual potential temperature  $\Theta$  and heat flux  $\mathcal{F}$  for the convectively-driven boundary layer. (a) Zero-order model (ZOM) by Lilly (1968); (b) first-order model (FOM) by Betts (1974). The strength of the capping inversion is denoted  $\Delta\Theta$ . The interfacial-layer thickness is designated  $\delta$  and  $\Gamma_{FA}$  is the  $\Theta$  lapse rate in the free atmosphere.

## 2 Structure of the convective boundary layer

The structure of the convective boundary-layer can be represented in varying the degree of complexity, especially regarding the structure of the interfacial layer. In the zero-order model (ZOM) proposed by Lilly (1968), the thickness of the interfacial layer is assumed infinitesimal, while the temperature profile exhibits a jump across that interface [see figure 1(a)]. In first-order models (FOMs), the finite thickness  $\delta$  of the IL is taken into account (e.g. Betts, 1974; vanZanten et al., 1999). Betts (1974) assumed that the mixed layer extends up to  $z_i$ , and that above  $z_i$  the heat flux increases linearly to zero up to  $z_i + \delta$  [see figure 1(b)]. Both zero- and first-order models are commonly used in many studies of mixed-layer growth and entrainment, though more refined versions that provide a more detailed representation of the interfacial layer do exist (e.g. Fedorovich & Mironov, 1995). The most important quantity which appears in the model here concerns the entrainment heat flux at  $z_i$ , denoted by  $\mathcal{F}_i$  (see figure 1). Indeed, the entrainment law is classically based on the ratio  $\mathcal{R} = -\mathcal{F}_i/\mathcal{F}_s$  of the negative of the heat flux at the interface to the heat flux at the ground surface. This ratio is usually taken constant, with value 0.2 (e.g. Stull, 1976).

## 3 Model for $\mathcal{R}$ in terms of mixing efficiency

In the regime of equilibrium entrainment (i.e. when the boundary-layer evolution is in a quasi-steady state), we proposed a parameterization for  $\mathcal{R}$  that involves the mixing efficiency, denoted by  $\gamma$ , and the thickness of the interface  $\delta$ , namely  $\mathcal{R} = [z_i/(z_i + \delta)] \gamma/(\gamma + 1)$ . This parameterization was actually found to match very well the numerical computation of  $\mathcal{R}$  from a high-resolution large-eddy simulation (Chemel et al., 2007). This result enabled us to derive modified expressions for the classical entrainment laws within the ZOM and FOM frameworks as a function of  $\gamma$ . We showed that within the ZOM framework, the scaling factor in the entrainment law, denoted by  $\mathcal{A}$ , is the flux Richardson number. This parameterization of  $\mathcal{A}$  is further improved within the FOM framework, as direct computation of  $\mathcal{A}$  from the aforementioned large-eddy simulation showed it.

## 4 Concluding remarks

From a practical point of view, the present parameterizations rely on the computation or measurement of  $\gamma$ . In this study,  $\gamma$  is computed directly from the large-eddy simulation results mentioned previously following the method proposed by Winters et al. (1995). The values of  $\gamma$  are between 0.26 and 0.30. This range is consistent with the one found in turbulent stratified flows (e.g. Staquet, 2004). The understanding of the approximate 0.2 value found for  $\gamma$  in stably-stratified flows would provide the final closure but this question is still a challenge. Some boundary-layer flow parameterization schemes within meso-scale models do not represent explicitly the entrainment process. This is for instance the case for the  $K$ -profile model (Troen & Mahrt, 2003), for which the vertical mixing and the growth of the boundary layer are modelled separately. Hence one may wonder whether our formulation of convective entrainment in terms of mixing efficiency could not be applied to represent the entrainment process in such parameterization schemes. Since Winters & D'Asaro (1996) provided a *recipe* on how to infer  $\gamma$  from vertical temperature soundings, it should be possible. Further research is under way to take the step forward by testing this parameterization.

## Further reading

Chemel, C. & C. Staquet 2007 A formulation of convective entrainment in terms of mixing efficiency. *J. Fluid Mech.* In press. *A part of this paper is adapted here thanks to CUP.*

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