Generation of Inertia-Gravity Waves by Pulsating Lens-like Axisymmetric Vortices

Georgi Sutyrin & Vladimir Zeitlin

Graduate School of Oceanography
University of Rhode Island,
Narragansett, RI 02882 USA
gsutyrin@gso.uri.edu

Laboratoire de Meteorologie Dynamique
Ecole Normale Superieure
24 Rue Lhomond 75231 Paris FRANCE
zetlin@lmd.ems.fr

Abstract :

We consider interactions between the two most important components of the atmosphere and ocean dynamics: slowly evolving vortical motion and inertia-gravity waves in rotating stratified axisymmetric flows. Any steady axisymmetric solution for a finite volume anticyclonic vortex with outcropping isopycnals is known to correspond to a set of self-similar analytical time-periodic pulson solutions assuming flows in surrounding fluid is negligible. Here we analyze the flow patterns generated in homogeneous fluid below stratified pulsating lens-like vortex and its feedback on the upper layer vortex.

Résumé :

Nous considérons les interactions entre les deux composantes de la dynamique océanique et/ou atmosphérique les plus importantes: les mouvements tourbillonnaires lentes et les ondes d'inertia-gravité rapides dans les écoulements stratifiés en rotation. Il est connu que chaque solution stationnaire axisymétrique pour un vortex anti-cyclonique aux surface isopycnes en intersection avec la surface libre engendre une série de solutions auto-similaires, périodiques en temps ("pulsons"), si le mouvement du reste du fluide, supposé homogène, est négligée. Nous analysons ci-dessous les écoulements générées dans le fluide profond homogène au-dessous d'un pulson, et sa réaction inverse sur le vortex dans les couches supérieures.

Key-words : baroclinic vortex ; rotating fluid ; inertia-gravity waves

1 Introduction

Hydrostatic, stratified Boussinesque primitive equations (PE) are widely used for modeling large- and meso-scale variability in planetary atmospheres and oceans. Solutions for the PE include the two most important components of the atmosphere and ocean dynamics: slowly evolving vortical motion and inertia-gravity waves (IGW). At the synoptic scale and larger, the later are relatively weak, giving rise to a nearly "balanced" vortical dynamics, usually related to the potential vorticity inversion (e.g., see Sutyrin 2004 and references therein). The problem of emitting IGW and adjustment to balanced state has been considered in a number of publications. In particular, the nonlinear geostrophic adjustment of single-scale vortex-like disturbances in rotating shallow water was analyzed in detail by Reznik et al. (2001) and in continuously stratified fluid by Zeitlin et al. (2003) by means of the multi-time-scale perturbation theory in the Rossby number. Although the classical scenario of adjustment was, generally, confirmed, it was also demonstrated that large-scale large-amplitude initial perturbations contain near-inertial oscillations which stay coupled to the slow vortical component of the flow for a long time.

Exact analytic nonlinear solutions for finite-area lens-like vortices pulsating with inertial frequency (pulsons) were recently described in a self-similar form (see Sutyrin 2006 and references therein). Here we analyze the effect of deep lower layer on the evolution of the upper-layer pulson using asymptotic expansions. The rest of the paper is organized as follows. In section 2 we formulate primitive equations for axisymmetric flows in the stratified upper layer overlying the homogeneous lower layer. In section 3 we discuss a special case of inertially pulsating vortices when the solution remains self-similar with the lower layer at rest. In section 4 we analyze the case with active lower layer. Section 5 provides a summary and conclusions.

2 Model Formulation

We consider a stratified, Boussinesq fluid on the rotating plane. Assuming axisymmetry, we write the governing equations for an inviscid flow with the velocity (*U, V, W*) in the cylindrical coordinates (*r*, θ, *z*),

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial r}+W\frac{\partial U}{\partial z}+\frac{\partial (P+p)}{\partial r}=\frac{V^2}{r}+fV=\frac{M^2}{r^3}-\frac{f^2 r}{4}, \tag{1}$$

$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial r}+W\frac{\partial V}{\partial z}+(\frac{V}{r}+f)U=\frac{\partial M}{\partial t}+U\frac{\partial M}{\partial r}+W\frac{\partial M}{\partial z}=0, \tag{2}$$

$$\frac{U}{r}+\frac{\partial U}{\partial r}+\frac{\partial W}{\partial z}=0, \tag{3}$$

$$\left(\frac{\partial}{\partial t}+U\frac{\partial}{\partial r}+W\frac{\partial}{\partial z}\right)\frac{\partial P}{\partial z}=0\ , \tag{4}$$

where *f* is the Coriois parameter, $M = Vr + fr^2/2$ is the absolute angular momentum which is conserved by fluid parcels as well as the buoyancy, $g(\rho-\rho_0)/\rho_0 = \partial P/\partial z$ related to the geopotential, $P + p$, by the hydrostatic approximation, *g* is the gravity acceleration, ρ is the density, ρ_0 is its reference value, and *p* is the geopotential in the deep homogeneous layer where the horizontal velocity (*u, v*) does not depend on the depth:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{\partial p}{\partial r}=\frac{v^2}{r}+fv, \qquad \frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+(\frac{v}{r}+f)u=0. \tag{5}$$

In the rigid-lid approximation the deep radial velocity can be expressed by the velocity in the upper stratified layer from the total mass balance by integrating (3)

$$(D-h)u=-\int_0^h U dz\ , \tag{6}$$

where $h(r, t)$ is the depth of the upper stratified layer, and *D* is the total depth. Then the geopotential gradient $\partial p/\partial r$ in the axisymmetric flow can be expressed from (5) to include the deep flow effects in (1). In this way we consider the deep flow feedback on inertially pulsating vortex in the upper layer.

3 The Structure of Pulson Solutions

First we consider a flow between horizontal level z = 0 and an isopycnal (i.e., constant buoyancy) surface $z = h$ assuming $h << D$ so that the pulson solution can be described in a self-similar form following Sutyrin (2006)

$$M = M_0(R,Z)\ , \qquad \frac{\partial P}{\partial z} = -B_0(R,Z)\ , \qquad (R,Z) = (\frac{r}{\sqrt{S}}, Sz)\ , \tag{7}$$

where $S = 1 + \gamma \sin(ft+\lambda)$ – pulsates with inertial frequency, γ is the amplitude of pulsations, and λ is the phase. Then the radial velocity does not depend on the vertical coordinate

$$U = U_0 = \frac{\dot{S}r}{2S} = \frac{rf}{2}\frac{\gamma\cos(ft+\lambda)}{1+\gamma\sin(ft+\lambda)}, \tag{8}$$

while $SP = \Phi_0(R,Z)$, and $Sh = Z_0(R)$ are defined by the spatial vortex structure. The relation between M_0 and Φ_0 depends on the pulson amplitude γ

$$\frac{M_0^2}{R^4} - \frac{1}{R}\frac{\partial \Phi_0}{\partial R} = \frac{f^2}{4}(1-\gamma^2)\,. \tag{9}$$

Such exact solutions of PE describe inertially pulsating combinations of a vortical flow and IGW which do not adjust to balanced state. The spatial distribution of $P = S^{-1}\Phi_0(R,Z)$ in coordinates (R, Z) is the same as for the stationary solution with $\gamma = 0$, except its amplitude pulsates inversely proportional to S in order to provide the mass conservation described by Eq. (3). Correspondingly, the azimuthal velocity calculated from Eq. (9) for $\gamma > 0$ deviates from stationary gradient balance to compensate impact of pulsating radial velocity. Such solution describes anticyclonic (warm-core) lens-like vortex with all isopycnals outcropping at the level $z = 0$ at variable radial distances. The actual maximum vortex radius at this level pulsates with time as $r_0 = R_0\sqrt{S(t)}$, where R_0 is defined by $Z_0(R_0) = 0$.

In particular, for a parabolic radial profile of $Z_0 = Z_m(1 - R^2/R_0^2)$ and constant B_0, from Eq. (7) we obtain $\Phi_0 = B_0(Z_0 - Z)$ so that from Eq. (9) the azimuthal velocity has the form

$$VRS^{1/2} = M_0 - \frac{fR^2}{2}S = \frac{fR^2}{2}\left[\sqrt{1-\gamma^2-A_m} - S\right] \tag{10}$$

where $A_m = 8Z_mB_0/f^2R_0^2$ characterizes the nondimensional vortex amplitude and such solution exists for any $\gamma < \sqrt{1-A_m}$. At the moment of maximum contraction $S = 1-\gamma$, the radial velocity becomes zero in (8), and the azimuthal velocity becomes zero for a particular value of $\gamma = (1-\sqrt{1-2A_m})/2$. In this case the solution describes the evolution of the lens-like perturbation without initial velocity.

4 Deep Flow Feedback on the Upper Layer Vortex

Using asymptotic expansion in Z_m / D, in the leading order the deep velocity is obtained from (5), (6) and (8)

$$u = -\frac{r\dot{S}}{2S^2}\frac{Z_0}{D}, \qquad v = -f\int_0^t u dt, \tag{11}$$

To calculate the azimuthal velocity in (11), it is convenient to start at the moment of maximum contraction setting $\lambda = -\pi/2$ so that $S = 1-\gamma\cos(ft)$, then for a parabolic profile we obtain

$$v = \frac{fr}{2}\frac{Z_m}{H}\left(\frac{1}{F}-\frac{1}{S}\right)\left(1-\frac{r^2}{2R_0^2}\left(\frac{1}{F}+\frac{1}{S}\right)\right)H(S-F), \tag{12}$$

where $F(r) = \max(1-\gamma, r^2/R_0^2)$ and H is the Heaviside function. Note, only periods when expanding lens generates the radial velocity in deep layer are taken into account in this expression for $R_0\sqrt{1-\gamma} < r < R_0\sqrt{1+\gamma}$. Finally, the deep geopotential gradient is found from

$$\frac{\partial p}{\partial r} = fv - \frac{\partial u}{\partial t}. \tag{13}$$

The correction to the upper layer motion is calculated by linearizing (1) – (4) relative to the pulson solution using the deep geopotential gradient (13) as a forcing term.

5 Conclusions

Here we calculated the flow patterns generated in homogeneous fluid below stratified pulsating lens-like vortex described by (11). The feedback on the upper layer vortex is provided by the geopotential gradient according to (13).

References

Reznik, G. M., Zeitlin, V. & Ben Jelloul, M. 2001 Nonlinear theory of geostrophic adjustment. Part 1. Rotating shallow-water model. *J. Fluid Mech.* **445**, 93–120.

Sutyrin, G.G. 2004 Agradient velocity, vortical motion and gravity waves in rotating shallow water model. *Quart. J. Roy. Meteorol. Soc.*, **130**, 1977-1989.

Sutyrin, G.G. 2006 A self-similar axisymmetric pulson in rotating stratified fluid. *J. Fluid Mech.*, **560**, 243-248.

Zeitlin, V., Reznik, G.M. & Ben Jelloul, M. 2003 Nonlinear theory of geostrophic adjustment. Part 2. Two-layer and continuously stratified primitive equations. *J. Fluid Mech.* **491**, 207–228.