Passive Scalars in Stratified Turbulence

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Abstract :

The statistics of a passive scalar in randomly forced and strongly stratified turbulence is investigated by numerical simulations including a horizontal passive scalar mean gradient. We observe that horizontal isotropy of the passive scalar spectrum is satisfied in the inertial range. The spectrum has the form $E_\theta(k_h) = C_\theta \varepsilon_\theta \varepsilon_K^{-1/3} k_h^{-5/3}$*, where* $\varepsilon_\theta, \varepsilon_K$ *are the dissipation of scalar variance and kinetic energy respectively, and* $C_\theta \simeq 0.5$ *is a constant. This spectrum is consistent with atmospheric measurements in the mesoscale range. The calculated passive scalar structure functions show that intermittency effects are significant.*

Key-words :

stratification ; turbulence ; passive scalar

1 Introduction

According to the Obukhov-Corrsin theory of isotropic turbulence, the one-dimensional spectrum of the variance of a passive scalar fluctuation θ, the structure function of an arbitrary order n, and the second-order structure function in particular, in the inertial range are given by (Obukhov 1949, Corrsin 1951)

$$E_\theta(k) = C_\theta \varepsilon_\theta \varepsilon_K^{-1/3} k^{-5/3}\,, \tag{1}$$
$$\langle \delta\theta^n \rangle \propto r^{n/3}\,, \tag{2}$$
$$\langle \delta\theta\delta\theta \rangle = C'_\theta \varepsilon_\theta \varepsilon_K^{-1/3} r^{2/3}\,. \tag{3}$$

Here ε_θ and ε_K are the dissipation of scalar variance and kinetic energy respectively, $\delta\theta = \theta' - \theta$ is the difference between the scalar values at two points separated by a vector ***r***, $\langle\rangle$ denotes an ensemble average, and C_θ, C'_θ are dimensionless constants with $C'_\theta = 0.25 C_\theta$.

The spectrum (1) and the second-order structure function relation (3) have been confirmed by several studies considering passive scalars in (nearly) isotropic turbulence (Sreenivasan 1996, Warhaft 2000). Horizontal spectra (Nastrom *et al.* 1986, Bacmeister *et al.* 1996, Cho *et al.* 1999) and second-order structure functions (Lindborg & Cho 2000) of passive scalars measured in the mesoscale range of the middle atmosphere appear to correspond with (1) and (3) as well. The correspondence with (1) and (3) is remarkable because mesoscale eddies are strongly influenced by stratification and can therefore not be isotropic.

The origin of the mesoscale scalar spectra in the middle atmosphere is not fully clear. Nastrom *et al.* (1986) suggested that the spectra are produced by two-dimensional turbulence. However, the existence of two-dimensional turbulence in the middle atmosphere has been questioned by Lindborg (1999). Mixing by gravity waves has also been mentioned as an explanation for the observed spectra (e.g. Cho *et al.* 1999), but Bacmeister *et al.* (1996) concluded that arguments for mixing by gravity waves are not convincing. We suggest an alternative explanation for the measured spectra.

In strongly stratified fluids, horizontal layers are commonly observed. Billant & Chomaz (2001) argued that the thickness of these layers scales as $l_v \sim U/N$, where U is a characteristic horizontal velocity scale and N the Brunt-Väisälä frequency. Lindborg (2006) and Brethouwer *et al.* (2007) argue that these thin layers structures break up into smaller structures and in this way energy is transferred from large to small scales. This nonlinear cascade is three-dimensional but strongly anisotropic. This particular kind of dynamics, with the scaling $l_v \sim U/N$ and the forward energy cascade, was observed in numerical simulations (Brethouwer *et al.* 2007, Lindborg 2006) and has been called stratified turbulence. Computed horizontal one-dimensional spectra of kinetic and potential energy in the inertial range closely approximated

$$E_K(k_h) = C_K \varepsilon_K^{2/3} k_h^{-5/3}\, , E_P(k_h) = C_P \varepsilon_P \varepsilon_K^{-1/3} k_h^{-5/3}\, , \quad (4)$$

where k_h is the horizontal wave number, ε_P is the potential energy dissipation and $C_K \simeq C_P \simeq 0.5$. The computed spectra agreed well with measured spectra in the atmosphere.

A reasonable hypothesis is that the observed mesoscale kinetic and potential energy spectra in the atmosphere are the result of stratified turbulence. The logical following question is if stratified turbulence can explain the passive scalar statistics in the mesoscale range of the middle atmosphere as well. Our objective is to address this question by studying passive scalar statistics in stratified turbulence through numerical simulations and to study scalar intermittency.

2 Simulations

Numerical simulations of homogeneous turbulence with a strong and uniform stratification are carried out employing a pseudospectral code with periodic boundary conditions in all three directions. The code solves the three-dimensional Boussinesq equations and the transport equation for a passive gradient with a mean gradient. Diffusion is modelled with hyperviscosity and the forcing is of the large scale velocity modes is purely horizontal. Because the vertical length scales which develop in the simulations are much smaller than the horizontal length scales we use a computational domain that is much larger in the horizontal direction than in the vertical direction. More details on the numerical methodology can be found in Lindborg & Brethouwer (2007) who used the same approach.

The transport equations for the active and passive scalar are identical if the passive scalar gradient is vertical. Hence, in this case the passive and active scalar statistics must be the same and therefore we take the mean passive scalar gradient in the horizontal direction instead, since this is a non-trivial case.

3 Results

Starting with small initial perturbations, the flow field reaches a statistically stationary state in the simulations after an initial period when energy grows. In the stationary state, energy injected at the large scales is transfered to the small scales and dissipated, and the kinetic and potential energy approximately stay constant. The same features were observed by Lindborg (2006) and Brethouwer *et al.* (2007). The velocity and buoyancy fields reach a statistical stationary state relatively fast, in contrast to the scalar field. Even after very long integration times, the scalar variance still grows in both simulations implying that the mean production of scalar variance is not balanced by its dissipation. To reach a fully stationary state for the scalar field requires tremendous computational costs, but the ratio of the dissipation and production showed that the simulations were approaching it. The simulation time was long enough so that an inertial passive scalar range could develop as will be shown later and this was sufficient for our purpose in this study.

Figure 1 shows compensated horizontal one-dimensional spectra of the kinetic and potential energy. The largest scales are affected by the forcing, but for wave numbers between $k_h = 3$

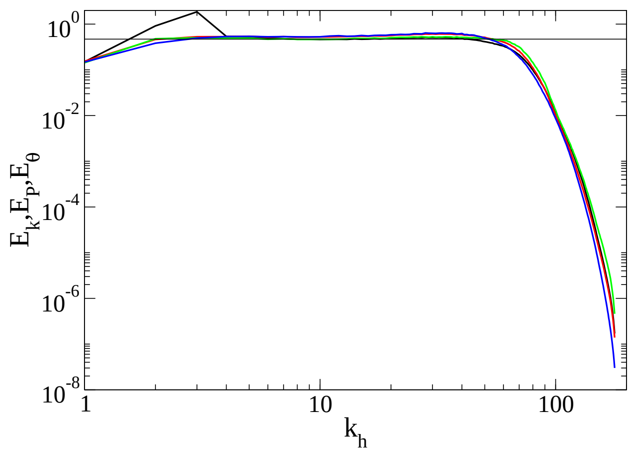

Figure 1: Compensated horizontal one-dimensional kinetic energy spectra $E_K(k_h)k_h^{5/3}/\varepsilon_K^{2/3}$ (black line), potential energy spectra $E_P(k_h)k_h^{5/3}\varepsilon_K^{1/3}/\varepsilon_P$ (red line) and passive scalar variance $E_\theta(k_h)k_h^{5/3}\varepsilon_K^{1/3}/\varepsilon_\theta$ with k_h parallel to the mean scalar gradient (green line) and perpendicular (blue line). The straight line is $C = 0.47$.

and $k_h = 40$ the kinetic and potential energy spectra are very close to the straight horizontal line. We find $C_K \simeq C_P \simeq 0.47$ in close agreement with Lindborg (2006). The compensated horizontal one-dimensional spectra of the scalar variance with k_h parallel or perpendicular to the mean scalar gradient, plotted in the same figure, also show an inertial $k_h^{-5/3}$-power-law range. All spectra approximately fall on top of each other giving the Obukhov-Corrsin constant $C_\theta \simeq 0.47$. Figure 2 displays the ratio of the scalar variance spectra with k_h parallel and perpendicular to the mean scalar gradient. We can see that the large-scale anisotropy caused

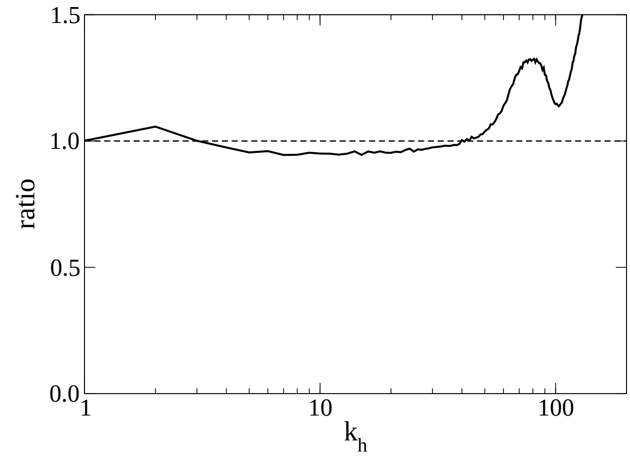

Figure 2: Ratio of the horizontal scalar variance spectra with k_h parallel to the mean scalar gradient and perpendicular.

by the passive scalar mean gradient leads to small or negligible anisotropy in the inertial range. Because a vertical passive mean scalar gradient also results in the same spectrum and $C_\theta \simeq 0.47$ we can conclude that they are independent of the mean gradient direction and thus universal.

Figure 3 shows the scaled second-order passive scalar structure functions. Only the structure function with ***r*** horizontal and parallel to the mean scalar gradient is displayed because the difference with ***r*** perpendicular to the mean scalar gradient is insignificant in the inertial range. The inertial $r^{2/3}$-range as predicted by (3) and corresponding to the $k_h^{-5/3}$-range of the

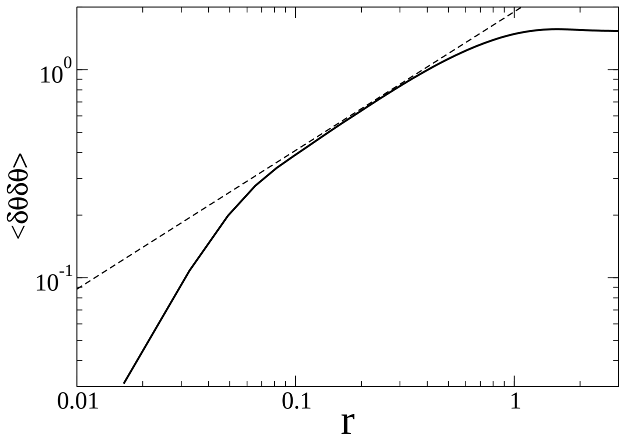

Figure 3: Scaled second-order structure function of the passive scalar $\langle\delta\theta\delta\theta\rangle/\varepsilon_\theta\varepsilon_K^{-1/3}$ with ***r*** horizontal and parallel to the mean scalar gradient. Solid line, simulation; dashed line, $1.9r^{2/3}$.

horizontal spectrum is evident. The results show that C'_θ is about 1.9 and this corresponds to $C_\theta = 0.25C'_\theta \simeq 0.48$, similar to C_θ estimated from the scalar variance spectra. We note that the spectra display a wider inertial range than the structure function, but this is a common observation.

In (nearly) isotropic turbulence it has been observed that for $n \geq 3$ passive scalar structure functions scale as $\langle\delta\theta^n\rangle \sim r^{\zeta_n}$, whereby the scaling exponent ζ_n deviates from $n/3$ as predicted by the Obukhov-Corrsin theory. This deviation is caused by intermittency (Warhaft 2000). We have computed the fourth-order scalar structure functions and observed that $\zeta_4 \simeq 1.15$ which is smaller than the Obukhov-Corrsin theory prediction $\zeta_4 = 4/3$. In the atmospheric boundary layer, VanAtta & Chen (1970) and Dhruva *et al.* (1997) measured scaling exponents for the fourth-order structure function of the velocity between 1.2 and 1.27, comparable with our results.

The flatness factor of the scalar defined as

$$F = \frac{\langle\delta\theta^4\rangle}{\langle\delta\theta\delta\theta\rangle^2} \tag{5}$$

is displayed in figure 4. From (2) and (3) it follows that according to the Obukhov-Corrsin theory F is constant in the inertial range. We see that in our simulations F is not constant but is proportional to $r^{-0.2}$ in the inertial range, which indicates intermittency. At small separation distances F is of order 10 suggesting intermittent high amplitude differences in the scalar field at small scales. Measurements in the atmospheric boundary layer show that the flatness factor of the velocity behaves as $r^{-0.12}$ in the inertial range (VanAtta & Chen 1970, Dhruva *et al.* 1997),

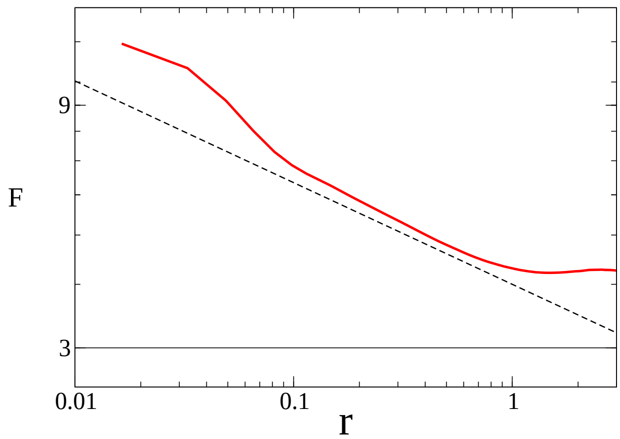

Figure 4: Flatness factors of the scalar (red line) with ***r*** parallel to the mean gradient. Black dashed line: $r^{-0.2}$.

but in the middle atmosphere the flatness factor increases much faster with decreasing scales (Lindborg 1999) revealing a much extremer intermittency than we observe in our simulations.

4 Conclusions

We conclude that the results of the numerical simulations are consistent with the Obukhov-Corrsin theory for the passive scalar spectrum (1) and second-order structure function (3). They are also consistent with atmospheric observations at mesoscales. Our study suggests that stratified turbulence may play an important role in passive scalar dispersion and transport in the mesoscale range of the middle atmosphere.

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