

## Over-Reflection in Lab: The All-Sufficient Cause of Instability of an Annular Supersonic Shear in Simulations on Free-Surface Shallow Water

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### Abstract:

*Presented are results of the pioneering research on the over-reflection instability of an annular 'supersonic' shear in experiments on free-surface shallow water covering a differentially rotating and properly shaped bottom (characteristic waves on shallow water play the role of sound, all alternative shear instabilities are suppressed due to specificity of the rotation profile and experimental procedure). The consideration focuses upon distinctive features of the structures generated by the instability as perturbations of shallow-water thickness. The features of the structures observed are compared with those predicted by an original theory. The structures are also readily interpreted as a superposition of Huygens-Mach fronts that are multiply over-reflected from the shear, having been induced by a supersonic disturbance moving along it. Owing to the annular geometry, the instability in the experiments develops even in absence of external boundaries that are universally included in traditional theoretical schemes for feedback necessary for the wave generation.*

### Key-words:

**Over-reflection; shear instability; shallow water**

### 1 Introduction

Theoretical studies on wave over-reflection (reflection with amplification) from a supersonic flow date back half a century to Ribner (1957) and Miles (1957). The whole bulk of analytical and computational data accumulated by now predicts the effect to play an essential role in significant applications and natural processes. The contributions of the over-reflection to destabilisation and turbulisation of flows and jets in aerodynamics and of accretion discs in astrophysics (see, e.g. Fridman *et al.* (2003)) are of principal importance. The theoretical predictions, however, have not yet been gained by direct experimental verification. Neither the instability based on the over-reflection (Kolykhalov (1984)), nor the over-reflection on its own has ever been observed in reality.

Recently Fridman *et al.* (2006, 2006) put forward a detailed concept of laboratory experiments aimed at closing the gap. It was suggested to consider the phenomena in their association with an annular (rather than straight) sheared flow, simulating it in fast rotating shallow water by the experimental method developed by Nezlin & Snezhkin (1993). According to the concept, a liquid layer is to be placed on a differentially rotating bottom, with its upper surface left free so that shallow-water waves can spread in the layer like sound in 2D gas. With the bottom shaped properly, the layer can be made thin enough for the velocity of these surrogate 'sonic waves',  $(gH)^{1/2}$ , to be lower than the local velocity of the rotation ( $g$  - acceleration of gravity,  $H$  - thickness of the shallow-water layer). Angular velocity of the rotation is to increase with radius since the forced flow will be otherwise destroyed by the strong centrifugal instability (Fridman *et al.* (1985)). The other instability able to hamper the experiment is Kelvin-Helmholtz instability (KHI). It does not develop in

excessively ‘supersonic’ regimes (Antipov *et al.* (1983)) including the wanted in which the over-reflection phenomena are expected to take place. However, at any attempt to bring the system into these regimes directly ‘from below’, KHI will inevitably break the flow as early as at a ‘subsonic’ stage. The concept left room for solving this particular problem (Sec. 2).

The following is a description of the first translation of the concept into reality.

## 2 Experiments and their results

In our experimental set-up (Fig. 1), a thin layer of liquid (ordinary water dyed with  $NiSO_4$ ) covered the bottom of a round pan-like vessel on a turntable. The outer part of the bottom, rotating with the turntable at angular velocity  $\Omega$ , was conic. The inner part, remaining still due to firm fastening to a bedplate, was flat. The two were separated by a 0.4-mm-wide annular gap of radius  $R_0$ . Mechanical interaction of the liquid with the bottom provided forcing of rotation with a sharp velocity jump in a zone between the corresponding parts of the layer, the outer ‘periphery’ and inner ‘core’. The set-up was equipped with a b/w CCIR camera and top side illumination. A technique of optical densitometry against the background of diffusely reflected bottom (Nezlin & Snezhkin (1993), Snezhkin & Sommeria (1998), Rylov *et al.* (2004)) was used for obtaining fields of layer thickness perturbations.

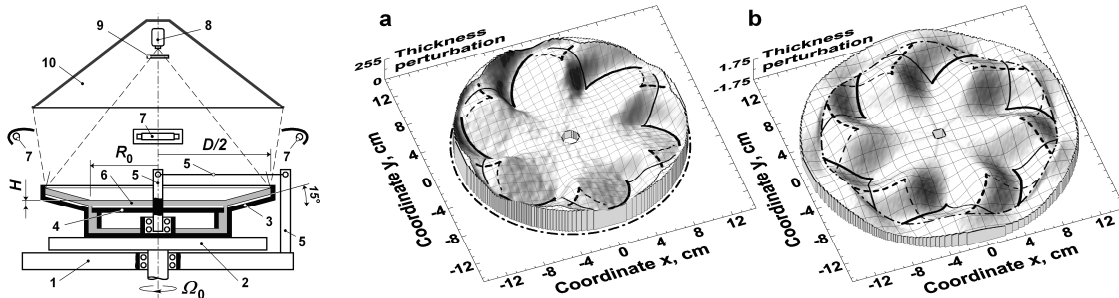


FIG. 1 – Schematic of the set-up: **1** - bedplate, **2** - turntable rotating at velocity  $\Omega$ , **3** - rotating vessel with conic bottom, **4** - unmoving flat bottom, **5** - fixing devices, **6** - free-surface layer of ‘short-pass’ green liquid, **7** - white halogen lamps, **8** - camera, **9** - long-pass red filter, **10** – matte-black screen;  $R_0 = 12$  cm - radius of the gap between the two parts of the matte-white bottom,  $D = 41$  cm and  $H$  - external diameter and thickness of the layer.

FIG. 2 – Typical structure generated by the mode  $m = 6$  of the flow instability: perturbation of the liquid-layer / shallow-water thickness in arbitrary units, **dash-and-dot line** - the gap between the outer and inner parts of the bottom / locus of the shear, **solid and dash lines** - troughs and crests of waves incident on the flow (**thin lines**) and over-reflected by it (**thick lines**), **a** - experiment ( $H \approx 4$  mm,  $\Omega = 3.98$  rad/s ( $M_0 \approx 2.4$ ) and  $\Omega_p = 3.13$  rad/s ( $M_{p0} \approx 1.9$ )), **b** - theory ( $H = 5.0$  mm,  $\Omega = 4.83$  rad/s ( $M_0 = 2.62$ ),  $\Omega_p = 4.25$  rad/s ( $M_{p0} = 2.30$ )).

The experiment runs started with a stationary regime in which the liquid covered the entire bottom with a layer as thin as possible. The core was still, the periphery rotated as a rigid body, therewith a narrow in-between shear was stable since the ‘supersonic’ jump in velocity and losses due to bottom friction were exceedingly high. Then the system was brought into the wanted regimes ‘from above’: we very slowly lowered the Mach number of the jump,  $M_0 = \Omega R_0 / (gH)^{1/2}$ , by either decreasing the speed  $\Omega$ , or adding liquid into the vessel, or both ( $H$  was measured near the centre). At a threshold value of  $M_0$ , the layer thickness became perturbed in a zone of the core adjoining the periphery. The perturbations appeared as a variety of trains of almost radial ‘spokes’ which altering in length and width ran azimuthally at different velocities after the periphery. After a time, even if the current value of  $M_0$  remained unchanged, the mess turned to a stable single structure (pattern) rotating in the same direction at constant angular velocity  $\Omega_p$ . Having rotational symmetry of order  $m$ , the structure was obviously a manifestation of a hydrodynamic instability developing as a mode with azimuthal wave number  $m$  (Fig. 2a shows a typical structure with  $m = 6$ , structures with other  $m$  appeared correspondingly). On further decreasing  $M_0$ , the system

rearranged itself again and again with successive formation of kindred structures with higher and higher  $m$ . Reverse changes in  $M_0$  caused rearward transitions to the structures with progressively lower  $m$ , all these processes therewith exhibiting a pronounced hysteresis. Transition stages lasted up to several tens of the periphery rotation periods. The wave numbers of the modes accessible ranged from three to ten. Appearing more tolerant of small variations in experimental conditions, the modes with  $m = 5, 6$  tended to spontaneous transitions less than the others did.

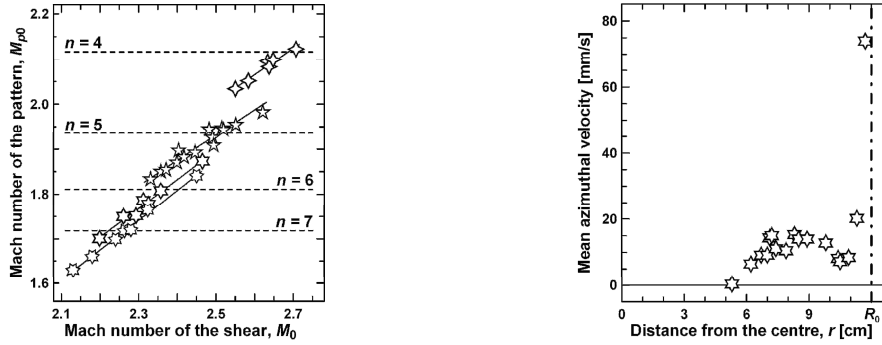


FIG. 3 – Velocity of the structures (patterns) generated by different modes of the instability as a function of the flow velocity in terms of Mach numbers at radius  $R_0$ : **stars** - experimental data (number of points of the star gives the mode number  $m$ ), **solid lines** - linear fits of the data and existence ranges of the modes, **dot lines** - theoretical values of  $M_{p0}$  from the model of the resonant self-organisation of the fronts multiply ( $n$  times) over-reflected by the flow.

FIG. 4 – Mean azimuthal velocity on the liquid-layer surface versus distance from the centre in conditions much like those in Fig. 2a.

Kinematic characteristics of the structures and ranges of existence of some modes are shown in Fig. 3 where Mach numbers characterising rotation velocity of the structures,  $M_{p0} = \Omega_p R_0 / (gH)^{1/2}$ , are plotted against  $M_0$ . Some notion of velocity fields in the structures can be gained from the data in Fig. 4 obtained by velocimetry of floating tracers.

When  $M_0$  reached sufficiently low values, the system came under the action of KHI developing here as a mode with the azimuthal wave number one or two.

### 3 Discussion

To identify the instability that took place in the experiment, the structure generated by it (Figs 2a) is to be compared with the eigenfunction of the over-reflection instability (ORI) calculated in line with the theory by Fridman *et al.* (2006, 2006) for conditions physically close to the experimental (Fig. 2b). The eigenfunction appears as a superposition of two coherent systems: of twelve wave crests and twelve wave troughs. In its turn, each of the systems consists of six (leading) spiral waves travelling inwards, i.e. from the shear, and six (trailing) spiral waves travelling outwards, i.e. towards the shear. On the same distance from the shear, the inward waves are more intensive than the outward waves. It is precisely what should be expected: the shear over-reflects the incident outward waves, i.e. reflects them inwards with amplification. In the case that the reflection coefficient is complex, the over-reflected waves can be shifted in phase with respect to the incidents. That is very likely the reason why the waves of the same signs do not meet each other on the shear. Conversely, they join in pairs, connecting with their opposite ends facing the centre. Each of the pairs forms a tick-shaped pattern, with overall maximal magnitude of the joined crest or trough in the connection point, in the tip of the tick. It is easy to see that the real structure in Figs. 2a features all the above too. The resemblance between Fig. 2a and Fig. 2b is quite remarkable, taking into account that the compared are results of an a-priori non-linear viscous experiment and de-facto linear inviscid theory. As for the alternative ('neighboring') instability, there was clear-cut visual distinction between the structures generated by ORI and KHI. The ORI perturbations were maximal in the core away from the shear zone. The zone itself

was perturbed relatively weakly and the banana-like vortices expected to exist there were so narrow that they could be hardly distinguished in the experiments (see also Fig. 4). By contrast, the KHI perturbations were maximal in the shear zone and the fluid-trapping vortices were wide and very well pronounced.

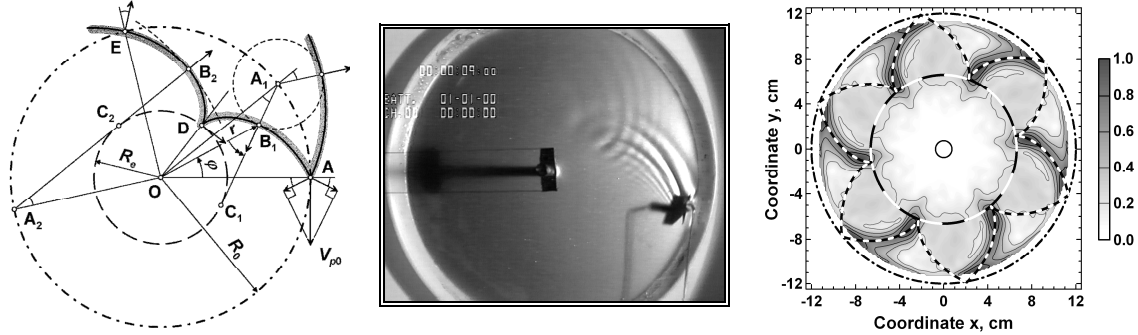


FIG. 5 – Involute as a locus of the Huygens-Mach shock front (**shaded solid line**) from a point disturbance **A** moving in shallow water on the circle of radius  $R_0$  (**dash-and-dot line**) at ‘supersonic’ velocity  $V_{p0}$ ; **dash line** - evolute of the front as a circle of radius  $R_e$ , **unlabeled straight arrows** – local velocity vectors of the ‘sonic’ motion of the fronts, **dot lines** - elementary circular fronts,  $r$  and  $\varphi$  – polar coordinates, **double-head arrow** shows the angle  $\theta_p$ , **arcs** show angles equal to the Mach angle  $\theta_{p0}$ ;  $M_{p0} = 2.25$ .

FIG. 6 – Waves from a small irregularly shaped disturbance dipped into a still free-surface liquid layer and moving in it on a circle near an annular rigid wall under conditions physically comparable to those in Fig. 5, with no flow in the layer (visualisation with aluminium powder).

FIG. 7 – Magnitude of the thickness gradient in the structure in Fig. 2a overlaid with a theoretical pattern (‘rosette’) of six tick-shaped fronts (**black-and-white short-dot lines**) resonantly self-organised in their over-reflections from the flow: **dash-and-dot line** - gap between the outer and inner parts of the bottom and the trajectory of the initial disturbance in the theoretical model, **black-and-white long-dash line** – common evolute of the fronts as a circle of radius  $R_e$ .

Some features of the structure and the very origin of it can be clarified by considering shock fronts from point disturbances. Let such a disturbance (A in Fig. 5) to move in shallow water on the circular trajectory of radius  $R_0$  at ‘supersonic’ velocity  $V_{p0} = \Omega_p R_0$ . Not counting effects of the rotating periphery for now, the front the disturbance produces in the core can be build on Huygens’ Principle as an envelope of elementary circular ‘sonic’ fronts which, having been sequentially emitted by the disturbance on its way along the trajectory, expand laterally at the speed  $(gH)^{1/2}$ . Clearly, two features of the envelope are as they would be if the disturbance moved rectilinearly. First, the angle made by the envelope with the trajectory at the point A is the Mach angle  $\theta_{p0}$  defined by the Mach number of the disturbance motion,  $M_{p0} = V_{p0}/(gH)^{1/2}$ , via the relation  $\sin\theta_{p0} = 1/M_{p0}$ . Second, all straight lines normal to the envelope cross the trajectory at the same angle  $\pi/2 - \theta_{p0}$ . All such perpendiculars are tangential to the circle of radius  $R_e = R_0 \sin\theta_{p0} = \Omega_p/(gH)^{1/2}$  co-centred with the trajectory. The existence of these dual-feature straight lines indicates that the desired envelope and the circle inside the trajectory are respectively an involute and evolute of each other. Based on the handbook equations of a circle involute, it is possible to represent the envelope segment ADE in the suitable parametric form:

$$r = R_0 \sin\theta_{p0} / \sin\theta_p, \quad \varphi = \theta_{p0} + \text{ctg}\theta_{p0} - \theta_p - \text{ctg}\theta_p, \quad \theta_{p0} \leq \theta_p \leq \pi - \theta_{p0},$$

where  $r$  - distance from the centre,  $\varphi$  - polar (azimuthal) angle counted from the current location of the disturbance,  $\theta_p$  - angle between the direction to the centre and the local direction of the envelope (front) motion. The parts AD and DE of the tick-shaped segment make up two fronts that move respectively from and to the trajectory. The parts are axisymmetric about the radius passing through the point D, at which  $\theta_p = \pi/2$ , so that the angle of incidence of the part DE at the trajectory is equal to the angle of obliqueness of the part AD from the trajectory, i.e. equal to the Mach angle  $\theta_{p0}$ . In our supplementary experiment, shock fronts in a real liquid layer behaved

exactly in the above way: having passed some distance to the centre, they suffered a sharp kink and thence went from the centre (Fig. 6). [Noticed for the first time in practical aeronautics, such reverses of shock fronts from supersonic jet planes were similarly interpreted by Meyer (1973). Noteworthy also are similar effects encountered in studies of radiation at superluminal motion of charges, e.g. by Ardavan (1989), Bolotovskii & Bykov (1990).]

As for the effects of the rotation of the periphery, the velocity jump at  $r = R_0$  is to reflect the incident front with the same  $\theta_{p0}$  as the angle of reflection. By applying Huygens' Principle again, it is easy to verify that the reflected front will be a copy of the initial front ADE produced directly by the disturbance. This primary copy will also be reflected by the shear, giving rise to a secondary copy, then a tertiary copy will be produced in the same way, and so on. Thus, in the presence of the shear, the disturbance will produce in the core a chain of tick-shaped segments equiform to ADE. If the circumference of the chain is a whole number of the links, resonant phenomena are clearly possible: at proper rotation of the periphery, when the shear over-reflects (rather than just reflects) with amplification high enough to compensate losses, the disturbance can trigger generation of a stationary rotating azimuthally symmetric structure. The shallow-water system in such regimes is similar to a peculiar kind of acoustic resonator with permanent pumping all along its annular boundary. For simplicity not counting the mentioned phase shift at the over-reflection, the number of the links in a closed chain,  $n$ , is unambiguously related to a 'resonant' value of  $\theta_{p0}$  by the equation  $\angle AOE = 2\pi/n$ , where  $\angle AOE = 2(\theta_{p0} + \text{ctg}\theta_{p0}) - \pi$ . Corresponding resonant values of  $M_{p0}$  are in reasonably good agreement with the experimental kinematic data in Fig. 3. The chain in the form of a rosette expected to be resonantly pumped up with  $n = 6$  is shown in Fig. 7. The rosette evidently fits well with the experimental data on the mode  $m = 6$  underlaid it as a scalar field of the perturbed thickness gradient.

It is clear that the wave perturbations singled out by the azimuthal quantisation and resonant amplification at the shear are not to penetrate the central area of radius  $R_e$  but only to touch the bounding evolute, sliding over it at the speed  $(gH)^{1/2}$ . The zone between the shear and evolute resembles a waveguide with the evolute acting as a kind of virtual self-produced wall that oddly reflects the waves impinging onto it tangentially. This correlates well with the fact that the thickness in the experimental and theoretical structures was not perturbed within the corresponding central area (see Figs 2 and 7). In the experiments, any obstacle put in the liquid within this 'stagnant' area had no effect on the generation and features of the structures.

The situation reversed if the obstacles (thin walls) were placed closer to the shear. Oriented radially, the walls there deformed the structures, quenching them even to the point of disappearance. Oriented azimuthally, they did not change drastically the appearance of the generated structures, altering, nevertheless, the essence of the generating instability. In its pure state, the latter case took place with a wall shaped as a coaxial ring larger in diameter than the stagnant area. Being a real reflector of waves, the ring provided the feedback necessary for developing ORI of the 'traditional' type (Kolykhalov (1984)). The structures generated in this case resembled much those above, therewith the closer in diameter the ring was to the stagnant area, the stronger the resemblance was. All other factors being equal, the shear with the ring lost its stability at higher values of  $M_0$ , which could be expected due to lower losses suffered by the waves traversing the narrower waveguide. In some regimes, we also observed large-scale wave structures generated by the same type of the instability in the periphery where over-reflection of the waves from the shear was accomplished by reflection of them from the wall of the vessel (generation of such structures was pre-studied theoretically by Fridman *et al.* (2006, 2006)).

#### 4 Conclusions

The notion of an involute was first introduced by Huygens in application to pendulums and clock movements with no relation to wave dynamics. Though now involutes have been realised to be directly associated with wave fronts and other objects of Huygens' Principle, yet they are

primarily known through the involute gear that is by far the most common system for gearing not only in fine mechanics, but about everywhere in machinery. Curious is the fact that the profile of the gear (see any handbook on mechanical engineering) coincides geometrically with the rosettes that we offer above for describing the structures generated by the ORI in our laboratory hydrodynamic simulations.

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