The role of a movable sandy-bed in modelling open-channel flow

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Abstract :

Sediment transport in rivers is important regarding to environmental and hydro-engineering issues. The complexity of this phenomenon derives on several assumptions for its theoretical analysis. One of these assumptions is to consider the water density and viscosity as constants; another is to neglect the influence of the flow through the porous medium on the water velocity profile. The presence of a movable sandy-bed is determinant for the variation of the fluid's density and viscosity because of the processes that take place on the boundary between the porous medium and the fluid in motion. This paper describes at a glance some experimental investigations on how the water density varies as a function of the depth, velocity and channel roughness. In addition, the flow through the sandy layer is not neglected because a numerical model is developed to find the velocity field on the boundary between the bed and the water.

Key-words :

Suspended sediment; hydrodynamic modelling; density variation

1 Introduction

Sedimentation processes in rivers are considered very complex phenomena that the scientists have been researched since the beginning of the civilization in order to enhance the quality of life of the people living close to streams, reservoirs and rivers. The material that is transported by the river flow not only affects the geometry of the cross sections along the waterways, but also the hydrodynamic conditions of the fluid in motion that becomes a mixture between clean water and suspended sediment. The suspended load provokes changes of the fluid's density that normally is taken as constant for hydrodynamic modelling. Hence, the kinematic viscosity can neither be considered as a constant. In addition, when the upper layer of any river bed is a porous medium; the water depth of the channel represents a hydraulic head that can derive in flow through this porous medium. Thus, a velocity field exists within the river bed and it can be estimated to check whether the velocity of the water in the boundary is significant or can be neglected for modelling. This paper describes at a glance the classical approaches to model free surface flow and sediment transport in open channels highlighting the importance that the soil–water interaction has. Two main influences of a sandy bed in the hydrodynamic conditions are described: the variation of the liquid's density and the existence of the water velocity in the boundary between the fluid and the channel bed.

2 Modelling flow and sediment transport in open channels

Several models and methodologies have been developed in order to simulate flow and sediment transport mechanisms. The simplest methodologies were based on field observations and physical modelling; nowadays, complex models are developed on the basis of numerical and mathematical computations. Free surface flow and sediment transport is characterized by turbulence, free-surface variation, bed change, etc. Thus, most of the models adopt several assumptions to simplify its theoretical analysis. Wang (2004) summarized these assumptions as:

a) The interaction between flow and sediment movement can be neglected because the sediment concentration is low;
b) The flow can be calculated assuming a fixed bed because the bed change is much slower than the flow movement and
c) The interactions among different size classes of moving sediments are ignored.

The simplest equation to model free surface flow is *the equation of motion for a stream point following a trajectory s*, better known as the equation of conservation of energy (Sotelo – Ávila, 1997); its differential form is presented below:

$$\frac{\partial E}{\partial s} = \frac{\partial}{\partial s}\left(z + \frac{p}{\gamma} + \frac{v^2}{2g} + h_r \right) = -\frac{1}{g}\frac{\partial v}{\partial t} \qquad (1)$$

Where the z is the elevation of the liquid vain with respect an established datum, p is the pressure acting on it, γ is the specific volume of the fluid, v is the velocity of the vain, h_r is a coefficient that represents the loss of energy and g is gravitational acceleration. The specific weight (γ)is the product of the density and the gravitational acceleration, thus if the density vary as a function of z, the specific volume cannot be considered as constant. This equation neglects the viscous effects of the fluid in motion; nevertheless it is commonly used as the base of one-dimensional models including HEC-RAS. Modelling sediment transport should consider the viscous effects; several 1D models were developed based on the St. Venant equations; while 2D and 3D models have the Reynolds average and Navier-Stokes equations as governing equations. Derived from the Newton's second law of motion, the Navier-Stokes set of equations is the basis for modelling viscous flow. The general form of the set of equations can be written as follows:

$$\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = \frac{1}{\rho}\frac{\partial}{\partial x_j}\left(-P\delta_{ij} + \rho\upsilon\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)\right) \qquad (2)$$

Where U = the velocity vector; ρ = density of the viscous fluid; υ = the kinematic viscosity and P = the pressure acting on the system. The Navier-Stokes equations are the basis of the majority of the hydro-dynamic models; nevertheless and due to the fact that natural streams, channels and rivers flow invariably in turbulent regime, the validity of the Navier-Stokes equations is limited. Highlighting that the density and viscosity are to be defined for modelling purposes, the assumption of considering these two parameters are constant is not valid for multiphase flow or when there are external agents that can modify these parameters.

3 The variation of the fluid properties due to turbulent and viscous shear stresses

The first and second assumptions summarized by Wang (2004) demonstrate that the role of a movable bed in modelling open channel flow and sediment transport is neglected, above all when the material is composed by coarse and sandy soils. The FIG. 1 depicts the traditional velocity profile for open channels. This figure shows as well, the scientific flow layer classification according to the kind of shear stresses and types of flow that take place within the fluid. Two kinds of shear stresses are identified: the viscous and the turbulent shear stresses. Close to the river bed, where the flow tends to be laminar, a viscous shear stress can be defined using the Newton's law of viscosity. In addition, and considering that the majority of the flows in nature are turbulent, a typical phenomenon takes place, called the fluctuation of velocity, denoted by *u'* and *w'* (Liu, 2001) in the *x* and *z* direction. This fluctuation of velocities causes turbulent shear stresses. The sum of both stresses is expressed in the equation (3):

$$\tau = \tau_\upsilon + \tau_t = \rho\upsilon\frac{du}{dz} + (-\rho\overline{u'w'}) \tag{3}$$

Where τ_υ is the viscous shear stress, τ_t is the turbulent shear, $\overline{u'w'}$ is the velocity fluctuation vector. Shear stresses acting on the area of the material from the bed exert forces that drag or lift the solid particles.

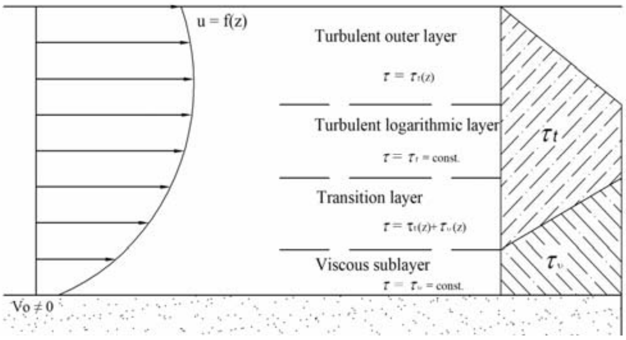


FIG. 1 – The velocity profile u = f (z) and the flow layer classification

Lifting forces acting on the bed provoke that the finest and part of the coarse material will be transported as suspended load. This Lifting force F_L is defined as (Liu, 2001):

$$F_L = \frac{1}{2}\rho C_L A U^2 \tag{4}$$

Where the C_L is the lift coefficient, which depends on the shape and surface roughness of the body as well as on the Reynolds number, *A* is the projected area of the body perpendicular to the plane of the flow direction and *U* is the average velocity of the flow in steady uniform state. The shear stresses and lifting forces acting on the bed material can change the value of the density and viscosity as a function of the water depth. Suspended sediment concentration has been a well-researched topic. Nevertheless, a relation that describes the variation of the water density when it is affected by this material in suspension is a topic to be researched.

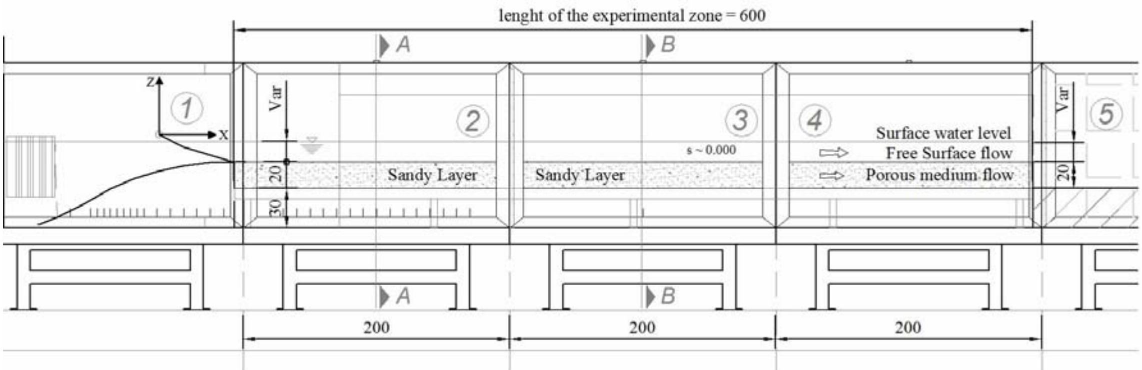
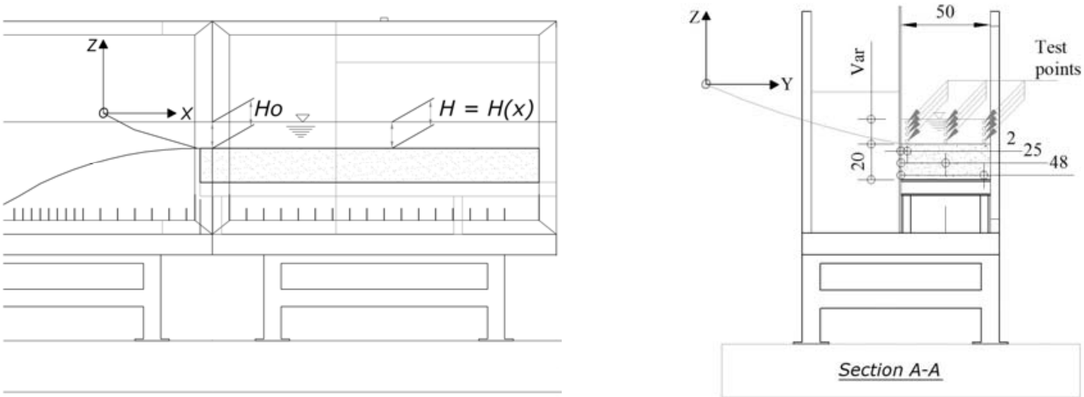


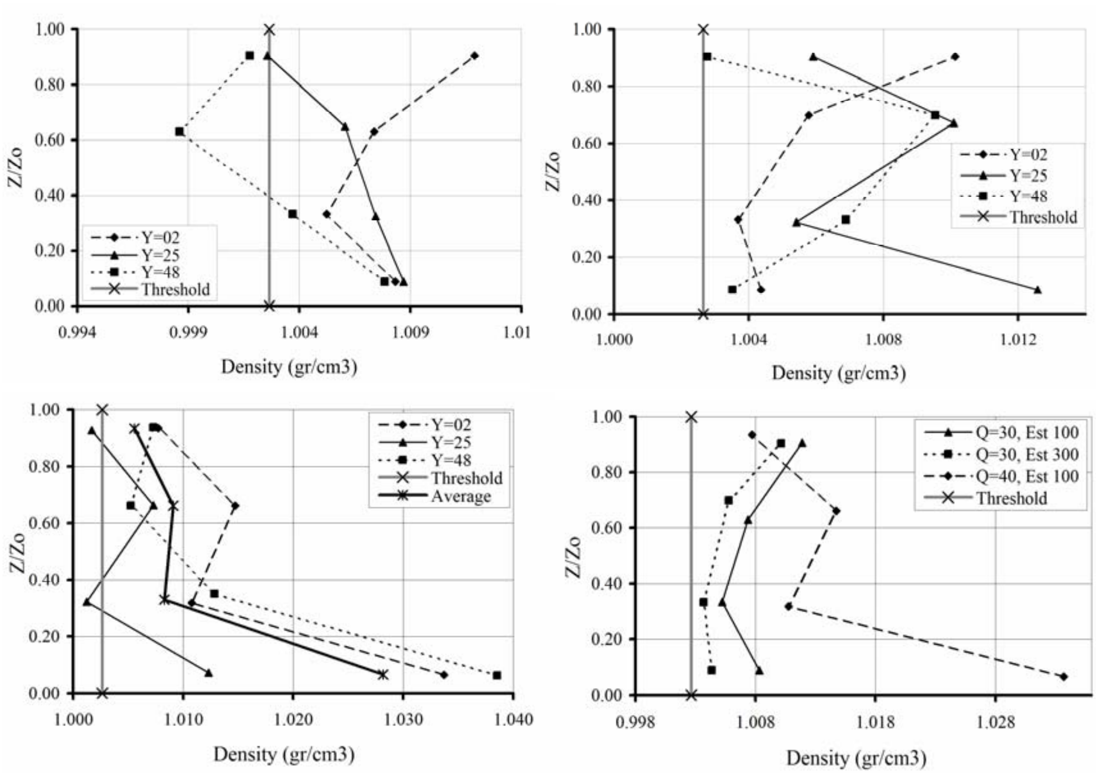
FIG. 2 – The flume at the open air laboratory

A series of experimental tests were carried out at the laboratory of the WUT Institute of Geotechnics and Hydro-engineering to verify whether there is a significant variation of this water property as a function of the depth and flow velocity. The open air flume shown in the Fig. 2 (6.0 meters long, 0.5m width, and 1.0 m high) was used for this purpose; this channel was provided with a special structure to allocate a 0.20 m sandy bed in order to observe and measure the water density variations. The FIGS. 3 & 4 depict the first sector of the flume and the cross section A-A, where many of the experiments were carried out.



FIGS. 3 & 4 – The sector 2 and the cross section A-A of the flume

Three different flow rates were used: 20, 30 and 40 l/s. The FIGS. 3 & 4 indicate the arbitrary origin of the channel.



FIGS. 5, 6, 7 & 8 Experimental results for obtaining water density distributions

The tests were performed for three different cross sections (station +100 where the section A-A is, station +300, and station +500) in twelve different points per cross section, these points are shown in the FIG. 4. The results of the first ten series of experiments are presented in the FIGS. 5, 6, 7 & 8 for chosen profiles (FIG. 5 Q =30 l/s station = 100, FIG. 6 Q =30 l/s station = 300, FIG. 7 Q =40 l/s station =100, FIG. 8 Comparison of the profiles with Y = 48). The series of experiments will continue during the spring 2007 to perform a more adequate statistical analysis and to decrease the sensitivity of the results. A normalized water depth was used (z/z_o), where z_o is the total depth and z is the depth of the test in order to have the same vertical scale for different flow rates and cross sections. Tests of the water were taken with a siphon to be weighted. The volume of each probe was measured and using the definition of density, the experimental profiles were determined. In addition, water clean tests carried out to establish a threshold value in order to compare the value of the clean water density with the value of the mixture between suspended sediment and water. This threshold value is represented by the grey line that appears in every profile. The density profiles that correspond to the flow rate 40 l/s are higher for higher velocities; a fact that results evident taking the consideration that the lifting force is proportional to the average velocity (formula 4). The theoretical value of the density for 19 C is equivalent to 0.998 gr/cm^3 (Strzelecki & Kostecki, 2007) while the average value of the clean water tests was 1.0026 gr/cm^3.

The hydro-dynamic models consider as well the kinematic viscosity (υ) as constant. Once it was proved that the density of the fluid varies with respect the water depth, the kinematic viscosity vary as well and should not be consider as constant. This condition is fulfilled, when suspended load is transported by the fluid in turbulent motion.

4 The flow trough the porous media

The FIG. 1 depicts the scheme of the water velocity profile along the z direction. This figure considers the case when the channel bed is constituted by a porous medium, such as the case of the movable sandy bed of the flume shown of the FIG. 2. The velocity of the water in the boundary between the sandy bed and the water is considered as zero for hydrodynamic modelling. A numerical model was built using the finite element program Flex PDE in order to estimate how big is the velocity in this boundary and to check whether this value can be neglected for modelling open-channel flow of the flume at the laboratory. The Darcy law was the base of this model. This law established the basis of the filtration flow modelling through porous media. Forchheimer (Juarez, 2004) demonstrated the hydraulic head is a harmonic function which fulfils the Laplace equation:

$$v_f = -k_f \cdot \operatorname{grad}\left(\frac{p}{\gamma} + z\right) = -k_f \cdot \operatorname{grad} H \qquad (5)$$

$$\nabla^2 \varphi = \nabla^2 H = \frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial z^2} = 0 \qquad (6)$$

Where ϕ is the potential function of the flow represented in this case by the total hydraulic head. In the case of the open air flume, this hydraulic head varies linearly along the x-axe. The equation (1) was used to determine this hydraulic head and to establish the potential function. It is possible to expect a very slow velocity field due to the fact that the variation of the head is too small in the system. The permeability of the sand that forms the bed of the flume was estimated with the Allen Hazen formula (Juarez, 2004). This formula establishes a relation between the D_{10} of the material and its permeability. After the sieve analysis of the sand that forms the bed of the flume, and its permeability was approximately 0.017 cm/seg. The FIG. 9 depicts the modelled velocity field through the sandy layer of the flume shown in the FIG. 2. The scale of the output demonstrates that this velocity in the whole layer and in the boundary with the water can be neglected for hydrodynamic modelling. Nevertheless, the flow through the porous medium exists. This situation can change for bigger channels, such as natural streams or rivers.

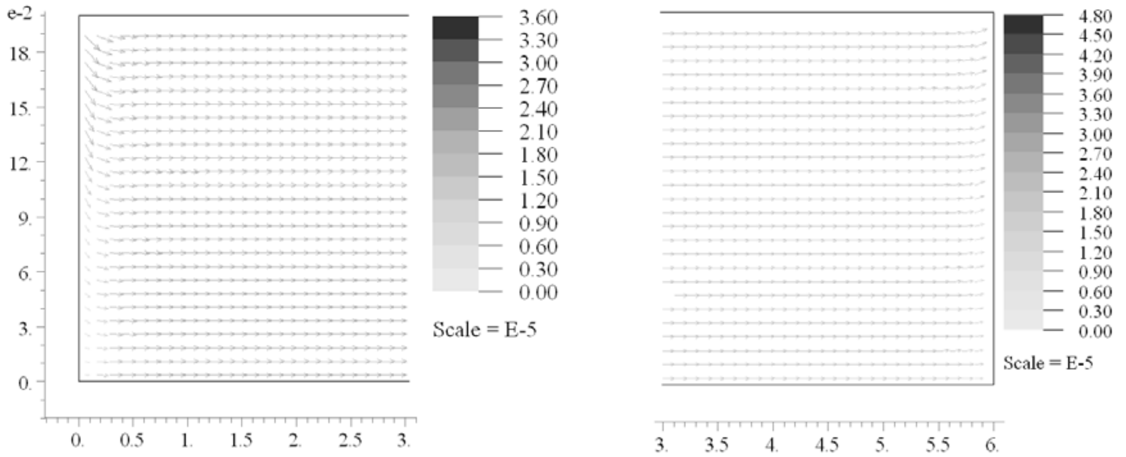


FIG. 9 – The velocity field through the sandy layer

5 Conclusions

Two main conclusions can be mentioned:

a) The presence of a sandy bed does affect the hydrodynamic parameters that are considered always as constants for modelling flow and sediment transport in open channels, namely the viscosity and the density. These properties are function of the water depth and the flow velocity and their theoretical formulations are to be researched. The experimental works at the laboratory of the WUT Institute of Geotechnics and Hydro-engineering can represent a first approach to study more in detail this fact. Nevertheless, the results of the laboratory are limited for certain hydrodynamic conditions.

b) A velocity in the boundary between the porous medium and the fluid in motion exists, nevertheless, the numerical simulation demonstrate that this velocity can be neglected for hydrodynamic modelling.

A model that would not neglect the stated conclusions is being built as the main topic of the author's PhD research.

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