Blind Multiuser Equalization using a PARAFAC-Subspace Approach

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Résumé – Dans cet article, nous utilisons la décomposition tensorielle PARAFAC (PARAllel FACtors) en vue de développer une nouvelle approche pour l'égalisation aveugle multi-utilisateur dans le cadre des systèmes de communications sans fil. Le système considéré est basé sur l'utilisation conjointe d'un réseau d'antennes et d'un sur-échantillonnage à la réception. Nous proposons tout d'abord un modèle tridimensionnel du type PARAFAC pour le signal reçu, dont les 3 dimensions sont l'*espace*, le *temps* et le *sur-échantillonnage*. Ensuite, nous présentons un nouveau récepteur aveugle multi-utilisateur pour la séparation des signaux et pour l'égalisation. Le récepteur proposé combine une modélisation PARAFAC, une méthode de sous-espace et l'exploitation de la propriété d'alphabet fini des symboles transmis. Des résultats de simulations sont montrés pour illustrer la performance du récepteur aveugle proposé.

Abstract – In this paper, we make use of the PARAFAC (PARAllel FACtors) tensor decomposition and propose a new blind multiuser equalization approach for wireless communications systems employing an antenna array and oversampling at the receiver. First, a tridimensional PARAFAC model for the received signal is proposed, the 3 dimensions being *space*, *time* and *oversampling*. Then, a blind receiver performing joint blind multiuser signal separation and equalization is formulated, combining PARAFAC modelling and a subspace method together with the use of Finite Alphabet (FA) property of symbols. Simulation results are provided to illustrate the performance of the proposed receiver.

1 Introduction

The blind multiuser equalization problem is an attractive research topic in the area of signal processing for wireless communications. It consists in recovering the information transmitted by several co-channel users with the assumption of a frequency-selective channel and without the knowledge of training sequences. Most of receiver algorithms deal with matrix (two-dimensional or 2-D) models for the received signal, exploiting its space and time dimensions as well as structural (problem-specific) properties of the transmitted signals (finite-alphabet, constant-modulus, etc) for signal separation and equalization [1, 2, 3].

Unlike the decompositions of 2-D arrays (matrices), which are generally nonunique for any rank greater than one (for rank one it is unique up to a scalar factor), low-rank decompositions of 3-D arrays (also called third-order *tensors*) are essentially unique. One of the most studied low-rank decompositions of 3-D (or higher dimensional) tensors is called PARAFAC (PARAallel FACtor) analysis, which was independently developed by Caroll and Chang [5] and Harshman [6] in the context of psychometrics and widely studied in the chemometrics area [7]. In the context of wireless communications, PARAFAC has recently appeared as a powerful tool for receiver signal processing, allowing to perform multiuser channel identification, beamforming and symbol recovery in a blind way. Most of research bringing PARAFAC to the context of signal processing for wireless communications were carried out by Sidiropoulos and his co-workers (see [8] and several references therein).

In this work, we present a new approach to the problem of blind multiuser equalization of single-input multiple-output (SIMO) wireless communication systems employing a receiver antenna array together with oversampling. We first show that the received signal can alternatively be represented as a tridimensional (3-D) PARAFAC model, the 3 dimensions being *space*, *time* and *oversampling*. After formulating the model, a new blind multiuser equalization receiver for joint blind multiuser signal separation and equalization is proposed, combining PARAFAC modelling and a subspace method together with the use of Finite Alphabet (FA) property of symbols. The key aspect of the proposed algorithm is that multiuser signal separation (PARAFAC stage) and equalization (Subspace+FA stage) are iteratively performed. Simulation results are provided to illustrate the performance of the proposed receiver with that of classical ones.

This paper is organized as follows. In Section 2, some background on the PARAFAC decomposition is given. Section 3 is dedicated to the signal modelling, where the proposed PARAFAC model is introduced. In Section 4, our PARAFAC receiver for blind multiuser equalization is formulated. Section 5 contains simulation results for performance evaluation. The paper is finalized in Section 6 with some conclusions and perspectives.

2 Parallel Factor (PARAFAC) analysis

For an $I \times J \times K$ third-order tensor $\mathcal{X}$, its Q-component PARAFAC decomposition is given by

$$x_{i,j,k} = \sum_{q=1}^{Q} a_{i,q} b_{j,q} c_{k,q}. \quad (1)$$

The standard PARAFAC model for a three-way (3-D) array expresses the original tensor as a sum of rank-one three-way factors, each one of which being an outer product of three vectors. By analogy with the definition of matrix rank, the rank of a third-order tensor is defined as the minimum number of rank-one three-way components needed to decompose $\mathcal{X}$.

The PARAFAC decomposition can also be represented in matrix notation. Define an $I\times R$ matrix $\mathbf{A}$, $J\times R$ matrix $\mathbf{B}$ and $K\times R$ matrix $\mathbf{C}$. Define also a set of matrices $\mathbf{X}_{i..}\in\mathbb{C}^{J\times K}$, $i=1,\cdots,I$, a set of matrices $\mathbf{X}_{.j.}\in\mathbb{C}^{K\times I}$, $j=1,\cdots,J$ and a set of matrices $\mathbf{X}_{..k}\in\mathbb{C}^{I\times J}$, $k=1,\cdots,K$. Based on these definitions, the model (1) can be written in three different ways. For each writing of the model a system of simultaneous matrix equations exists. The three writings of the model are:

$$\mathbf{X}_{i..} = \mathbf{B}D_i[\mathbf{A}]\mathbf{C}^T \quad i=1,\cdots,I, \tag{2}$$

$$\mathbf{X}_{.j.} = \mathbf{C}D_j[\mathbf{B}]\mathbf{A}^T \quad j=1,\cdots,J, \tag{3}$$

$$\mathbf{X}_{..k} = \mathbf{A}D_k[\mathbf{C}]\mathbf{B}^T \quad k=1,\cdots,K, \tag{4}$$

where the operator $D_i[\mathbf{A}]$ forms a diagonal matrix from the i-th row of $\mathbf{A}$. The matrices $\mathbf{X}_{i..}$, $i=1,\cdots,I$, $\mathbf{X}_{.j.}$, $j=1,\cdots,J$, and $\mathbf{X}_{..k}$, $k=1,\cdots,K$ can be interpreted as slices of the tensor along the first, second and third dimensions, respectively. Stacking the matrix slices $\mathbf{X}_{..k}$, $k=1,\cdots,K$ into a a matrix $\mathbf{X}_1\in\mathbb{C}^{IJ\times K}$, we have

$$\mathbf{X}_1=\begin{bmatrix}\mathbf{X}_{..1}\\ \vdots\\ \mathbf{X}_{..K}\end{bmatrix}=\begin{bmatrix}\mathbf{A}D_1[\mathbf{C}]\\ \vdots\\ \mathbf{A}D_K[\mathbf{C}]\end{bmatrix}\mathbf{B}^T=(\mathbf{C}\diamond\mathbf{A})\mathbf{B}^T, \tag{5}$$

where $\diamond$ is the Khatri-Rao (columnwise Kronecker) product. Two other matrices $\mathbf{X}_2\in\mathbb{C}^{JK\times I}$ and $\mathbf{X}_3\in\mathbb{C}^{KI\times J}$ containing the full tensor information can be similarly formed by stacking the matrix slices $\mathbf{X}_{.j.}$, $j=1,\cdots,J$, and $\mathbf{X}_{i..}$, $i=1,\cdots,I$. Uniqueness of the PARAFAC decomposition was studied by Harshman [6] and the proof was provided by Kruskal [4]. According to Kruskal, a trilinear PARAFAC decomposition over $\mathbb{R}$ is unique, except for trivial permutation and scaling ambiguity. The uniqueness theorem is now revisited. Consider a set of I matrices $\mathbf{X}_{i..}=\mathbf{B}\mathbf{D}_i[\mathbf{A}]\mathbf{C}^T$ $i=1,\cdots,I$, where $\mathbf{A}\in\mathbb{R}^{I\times R}$, $\mathbf{B}\in\mathbb{R}^{J\times R}$ and $\mathbf{C}\in\mathbb{R}^{K\times R}$. If $k_\mathbf{A}+k_\mathbf{B}+k_\mathbf{C}\geq 2(R+1)$ the matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are unique up to common permutation and scaling of columns. This means that, any matrices $\overline{\mathbf{A}}$, $\overline{\mathbf{B}}$ and $\overline{\mathbf{C}}$ satisfying the model $\mathbf{X}_{i..}$, $i=1,\cdots,I$, are linked to $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ by

$$\overline{\mathbf{A}}=\mathbf{A}\Pi\Delta_1,\ \ \overline{\mathbf{B}}=\mathbf{B}\Pi\Delta_2,\ \ \overline{\mathbf{C}}=\mathbf{C}\Pi\Delta_3, \tag{6}$$

where Π is a permutation matrix and Δ_1, Δ_2 and Δ_3 are diagonal matrices satisfying the condition $\Delta_1\Delta_2\Delta_3=\mathbf{I}$.

3 Signal modelling

Let us consider a linear and uniformly-spaced array of M antennas receiving signals from Q co-channel users. Assume that the signal transmitted by each co-channel user is subject to frequency-selective multipath propagation and arrives at the receiver via L specular paths. The length of the channel impulse response is K symbols long. At the output of each receiver antenna, the signal is sampled at a rate that is P times the symbol rate. Due to temporal oversampling, the resolution of the pulse-shaping filter response is increased by a factor P. Such an increase in the temporal resolution is interpreted here as an addition of a third axis (or dimension) to the received signal, called here the *oversampling dimension*. Let us organize the P oversamples of the signal received at the m-th antenna at the n-th symbol period in a vector $\mathbf{x}_m(n)=[x_m(n)x_m(n+1/P)\cdots x_m(n+(P-1)/P)]^T\in\mathbb{C}^P$. Its discrete-time baseband representation in absence of noise can be factored as

$$\mathbf{x}_m(n)=\sum_{q=1}^{Q}\sum_{l=1}^{L}b_{lq}a_m(\theta_{lq})\sum_{k=0}^{K-1}\mathbf{g}(k-\tau_{lq})s_q(n-k), \tag{7}$$

b_{lq} is the fading envelope of the l-th path of the q-th user, $a_m(\theta_{lq})$ is the phase response of the m-th antenna-element to the l-th path of the q-th user, θ_{lq} being the associated direction of arrival. Similarly, τ_{lq} denotes the propagation delay (in multiples of the symbol period T) and

$$\mathbf{g}(k-\tau_{lq})=\begin{bmatrix}g(k-\tau_{lq})\\ g(k-\tau_{lq}+1/P)\\ \vdots\\ g(k-\tau_{lq}+(P-1)/P)\end{bmatrix} \tag{8}$$

represents the k-th component of the oversampled pulse-shaping filter response evaluated at delay τ_{lq}. The channel length K is such that $K\geq max(\tau_{lq})$. This condition guarantees that all multipath energy is captured in our frequency-selective channel impulse response model. Finally, $s_q(n)$ is the information symbol transmitted by the q-th user at the n-th time symbol period. Depending on the type of signal processing used at the receiver, we may utilize either the above parametric channel model, with explicit description of angles and delays (narrowband assumption), or a non-parametric one, when we are not interested in characterizing angle and delay parameters of the channel. In this work we focus on the parametric model, which means that all the multipath parameters of all users are captured in our tensor model. Define

$$\mathbf{a}_{l,q}=[a_1(\theta_{lq})\,a_2(\theta_{lq})\cdots a_M(\theta_{lq})]^T\ \in\mathbb{C}^M \tag{9}$$

and

$$\mathbf{G}_{l,q}=[\mathbf{g}(0-\tau_{lq})\cdots\mathbf{g}(K-1-\tau_{lq})]\ \in\mathbb{C}^{P\times K} \tag{10}$$

as the spatial and temporal responses of the channel to the l-th multipath of the q-th user, $l=1,\ldots,L$, $q=1,\ldots,Q$. In order to rewrite (7) in a more compact form, let us concatenate the LQ spatial and temporal responses into equivalent matrices $\mathbf{A}=[\mathbf{a}_{1,1}\cdots\mathbf{a}_{l,q}\cdots\mathbf{a}_{L,Q}]\in\mathbb{C}^{M\times LQ}$ and $\mathbf{G}=[\mathbf{G}_{1,1}\cdots\mathbf{G}_{l,q}\cdots\mathbf{G}_{L,Q}]\in\mathbb{C}^{P\times KLQ}$, and define $\mathbf{b}=[b_{11}\cdots b_{lq}\cdots b_{LQ}]^T\in\mathbb{C}^{LQ}$ as a vector of multipath gains. Define also the overall channel impulse response matrix $\mathbf{H}\in\mathbb{C}^{P\times KLQ}$ as

$$\mathbf{H}=\mathbf{G}(diag(\mathbf{b})\otimes\mathbf{I}_K)\ \in\mathbb{C}^{P\times KLQ}, \tag{11}$$

where the operator $\otimes$ defines the Kronecker product. The matrix $\mathbf{H}$ is nothing but the temporal response matrix scaled by the complex multipath gains. The operator $diag(\cdot)$ forms a diagonal matrix out of its vector argument. Considering that a block of N transmitted symbols is processed at the receiver, we define $\mathbf{S}=[\mathbf{S}_1^T\cdots\mathbf{S}_Q^T]^T\ \in\mathbb{C}^{KQ\times N}$ a block-Toeplitz matrix concatenating Q Toeplitz symbol matrices, each one of which having its first row and column equal to $\mathbf{s}_q^{(r)}=[s_q(1)\,s_q(2)\cdots s_q(N)]$ and $\mathbf{s}_q^{(c)}=[s_q(1)\,0\cdots0]^T$, respectively.

TAB. 1: IPSP Algorithm

• $i=0$; Initialize $\widehat{\mathbf{A}}^{(0)}$ and $\widehat{\mathbf{B}}^{(0)}$
1. $i=i+1$;
2. Update $\widehat{\mathbf{C}}^{(i)} = \left[\left(\widehat{\mathbf{B}}^{(i-1)} \diamond \widehat{\mathbf{A}}^{(i-1)}\Psi\right)\Phi\right]^{\dagger}\mathbf{X}_1$;
3. **Subspace + FA projection stage** (Table 2)
4. Form $\widehat{\mathbf{C}}^{(i)}$ from $\widehat{\mathbf{C}}_1^{(i)}, \cdots, \widehat{\mathbf{C}}_Q^{(i)}$;
5. Update $[\widehat{\mathbf{A}}^{(i)}]^T = \left[\left((\Phi\widehat{\mathbf{C}}^{(i)})^T \diamond \widehat{\mathbf{B}}^{(i-1)}\right)\Psi^T\right]^{\dagger}\mathbf{X}_2$;
6. Update $[\widehat{\mathbf{B}}^{(i)}]^T = \left[\widehat{\mathbf{A}}^{(i)}\Psi \diamond (\Phi\widehat{\mathbf{C}}^{(i)})^T\right]^{\dagger}\mathbf{X}_3$;
7. Go to step 2 until convergence.

TAB. 2: Subspace + FA projection stage

for $q=1$ to Q,
- Determine $\mathbf{T}_q^{(i)}$ from $\widehat{\mathbf{C}}_q^{(i)}$ (subspace method [9]);
- $\widehat{\mathbf{S}}_q^{(i)} = [\widehat{\mathbf{T}}_q^{(i)}]^{-1}\widehat{\mathbf{C}}_q^{(i)}$;
- $\bar{\mathbf{s}}_q^{(i)} = \text{proj}\,[\widehat{\mathbf{S}}_q^{(i)}]$;
- $\widehat{\mathbf{C}}_q^{(i)} = \text{toeplitz}\,[\bar{\mathbf{s}}_q^{(i)}]$;
end

In absence of noise, the received signal is a 3-D tensor $\mathcal{X} \in C^{M\times N\times P}$ that can be expressed as a set of $M\times N$ space-time slices $\mathbf{X}_{..p}$, each one of which admitting the following factorization:

$$\mathbf{X}_{..p} = (\mathbf{A}\Psi)D_p(\mathbf{H})(\Phi\mathbf{S}), \quad p=1,\cdots,P, \tag{12}$$

where

$$\Psi = \mathbf{I}_{LQ}\otimes\mathbf{1}_K^T \ \in \mathbb{C}^{LQ\times KLQ}, \tag{13}$$
$$\Phi = \mathbf{I}_Q\otimes\mathbf{1}_L\otimes\mathbf{I}_K \ \in \mathbb{C}^{KLQ\times KQ}, \tag{14}$$

are constraint matrices, composed of 1's and 0's. The term $\mathbf{1}_K$ being a "all ones" column vector of dimension $K\times 1$. The operator $D_p(\mathbf{H})$ takes the p-th row of its matrix argument and forms a diagonal matrix out of it. Note that (12) follows a tridimensional (3-D) PARAFAC model. With respect to the PARAFAC decomposition in Section 2, Equation (12) can be interpreted as the p-th matrix slice of a (M,N,P)-dimensional tensor $\mathcal{X}$. According to (12), the received tensor is completely characterized by a set of three matrix components $\mathbf{A}\Psi$, $\mathbf{H}$ and $\Phi\mathbf{S}$. This tensor model is a PARAFAC model having a constrained structure, the constraints being given by matrices Ψ and Φ. According to (2), the received signal tensor can also be expressed as a set of $P\times M$ matrix slices $\mathbf{X}_{.n.} = \mathbf{H}D_n\left((\Phi\mathbf{S})^T\right)(\mathbf{A}\Psi)^T$, $n=1,\ldots,N$ or as a set of $N\times P$ matrix slices $\mathbf{X}_{m..} = (\Phi\mathbf{S})^T D_m(\mathbf{A}\Psi)\mathbf{H}^T$, $m=1,\ldots,M$. The three unfolded matrices $\mathbf{X}_{i=1,2,3}$, containing the full tensor information, are defined as $\mathbf{X}_1 = [\mathbf{X}_{..1}^T\cdots\mathbf{X}_{..P}^T]^T \in \mathbb{C}^{MP\times N}$, $\mathbf{X}_2 = [\mathbf{X}_{.1.}^T\cdots\mathbf{X}_{.N.}^T]^T \in \mathbb{C}^{PN\times M}$ and $\mathbf{X}_3 = [\mathbf{X}_{1..}^T\cdots\mathbf{X}_{M..}^T]^T \in \mathbb{C}^{NM\times P}$, respectively.

4 Receiver algorithm

A combined PARAFAC-Subspace receiver for joint blind multiuser signal separation and equalization is now presented. Multiuser signal separation is done in the 3-D tensor space, exploiting *oversampling*, *time* and *space* dimensions of the received signal in an alternating way. The alternating least squares (ALS) algorithm [6] is used for this purpose. Equalization is done in the 2-D matrix space, where the Toeplitz structure of users symbol matrices as well as the Finite-Alphabet (FA) property of the transmitted symbols are exploited for symbol estimation via a subspace method. The key aspect of the proposed algorithm is that multiuser signal separation (PARAFAC stage) and equalization (Subspace+FA stage) are iteratively performed. The goal of the PARAFAC stage is to estimate three component matrices from which the model parameters, i.e. the oversampled channel response matrix $\mathbf{H}$, the spatial signature matrix $\mathbf{A}$ and the transmitted symbols $\mathbf{S}$. In turn, the goal of the subspace+FA stage is to determine an ambiguity matrix that is inherent to the model as well as to estimate the transmitted symbols in the 2-D space, which are then used as an input to the PARAFAC stage to refine the estimates of model parameters in the 3-D space. In the following, we describe the proposed algorithm. This algorithm is called Iterative PARAFAC-Subspace with Projection (IPSP).

For the received signal tensor $\mathcal{X}\in\mathbb{C}^{M\times N\times P}$, multiuser signal separation consists in estimating in an alternating way three matrices $\widehat{\mathbf{A}}\in\mathbb{C}^{KLQ\times N}$, $\widehat{\mathbf{B}}\in\mathbb{C}^{M\times KLQ}$ and $\widehat{\mathbf{C}}\in\mathbb{C}^{P\times KLQ}$ from the matrix representations $\mathbf{X}_{i=1,2,3}$ of the received signal tensor. The multiuser signal separation problem can be formulated as a set of three independent nonlinear least squares problems:

$$\begin{aligned}
\widehat{\mathbf{B}} &= \underset{\mathbf{B}}{\text{argmin}}\ \|\mathbf{X}_1 - (\mathbf{C}\diamond\mathbf{A}\Psi)\Phi\mathbf{B}\|^2 \\
\widehat{\mathbf{A}} &= \underset{\mathbf{A}}{\text{argmin}}\ \|\mathbf{X}_2 - ((\Phi\mathbf{B})^T\diamond\mathbf{C})(\mathbf{A}\Psi)^T\|^2 \\
\widehat{\mathbf{C}} &= \underset{\mathbf{C}}{\text{argmin}}\ \|\mathbf{X}_3 - (\mathbf{A}\Psi\diamond(\Phi\mathbf{B})^T)\mathbf{C}^T\|^2
\end{aligned} \tag{15}$$

One iteration of the multiuser signal separation stage is composed of three steps. At each step one component matrix is updated while the others are fixed to the values obtained at the previous step. Assuming that identifiability conditions are satisfied, an estimate of the component matrices $\mathbf{A}$, $\mathbf{H}$ and $\mathbf{S}$ are related to $\widehat{\mathbf{A}}$, $\widehat{\mathbf{B}}$ and $\widehat{\mathbf{C}}$ in the following way

$$\widehat{\mathbf{B}} = \mathbf{T}(\Pi\otimes\mathbf{I}_K)\mathbf{S},\ \ \widehat{\mathbf{A}} = \mathbf{A}(\Pi\Delta\otimes\mathbf{I}_L),\ \ \widehat{\mathbf{C}} = \mathbf{H}(\Pi\otimes\mathbf{I}_{KL})\mathbf{T}^{-1}$$

where Π is a permutation ambiguity matrix, Δ is a scaling ambiguity matrix and $\mathbf{T}\in\mathbb{C}^{KQ\times KQ}$ is a block-diagonal (partial rotation) ambiguity matrix, which must be determined at the receiver in order to recover users symbol sequences. Since $\mathbf{T}$ is block-diagonal, the symbol sequences can be recovered by solving the following set of independent equations:

$$\widehat{\mathbf{B}}_1 = \mathbf{T}_1\mathbf{S}_1,\ \widehat{\mathbf{B}}_2 = \mathbf{T}_2\mathbf{S}_2,\ \ \ldots,\ \widehat{\mathbf{B}}_Q = \mathbf{T}_Q\mathbf{S}_Q. \tag{16}$$

The subspace+FA stage consists in estimating $\mathbf{T}_1,\cdots,\mathbf{T}_Q$ via a subspace method [9]. For reasons of space, we report the interested reader to [9] for further details on this algorithm. After determining the partial rotation ambiguity matrices, users symbol matrices can be estimated. An estimation of the symbol sequences can be obtained from the projection of the first row of the estimated symbol matrices $\widehat{\mathbf{S}}_1,\ldots,\widehat{\mathbf{S}}_Q$ onto the FA. Then, an updated (post-equalized) version of the received signal tensor, now free from the partial rotation ambiguity, is then formed and used as an input to the PARAFAC stage to refine user signal separation in the 3-D space. Tables 1 and 2 show the pseudo-code for the IPSP receiver algorithm. Table 1 enumerates the steps of the IPSP algorithm, with emphasis on the ALS stage while Table 2 shows the steps associated to the subspace+FA stage.

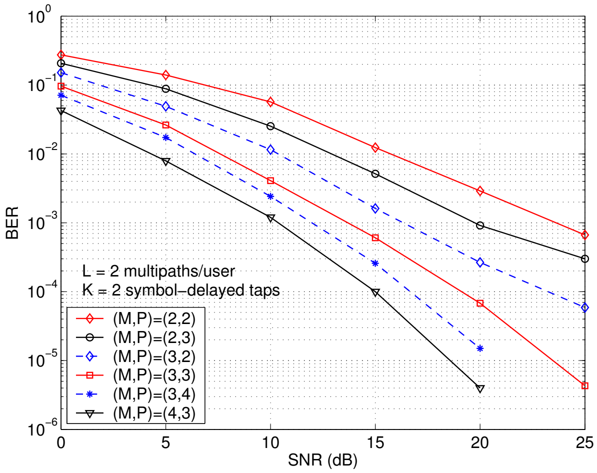

FIG. 1: BER versus SNR. L=2 and K=2.

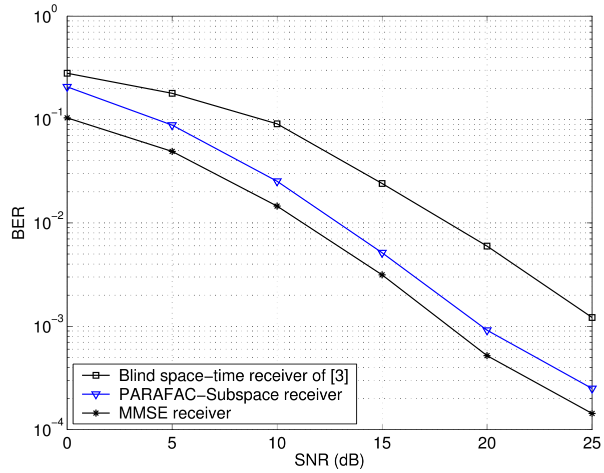

FIG. 2: Performance of the PARAFAC-Subspace receiver compared to those of the blind space-time receiver of [3] and MMSE receiver with perfect channel knowledge.

5 Simulation Results

The performance of the blind PARAFAC-Subspace receiver is evaluated through computer simulations. Results are shown in terms of bit-error-rate (BER) versus signal-to-noise ratio (SNR), averaged over 1000 Monte Carlo experiments. For each experiment, multipath fading gains are redrawn from an i.i.d. Rayleigh generator while user signals are redrawn from an i.i.d. distribution. Users symbols are modulated using binary-phase shift keying (BPSK). The number of users is fixed to Q = 2. For each experiment, a block of N = 50 received samples is processed at the receiver and the BER is averaged over the two users. At the beginning of the algorithm $\widehat{\mathbf{A}}^{(0)}$ and $\widehat{\mathbf{B}}^{(0)}$ are initialized as $\widehat{\mathbf{A}}^{(0)} = \mathbf{A} + \mathbf{E}_1$ and $\widehat{\mathbf{B}}^{(0)} = \mathbf{B} + \mathbf{E}_2$, with $\mathbf{E}_1$ and $\mathbf{E}_2$ being random error matrices, the entries of which are drawn from a normal distribution with standard deviation 10^{-1}. More sophisticated strategies exist but they are beyond the scope of this work. The performance results are compared by varying the number M of receiver antennas and the oversampling factor P. The number of multipaths/user is L = 2 and the length of the overall temporal channel response is K = 2. Multipath delays, gains and angles are respectively $(\tau_{11}, \tau_{21}) = (\tau_{12}, \tau_{22}) = (0, T)$, $(b_{11}, b_{21}) = (b_{12}, b_{22}) = (1, 0.5)$, $(\theta_{11}, \theta_{21}) = (0, 30^o)$ and $(\theta_{12}, \theta_{22}) = (-20^o, -40^o)$. Figure 1 shows the results for the proposed PARAFAC-Subspace receiver with the IPSP algorithm. As the number of antennas or the oversampling factor increases, the performance gradually improves. Note that an increase in the number M of antennas offers a greater performance improvement than an increase in the oversampling factor P. These results confirm that user signals are better distinguished in the space dimension than in the oversampling dimension. In order to provide a performance reference for our PARAFAC-Subspace receiver, we have also evaluated the performance of the blind space-time receiver proposed by Van der Veen et al. in [3], which is also based on a subspace method and FA projection. The performance of the Minimum Mean Square Error (MMSE) receiver with perfect knowledge is also considered as a reference. According to Figure 2, our receiver outperforms the blind space-time receiver of [3], and is close to the MMSE one, with a performance gap of 3 dB approximately.

6 Conclusions

In this work, a new blind multiuser equalization receiver has been proposed for joint blind multiuser signal separation and equalization. The receiver is based on a PARAFAC modelling of the received signal when an antenna array and oversampling are jointly employed at the receiver. The proposed Iterative PARAFAC-Subspace with Projection (IPSP) receiver combines PARAFAC modelling and subspace method with the use of FA-property of symbols in order to perform user signal separation and equalization iteratively. Our results have shown that the performance of the proposed blind receiver is better than that of the blind space-time receiver of [3] and is close to that of the MMSE receiver with perfect channel knowledge.

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