Structural vibration attenuation using a fractional order PD controller designed for a fractional order process

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Abstract: Structural vibration is a highly studied topic, especially in civil structures. Unwanted earth vibrations during seismic activity endanger life and often destroy buildings. In this paper, a Fractional Order Proportional Derivative controller is designed with the purpose of vibration mitigation in a three-story building. The experimental setup consists of a third floor building equipped with an active pendulum attached to the last floor. The controller is designed for the fractional order mathematical approximation of the structure by imposing frequency domain constraints such as gain crossover frequency, phase margin and robustness to gain variations. The validity of the controller is analyzed considering the simulated behavior of the compensated building to the El Centro earthquake and experimental disturbance rejection performance.

Keywords: fractional calculus; control engineering; active tuned mass damper; vibration suppression; structural dynamics.

1. INTRODUCTION

Buildings, bridges and civil structures are constantly threatened by ground motion during seismic activity. Several solutions have already been found involving passive tuned mass dampers such as the one from Poon et al. (2004) in order to improve safety and reduce loss. However, the high uncertainty in ground motion during earthquakes justifies the need for a more robust and reliable approach, achieved by active damping techniques.

Several damping strategies have been tested and validated such as the Internal Model Control validated on a mass-spring-damper benchmark presented in Keyser et al. (2017b). The IMC performs similar to the model based predictive approach tuned for an efficient disturbance model of the system presented by Copot et al. (2017).

Another viable option for active damping is fractional calculus. A highly studied branch of mathematics, fractional calculus, explores the possibility of differentiation and integration to an arbitrary order which is not limited to the integer numbers domain. In the last years, fractional calculus has been successfully used in the field of control engineering developing controllers with more degrees of freedom, offering increased stability and better performance as stated by Chen et al. (2009) and Monje et al. (2010).

Passive and active seismic mitigation of structures equipped with both Tuned Mass Damper (TMD) and Viscoelastic Damper (VED) is presented in Muresan et al. (2016). Active tuned mass dampers controlled with a fractional order PD controller successfully reduce oscillations in Prodan et al. (2016), Lin et al. (2015), Mackriell et al. (1997) proving that active tuned mass dampers drastically reduce the amplitude of the vibration when compared to passive TMDs. Robust tuning strategies involving the H_∞ controller have been studied in Wu et al. (2006). The robust controller has been validated on a steel building tested on a shake table. From the results obtained, the performance of the controller is remarkable and robust.

This paper presents the application of fractional order calculus in the field of active vibration suppression. A case study of an experimental three story building is selected to test active fractional-order vibration attenuation. The experimental setup is characterized as a fractional-order model for which a fractional order PD controller is tuned. The tuning strategy is based on imposing frequency domain specifications regarding gain crossover frequency, phase margin and robustness to gain variations. The obtained controller is validated through simulation and real life experimental tests.

The novelty of the presented work is the fractional order PD controller applied in the field of active vibration suppression of an experimental structure characterized by fractional-order structural dynamics.

The paper is structured as follows: Section 2 presents the frequency domain tuning method, Section 3 details the case study consisting of the building, Section 4 shows the obtained results, while Section 5 concludes the paper.

2. FRACTIONAL ORDER CONTROLLER TUNING STRATEGY

The transfer function of a fractional order Proportional Derivative controller is given by

$$H_{FO-PD}(s) = k_p(1 + k_d s^{\mu}) \tag{1}$$

where k_p and k_d are the proportional and derivative gains, while $\lambda \in (0, 2)$ is the derivative order. When $\mu = 1$, the transfer function from equation (1) describes an integer order PD controller.

Mapping the Laplace to the frequency domain and applying deMoivre's formula gives the trigonometric form of the fractional order PD controller

$$H_{FO-PD}(j\omega) = k_p[1 + k_d\omega^{\mu}(\cos\frac{\pi\mu}{2} + j\sin\frac{\pi\mu}{2})]. \tag{2}$$

The fractional PD controller is applied to a process characterized by fractional order dynamics described by

$$P(s) = \frac{k}{as^2 + bs^{\alpha} + c} \tag{3}$$

where α can be any real number. For $\alpha = 1$, the transfer function is the classical second order model involving the damping ratio and the natural frequency of the process.

The controller has three parameters that need to be determined: the proportional gain, the derivative gain and the fractional order of differentiation. The tuning procedure lies in solving a system of three frequency domain equations involving the three parameters needed for the controller and the transfer function of the process.

In the frequency domain, the magnitude of the open loop system is equal to 1 (0 dB) when the frequency ω is the gain crossover frequency denoted by ω_{gc}.

$$|P(j\omega_{gc})H_{FO-PD}(j\omega_{gc})| = 1 \tag{4}$$

The phase equation of the open loop system is expressed in

$$\angle P(j\omega_{gc}) + \angle H_{FO-PD}(j\omega_{gc}) = -\pi + \phi_m. \tag{5}$$

Since the control algorithm is intended for seismic mitigation where the ground motion is uncertain, an additional robustness condition involving the evolution of the phase near the gain crossover frequency is added to the constraints.

The robustness is imposed through a constant phase around the gain crossover frequency. A constant phase is a straight line on the Bode phase diagram which guarantees that at small gain variations the open loop phase remains the same. The constant phase is translated into frequency domain constraint by imposing the derivative of the phase around the frequency of interest as being 0.

$$\frac{d(\angle P(j\omega))}{d\omega} + \frac{d\angle H_{FO-PD}(j\omega)}{d\omega} = 0|_{\omega=\omega_{gc}}. \tag{6}$$

Replacing the trigonometric form of the fractional order PD controller from equation (2) in the frequency domain constraints from equations (4), (5) and (6) gives the system of equations that needs to be solved in order to determine the parameters of the controller.

$$\left|k_p\left[1 + k_d\omega_{cg}^{\mu}\left(\cos\frac{\pi\mu}{2} + j\sin\frac{\pi\mu}{2}\right)\right]\right| = \frac{1}{P(j\omega_{gc})} \tag{7}$$

$$\angle\left[1 + k_d\omega_{cg}^{\mu}\left(\cos\frac{\pi\mu}{2} + j\sin\frac{\pi\mu}{2}\right)\right] = -\pi + \phi_m - \angle P(j\omega_{gc}) \tag{8}$$

$$\frac{d\angle\left[1 + k_d\omega_{cg}^{\mu}\left(\cos\frac{\pi\mu}{2} + j\sin\frac{\pi\mu}{2}\right)\right]}{d\omega} = -\frac{d\angle P(j\omega)}{d\omega}|_{\omega=\omega_{gc}} \tag{9}$$

By solving the system of equations formed by (7), (8) and (9), the three parameters k_p, k_d and μ of the fractional PD controller are determined. Firstly, the derivative gain, k_d, and the fractional order of the differentiation, μ, can be obtained by solving the system composed by the equations of the phase and the derivative of the phase. Birs et al. (2016) presents several options for solving the system of equations. The proportional gain, k_p, is obtained by replacing k_d and μ in the magnitude equation:

$$k_p = \frac{1}{|P(j\omega_{gc})|}\frac{1}{\sqrt{1 + 2k_d\omega_{gc}^{\mu}\cos\frac{\pi\mu}{2} + k_d^2\omega_{gc}^{2\mu}}} \tag{10}$$

3. CASE STUDY

3.1 Description

The case study consists of a three story steel structure resembling a civil building. The experimental unit has been built at the Technical University of Cluj-Napoca and can be seen in Figure 1.

The height of the experimental building is 90 cm, the length is 45 cm and the width is 9 cm. The active vibration suppression algorithm is enacted using a pendulum attached to the last floor of the building excited by a servomotor. Passive vibration suppression has been previously realized using a pendulum in Oliveira et al. (2013). The drawback is that the pendulum must be custom built with the same natural frequency as the building, creating a dependency between the pendulum's length and the height of the structure. In the case of active seismic mitigation, the frequency of the pendulum can be programmed on the servomotor acting upon it, eliminating the need of a long and heavy metronome.

Each floor is furnished with an accelerometer that collects real-time vibration information in $LabVIEW^{TM}$ using dual-core ARM Cortex-A9 real-time NI myRIO board. The acceleration data is filtered and integrated twice in order to obtain the displacement of each floor. The command signal is computed real-time and sent to the servomotor using the same microcontroller used for data acquisition.

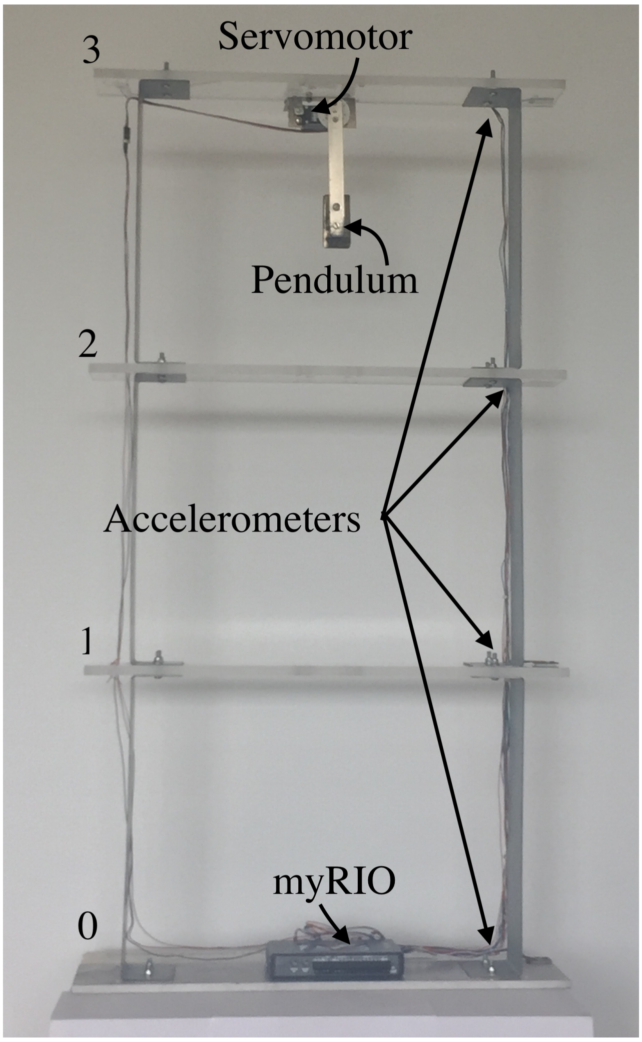

Fig. 1. Experimental three-story building equipped with a pendulum for active vibration suppression

3.2 Experimental fractional-order system identification

The input signal of the process is considered the amplitude of the pendulum's angle, while its frequency is kept constant at the natural frequency of the structure. Experimentally, the natural frequency of the building has been obtained at 3.45 Hz.

A fractional order second order transfer function expressing the relationship between the angular displacements of the pendulum versus the displacement of the third floor has been identified experimentally. The data used for the fractional order identification was obtained by exciting the pendulum with sine wave inputs of fixed frequency equal to the resonant frequency, 3.45 Hz, and different amplitudes. The identification was performed taking into consideration solely the movement of the last floor on which the pendulum is attached, while the data from the other floors has been ignored. By taking into consideration data from the rearmost floor, the obtained model characterizes the first flexural mode of the structure.

$$P(s) = \frac{1.1}{0.9978s^2 + 0.29064s^{0.5} + 441.46} \tag{11}$$

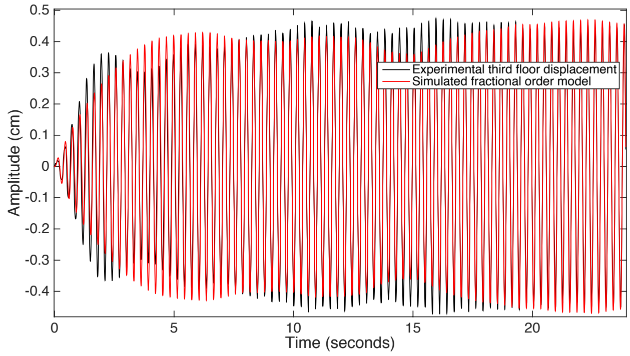

Fig. 2. Experimental fractional order model validation

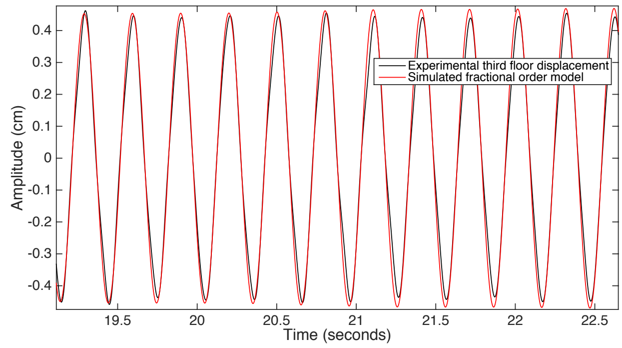

Fig. 3. Experimental fractional order model validation - zoomed

The fit of the obtained model is 84% based on data obtained by performing experiments around the first resonant frequency of the building. Figures 2 and 3 show the validation of the model.

The justification of using a fractional order approximation in favor of an integer transfer function is presented in Meral et al. (2010) and Zhou et al. (2015). Since ground motion is highly uncertain, the fractional order approximation of the model behaves better around the other resonant frequencies due to the viscoelastic character of the steel structure phenomenon more appropriately characterized by fractional calculus.

4. FRACTIONAL ORDER PD CONTROLLER TUNING

The controller is tuned with the purpose of completely eliminating the third floor oscillations. A simplified block diagram is shown in Figure 4. The seismic excitation $\ddot{x}_g$ is considered a disturbance and a properly tuned controller should keep the displacement of the structure at reference position 0. Reference 0 characterizes the natural state of the building without any displacement.

The control signal u(t) represents the angle between the free end of the pendulum and an imaginary perpendicular axis to the ground. The control signal is sent to the pendulum through the controlled servomotor.

In order to determine the parameters of the fractional order PD controller using the tuning strategy previously detailed, the desired gain crossover frequency of the open loop system is imposed at $w_{cg} = 25\ rad/s$, while the phase

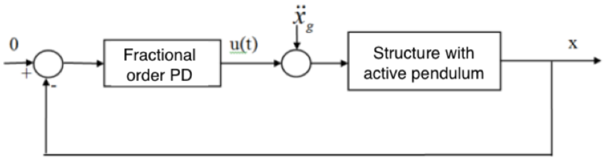

Fig. 4. Block diagram of the closed loop process

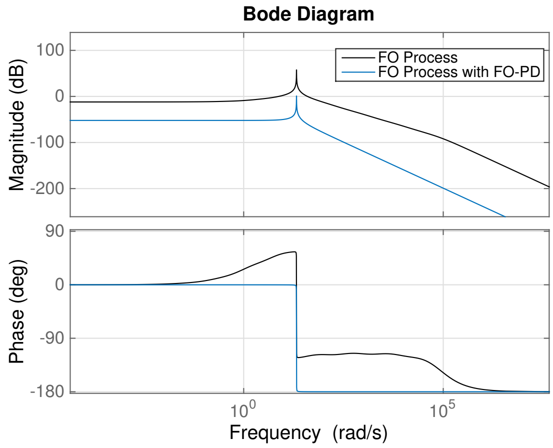

Fig. 5. Bode diagram of the open loop compared to the uncompensated system

margin $\phi_m = 60\ deg$. The constraints for the gain crossover frequency and phase margin have been chosen based on the existence conditions of fractional order controllers described by Muresan et al. (2018).

The controller that honors the constraints expressed in equations (7), (8) and (9) regarding magnitude, phase margin and robustness has been obtained as:

$$H_{FO-PD} = 0.2529(1 + 76.7641 s^{0.6639}). \qquad (12)$$

The fractional order of differentiation is obtained as being $\mu = 0.6639$.

The Bode diagram of the open loop system compared to the frequency response of the fractional order approximation shows that the imposed constraints are honored in Figure 5. The imposed robustness constraint is characterized by the straight line of the phase plot present between $\omega = 10^2\ rad/s$ and $\omega = 10^4\ rad/s$.

In order to implement the fractional order controller in real-life applications, it has to be approximated as a division of integer order polynomials. A discrete approximation method with sampling time $T_s = 0.025\ s$ based on Keyser et al. (2017a) has been used to obtained a fifth order approximation of the controller from (12). The obtained discrete form is implemented on the experimental setup with the purpose of actively suppress the structure's displacement.

5. RESULTS

The efficacy of the tuned controller is validated by simulating the closed loop response of the process when exposed to real seismic data recorded during the El Centro earthquake analyzed in Rockwell and Klinger (2013) and Ulrich (1941). Real life validation of the controller is also achieved by exposing the structure to impulse type disturbances and the closed loop response in terms of disturbance rejection settling time based on the ±2% criteria.

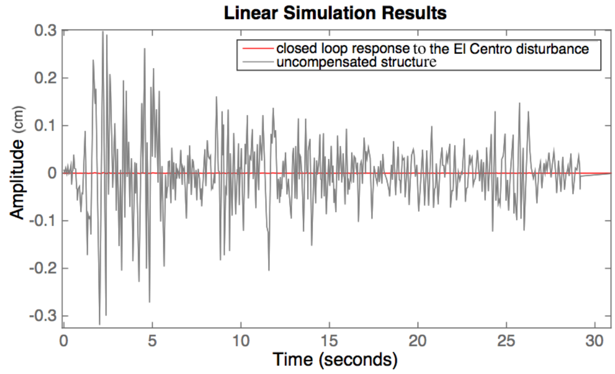

Fig. 6. Simulation of the closed loop response to the El Centro disturbance

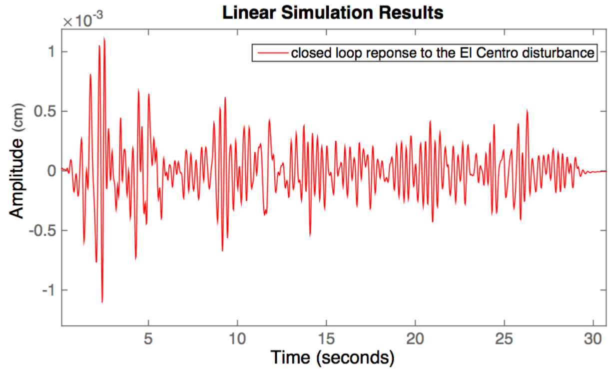

Fig. 7. Simulation of the closed loop response to the El Centro disturbance - zoomed

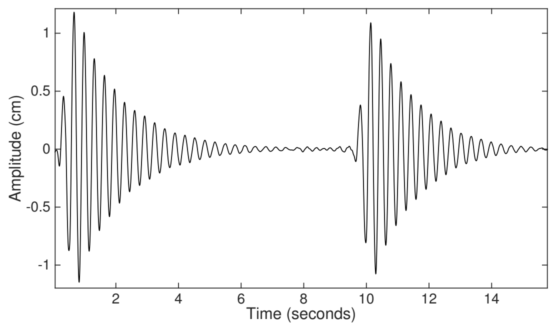

Fig. 8. Experimental result regarding the impulse disturbance response of the uncompensated structure

The simulated response of the structure at the El Centro disturbance is presented in Figure 6, while a zoomed perspective is presented in Figure 7. As can be seen, the attenuation of the oscillation amplitude is highly effective. In case of a real El Centro earthquake acting on the experimental building, the displacement of the third floor of the structure is reduced to less than 10^{-3} cm.

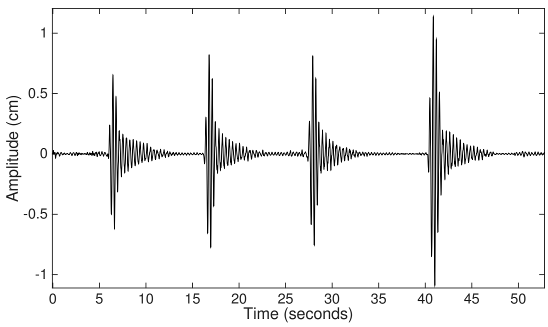

Fig. 9. Experimental closed loop impulse disturbance rejection with fractional order PD controller

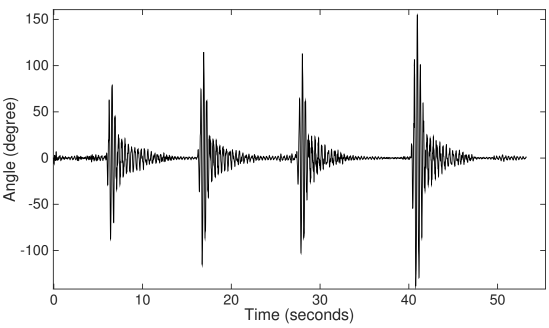

Fig. 10. Experimental control signal computed with the integer order approximation of the fractional order PD controller

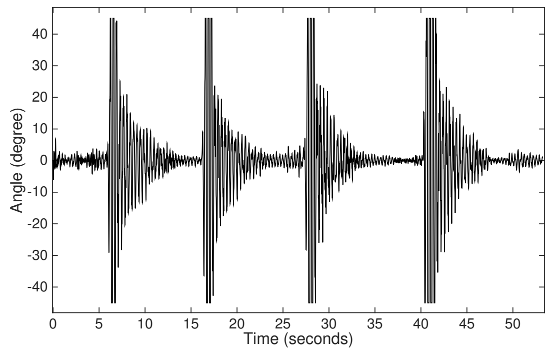

Fig. 11. Experimental saturated control signal applied to the structure's pendulum

Real time experiments consider impulse type disturbances of various amplitudes applied to the third floor of the structure. Figure 8 shows the response of the uncompensated structure to impulse type disturbances. The settling time of the open loop structure is 8 seconds.

The closed loop response to impulse disturbances of amplitudes varying between 0.6 cm and 1.2 cm is presented in Figure 9. It can be easily observed that the amplitude of the unwanted vibration is quickly reduced.

The command signal computed with the fifth order approximation of the fractional order PD controller is shown in Figure 10. It can be seen that for higher values of the disturbance amplitude, the command signal goes up to 80 degrees. In the real life implementation, the command signal applied to the servomotor is restricted between -45 and 45 degrees. Figure 11 presents the saturated command values.

The large amplitude of the command signal is justified by the fact that the amplitude of the disturbances varies between 0.6 and 1.2 cm. The displacement represents 0.54% to 1.08% movement when compared to the height of the building. Taking the real life situation of the previously mentioned Taipei tower, which has a 500 m height, a 0.54% displacement represents 270 cm in real life, while 1.08% represents 540 cm. Such great oscillation amplitudes are unrealistic in real life situations. The controller proves successful in attenuating the oscillation amplitude as well as the disturbance settling time.

The advantages of using a fractional order control over classical, integer order based control strategies are studied in Birs et al. (2016), Monje et al. (2010), Copot et al. (2013) and Chen et al. (2009). The advantages of the fractional approach are numerous including increased tuning flexibility, better transient and steady state responses, as well as easy encapsulation of more design parameters in the tuning process as described in Muresan et al. (2015).

6. CONCLUSION

Frequency domain constraints based on the gain crossover frequency, phase margin and robustness to gain variations can be used as a system of equations from which the parameters of a fractional order proportional derivative controller can be determined.

The parameters of the controller are obtained using a fractional order approximation of an experimental building equipped with an active pendulum. The structure's dynamics are identified in the form of a fractional order transfer function. The tuned controller is based on the fractional order model. Validation of the controller is realized both through simulation and real life experiments. The simulation is based on data registered during the El Centro earthquake, while the real life experiments involve impulse disturbance rejection. Both simulation and experimental responses of the closed loop system bring considerable improvements to the structure's behavior when exposed to unwanted displacement.

In addition, the present work proves that active, fractional-order controlled pendulums can be used to successfully reduce vibration in civil buildings.

Further research objectives involve determining analytic fractional-order models that encompass the effects of every floor on the structural dynamics of the entire building. The purpose is to accurately model the entirety of the flexural harmonics exhibited by the experimental structure.

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