Nonlinear Technique for the Analysis of the Free-Running Oscillator Phase Noise in Presence of an Interference Signal

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Abstract — **A new methodology for the prediction of oscillator phase noise under the effect of an interference signal is presented. It is based on a semi-analytical formulation in the presence of the noisy interferer, using a realistic oscillator model extracted from harmonic-balance simulations. The theoretical analysis of the phase process enables the derivation of key mathematical properties, used for an efficient calculation of the interfered-oscillator phase noise spectrum. The resulting quasi-periodic spectrum is predicted, as well as the impact of the interferer phase noise over each spectral component, in particular over the pulled oscillation frequency. It is demonstrated that under some conditions, the phase noise at this component is pulled to that of the interference signal. Resonance effects at multiples of the beat frequency are also predicted. The analyses have been validated with experimental measurements, obtaining excellent agreement.**

Index Terms — **Microwave oscillator, frequency-domain analysis, interferer, injection pulling, phase noise.**

I. INTRODUCTION

A well-known problem in communication systems is the pulling of the local-oscillator frequency by an interferer. In phase-locked loops (PLLs), this unwanted frequency shift can unlock the voltage-controlled oscillator (VCO) from its low frequency reference [1]. Synchronization to the interference may also occur, which in the PLL would lead to a mixing of the reference frequency and the synchronized-oscillation frequency. Though these effects have been studied in previous work, little effort has been devoted to investigate the impact of the interferer signal and its power spectral density on the interfered oscillator phase noise. This characteristic is of critical importance in communication systems, as it can give rise to demodulation errors. Predicting its degradation in the presence of an interferer will help correct the prototypes at the design stage, so as to make them more robust against the interferer action.

Here an analysis of the effect of an interferer on the phase-noise spectrum of an oscillator circuit is presented. To have an impact on this spectrum, the interferer frequency must be close to that of the interfered oscillator, as otherwise the two signals will have independent phase variations. The analysis is involved since in the general case of an oscillator that is not locked to the interferer, the solution will be quasi-periodic, with two fundamental frequencies: the oscillation frequency, affected by the interferer, and the interferer frequency. Ordinary phase-noise analysis methods of oscillator circuits are unable to consider an additional fundamental. The only way to address the problem would be through a two tone analysis, plus phase-noise analysis of the oscillator carrier, which has failed to converge in commercial harmonic balance

(HB). Instead, a semi-analytical method is applied here. A realistic model of the interfered oscillator is used, based on derivatives obtained through finite differences in HB, calculated about the free-running point [2-4]. The analysis will be based on the determination of the phase perturbation of the interfered oscillator in the presence of the undesired signal spectrum, as well as its own noise sources.

II. ANALYSIS OF THE QUASI-PERIODIC STATE

The analysis will be illustrated through its application to the FET-based oscillator at 2.5 GHz in Fig. 1. The interference signal is modeled as a current source, in parallel with the oscillator output resistor of 50 Ω. Interference signal entering through the output port is a realistic situation in front-end systems. However, the formulation can be equally applied for other locations of the interference equivalent source. It will be derived in terms of the voltage signal $v(t)$ at the oscillator drain terminal. Note that the analysis can be performed at a node different from the one where the interferer current source is connected, since its effect is modeled by means of transfer functions [2-4].
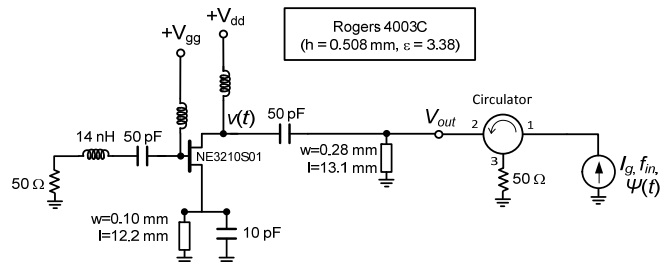

Fig. 1 Schematic of the interfered oscillator operating at 2.5 GHz based on the FET NE3210S01. The interference is modeled with a current source and introduced into the oscillator through a circulator

In this preliminary work, the interferer will be assumed to be a small amplitude single-tone current source with its own phase-noise process $\psi(t)$. This signal is expressed as $i_g(t) = 2\,\mathrm{Re}\left\{ I_g e^{j[\omega_{in}t + \psi(t)]} \right\}$ with $\omega_{in} = 2\pi f_{in}$. The introduction of the source $i_g(t)$ perturbs each k-th harmonic component of the signal $v(t)$:

$$v(t) = \sum_k v_k(t),\ v_k(t) = \left[V_k + \Delta V_k(t) \right] e^{j\phi_k(t)} e^{jk\omega_{in}t} = X_k(t) e^{jk\omega_{in}t} \quad (1)$$

where each $v_k(t)$ is a narrow band process centered at the frequency component kf_{in}. While the harmonic amplitudes V_k

are slightly perturbed by the interferer, the phase components $\phi_k(t)$ are unbounded in the non-synchronized state [1]. Now, following the semi-analytical formulation technique (SAF) proposed in [2-4], the Implicit Function Theorem is applied to obtain an envelope-domain model of the oscillator dynamics in the form of a single outer-tier equation:

$$Y\left[V_1+\Delta V_1, j(\omega_o+\Delta\omega)+s, I_g\cos\psi(t), I_g\sin\psi(t)\right]e^{j\phi(t)}=\frac{I_n(t)}{V_1+\Delta V_1} \quad (2)$$

where Y is the first harmonic total admittance at the observation node, V_1 is the first harmonic amplitude and $\Delta\omega = 2\pi\Delta f = \omega_o - \omega_{in}$, where $\omega_o=2\pi f_o$ is the free-running frequency. The phase $\phi(t) \equiv \phi_1(t)$ represents the first-harmonic phase shift from the interferer frequency f_{in}. Using the technique in [5], the effect of all the noise sources existing in the circuit has been modeled with an equivalent current generator $i_n(t) = Re\left\{I_n(t)e^{j\omega_{in}t}\right\}$ connected in parallel at the transistor drain. In the absence of noise sources, if the free-running oscillation at f_o does not synchronize to the interferer, system (2) provides a quasi-periodic solution with one autonomous component at f_o [6] Note that in the case of a phase-locked loop (PLL) or an injection-locked oscillator the situation would be different. In those cases, the admittance function Y has a dependence on the amplitude and phase components of the reference or external generator signal [5, 7]. This dependence removes the system autonomy of the interfered system quasi-periodic solution [8]. Therefore, the interfered system does not remain invariant under phase shifts, providing different phase noise results than the autonomous case, which will be the one analyzed here.

Assuming small interferer amplitude, equation (2) can be approximated by a first-order Taylor series about the free-running state, which yields the following equation for the phase shift [4]:

$$\dot{\phi} = \Delta\omega + K_s \sin(\phi-\psi) + K_c \cos(\phi-\psi) + \varepsilon(t),$$
$$\varepsilon(t) = H_r I_n^r + H_i I_n^i \quad (3)$$

In [4], the coefficients (K_c, K_s, H_r, H_i) are provided in terms of I_g and the derivatives of the admittance function (2) evaluated at the free-running solution, together with a technique to calculate these derivatives through finite differences in commercial HB. The coefficients K_c, K_s contain, among others, the admittance derivatives with respect to the real and imaginary parts of the current source phasor that models the interferer, acting as transfer functions. Equation (3) provided by the SAF technique can be used to calculate the oscillator phase noise in the presence of a noisy interferer. The study must be preceded by the analysis of the steady-state solution, in the absence of noise sources.

A. Phase dynamics in the absence of noise sources

In the absence of noise sources, Equation (3) represents an autonomous system that can be written as:

$$\dot{\phi} = \Delta\omega + K_s \sin\phi + K_c \cos\phi \equiv g(\phi) \quad (4)$$

The phase shift becomes a constant value $\phi=\phi_s$, fulfilling $g(\phi_s)=0$, only when the free-running frequency synchronizes to the interferer. Then, in the more general unsynchronized conditions $g(\phi) \neq 0$, $\forall\phi$ must be fulfilled, implying that $\phi(t)$ grows or decreases monotonically. Let us assume that $g(\phi) > 0$, $\forall\phi$. The case $g(\phi) < 0$, $\forall\phi$ is symmetric. The time required by the phase variable to pass through an arbitrary interval $[\phi_a,\phi_b]$ is given by:

$$T(\phi_a,\phi_b) = \int_{\phi_a}^{\phi_b}\frac{d\phi}{g(\phi)} = u_0(\phi_b - \phi_a) + q(\phi_b) - q(\phi_a),$$

$$u(\phi) \equiv \frac{1}{g(\phi)} = u_0 + \tilde{u}(\phi), \quad u_0 = \frac{1}{2\pi}\int_0^{2\pi}u(\phi)d\phi, \quad (5)$$

$$q(\phi) \equiv \int\tilde{u}(\phi)d\phi, \quad q(\phi) = q(\phi\pm 2\pi)$$

From the definition of $g(\phi)$, the phase shift fulfills:

$$T_b \equiv T(\phi,\phi+2\pi) = u_0 2\pi + q(\phi+2\pi) - q(\phi) = u_0 2\pi, \forall\phi \quad (6)$$

Property (6) implies that $\dot{\phi}(t)$ is periodic, since:

$$\dot{\phi}(t+T_b) = g\left[\phi(t+T_b)\right] = g\left[\phi(t)+2\pi\right] = g\left[\phi(t)\right] = \dot{\phi}(t) \quad (7)$$

Considering (6)- (7), the phase variable can be expressed as:

$$\phi(t) = \omega_b t + \sum_{k=-N}^{N} P_k e^{jk\omega_b t}, \quad \omega_b = 2\pi f_b = 2\pi / T_b \quad (8)$$

Now, introducing expression (8) in the process $v_1(t)$ in (1) one obtains:

$$v_1(t) = X_1(t)e^{j\omega_{in}t} \approx V_1 e^{j\phi(t)}e^{j\omega_{in}t}$$
$$= V_1 e^{j(\omega_{in}+\omega_b)t}\exp\left\{\sum_{k=-N}^{N}P_k e^{jk\omega_b t}\right\} \quad (9)$$
$$= \sum_k X_1^k e^{j(\omega_{in}+k\omega_b)t}$$

Equation (9) implies that, for a small amplitude interferer, $v_1(t)$ is a multi-tone signal containing the intermodulation products $f_k = f_{in} + kf_b$. The oscillator frequency is pulled from f_o to $f_o' \equiv f_1 = f_{in} + f_b$. This theoretical result has been verified by simulating the spectrum of the time-varying first harmonic component $X_1(t) \approx V_1 e^{j\phi(t)}$ using the envelope-transient method in commercial HB software. This simulation is compared in Fig. 2 with the result of equation (9). Here, the components $\{f_b, P_0,...P_N\}$ have been calculated in the frequency domain by introducing expression (8) in equation (4) and solving the resulting HB system.
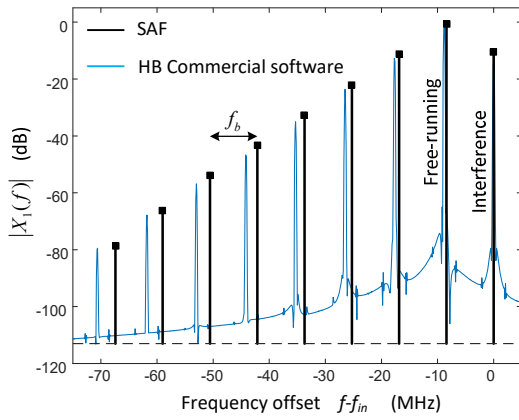
Fig. 2. Spectrum of the first harmonic component $X_1(t) \approx V_1 e^{j\phi(t)}$ in the presence of an interferer with I_g=0.2 mA and $\Delta f = -10$ MHz.

B. Phase noise analysis

Let $\phi(t) = \phi_0(t)$ be the phase shift in absence of noise sources. The noisy components introduced by the local sources and the interferer perturb the steady state $\phi_0(t)$ in the form $\phi(t) = \phi_0(t) + \Delta\phi(t)$. Assuming that $\Delta\phi(t)$ is small, equation (3) can be linearized about $\phi_0(t)$ obtaining:

$$\Delta\dot\phi(t) = b_0(t)\left[\Delta\phi(t) - \psi(t)\right] + \varepsilon(t), \quad b_0(t) \equiv \frac{dg[\phi_0(t)]}{d\phi} \quad (10)$$

Equation (10) shows that the phase perturbation is a cyclo-stationary process governed by the periodic function $b_0(t)$. Then, the phase perturbation can be expressed as:

$$\Delta\phi(t) = \sum_k \Phi_k(t) e^{jk\omega_b t} \quad (11)$$

where each time-varying harmonic $\Phi_k(t)$ is a narrow-band stochastic process. Equations (9) and (11) show that the phase noise about each component at f_k is the result of the contributions of all the components Φ_k. In particular, the main contribution to phase noise about the pulled frequency $f_o' \equiv f_1$ is given by Φ_0. This is because the contributions of the components Φ_k for $k \neq 1$ are scaled by the factors $\left|X_1^k / X_1^1\right| \ll 1$. Then, the phase noise about f_o' can be approximated by the power spectral density of the process $\Phi_0(t)$, assumed to be stationary. The conversion between the interferer phase noise $\psi(t)$ and $\Phi_0(t)$ is determined by the harmonic components $\{B_k\}$ of $b_0(t)$. In particular, the dc component of this signal is:

$$B_0 = \frac{1}{T_b}\int_0^{T_b} b_0(t)dt = \frac{1}{T_b}\int_0^{T_b} \frac{dg[\phi_0(t)]}{d\phi}dt = \frac{1}{T_b}\int_{g(0)}^{g(T_b)} \frac{dg}{g} =$$
$$= \frac{1}{T_b}\log\frac{g(T_b)}{g(0)} = \frac{1}{T_b}\log\frac{\dot\phi_0(T_b)}{\dot\phi_0(0)} = 0 \quad (12)$$

where $g=d\phi_0/dt$ has been applied. As a consequence, there is no direct conversion from the interferer phase noise $\psi(t)$ to $\Phi_0(t)$. The rest of the coefficients B_i determine the noise

conversion between all the harmonic components f_k, represented in Fig. 2, as gathered from (10). In this preliminary work, the analysis has been limited to the coefficient B_1, translating equation (10) to the frequency domain to approach the phase noise about f_o' by:

$$\left\langle|\Phi_0(f)|^2\right\rangle \approx \left(\frac{\alpha}{\alpha + 4\pi^2\left(f^2 - f_b^2\right)}\right)^2 \left\langle|\psi(f)|^2\right\rangle + \frac{\left\langle|\varepsilon(f)|^2\right\rangle}{4\pi^2 f^2} \quad (13)$$

where $\alpha = 2|B_1|^2$ acts as a sensitivity coefficient that determines the influence of the interferer phase noise, f is the frequency offset from the carrier at f_o' and the second term is the oscillator's own noise. In the absence of an interferer α becomes zero and equation (13) agrees with the free-running oscillator phase noise. The introduction of the interferer provides $\alpha > 0$, increasing the level of the phase noise characteristic, since it adds a term proportional to the interferer phase noise. The interferer influence is most noticeable when its phase noise characteristic is higher than that of the free-running oscillator. In that case, equation (13) predicts that the phase noise characteristic about f_o' is pulled towards the interferer phase noise curve. This behavior has been verified in Fig. 3, where the measured phase noise spectrum and the one predicted by equation (13) (SAF) have been compared. The interferer is not represented. Instead, its measured phase noise from zero offset frequency is traced for comparison. As predicted by (13), the oscillator phase noise is pulled to that of the interferer. Equation (13) also demonstrates a resonance effect, with maximum phase noise at an offset frequency about f_b. Note that the singularity predicted by (13) at f_b is fully consistent with the presence of a steady-state spectral line at $f_2 = f_o' + f_b$. The noise resonances at frequency offsets kf_b for $k>1$ could be obtained by considering the corresponding coefficients B_k when translating equation (10) to the frequency domain. Measurements are superimposed with a very good agreement.
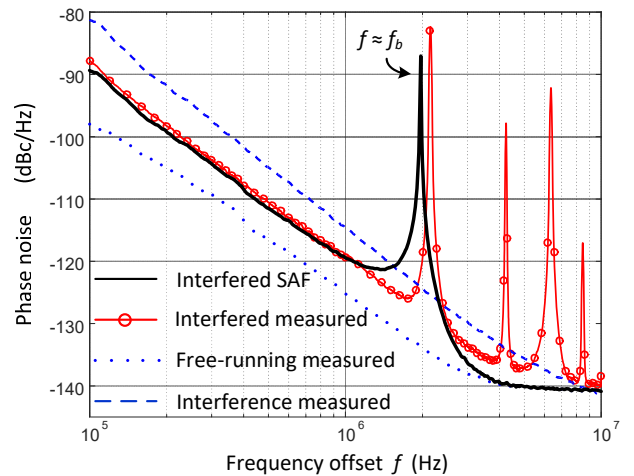

Fig. 3. Phase noise in the presence of an interferer with P_{in} = -29.6 dBm and $\Delta f \approx -2$ MHz. The phase noise characteristic about f_o' is pulled towards the interferer phase noise.

III. Conclusion

The influence of the phase noise of an interferer on free-running oscillator phase noise has been studied. A semi-analytical formulation has been derived, which leads to a simple equation that predicts the pulling effect of the interfered oscillator phase noise towards that of the interference signal. This prediction has been verified by measurements in a FET-based oscillator at 2.5 GHz in the presence of an interferer at 2 MHz offset.

Acknowledgement

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