Benchmark Challenge: a robust fractional order control autotuner for the Refrigeration Systems based on Vapor Compression

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Abstract: This paper proposes fractional order autotuner controller for the benchmark refrigeration system. The method is an extension of a previously presented autotuning principle and produces a robust fractional order PI controller to gain variations. Fractional order PI controllers are generalizations of the integer order PI controllers, which have a supplementary parameter that is usually used to enhance the robustness of the closed loop system. The method is not restricted to robustness to gain variations and can be adapted to obtain robust fractional order controllers to time delay or time constant variations, for example. The autotuning method presented in this paper has several advantages such as the need for a single sine test to be applied to the process to extract the necessary information and the elimination of complex nonlinear equations in the tuning procedure for fractional order controllers. The results obtained on the benchmark system indicate the method has high potential for real-life applications.

Keywords: benchmark challenge, fractional order controllers, autotuning method, robustness

1. INTRODUCTION

Using the emerging tools from fractional calculus and acknowledged success of fractional order controllers in practice (Vilanova and Visioli, 2012), this paper presents a solution for the control of the benchmark system proposed at the 3rd IFAC Conference on Advances in Proportional-Integral-Derivative Control (PID18), held in Ghent, Belgium. The originality of the approach is to use a simple test and detect automatically controller parameters that make the closed loop system robust to gain variations. The robustness can be addressed also in terms of time delay or time constants variations by modifying the tuning rules. There is no need to determine the process model and the complicated set of nonlinear equations involved in fractional order controller tuning is completely eliminated.

A few autotuning methods have been developed so far for fractional order controllers. In (Chen et al., 2004, Chen and Moore, 2005) the so-called phase shaper is designed, consisting in an integer order PID combined with a fractional order integrator or differentiator s^{α}, with $\alpha \epsilon (-1,1)$. The design of the phase shaper is based on the achieving closed loop robustness to gain variations. A relay test is used to tune fractional order controllers in (Monje et al., 2008). The design produces first a fractional order PI (FO-PI) controller, and then a fractional order PD controller with a filter. In this case, the performance specifications refer to iso-damping, gain crossover frequency and phase margin. The Ziegler-Nichols tuning procedure is used in (Yeroglu et al., 2009) to determine the proportional and integral gains of the fractional order controller, then the Åström-Hägglund method (Åström and Hägglund, 1984;2004) is used to compute the initial value of the derivative gain. The performance specifications refer to gain crossover frequency, phase margin and iso-damping property and an optimization procedure is required to solve the resulting nonlinear equations. In (De Keyser et al., 2016) the same three performance specifications are used in an autotuning procedure to determine either fractional order PI or PD controllers for stable, integer order or fractional order processes. A simple sine test is used to determine the required information for the tuning: the process magnitude, phase and phase slope at the gain crossover frequency. Using this information, either a graphical approach or an optimization routine is required to solve the resulting system of nonlinear equations.

In this paper, a previously designed autotuning method for integer order PID controllers (the KC autotuner) is extended to fractional order controllers and used to tune the controllers for the benchmark process. In the KC autotuning method (De Keyser et al., 2017a), a ‘forbidden region’ that includes the -1 point in the Nyquist plane is defined. To determine the forbidden region two design constraints referring to the phase and gain margins are used. The core idea of the KC autotuner is that the integer order PID parameters are computed such that the loop frequency response touches the border of the forbidden region. The extension of this KC autotuner to fractional order controllers (the FO-KC autotuner) ensures that a certain open loop gain crossover frequency, phase margin and iso-damping are obtained. In this case, the gain crossover frequency and phase margin specifications are used to compute the forbidden region. Finally, the optimal fractional order PI controller parameters are determined such that the slope-difference between the forbidden region border and the loop frequency response is minimal (i.e. tangent). A FO-PI controller is preferred instead of its integer order equivalent due to the improved closed loop dynamics achievable with the fractional order controllers (Monje et al., 2010).

2. THE FRACTIONAL ORDER (KC) AUTOTUNER

The original version of the (integer-order) KC autotuner has been presented in (De Keyser et al., 2017a). The main idea is to define a forbidden region, represented by a circle including the -1 point in the Nyquist plane. The center and radius of this circular region are computed according to two design constraints: the gain margin (*GM*) and the phase margin (*PM*). In the autotuning method, a (minimum) *GM*=2 and a (minimum) *PM*=45^o are selected. The robust PID controller is determined as the controller that makes the slope-difference between the circle border and the loop $L(s)=P(s)C(s)$ frequency response minimum, with $P(s)$ – the process transfer function and $C(s)$ – the controller transfer function. Fig. 1 shows the forbidden region and the loop frequency response in a general case.

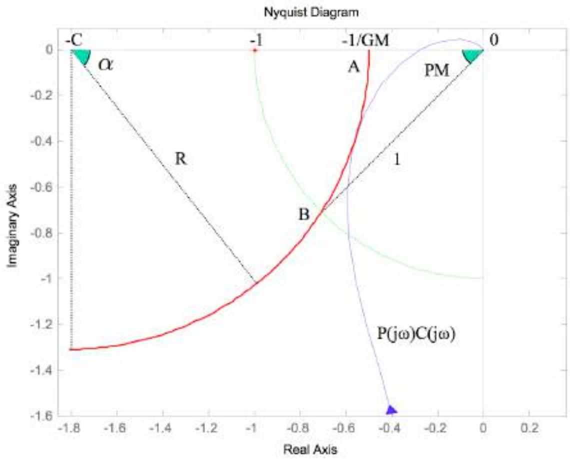

Figure 1. The loop frequency response (blue line) and the 'forbidden region' (red circle)

In the KC autotuning procedure, any user specified frequency $\bar{\omega}$ can be used used to tune the PID controller. In this case, the loop frequency response point $L(j\bar{\omega})=P(j\bar{\omega})\cdot C(j\bar{\omega})$ is defined as a point on the circular region border. Its derivative, as the slope of the loop frequency response, needs to be evaluated as well. For this, the process frequency response, $P(j\bar{\omega})$ and its derivative, have to be estimated via a simple sine test (De Keyser et al., 2017b).

The main idea of the KC autotuner is to minimize the difference between the slope of the loop frequency reponse and the slope of the region border at the user specified frequency $\bar{\omega}$. Obviously, as indicated in Fig. 1, the slope of the region border depends on the angle α. Then, the minimization problem can be simply solved with a single *for*-loop where α varies from 0 to α_{max} in 1° steps. The set of equations for determining the point of tangent between the forbidden region and the Nyquist curve is given in (De Keyser et al., 2017a).

The FO-KC approach attempts to determine the parameters of the fractional order controllers. In this paper, fractional order PI controllers will be designed, described by the following transfer function:

$$C_{PI}(s) = k_p\left(1 + k_i s^{-\lambda}\right) \tag{1}$$

where k_p and k_i are the proportional and integral gains and λ is the fractional order, with $\lambda_{min} < \lambda < 2$. Based on the iso-damping property and certain phase margin *PM* requirement, the forbidden region center, *C*, and its radius, *R*, are determined. Figure 2 illustrates the forbidden region in the case of the FO-KC autotuner.

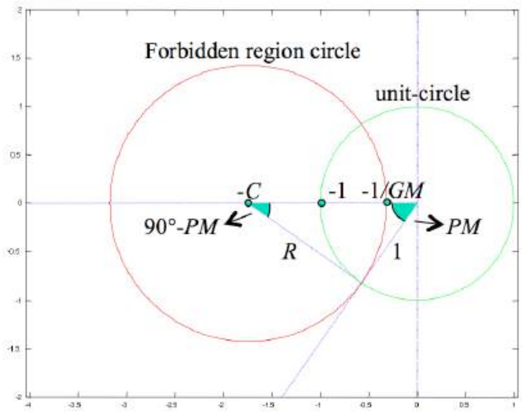

Figure 2. Computation of the forbidden region center and radius for the FO-KC autotuner

Using some trigonometric relations in Fig. 2, the following results are obtained:

$$C = 1\Big/\cos\left(PM\right)\ ,\quad R = \sqrt{C^2 - 1} \tag{2}$$

$$\alpha = 90^o - PM \tag{3}$$

and the optimization angle α of the KC autotuner is now fixed. The design of the fractional order PI controller using the FO-KC method is defined as a minimization problem with the following *flowchart steps*.

1. For a set of performance specifications referring to phase margin (*PM* and iso-damping, compute the forbidden region circle and radius using (2).
2. Compute the slope of the forbidden region $\frac{d\,Im}{d\,Re}$ based on specified *PM*.
3. Perform a single sine test on the process with frequency equal to the gain crossover frequency ω_c, defined as a third performance specification, and determine the process frequency response $P(j\omega_c)$ and

the slope of its frequency response $\left.\frac{dP(j\omega)}{d\omega}\right|_{\omega=\omega_c}$ (see for details (De Keyser et al., 2017b).

4. Using the performance specifications determine the open loop frequency response $L(j\omega_c)$ at the gain crossover frequency, considering the modulus $M_L(j\omega_c)=1$ and phase $\varphi_L=-180^o+PM$.

The frequency response of the fractional order PI controller can be easily computed:

$$C(j\omega_c)=\frac{L(j\omega_c)}{P(j\omega_c)}=a+jb. \tag{4}$$

The following relations hold:

$$a=k_p\left(1+k_i\omega^{-\lambda}cos\frac{\lambda\pi}{2}\right) \text{ and } b=-k_pk_i\omega^{-\lambda}sin\frac{\lambda\pi}{2}. \tag{5}$$

From (5), the controller parameters follow as:

$$k_i=-\frac{b}{\omega^{-\lambda}x} \text{ and } k_p=\frac{1}{sin\frac{\lambda\pi}{2}x} \tag{6}$$

with $x=a\,sin\frac{\lambda\pi}{2}+b\,cos\frac{\lambda\pi}{2}$. For a and b known, the controller parameters in (6) depend solely on λ.

5. Then, for different values of λ, in small increments in the range $\lambda_{min}<\lambda<2$, the gains k_p and k_i can be computed based on (6). The minimum value λ_{min} is computed according to (Muresan et al., 2017).

Once the controller parameters are determined, compute the slope of the fractional order PI controller frequency response $\left.\frac{dC(j\omega)}{d\omega}\right|_{\omega=\omega_c}$, either analytically or numerically. Compute the derivative of the open loop frequency response at the gain crossover frequency:

$$\left.\frac{dL(j\omega)}{d\omega}\right|_{\omega=\omega_c}=P\left(j\omega_c\right)\left.\frac{dC(j\omega)}{d\omega}\right|_{\omega=\omega_c}+C\left(j\omega_c\right)\left.\frac{dP(j\omega)}{d\omega}\right|_{\omega=\omega_c} \tag{7}$$

and determine the real and imaginary parts:

$$\left.\frac{dL\left(j\omega\right)}{d\omega}\right|_{\omega=\omega_c}=\left.\frac{d\Re_L}{d\omega}\right|_{\omega=\omega_c}+j\left.\frac{d\Im_L}{d\omega}\right|_{\omega=\omega_c}. \tag{8}$$

Next, the slope of the loop frequency response in the Nyquist plane can be computed as the ratio: $\left.\frac{d\Im_L}{d\Re_L}\right|_{\omega=\omega_c}$.

6. Compute $\left\|\frac{d\,Im}{d\,Re}-\left.\frac{d\Im_L}{d\Re_L}\right|_{\omega_c}\right\|$ for all values of the fractional order λ, in small increments in the range $\lambda_{min}<\lambda<2$ and select the minimum value. This then results in the optimal FO-PI controller parameters.

4. APPLICATION TO THE BENCHMARK SYSTEM

The benchmark system is described in detail in (Bejarano *et al*, 2017). The schematic of the process is given in Fig. 3, where the manipulated variables are the compressor speed N and the expansion valve opening A_v, respectively the controlled signals are the outlet temperature of the evaporator secondary flux $T_{e,sec,out}$ and the degree of superheating T_{SH}. The remaining signals, such as mass flow $\dot{m}_{e,sec}$ and inlet temperature $T_{e,sec,in}$ of the evaporator secondary flux, mass flow $\dot{m}_{c,sec}$ and the inlet temperature $T_{c,sec,in}$ of the condenser secondary flux are considered as disturbances. Additional disturbances are represented by the inlet pressure of the condenser secondary flux $P_{c,sec,in}$, inlet pressure of the evaporator secondary flux $P_{e,sec,in}$ and compressor surroundings temperature T_{surr}. This 2x2 system has been identified by applying Pseudo Random Binary Signals (PRBS) to one input of the system, while keeping the other inputs constant at the initial operating point, namely 48.79% expansion valve opening and 36.45Hz compressor speed. The 2x2 system is defined by $u_1=N$, $u_2=A_v$, $y_1=T_{e,sec,out}$ and $y_2=T_{SH}$ with the identified transfer functions as:

$$G_{11}(s)=\frac{-0.5658s-0.01116}{s^2+14.87s+0.5584} \tag{9}$$

$$G_{12}(s)=\frac{-0.006655s-0.0005089}{s^2+9.931s+0.2901} \tag{10}$$

$$G_{21}(s)=\frac{-2.821s-0.8192}{s^2+6.576s+0.2569} \tag{11}$$

$$G_{22}(s)=\frac{0.9626s+0.02772}{s^2+5.928s+0.184}. \tag{12}$$

We propose to steady state decouple the 2x2 system, using the inverse of the steady state matrix:

$$D=\begin{pmatrix} -17.5 & -0.204 \\ -370.6 & 2.32 \end{pmatrix}. \tag{13}$$

This implies that the diagonal elements of the decoupled process are approximated with:

$$G_{d11}(s)=\frac{12.06s^2+0.7963s+0.01434}{s^3+13.23s^2+0.8726s+0.01434} \tag{14}$$

$$G_{d22}(s)=\frac{2.81s+0.2395}{s^2+6.055s+0.2395} \tag{15}$$

approximated with a fit of 99.7% and 99.5%, respectively (in absence of noise). For these two transfer functions, two fractional order PI (FO-PI) controllers are designed such as to ensure iso-damping, a phase margin of PM=70° in both cases and a gain crossover frequency ω_{c1}=5 rad/s and ω_{c2}=20 rad/s, respectively. An increased value for the PM specification ensures a low overshoot and a stable system. The gain crossover frequencies have been selected similarly to the decentralized benchmark controller (Bejarano et al., 2017) and then gradually increased for a faster settling time.

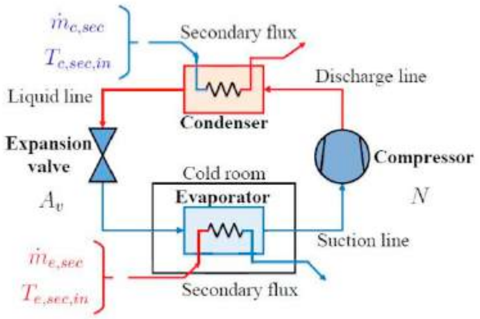

Figure 3. Refrigeration system based on vapor compression

The forbidden region center and radius are computed as:

$$C = \frac{1}{cos(PM)} = 2.92 \quad , R = \sqrt{C^2 - 1} = 2.74 \tag{16}$$

and the angle α is:

$$\alpha = 90^o - PM = 20^o . \tag{17}$$

The slope of the forbidden region is given by:

$$\frac{d\,Im}{d\,Re} = -\frac{Re(\alpha) + C}{Im(\alpha)} = \frac{cos(\alpha)}{sin(\alpha)} = 2.74 . \tag{18}$$

For the first loop, the sine test with frequency equal to ω_{c1}= 5 rad/s, yields the process frequency response and the slope of the process frequency response:

$$G_{d11}(j\omega_{c1}) = 0.8- 0.3\cdot j \tag{19}$$

$$\left.\frac{dG_{d11}(j\omega)}{d\omega}\right|_{\omega=\omega_{c1}} = -0.04 -0.045j . \tag{20}$$

The open loop frequency response is:

$$L(j\omega_{c1}) = -0.342 - 0.939j . \tag{21}$$

And the frequency response of the FO- PI controller is:

$$C(j\omega_{c1}) = 0.016 - 1.167j \tag{22}$$

with a=0.016 and b=-1.167. The minimum value for the fractional order is computed according to previous research results (Muresan et al., 2017):

$$\lambda_{min} = 0.99 . \tag{23}$$

Now all ingredients are present for the tuning procedure at *step 5* in the flowchart. For each λ in the range $\lambda_{min} < \lambda < 2$, the FO- PI proportional and integral gains are computed based on (6), then the $\left.\frac{dC(j\omega)}{d\omega}\right|_{\omega=\omega_c}$ is estimated, allowing for the final computation of the derivative of the open loop frequency response $\left.\frac{dL(j\omega)}{d\omega}\right|_{\omega=\omega_c}$ and its slope $\left.\frac{d\Im_L}{d\Re_L}\right|_{\omega=\omega_c}$. The method searches through all values of the fractional order λ and computes the difference $\left\| \frac{d\,Im}{d\,Re} - \left.\frac{d\Im_L}{d\Re_L}\right|_{\omega_c} \right\|$ with $\frac{d\,Im}{d\,Re}$=2.74, as computed in (18). The minimum difference is obtained for a fractional order λ=1.1711, yielding a proportional gain k_p=0.34 and an integral gain k_i=23.6.

For the second loop, the sine test with frequency equal to ω_{c2}= 20 rad/s, yields the process frequency response and the slope of the process frequency response:

$$G_{d22}(j\omega_{c2}) = 0.038-0.0129\cdot j \tag{24}$$

$$\left.\frac{dG_{d22}(j\omega)}{d\omega}\right|_{\omega=\omega_{c2}} = -0.0035 - 0.0053 \cdot j . \tag{25}$$

The open loop frequency response remains the same, as for the first loop, since the same performance requirements are addressed:

$$L(j\omega_{c2}) = -0.342 - 0.939j . \tag{26}$$

The frequency response of the FO- PI controller for the second loop is:

$$C(j\omega_{c2}) = 5.96 - 4.43 \cdot j \tag{27}$$

with a=5.96 and b=-4.43. The minimum value for the fractional order is:

$$\lambda_{min} = 0.4067 . \tag{28}$$

A minimum value for the slope difference $\left\| \frac{d\,Im}{d\,Re} - \left.\frac{d\Im_L}{d\Re_L}\right|_{\omega_c} \right\|$ is obtained for a fractional order λ=0.7767, with the proportional gain k_p=4.34 and the integral gain k_i=11.13.

These FO-PI controllers are now approximated in an integer order form of order 5, for the purpose of simulation in closed loop (De Keyser et al., 2018). Their equivalent forms are tested for reference tracking and disturbance rejection. The output signals are represented in Fig. 4, and the corresponding input signals are shown in Fig. 5. There is a slight overshoot and a relatively fast settling time, combined with a decreased interaction between the two control loops. The input signals reach acceptable values. The associated condenser and evaporator pressures are pictured in Fig.6, while Fig. 7 shows the thermal power at each component and refrigerant mass flow. The compressor efficiency and coefficient of performance are presented in Fig. 8.

A qualitative comparison with the benchmark controller (Bejarano et al., 2017) is presented in Figures 9-10, where Controller 1 stands for the multivariable PID benchmark controller and Controller 2 stands for the proposed control strategy. It is obvious that the designed controller provides better closed loop performance than the benchmark controller, reducing significantly the interaction between the control loops. The overshoot and the settling time are slightly larger. The quantitative comparison between the two control strategies is performed in terms of ratios of Integrated Absolute Error (*RIAE*), taking into account that both output signals should follow their respective references. The Ratio of Integrated Time multiplied Absolute Error (*RITAE*) is evaluated for the first controlled variable, taking into account that the simulation includes one sudden change in its reference. The other indices are the Ratios of Integrated Time multiplied Absolute Error (*RITAE*) for the second controlled variable, taking into account that the simulation includes three sudden changes in its reference, and the Ratios of Integrated Absolute Variation of Control signal (*RIAVU*) for the two manipulated variables. The results are included in Table 1. A combined index is also computed as the mean value of the eight individual indices using a weighting factor for each index. The quantitative results in Table I show that the proposed control strategy is overall a better choice than the multivariable PID benchmark controller.

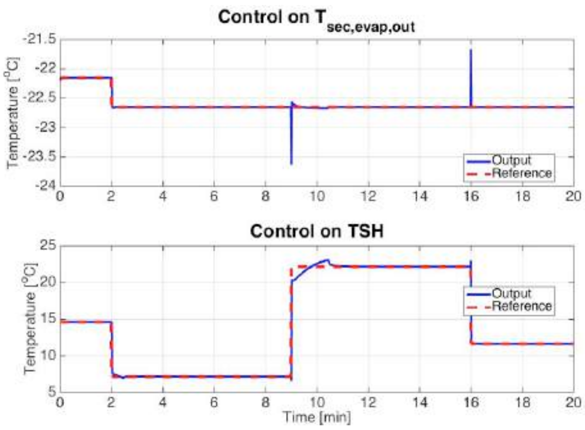

Figure 4. Closed loop response of the two output signals

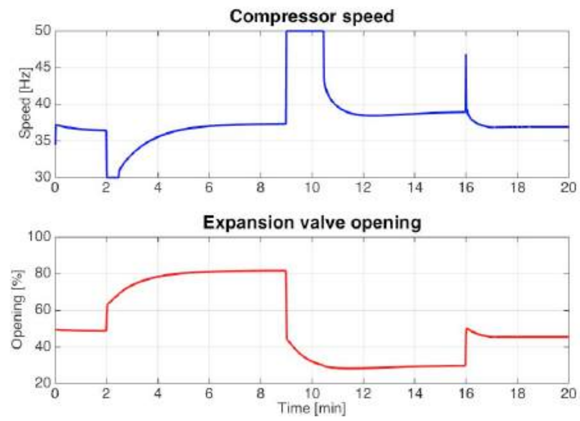

Figure 5. Corresponding input signals in closed loop

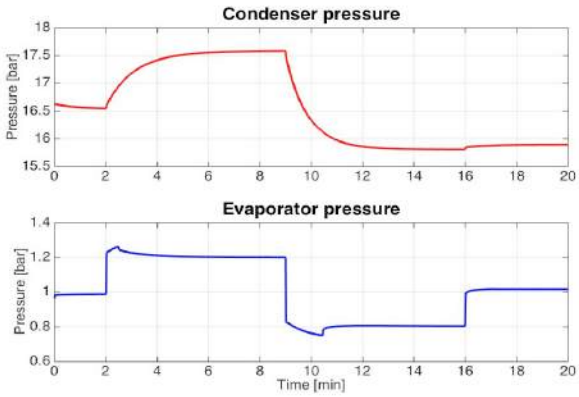

Figure 6. Corresponding condenser and evaporator pressures

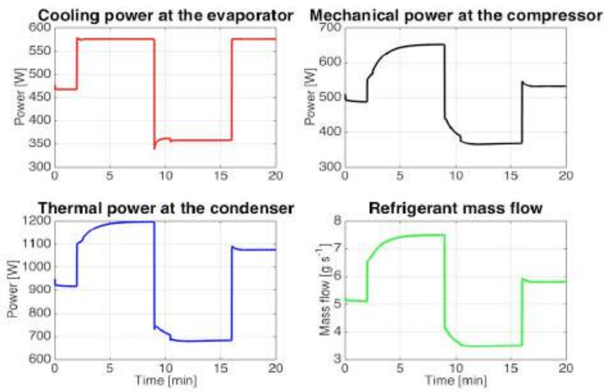

Figure 7. Thermal power at each component and refrigerant mass flow

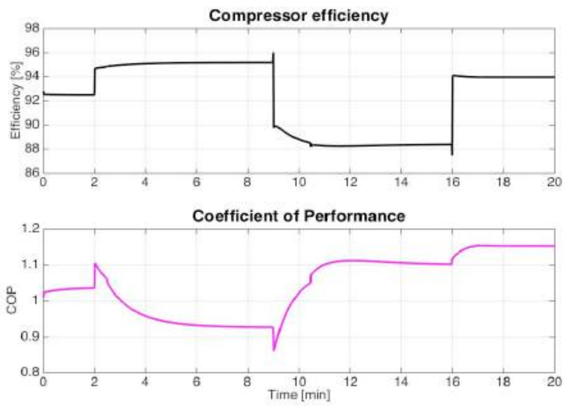

Figure 8. Compressor efficiency and Coefficient of Performance: comparison between controllers

4. CONCLUSIONS

The quantitative results in Table 1 suggest that the proposed control strategy (Controller 2) improves the closed loop performance compared to the multivariable PID benchmark controller (Controller 1). Overall the combined index *J* shows that the proposed control strategy is a better choice for controlling the refrigeration system based on vapor compression. A quantitative comparison has been performed with the decentralized benchmark controller, with J=0.2782, thus the proposed control strategy is a valuable option for controlling the benchmark process. The improved results are a direct consequence of the optimal tuning of a robust FO-PI controller using the FO-KC autotuning method.

Index	Value
$RIAE_1(C_2,C_1)$	0.3609
$RIAE_2(C_2,C_1)$	0.5935
$RITAE_1(C_2,C_1,t_{c1},t_{s1})$	0.0270
$RITAE_2(C_2,C_1,t_{c2},t_{s2})$	2.8161
$RITAE_2(C_2,C_1,t_{c3},t_{s3})$	1.6339
$RITAE_2(C_2,C_1,t_{c4},t_{s4})$	0.4625
$RIAVU_1(C_2,C_1)$	0.9270
$RIAVU_2(C_2,C_1)$	0.6868
$J(C_2,C_1)$	0.7837

Table 1. Quantitative comparison of the two controllers (C_1- Controller 1, C_2- Controller 2)

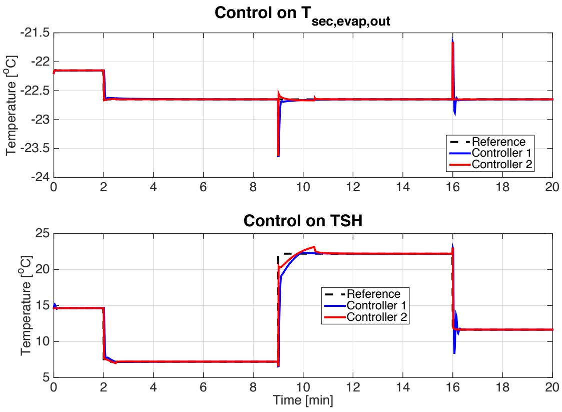

Figure 9. Closed loop response of the two output signals: qualitative comparison

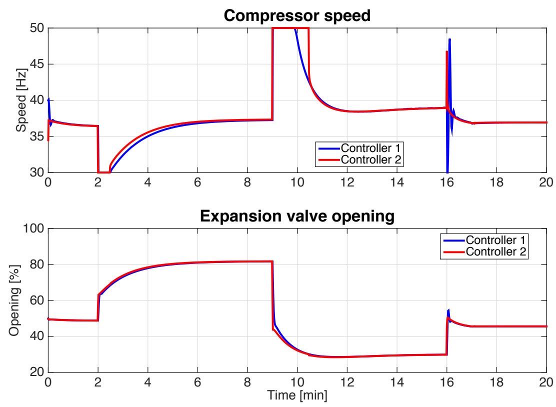

Figure 10. Corresponding input signals in closed loop: qualitative comparison

ACKNOWLEDGMENT

This work was supported by a grant of the Romanian National Authority for Scientific Research and Innovation, CNCS – UEFISCDI, project number PN-III-P1-1.1-TE-2016-1396.

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