Optimization-based analysis of last-mile one-way mobility sharing

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by

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Abstract

One-way carsharing is becoming increasingly popular because it is more convenient for users than conventional round-trip carsharing. However, a one-way service suffers from a mismatch between user demand and the distribution of vehicles (supply). A number of methods have been studied in attempts to resolve this issue. In particular, "last-mile mobility", a type of one-way carsharing providing short trips between stations at public transport hubs and origins/destinations in local areas, reserves destination parking spaces in addition to vehicles at origins, and it needs to control both resources carefully.

Here, we develop a mixed integer linear programming (MILP) model of an existing short-trip last-mile mobility service (Ha:mo RIDE) in Japan and optimize the vehicle distribution and relocations to maximize the operator's profit. The optimal solution indicates that the operator should reduce relocation disproportionately because of the low revenue per short trip. However, a low satisfied demand rate would result in customer defection and lead to a long-term loss. As solution, we propose autonomous relocation by low-speed automated driving and show that slow but frequent relocations would yield both a benefit to the user (high satisfied demand) and profit for the operator as well as a virtual increase in parking capacity for last-mile mobility operations with many small stations.

1 Introduction

The Japanese government is promoting "Society 5.0" [1], a "super smart" society as a successor to hunter-gatherer, agricultural, industrial, and information societies that came before it. Society 5.0 is described in the 5th Science and Technology Basic Plan [2]; one of its pillars is MaaS, mobility as a service, such as carsharing and ridesharing.

Carsharing has become a popular means of transportation in the 21st century. For instance, Zipcar, founded in 2000 in the US, has expanded to Canada and Europe and provided services to more than 1 million members in 2017. Members can find and reserve vehicles with a smartphone app and get access to vehicles simply by using their member's cards as entry badges. In 2007, car2go started as a free-floating one-way carsharing experiment in Ulm, Germany, and it is now the world's largest carsharing company

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with operations all over the globe. Car2go has a convenient one-way service which allows users to return vehicles anywhere within the service area, while a round-trip service requires vehicles to be returned their original points. Autolib', initiated in Paris in 2011, is a one-way carsharing scheme that provides electric vehicles (EVs). Autolib' is a non-floating (station-base) system and EVs have to be returned to Autolib' stations to be recharged.

Station-base one-way carsharing using ultra-compact EVs started in Japan in 2012. Yokohama city and Nissan launched "Choimobi Yokohama", a two-person EV sharing service, whose main objectives are to encourage low-carbon emission transport and promote tourism. One-way service was available for the first two years, and round-trip service is currently available. In the same year, Toyota launched "Ha:mo RIDE" [3], a new type of mobility service for last-mile transport. Providing one-way sharing service with ultra-compact EVs such as i-ROAD [4] and COMS [5], Ha:mo is intended to be a stress-free and eco-friendly "Harmonious Mobility Network" that utilizes both personal and public transport. Since its launch in Toyota city, Ha:mo RIDE has expanded to Tokyo, Okinawa, and Okayama in Japan, Grenoble in France, and Chulalongkorn University in Thailand.

One-way carsharing provides more flexibility for users, but has an issue of imbalanced user demand and vehicle supply. To increase satisfied demand, many carsharing operators hire personnel to relocate vehicles from lower demand areas to higher demand areas in a time dependent manner, but the labor cost of the relocation staff puts pressure on their profitability. Another approach to ease the imbalance is adaptive pricing, which controls user demand by placing premiums on trips from high-demand areas (or giving discounts for trips to high-demand areas). There are several studies on adaptive pricing for on-demand mobility, such as that of Drwal, Gerding, Stein, Hayakawa, and Kitaoka [6]. Ridesharing services such as Uber and Lyft use dynamic pricing to control both demand and supply [7]. To increase both the number of demands served and revenue, companies have placed premiums of attracting drivers (increasing supply) as well as discouraging potential users (decreasing demand). By allying with car makers, ridesharing companies are developing self-driving vehicles [8] to maximize vehicle utilization.

In this paper, we focus on last-mile mobility, one-way carsharing designated for short trips within a city, and develop an optimization model for making optimal operation decisions. The next section 2 reviews previous research related to the one-way carsharing optimization. Section 3 explains the characteristics of last-mile mobility and develops an optimization model for Ha:mo RIDE. Section 4 describes our application and the developed model for Ha:mo RIDE in Toyota city (Ha:mo RIDE Toyota) and it compares the optimal solution with the actual operation. It also describes two case studies involving modifications to the model: (1) a "premium service" guaranteeing reservations in return for price premiums and (2) autonomous relocation by slow-speed automated self-driving. The last section 5 provides conclusions and ideas for future research.

2 Recent research on optimization-based analysis of carsharing

There is a growing body of research on optimization-based analysis of carsharing and other kinds of mobility sharing. Jorge and Correia [9] reviewed the history of carsharing, from its beginnings as a cooperative initiative to recent one-way services and related research. Boyaci, Zografos, and Geroliminis [10] classified models related to planning and operation of carsharing systems into two categories: (i) models addressing strategic planning decisions and (ii) models supporting operational decisions. In order to maximize performance metrics such as the number of uses, strategic planning decides the locations and sizes of stations and the number of vehicles assigned to each station, while operational decisions relocate the vehicles among the stations.

Correia and Antunes [11] presented an optimization approach for determining the location of stations to maximize one-way carsharing operator's profit including revenues from users and costs of maintaining vehicles, parking space, and relocating vehicles. The interest of the study was strategic decision making, and neither detailed relocation operations nor constraints on maximum parking space per station or the total number of vehicles was considered. Later, Jorge and Correia [12] incorporated the relocation operation into the optimization models for maximizing profit and examined relocation policies in simulations. They found that a dynamic (frequent) relocation policy significantly improved profitability.

Correia, Jorge, and Antunes [13] introduced a notion of user flexibility by offering a choice of disembarking at the second or third closest stations. When exploited in combination with real-time information about vehicle availability, such user flexibility increases the number of demands served and raises the operator's profit. Although this notion reflects the situation of actual carsharing services that provide information on nearby vehicles, an actual user's willingness to use alternative stations can only be supposed until he or she states it.

Kek, Cheu, Meng, and Fung [14] proposed a three-phase optimization-trend-simulation (OTS) decision support system for the staff-based vehicle relocation problem. Phase one of the OTS *Optimizer* returns the lowest-cost resource allocation, phase two *Trend Filter* filters the output through a series of heuristics, and phase three *Simulator* evaluates the performance of the result. In the optimization, the problem is modeled as an MILP on a time-space network in which nodes represent staff behaviors. It was found that an increase in variables such as number of stations and time steps increased the computational complexity of the model, and no effective solution was presented in their paper.

Boyaci, Zografos, and Geroliminis [15] proposed an integrated multi-objective mixed integer linear programming (MMILP) optimization and discrete event simulation framework for vehicle and personnel relocation in a one-way carsharing system with reservations. By clustering stations, the number of variables is reduced and the computational complexity of the optimization decreases as result. The optimization model was developed on a time-augmented network, using time intervals (15 minutes) during which each vehicle and each personnel had only one status. This network contributes to a clear formulation including the flow conservations of the vehicles and relocation staff. Our optimization modeling for last-mile mobility in section 3.2 uses the same network structure.

As shown above, there has been research pursuing optimal vehicle distributions and relocation operations for one-way carsharing. However, the developed models are inappropriate for last-mile mobility because of the characteristics described in section 3.1.

3 Framework and modeling

3.1 Last-mile mobility sharing

Last-mile mobility sharing has different aspects from the usual one-way carsharing. Here, taking Ha:mo RIDE as an example, we describe the characteristics of last-mile mobility. To be used as last-mile transport, vehicles are parked at stations close to public transport stations, bus stops, and other points of interest such as shopping malls, office, and homes (Fig.1). In last-mile mobility, users typically travel from the origin to the destination without taking any detours, as in the case of other kinds of last-mile transportation such as bus or taxi. Round-trips account for only a few rides, and most last-mile one-way trips end in a shorter time than other kinds of one-way carsharing. The actual data on use of Ha:mo RIDE Toyota clearly reflects these features.

A characteristic of last-mile mobility is that users reserve parking spaces at destinations as well as

Fig. 1: Last-mile mobility network - Ha:mo RIDE [3]

vehicles at origins. Assigning parking spaces before departure allows users to end their trips on arrival, just as other last-mile transport such as bus and taxi do. This feature is necessary for last-mile mobility: while it is frustrating if it takes 10 minutes to find a parking space after driving for 1 hour, it is nearly intolerable if it takes 10 minutes to park after driving for only10 minutes. On the other hand, users cannot make reservations if they cannot find available parking spaces at the destinations, even if they find vehicles at the origin station (Fig.2). In last-mile mobility with parking assignments, both vehicles and parking spaces need to be managed more carefully than in carsharing without parking assignments in order to meet demand.

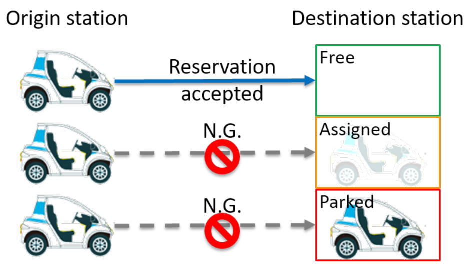
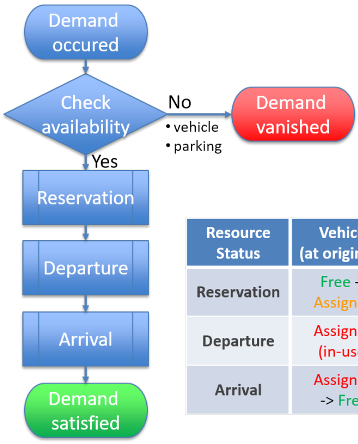


Fig. 2: No parking, no reservation. Users reserve parking spaces as well as vehicles. Users don't need to look for available parking spaces when they arrive, but parking spaces have to be available when they make reservations.

Ha:mo RIDE is operated by local operators in each area a under common service procedure as follows. First, users reserve vehicles with smartphone apps or through a web page by registering both origin and destination stations. If there is no "free" (available) vehicle at the origin or no "free" parking space at the destination, the applications fail (vanish). Once made, reservations are valid for 20 to 30 minutes depending on each service, and both vehicles and parking spaces become "assigned", i.e., not available to other users. When a registered user depart from an origin station, a parking space becomes "free", i.e., available to other users and vehicles. When the user arrives at the destination station, the driven vehicle becomes "free" again. Fig. 3 shows this process together with the changes in state of the resources.

To avoid imbalances between supply and demand and at the same time increase the number of uses, operators set vehicle distribution targets (relocation thresholds at each station) based on use-history



Resource Status	Vehicle (at origin st.)	Origin Station	Destination Station
Reservation	Free -> Assigned	Parked	Free -> Assigned
Departure	Assigned (in-use)	Parked -> Free	Assigned
Arrival	Assigned -> Free	Free	Assigned -> Parked

Fig. 3: Service procedure and changes in resource status. Note that optimization modeling assumes that users depart the origin station immediately after making a reservation; i.e., the resource state transitions reflecting reservation and departure occur at the same time.

analyses and experience. In practice though, actual operations depend on the on-site relocation staff. Despite the operators' efforts and costs, there still remains customer dissatisfaction due to imbalances, which means lost business and if, not corrected, long-term reductions in the number of customers. In the next section 3.2, we develop a mixed integer linear programming (MILP) model that optimizes the vehicle distribution and relocations to maximize the operator's profit.

3.2 Optimization modeling

Our optimization model is based on the idea described in Boyaci et al. [15], but it is more restrictive than theirs [15]. In particular, it puts specific assumptions on last-mile mobility sharing, as follows.

1. Last-mile mobility is modeled on a time-augmented network, which consists of stations and time intervals. Note that each time interval is 5 minutes, shorter than in previous research (e.g., 15 minutes), because last-mile mobility is used for relatively short trips.
2. The demand for the whole day is given at once, though actual reservations are made 30 minutes before rides. There is no time gap between reservation and departure, and there are no cancellations.
3. For a trip to occur, there have to be vehicles at the origin and parking spaces at the destination at the time of departure.
4. Travel time is the trip duration plus a constant period, assuming one-way short trips without making any detours. The constant period is thus a margin for departure and arrival that includes the time spent getting into and out of the vehicle and getting the vehicle ready to move. The travel time of moving staff and relocating vehicles is calculated in the same way.
5. The trip duration is expressed in terms of parameters depending on the traffic conditions as well as the distance between stations. Travel times between the same stations at different times of the day can be different.
6. Every trip or relocation takes at least 1 time interval.
7. Each node of the network is represented as (j, t), where j identifies the station and t the time interval, is a state including the number of parked vehicles, number of assigned parking spaces,

and number of relocation staff.
8. In each time interval, each vehicle can have only one status: “free” or “assigned”. “Assigned” includes rides of users and relocation of staff.
9. In each time interval, each parking space can have only one status: “free” (available) or “assigned” or “parked” (both unavailable).
10. In each time interval, each relocation staff can have only one status: “idle”, “moving”, or “relocating”.
11. Relocation staff work for one shift, and each shift has a beginning time and a finishing time. The manager must work a specific shift (the shift in dark green in Fig. 4), while other staff work when relocation becomes necessary.

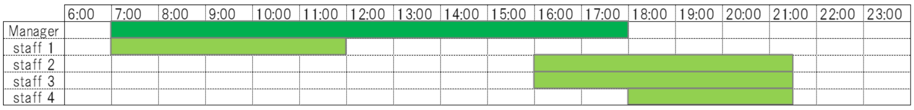


Fig. 4: Staff shift

12. The relocation staff move on a staff vehicle as a team (a two-man team), and they relocate vehicles by one driving the vehicle that was used and the other driving the staff vehicle (Fig.5). Previous research did not clearly define how the staff moves or otherwise assumed they did so by bicycle over a long distance. Instead, we followed the actual operation procedure of Ha:mo RIDE Toyota [*1].

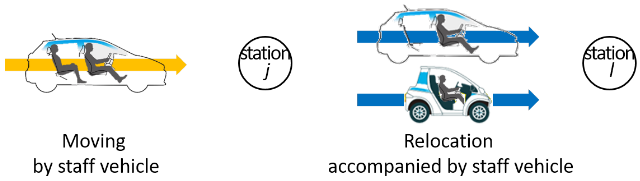


Fig. 5: Moving and relocating using staff vehicle

13. The state of charge (SOC) of the vehicle’ s battery is not considered. Last-mile mobility is mostly intended for one-way short trips, and electricity consumption is low enough for the battery to be recharged during time between one trip and the next. In the Ha:mo RIDE Toyota case study, we ran simulations on a simulator)updated by Shimazaki, Kuwahara, Yoshioka, Homma, Yamada, and Matsui [16]) and found that no vehicles became unavailable because of a low SOC.
14. The number of vehicles and the number, capacity (number of parking spaces), and locations of stations are given as parameters.

Assumptions 2, 3, 4, 5, 9, 12, 13, 14 are different from previous research.

Fig. 6 shows the time-augmented network and arcs representing trips and relocations.

- The arcs represent movements of vehicles between nodes. There are four vehicle movements, three for trips and one for relocation: three vehicles driven by users from node $(2, 2)$ to (j, t) (from

[*1] Staff walk between neighboring stations within walking distance, but for the sake of simplicity, we assume that they walk between all stations.

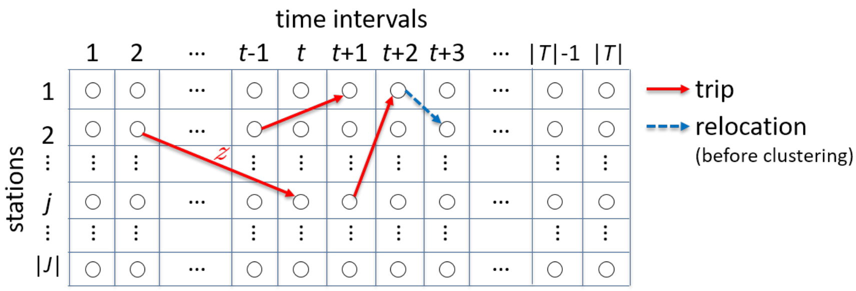


Fig. 6: Time-augmented network and arcs representing vehicle movements

station 2 at time interval 2 to station j at time interval t), from $(2, t-1)$ to $(1, t+1)$, from $(j, t+1)$ to $(1, t+2)$, and one vehicle driven by relocation staff from $(1, t+2)$ to $(2, t+3)$.

- Because each vehicle can have only one status during each time interval, a vehicle starting at station j in time interval t must be ready by the end of the previous time interval $t-1$. Moreover, a vehicle arriving at station j in time interval t must be ready starting from the next time interval $t+1$. The same is true for parking spaces and relocation staff.
- Parking spaces must be assigned to each trip from the beginning of the trip. For example, for a trip from $(2, 2)$ to (j, t), a parking space at station j must be assigned from the beginning of time interval 2.
- Relocation must be accompanied by relocation staff, obviously because relocating vehicles are driven by staff. During the relocation from $(1, t+2)$ to $(2, t+3)$, the relocation staff move just the same as the vehicle.
- Besides using the staff vehicle for relocation, staff also move between stations in it, as described in section 3.2. The operation is limited to serial relocation from a station where the staff start working when only one member is working. For example, after relocation to $(2, t+3)$, the staff cannot move from station 2 without the staff vehicle.

For staff-moving and vehicle-relocating, stations are clustered in order to decrease the number of variables and reduce computational complexity. Fig. 7 shows the moving and relocating arcs on the time-augmented network with clusters of stations.

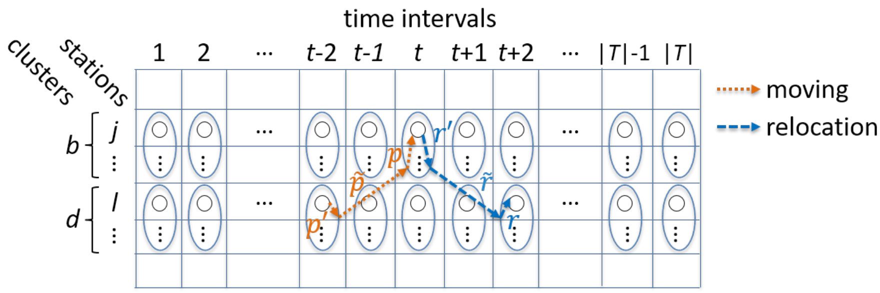


Fig. 7: Time-augmented network with clusters of stations and moving and relocating arcs

- The moving and relocating operations are divided into three parts. Taking relocation from node (j, t) to $(l, t+2)$ as an example, part (i) is from origin station j to the origin station's cluster b in time interval t, part (ii) is from cluster b at time interval t to the destination station's cluster d in

time interval $t+2$, while part (iii) is from cluster d to destination station l in time interval $t+2$.
- Stations are clustered by using an algorithm based on Boyaci et al. [15]. The station clustering algorithm is described in section 3.3.

3.2.1 Sets and indices

- $i \in \boldsymbol{I}$: trips
- $j, l \in \boldsymbol{J}$: stations
- $t \in \boldsymbol{T}$: time interval
- $s \in \boldsymbol{S}$: relocation staff shift
- $b, d \in \boldsymbol{B}$: station clusters
- $\boldsymbol{J}_b$: set of stations in cluster b, i.e. $\boldsymbol{J} = \bigcup\limits_{b \in \boldsymbol{B}} \boldsymbol{J}_b$

3.2.2 Parameters

- $\mathrm{begin}_s, \mathrm{fin}_s \in \boldsymbol{T}$: beginning and finishing time intervals of shift s
- $\mathrm{start}_i, \mathrm{end}_i \in \boldsymbol{T}$: start and end time intervals of trip i
- $\mathrm{origin}_i, \mathrm{dest}_i \in \boldsymbol{J}$: origin and destination stations of trip i
- $\mathrm{travel}_{tbd} \in \mathbb{N}$: travel time starting at time interval t from cluster b to cluster d
- $\mathrm{t.end}_{tbd} \subset \boldsymbol{T}$: set of departing time intervals of driving or relocating from cluster b to cluster d arriving during time interval t [*2]
- P_i: price for trip i
- PC_s: personnel cost of relocation staff for shift s
- $\tilde{\mathrm{RC}}_{bd}$: relocating cost from cluster b to cluster d (fuel cost)
- $\tilde{\mathrm{MC}}_{bd}$: moving cost from cluster b to cluster d (fuel cost of staff vehicle)
- Lcost: maximum acceptable labor cost per day in the service
- NV: maximum number of vehicles available in the service
- cap_j: number of parking spaces (capacity) at station j
- NW: total number of relocation staff able to work in the service

3.2.3 Variables

The decision variables are as follows.

- $z_i \in \{0, 1\},\ \forall i \in \boldsymbol{I}$: binary variables, if trip i is served 1, otherwise 0.
- $v_s \in \mathbb{Z}^+,\ \forall s \in \boldsymbol{S}$: number of staff in shift s.
- $n_{jt} \in \mathbb{Z}^+,\ \forall j \in \boldsymbol{J},\ t \in \boldsymbol{T}$: number of vehicles at station j at the beginning of time interval t.

⋆ $n'_{jt} \in \mathbb{Z}^+,\ \forall j \in \boldsymbol{J},\ t \in \boldsymbol{T}$: number of parking spaces at station j at the beginning of time interval t that are assigned to vehicles coming to the stations.

- $m_{sjt} \in \mathbb{Z}^+,\ \forall s \in \boldsymbol{S},\ j \in \boldsymbol{J},\ t \in \{\mathrm{begin}_s, ..., \mathrm{fin}_s\}$: number of staff in shift s at station j at the beginning of time interval t.
- $r'_{sjt} \in \mathbb{Z}^+,\ \forall s \in \boldsymbol{S},\ j \in \boldsymbol{J},\ t \in \{\mathrm{begin}_s, ..., \mathrm{fin}_s\}$: number of staff of shift s relocating vehicle(s) <u>from</u> station j during time interval t.
- $r_{sjt} \in \mathbb{Z}^+,\ \forall s \in \boldsymbol{S},\ j \in \boldsymbol{J},\ t \in \{\mathrm{begin}_s, ..., \mathrm{fin}_s\}$: number of staff of shift s relocating vehicle(s) <u>to</u>

[*2] Travel time depends not only on OD (origin b and destination d) but also on time of departure t. For example, suppose that each time interval duration is 1, $\mathrm{travel}_{1bd} = 4$ and $\mathrm{travel}_{2bd} = 3$; then $\mathrm{t.end}_{5bd}$ is {1, 2}.

station j during time interval t.

- $p'_{sjt} \in \mathbb{Z}^+,\ \forall s \in \boldsymbol{S},\ j \in \boldsymbol{J},\ t \in \{\text{begin}_s, ..., \text{fin}_s\}$: number of staff of shift s moving from station j during time interval t.
- $p_{sjt} \in \mathbb{Z}^+,\ \forall s \in \boldsymbol{S},\ j \in \boldsymbol{J},\ t \in \{\text{begin}_s, ..., \text{fin}_s\}$: number of staff of shift s moving to station j during time interval t.
- $\tilde{r}_{sbdt} \in \mathbb{Z}^+,\ \forall s \in \boldsymbol{S},\ b, d \in \boldsymbol{B},\ t \in \{\text{begin}_s, ..., \text{fin}_s\}$: number of staff of shift s relocating vehicle(s) from cluster b to cluster d during time interval t.
- $\tilde{p}_{sbdt} \in \mathbb{Z}^+,\ \forall s \in \boldsymbol{S},\ b, d \in \boldsymbol{B},\ t \in \{\text{begin}_s, ..., \text{fin}_s\}$: number of staff in shift s moving from cluster b to cluster d during time interval t.
- $l_{jt} \in \{0,1\},\ \forall j \in \boldsymbol{J},\ t \in \boldsymbol{T}$: binary variables, if relocation staff vehicle exists at station j during time interval t 1, otherwise 0.
- $u'_{jt} \in \{0,1\},\ \forall j \in \boldsymbol{J},\ t \in \boldsymbol{T}$: binary variables, if relocation staff vehicle moves out from station j during time interval t 1, otherwise 0.
- $u_{jt} \in \{0,1\},\ \forall j \in \boldsymbol{J},\ t \in \boldsymbol{T}$: binary variables, if relocation staff vehicle moves to station j during time interval t 1, otherwise 0.

3.2.4 Formulation

The mixed integer linear programming problem is formulated as follows.

$$\text{maximize} \sum_{i \in \boldsymbol{I}} \mathrm{P}_i z_i - \sum_{s \in \boldsymbol{S}} \mathrm{PC}_s v_s - \sum_{s \in \boldsymbol{S}} \sum_{b,d \in \boldsymbol{B}} \sum_{t \in \{\text{begin}_s, ..., \text{fin}_s\}} (\tilde{\mathrm{RC}}_{bd} \tilde{r}_{sbdt} + \tilde{\mathrm{MC}}_{bd} \tilde{p}_{sbdt}) \tag{1}$$

such that

$$\begin{aligned} n_{jt+1} = n_{jt} &- \sum_{\substack{i:\text{origin}_i = j \\ \text{start}_i = t}} z_i - \sum_{s:\text{begin}_s \le t \le \text{fin}_s} r'_{sjt} \\ &+ \sum_{\substack{i:\text{dest}_i = j \\ \text{end}_i = t}} z_i + \sum_{s:\text{begin}_s \le t \le \text{fin}_s} r_{sjt} \qquad \forall j \in \boldsymbol{J}, t \in \{0, ..., t_{\text{last}-1}\} \end{aligned} \tag{2}$$

$$n_{jt} \ge \sum_{\substack{i:\text{origin}_i = j \\ \text{start}_i = t}} z_i + \sum_{s:\text{begin}_s \le t \le \text{fin}_s} r'_{sjt} \qquad \forall j \in \boldsymbol{J}, t \in \boldsymbol{T} \tag{3}$$

$$\sum_{j \in \boldsymbol{J}} m_{sj\text{begin}_s} = v_s$$

$$\sum_{j \in \boldsymbol{J}} m_{sj\text{fin}_s} + r_{sj\text{fin}_s} + p_{sj\text{fin}_s} = v_s \qquad \forall s \in \boldsymbol{S} \tag{4}$$

$$m_{sjt+1} = m_{sjt} - p'_{sjt} - r'_{sjt} + p_{sjt} + r_{sjt} \qquad \forall s \in \boldsymbol{S}, j \in \boldsymbol{J}, t \in \{\text{begin}_s, .., \text{fin}_s - 1\} \tag{5}$$

$$m_{sjt} \ge p'_{sjt} + r'_{sjt} \qquad \forall s \in \boldsymbol{S}, j \in \boldsymbol{J}, t \in \{\text{begin}_s, ..., \text{fin}_s\} \tag{6}$$

$$\sum_{d\in\boldsymbol{B}}\tilde{r}_{sbdt} = \sum_{j\in\boldsymbol{J}_b} r'_{sjt}$$

$$\sum_{d\in\boldsymbol{B}}\sum_{t'\in \text{t.end}_{tdb}\cap\{\text{begin}_s,...,\text{fin}_s\}}\tilde{r}_{sdbt'} = \sum_{j\in\boldsymbol{J}_b} r_{sjt}$$

$$\forall s\in\boldsymbol{S}, b\in\boldsymbol{B}, t\in\{\text{begin}_s,...,\text{fin}_s\} \tag{7}$$

$$\sum_{d\in\boldsymbol{B}}\tilde{p}_{sbdt} = \sum_{j\in\boldsymbol{J}_b} p'_{sjt}$$

$$\sum_{d\in\boldsymbol{B}}\sum_{t'\in \text{t.end}_{tdb}\cap\{\text{begin}_s,...,\text{fin}_s\}}\tilde{p}_{sdbt'} = \sum_{j\in\boldsymbol{J}_b} p_{sjt}$$

$$\forall s\in\boldsymbol{S}, b\in\boldsymbol{B}, t\in\{\text{begin}_s,...,\text{fin}_s\} \tag{8}$$

$$\sum_{j\in\boldsymbol{J}} n_{j0} \leq \text{NV} \tag{9}$$

$$n_{jt} \leq \text{cap}_j \qquad \forall j\in\boldsymbol{J}, t\in\boldsymbol{T} \tag{10}$$

$$n'_{jt+1} = n'_{jt} - \sum_{\substack{i:\text{dest}_i=j\\ \text{end}_i=t}} z_i + \sum_{\substack{i:\text{dest}_i=j\\ \text{start}_i=t}} z_i \qquad \forall j\in\boldsymbol{J}, t\in\{0,...,t_{\text{last}-1}\} \tag{11}$$

$$n_{jt} + n'_{jt} \leq \text{cap}_j \qquad \forall j\in\boldsymbol{J}, t\in\boldsymbol{T} \tag{12}$$

$$\sum_{s\in\boldsymbol{S}} v_s \leq \text{NW} \tag{13}$$

$$\sum_{s\in\boldsymbol{S}} (\text{fin}_s - \text{begin}_s) v_s \leq \text{Lcost} \tag{14}$$

$$v_0 \geq 1 \tag{15}$$

$$\sum_{s:\text{begin}_s\leq t\leq \text{fin}_s} p'_{sjt} \leq \text{NW}u'_{jt}$$

$$\sum_{s:\text{begin}_s\leq t\leq \text{fin}_s} p_{sjt} \leq \text{NW}u_{jt} \qquad \forall j\in\boldsymbol{J}, t\in\boldsymbol{T} \tag{16}$$

$$\sum_{s:\text{begin}_s\leq t\leq \text{fin}_s} p'_{sjt} \geq u'_{jt}$$

$$\sum_{s:\text{begin}_s\leq t\leq \text{fin}_s} p_{sjt} \geq u_{jt} \qquad \forall j\in\boldsymbol{J}, t\in\boldsymbol{T} \tag{17}$$

$$l_{jt+1} = l_{jt} - u'_{jt} + u_{jt} \qquad \forall j\in\boldsymbol{J}, t\in\boldsymbol{T} \tag{18}$$

$$\sum_{j\in\boldsymbol{J}} l_{j0} \leq 1 \tag{19}$$

The objective function (1) maximizes the total daily profit including revenues from users' served trips, personnel costs of relocation staff, and relocating and moving costs (cost of fueling the staff vehicle) between clusters. While carsharing operators want to increase the number of trips, relocation costs put pressure on their profitability (an easy way to increase the number of trips is to hire hundreds of relocation staff). We propose to find a way to maximize profits under certain costs restrictions.

Constraints (2) are the flow conservation equations for vehicles for each station and each time interval.

The number of vehicles at station j at the beginning of time interval $t+1$ (n_{jt+1}) is equal to the number of vehicles at the beginning of the previous time interval t (n_{jt}) minus the number of vehicles departing from station j during time interval t on trips ($\sum_{\substack{i:\text{origin}_i=j\\ \text{start}_i=t}} z_i$) and relocations ($\sum_{s:\text{begin}_s \le t \le \text{fin}_s} r'_{sjt}$) plus the number of vehicles arriving at j during t on trips ($\sum_{\substack{i:\text{dest}_i=j\\ \text{end}_i=t}} z_i$) and relocations ($\sum_{s:\text{begin}_s \le t \le \text{fin}_s} r_{sjt}$). This flow is shown in Fig. 8.

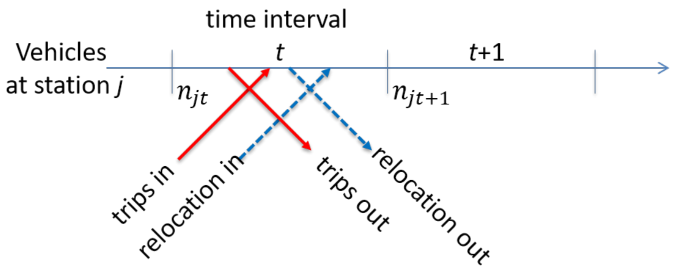


Fig. 8: Flow diagram of vehicles at station j at time interval t

Constraints (3) represent that there are enough vehicles for trips and relocations at each station during each time interval. Vehicles arriving at station j during t ($\sum_{\substack{i:\text{dest}_i=j\\ \text{end}_i=t}} z_i + \sum_{s:\text{begin}_s \le t \le \text{fin}_s} r_{sjt}$) cannot be counted during the same time interval t. Because of these constraints, every vehicle can have one and only one status ("free", "assigned") during each time interval.

Constraints (4) show the boundary of the number of staff at the beginning shift and finishing shift time intervals. The sum of staff in shift s over all stations at the beginning shift time interval begin_s must equal the number of staff from shift v_s. Moreover, at the finishing shift time interval fin_s as well, the total number of staff in shift s at all stations must equal the number of staff from shift v_s.

Constraints (5) are the flow conservations of staff for each shift, each station, and each time interval. The number of staff from shift s at station j at the beginning of time interval $t+1$ (m_{sjt+1}) is equal to the number of staff at the beginning of the previous time interval t (m_{sjt}) minus the number of staff departing from station j during time interval t for moving (p'_{sjt}) and relocating (r'_{sjt}) plus the number of staff arriving at station j during time interval t for trips (p_{sjt}) and relocations (r_{sjt}). This flow is shown in Fig.9.

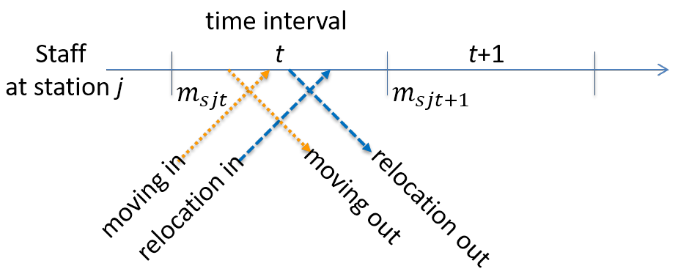


Fig. 9: Flow diagram of relocation staff at station j during time interval t

Constraints (6) on the number of staff, similar to the constraints (3) on vehicles, represent whether there are enough staff for moving and relocating operations at each station during each time interval. Staff arriving at station j in time interval t ($p_{sjt} + r_{sjt}$) cannot be counted during the same time interval t. Because of these constraints, every staff can have one and only one status ("idle", "moving", or "relocating") during each time interval.

Constraints (7) and (8) are respectively the relocating and moving staff flow conservations between clusters and stations in the clusters. The left equations indicate that the number of staff in shift s starting to relocate/move from cluster b during time interval t must be equal to the sum of staff from shift s relocating/moving from all stations in cluster ($\boldsymbol{J}_b$) during t. Likewise, the right equations represent that the number of staff in shift s starting to relocate/move into cluster b during t must be equal to the sum of staff in shift s relocating/moving to all stations in cluster ($\boldsymbol{J}_b$) during t.

Constraint (9) indicates that the total number of vehicles at all stations at the beginning of the day must be equal to or less than the maximum number of vehicles available in the service.

Constraints (10) show the parking capacity restriction at each station during each time interval t. The number of vehicles at station j during t must be equal to or less than the number of parking spaces at station j.

Constraints (11) are the flow conservation equations for assigned (unavailable) parking spaces at each station and each time interval. The number of assigned parking spaces at station j at the beginning of time interval $t+1$ (n'_{jt+1}) is equal to the number of assigned parking spaces in the previous time interval t (n'_t) minus the number of (parking spaces assigned for) vehicles arriving at station j during time interval t on trips ($\sum_{\substack{i:\text{dest}_i=j\\ \text{end}_i=t}} z_i$) plus the number of (parking spaces assigned for) vehicles departing from station j during t on trips ($\sum_{\substack{i:\text{origin}_i=j\\ \text{start}_i=t}} z_i$). Note that parking spaces are only assigned to user trips and not for staff relocation, so the equations don't include the number of relocations (r'_{sjt} or r_{sjt}), unlike constraints (2).

Constraints (12) show the parking capacity restrictions including assigned parking spaces at each station in each time interval t. The sum of vehicles and assigned parking spaces at station j during t must be equal to or less than the number of parking spaces at station j.

Constraint (13) represents that the total number of staff in all shifts must be equal to or less than the total number of relocation staff able to work in the service.

Constraint (14) represents the maximum labor cost per day.

Constraint (15) indicates that at least a manager must work in shift 0, which is the full-time shift.

Constraint (16) indicates that if u_{jt} or u'_{jt} is equal to 0, no staff move from/to station j during time interval t. On the contrary, constraint (17) represents that if u_{jt} or u'_{jt} equals 1, there must be staff moving from/to station j during t. These constraints mean that relocation staff can only move by using the staff vehicle.

Constraint (18) is the flow conservation of staff vehicles at each station during each time interval. Finally, constraint (19) is the maximum number of staff vehicles. In this study, it is assumed there is only one staff vehicle.

Note that the objective function (1) is different from that of Boyaci et al. [15], and the constraints (9), (10), (11), (12), (13), (14), (15), (16), (17), (18), (19) are not used in [15].

3.3 Station clustering

To decrease the number of variables and reduce the computational complexity, the stations used for staff-moving and vehicles-relocating are clustered.

Without clustering, the number of variables for moving and relocating is on the order of $\mathcal{O}(|\boldsymbol{S}||\boldsymbol{J}|^2|\boldsymbol{T}|)$, where $\boldsymbol{S}$ is the set of relocation staff shifts, $\boldsymbol{J}$ is the set of stations, and $\boldsymbol{T}$ is the set of time intervals (the number of variables for trip $|\boldsymbol{I}|$ is ignored here). As described in section 3.2, by clustering stations and assuming staff moving and relocation operations are divided to intra-cluster and inter-cluster parts (Fig. 7), the number of the variables changes to on the order of $\mathcal{O}(|\boldsymbol{S}|(|\boldsymbol{J}||\boldsymbol{T}|+|\boldsymbol{B}|^2|\boldsymbol{T}|))$. By setting the number of clusters as $|\boldsymbol{B}| \leq \sqrt{|\boldsymbol{J}|}$, the number of variables decreases to on the order of $\mathcal{O}(|\boldsymbol{S}||\boldsymbol{J}||\boldsymbol{T}|)$.

As a result of clustering, the travel time from station j to station l (δ_{tjl}) is replaced with the travel time from cluster b of station j to cluster d of station l (travel_{tbd}), whose difference remains as a clustering error (dev_{tjl}).

$$\mathrm{dev}_{tjl} = \mathrm{travel}_{tbd} - \delta_{tjl} \qquad \forall j \in \boldsymbol{J}_b, l \in \boldsymbol{J}_d \tag{20}$$

To minimize the sum of clustering errors dev_{tjl}, we developed an algorithm based on Boyaci et al. [15]. The algorithm is similar to the k-Medoid algorithm, which minimizes the sum of dissimilarities within clusters. We define the travel time from cluster b to cluster d (travel_{tbd}) as the maximum travel time between any stations in each cluster as (21) and set the objective function as (22), the sum of the clustering errors to be minimized. Because of the definition (21), the clustering algorithm always returns feasible solutions (travel_{tbd} is always equal to or larger than the actual travel time).

$$\mathrm{travel}_{bd} = \max\{\delta_{tjl} \mid j \in \boldsymbol{J}_b, l \in \boldsymbol{J}_d\} \tag{21}$$

$$\mathrm{F}_{\boldsymbol{B}} = \sum_{b,d\in\boldsymbol{B}} \sum_{j\in\boldsymbol{J}_b} \sum_{l\in\boldsymbol{J}_d} \mathrm{dev}_{jl} \tag{22}$$

In the objective function (22), the clustering error dev_{tjl} is replaced with dev_{jl} (without t). Because the set of clusters $\boldsymbol{B}$ is unique in the optimization formulation, the station clustering requires time-independent representatives of dev_{tjl}. We set dev_{jl} as the median of dev_{tjl} in the case studies described later in section 4.

The station clustering algorithm 1 is based on Boyaci et al. [15].

- $\boldsymbol{J}$: set of stations
- N_k: number of clusters
- n: maximum number of iterations
- $\mathrm{F}_{\boldsymbol{B}}$: objective function to be minimized
- $\boldsymbol{B}^*$: set of clusters with the best objective function
- Z^*: value of the best objective function

1. Create an initial solution (lines from 1 to 15). For all N_k stations, assign each station to each

cluster one by one, and add the station to a cluster which gives the best (minimum) objective function (22). Note that the initial solution depends on the order of stations added.

2. Iteratively search for a better solution by exchanging stations between clusters (line 16 below).
   (a) Find a station to be removed (lines 21 to 31). For all stations, temporally remove each station from its cluster one by one, and find a removal which improves the objective function the most. We avoid selecting the same stations repeatedly by remembering the stations selected to be removed in variable $\boldsymbol{C}$ and excluding them from subsequent searches.
   (b) Find a cluster to which the removed station can be added (lines 32 to 40). For all clusters except the one the station was removed from, assign the station to each cluster one by one, and find an addition which gives the minimum objective function.
   (c) If the new solution has a better (smaller) objective function than the best solution, replace the best solution with the new solution (line 41). If the best solution is not updated after every station has been check to see if it is to be put in a different cluster, run the algorithm for swapping stations between any two clusters to seek a new set of clusters with the smallest objective function (Algorithm 2). If the new solution has better (smaller) objective function than the best solution, replace the best solution with the new solution.

Algorithm 1 Clustering stations

Input: $\boldsymbol{J}, \mathrm{N}_k, n, \mathrm{F}_{\boldsymbol{B}}$
Output: $\boldsymbol{B}^*, Z^*$

for $k \leftarrow 1$ **to** N_k **do**
  $\boldsymbol{B}_b \leftarrow \emptyset$
end for
for $j \leftarrow 1$ **to** $|\boldsymbol{J}|$ **do**
  $Z \leftarrow \infty$
  for $b \leftarrow 1$ **to** N_k **do**
    $\boldsymbol{B}_b \leftarrow \boldsymbol{B}_b \cup \{j\}$
    if $\mathrm{F}_{\boldsymbol{B}} < Z$ **then**
      $b^* \leftarrow b; Z \leftarrow \mathrm{F}_{\boldsymbol{B}}$
    end if
    $\boldsymbol{B}_b \leftarrow \boldsymbol{B}_b \setminus \{j\}$
  end for
  $\boldsymbol{B}_{b^*} \leftarrow \boldsymbol{B}_{b^*} \cup \{j\}$
end for
$\boldsymbol{B}^* \leftarrow \boldsymbol{B}; Z^* \leftarrow \mathrm{F}_{\boldsymbol{B}}$
$\boldsymbol{C} \leftarrow \emptyset$
for $i \leftarrow 1$ **to** N **do**
  if $\boldsymbol{C} = \boldsymbol{J}$ **then**
    break
  end if
  $Z \leftarrow \infty$
  for $b \leftarrow 1$ **to** N_k **do**
    for $j \in \boldsymbol{J}_b \setminus \boldsymbol{C}$ **do**
      $\boldsymbol{B}_b \leftarrow \boldsymbol{B}_b \setminus \{j\}$
      if $\mathrm{F}_{\boldsymbol{B}} < Z$ **then**
        $j^* \leftarrow j; b^* \leftarrow b; Z \leftarrow \mathrm{F}_{\boldsymbol{B}}$
      end if
      $\boldsymbol{B}_b \leftarrow \boldsymbol{B}_b \cup \{j\}$
    end for
  end for
  $\boldsymbol{B}_{b^*} \leftarrow \boldsymbol{B}_{b^*} \setminus \{j^*\}; \boldsymbol{C} \leftarrow \boldsymbol{C} \cup \{j^*\}$
  $Z \leftarrow \infty$
  for $b \leftarrow 1$ **to** N_k **do**
    $\boldsymbol{B}_b \leftarrow \boldsymbol{B}_b \cup \{j^*\}$
    if $\mathrm{F}_{\boldsymbol{B}} < Z$ **then**
      $b^* \leftarrow b; Z \leftarrow \mathrm{F}_{\boldsymbol{B}}$
    end if
    $\boldsymbol{B}_b \leftarrow \boldsymbol{B}_b \setminus \{j^*\}$
  end for
  $\boldsymbol{B}_{b^*} \leftarrow \boldsymbol{B}_{b^*} \cup \{j^*\}$
  if $\mathrm{F}_{\boldsymbol{B}} < Z^*$ **then**
    $\boldsymbol{B}^* \leftarrow \boldsymbol{B}; Z^* \leftarrow \mathrm{F}_{\boldsymbol{B}}; \boldsymbol{C} \leftarrow \emptyset$
  else if $\boldsymbol{C} = \boldsymbol{J}$ **then**
    $\boldsymbol{B} \leftarrow$ Swaping stations$(\boldsymbol{B}, Z)$
    if $Z < Z^*$ **then**
      $\boldsymbol{B}^* \leftarrow \boldsymbol{B}; Z^* \leftarrow \mathrm{F}_{\boldsymbol{B}}; \boldsymbol{C} \leftarrow \emptyset$
    end if
  end if
end for

Algorithm 2 Swapping stations

Input: $\boldsymbol{B}, Z$
Output: $\boldsymbol{B}$
$Z_0 \leftarrow Z$
for $b \leftarrow 1$ **to** $\mathrm{N}_k - 1$ **do**
  for $d \leftarrow b + 1$ **to** N_k **do**
    for $j \in \boldsymbol{J}_b$ **do**
      for $l \in \boldsymbol{J}_d$ **do**
        $\boldsymbol{B}_b \leftarrow \boldsymbol{B}_b \setminus \{j\}; \boldsymbol{B}_d \leftarrow \boldsymbol{B}_d \setminus \{l\}; \boldsymbol{B}_b \leftarrow \boldsymbol{B}_b \cup \{l\}; \boldsymbol{B}_d \leftarrow \boldsymbol{B}_d \cup \{j\}$
        if $\mathrm{F}_{\boldsymbol{B}} < Z$ **then**
          $b^* \leftarrow b; d^* \leftarrow d; j^* \leftarrow j; l^* \leftarrow l; Z \leftarrow \mathrm{F}_{\boldsymbol{B}}$
        end if
        $\boldsymbol{B}_b \leftarrow \boldsymbol{B}_b \setminus \{l\}; \boldsymbol{B}_d \leftarrow \boldsymbol{B}_d \setminus \{j\}; \boldsymbol{B}_b \leftarrow \boldsymbol{B}_b \cup \{j\}; \boldsymbol{B}_d \leftarrow \boldsymbol{B}_d \cup \{l\}$
      end for
    end for
  end for
end for
if $Z < Z_0$ **then**
  $\boldsymbol{B}_{b^*} \leftarrow \boldsymbol{B}_{b^*} \setminus \{j^*\}; \boldsymbol{B}_{d^*} \leftarrow \boldsymbol{B}_{d^*} \setminus \{l^*\}; \boldsymbol{B}_{b^*} \leftarrow \boldsymbol{B}_{b^*} \cup \{l^*\}; \boldsymbol{B}_{d^*} \leftarrow \boldsymbol{B}_{d^*} \cup \{j^*\}$
end if
return $\boldsymbol{B}$

4 Case study

This section describes case studies applying the framework developed in section 3 to Ha:mo RIDE Toyota, which has been operating in Toyota, Japan since 2012. The models were implemented in an E5-1630 v3 (3.70GHz), 64GB computer running the Windows 8.1 Pro OS and the Gurobi Optimizer 7.0.2 tool.

4.1 Ha:mo RIDE (current service)

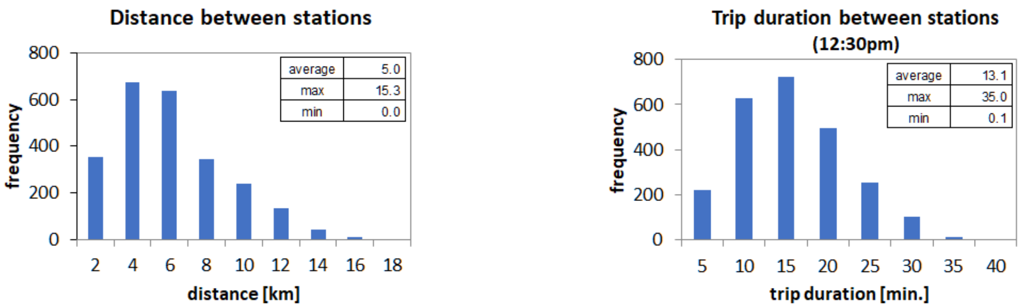
The use history for one year from April 1st, 2016 to March 31st, 2017 and station data were provided by Toyota Motor Corporation[*3].

4.1.1 Station clustering

There were 55 stations operating during the period when the use data was acquired, including one depot that was off-limits to users. The travel time δ_{tjl}, starting from time interval t between station j and station l, was estimated using the Google Maps Distance Matrix API [17]. By setting the travel mode option to "driving" and specifying the departure time t, the API returns a route distance and trip duration between two points, taking traffic conditions into account. Deviations in trip duration of up to a maximum of 20 minutes were observed in an OD throughout the day, and we used the travel time at $h : 30$ as a representative departure time t during [h:00, ($h + 1$):00). We added a 3 minute margin (described in 3.2) to the trip durations] to make the travel time δ_{tjl}. These travel times were applied to the user as well as relocation staff travels. Note that the travel times between the same stations in

[*3] Not allowed to be publicly shared.

opposite directions are generally not the same ($\delta_{tjl} \neq \delta_{tlj}$). Fig. 10 shows histograms of route distances and trip durations returned by the Google Maps Matrix API.



(a) Distance **(b)** Trip duration

Fig. 10: Distribution of (a) distance and (b) trip duration returned by Google Maps Distance Matrix API

As described in section 3.3, we set the number of clusters so that $|\boldsymbol{B}| \leq \sqrt{|\boldsymbol{J}|}$ holds, where $\boldsymbol{B}$ is the set of clusters and $\boldsymbol{J}$ is the set of stations. For the 55 stations, the number of clusters turned out to be equal to or less than $\sqrt{55}$, or around 7.4 to reduce computational complexity. Fewer clusters would result in more errors in the travel time for moving or relocating.

We tried various numbers of clusters to see how varying the number would affect the objective function. Because clustering results depend on the initial solutions, i.e. on the order of stations added, we shuffled the stations randomly and ran the clustering algorithm 25 times per condition and picked the best result. The maximum number of iterations n was 1,000. As shown in Fig.11, the objective function was sharply worse at less than five clusters. As a result, we decided to use seven clusters, shown in different colors in Fig.12.

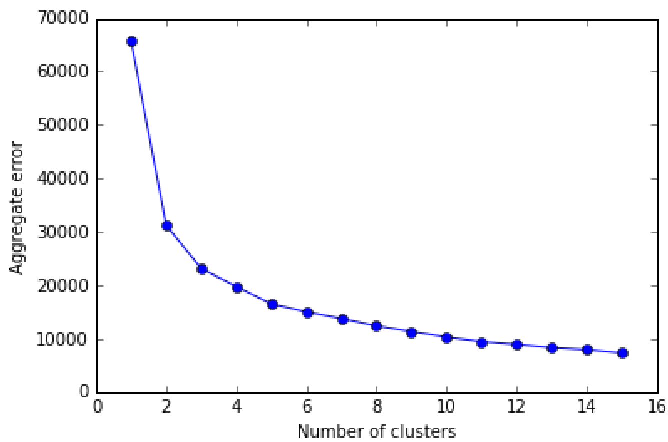


Fig. 11: Aggregate error of different numbers of clusters

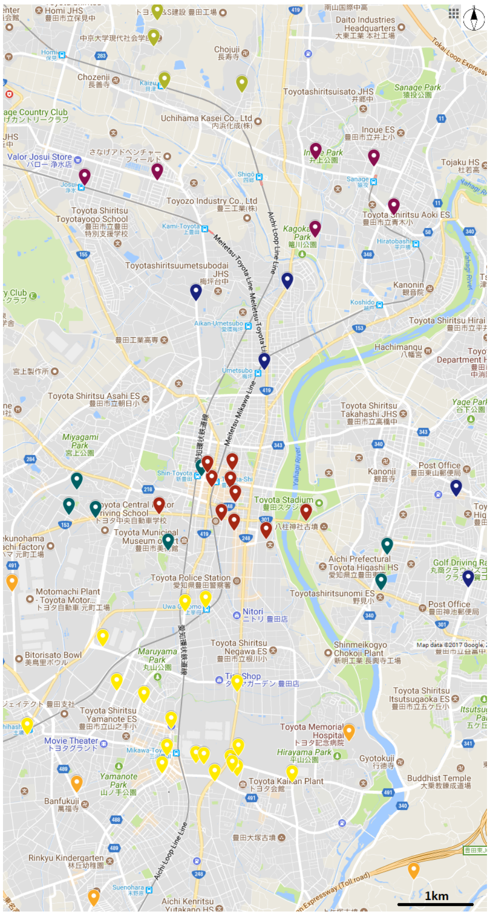


Fig. 12: Ha:mo RIDE Toyota station map. The colors indicate clusters of seven stations.

4.1.2 Optimization settings

As mentioned above, the number of stations was 55, and the total number of parking spaces $\sum_{j \in \boldsymbol{J}} \mathrm{cap}_j$ was 266. The maximum number of vehicles NV was 90. The time period for optimization was 6 am to 12 am, when demand was relatively high and all stations were open. Fig. 13 shows trip data taken from a 2011 person-trip survey conducted in Toyota city, which have a similar distribution to the Ha:mo RIDE Toyota use data.

The relocation staff shifts are shown in Fig. 14a. It was necessary to set the number of shifts to 2 or less in order to avoid excessive computational time. Shift 0 is a full-time shift in which a manager has to work, and shift 1 is a part-time shift that covers high-demand hours in the evening. The total number of relocation staff NW was 5, the personnel costs of relocation staff PC_s was ¥900 per hour for any shift s, and the maximum labor cost Lcost was ¥27,000 per day (equivalent to 30 man-hours per day).

Instead of estimating potential demand, we generated demand values from the use history according to a Poisson distribution [*4] to run the optimization a large number of times. From the use history,

[*4] For events with an expected occurrence rate of λ, the Poisson distribution describes the probability of x events

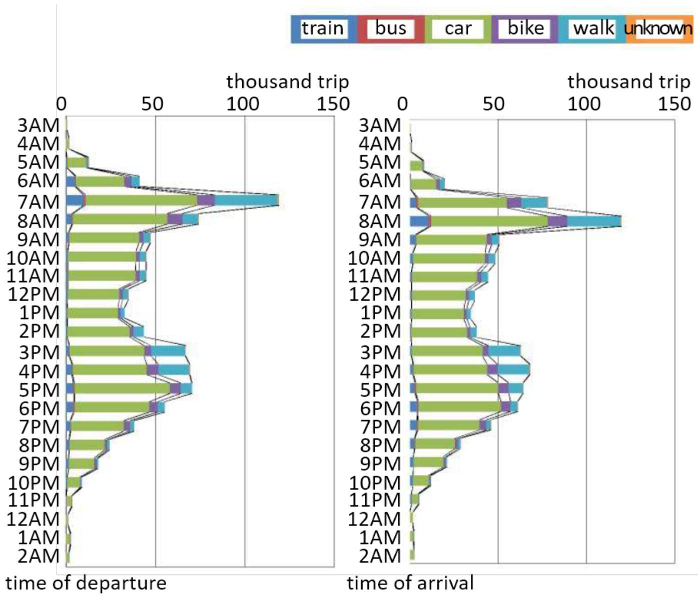
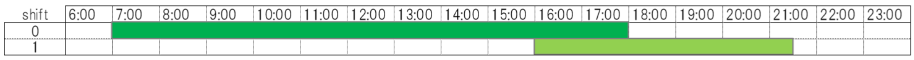
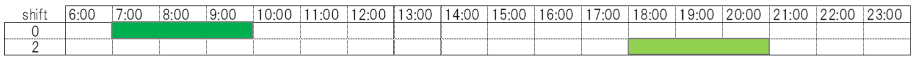


Fig. 13: Number of trips from/to Toyota City: data from 2011 person-trip survey



(a) staff shift #1



(b) staff shift #2
(only high-demand hours)

Fig. 14: Staff shift

we aggregated the one-way use data except the data gathered on holidays and calculated the average number of uses per hour h per OD from station j to l, λ^h_{jl}. The number of trips departing on h:m ($h \in [6,23]$, $m \in [0,5,10,...,55]$) from station j to l is given by a Poisson distribution with an average $\lambda^h_{jl} \times \frac{5}{60}$. The end time of the trip is given by adding δ_{tjl} to departing time and rounding up to the nearest five-minute interval.

The price for trip i (P_i) was taken to be the actual price: fare of ¥200 for up to 10 minutes and ¥20 per minute after that. The price for a trip from station j to l was calculated as follows:

$$\mathrm{P}_{jl} = \max\{\mathrm{P}_0, \mathrm{P}_0 + (\delta_{jl} - \mathrm{t}_0) \times \mathrm{dP}\} \quad \mathrm{P}_0 = 200, \mathrm{t}_0 = 10, \mathrm{P}_0 = 20$$

occurring within a fixed interval of time.

$$P(x) = \frac{\lambda^x}{x!} \mathrm{e}^{-\lambda x}$$

Thus, for demands with an average λ^h in 1 hour (60 minutes), the Poisson distribution with an average $\lambda = \lambda^h \times \frac{5}{60}$ gives the number of demands in 5 minutes.

Finally, the relocating costs and moving costs $\tilde{\mathrm{RC}}_{bd}$ (1.0) and $\tilde{\mathrm{MC}}_{bd}$ (1.1) reflected the use of fuel-efficient compact cars.

4.1.3 Optimization results

Table 1 shows the results of the optimization compared with data gathered during actual operation. The numbers are averages of 50 runs with different demand samples (satisfied demand and profit related numbers are shown as ratios of the optimization results to the data gathered during actual operation).

Table. 1: Optimization results compared with data in actual operations that served all trips.

	Shift #1	Shift #2	1.5×demand	Actual
Satisfied demand ratio [%]	83.5	81.3	105.9	100.0
Number of staff working	1.0	1.0	1.0	5.0
Number of vehicles used	56.1	55.3	69.8	90.0
Number of relocations	4.4	2.0	2.2	21.8
Optimized profit compared with actual	1.97	2.54	3.53	1.0
Revenue ratio	0.84	0.82	1.12	1.0
Personnel costs ratio	0.37	0.10	0.10	1.0
Relocation and moving costs ratio	0.37	0.15	0.19	1.0

In spite of using a demand generated from the actual history, the satisfied demand was only 83.5%. While the actual operation relocated vehicles with 5 staff in total, the optimal solution had only 1 staff (manager, forced to work by constraint (15)), thereby cutting personnel costs to maximize profit. The average revenue was a little over ¥300 because of the average approximate 15-minute travel time. One set of relocations often consisted of two trips: moving to a station from which a vehicle was to be removed and the actual relocation, as shown in Fig. 5. The number of relocations per hour was 2 on average[*5]. A two-man team relocated 2 vehicles and cost ¥1,800 per hour, i.e. ¥900 per relocation, a three-man team relocated 4 vehicles and cost ¥2,700 per hour, i.e. ¥675 per relocation, and so on[*6]. The larger the team was, the less expensive each relocation became, by decreasing the driver staff cost per relocation, ¥450 (¥900 per 2 sets of relocations).

To confirm the above results, we added another case "staff shift #2" (Fig. 14b) with a staff shift focusing only on high-demand hours and a case with a staff shift and 1.5 times the demand ("Shift#2" and "1.5×demand" in Table 1). Both cases had only 1 staff and infrequent relocations, which indicates that staff-based relocation does not lead to additional revenue.

In "staff shift#1," the number of vehicles used was 56.1, about 2/3rds of the maximum number of available vehicles. This result suggests that there is a best ratio of the number of vehicles to the number of parking spaces depending on the relocation frequency. Obviously, the more frequently vehicles are relocated, the more vehicles should be added, but when vehicles are relocated often, it is better for fewer vehicles to leave parking spaces available for incoming trips. We will discuss this parking to vehicle rate in section 4.3.

The optimal solution maximizing profit indicates that the operator should reduce relocation staff. Additional revenues from the short trips of last-mile mobility are too low to cover relocation personnel costs, and the best operation is distributing an appropriate number of vehicles to stations and leaving them

[*5] 60 minutes / (15 minutes × 2) = 2.
[*6] Note that one of staff in the team has to drive the staff vehicle and cannot relocate vehicles.

there. However, in doing so, the low satisfied demand rate would result in customer dissatisfaction and long-term defection. The operator must relocate vehicles not to increase profit but to maintain certain levels of satisfied demand and retain users.

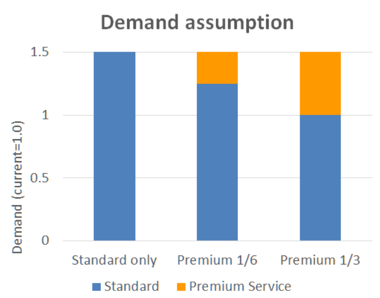
4.2 Premium Service

Here, in an attempt to cover the costs of relocating personnel, we introduced a "Premium Service" guaranteeing reservations in return for price premiums. As mentioned in section 1, ridesharing companies such as Uber and Lyft are implementing special pricing for high-demand hours, called "surge pricing" and "Premium Time" [7], respectively.

4.2.1 Settings

Here the idea was that demand for premium service must be satisfied before demand for the standard service is satisfied. Vehicles were relocated if necessary.

Although premium service should be more popular during high-demand hours and ODs, it is quite challenging to estimate the potential demand accurately as it depends on pricing. Instead, we assumed a 1.5 times total demand and regenerated trips from the average $1.5 \times \lambda_{jl}^{h}$ in the same manner as in section 4.1.2. From these trips, we picked trips randomly at a uniform rate of p per hour and OD, and formed a premium service trip set $\boldsymbol{I}_p$ ($\boldsymbol{I}_p \subseteq \boldsymbol{I}$). As for pricing, we set the starting fare higher than the standard and lower than that of a taxi, ¥300. We ran the optimization at different selection ratios p: 1/6 and 1/3.



(a) Demand setting

(b) Price setting

Fig. 15: Assumptions of Premium Service

The premium service formulation added the following constraints to those in section 3.2.

$$z_i = 1 \qquad \forall i \in \boldsymbol{I}_p \tag{23}$$

Constraints (23) represent that every premium service demand must be satisfied.

4.2.2 Results

Table 2 shows the results of premium service optimization. The numbers for 1/6 and standard-only cases are averages of 50 runs with different demand samples. The numbers for the 1/3 case are averages of 31 runs out of 50, terminated by a 20,000-second timeout; the remaining 19 runs are infeasible, as described later. (The satisfied demand and profit related numbers are ratios of the results of Premium Service to those of the standard service for 1.5-time demand.)

Table. 2: Premium service compared with current service (standard only)

Premium service demand ratio p	Standard only	1/6	1/3
Satisfied demand ratio [%]	72.2	77.1	73.2
Premium service	-	100.0	100.0
Standard service	72.2	71.9	59.8
Number of staff working	1.0	1.0	1.3
Number of vehicles used	70.0	68.4	70.1
Number of relocations	5.5	5.3	6.3
Profit ratio compared with standard only	1.00	1.00	1.59
Revenue ratio	1.00	1.00	1.49
Personnel costs ratio	1.00	1.00	1.16
Relocation and moving costs ratio	1.00	0.96	1.22

In the case of $p = 1/6$, no big differences from the standard service were observed. The number of staff remained 1 (manager only).

In the case of $p = 1/3$, the total satisfied demand ratio was almost the same as in the standard case, but the satisfied demand ratio of the standard service decreased by 12.4%, which means standard service users were pushed out by premium service users, though some runs returned solutions with 2 or 3 relocation staff working to meet the demand. Also, 19 out of 50 runs (38%) were infeasible because of constraints (23), i.e., the requirement of satisfying premium service demand, and other constraints about vehicles, parking spaces, and relocation staff. In other words, premium service could not guarantee rides due to the shortage of resources such as vehicles, parking, and staff, even under assumption 2 that the demand for the whole day is given in advance.

Although the profit might increase in the short-term, this service would result in a long-term loss or become infeasible. We conclude that premium service is not worth implementing in our setting.

4.3 Autonomous relocation

While the previous section 4.2 investigated premium pricing to cover relocation costs, in this section, we consider self-driving as a way of low-cost relocation. The majority of the relocation costs consist of personnel costs, which exceed additional revenues from trips afforded by the staff-based relocation. We supposed that unstaffed autonomous relocation could increase the number of trips and cover the relocation costs (Note that we did not consider fixed costs of self-driving such as vehicle remodeling or infrastructure investments).

4.3.1 Settings

Although car makers and ridesharing providers are developing self-driving systems for drivers (Advanced Driver Assistance Systems or ADAS) and passengers (so-called “robot taxis”), we assumed that self-driving would only be for autonomous relocation, not for providing rides for users.

Because of it being a not-for-passenger kind of transport, vehicles being relocated do not need to drive as fast as human would drive them. Moreover, the vehicles would be required to drive on designated paths at low speed. Assuming a 10 km/h speed limit for automatically commanded steering functions [18][*7] or 6 km/h walking speed [*8], low-speed automated relocation would take approximately five times longer than manual driving. (Fig. 16 shows “walking” trip duration calculated from the Google Maps Distance Matrix API).

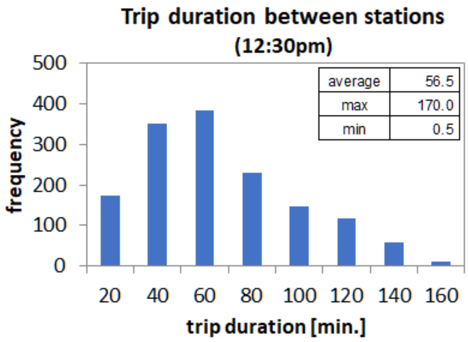


Fig. 16: Trip duration on foot, approx. five times longer than by car in Fig.10.

We denoted a trip duration of five times the duration of driving 4.1 (with the same margin of 3 minutes) as “walking speed” and a trip duration of 50 times that of driving as “super slow”. Although automated driving might actually be allowed only at limited times and in limited areas, we did not limit the autonomous relocation time or placement of stations in this case study. We set three levels of potential demand, 1.2, 1.5, 2.0 times higher than actual demand (equal to the number of uses), under the expectation that the autonomous relocation would increase demand. Vehicle equipment or infrastructure for autonomous relocation were not included in the formulations, as they were considered to be sunk.

The transformed objective function shown below does not include terms related to personnel costs.

$$\text{maximize} \quad \sum_{i \in \boldsymbol{I}} \mathrm{P}_i z_i - \sum_{b,d \in \boldsymbol{B}} \sum_{t \in \boldsymbol{T}} \tilde{\mathrm{RC}}_{bd} \tilde{r}_{bdt} \tag{24}$$

In addition, constraints on shifting relocation staff (4), (5), (6), (13), (14), (15) and moving relocation staff (8), (16), (18), (19) are omitted, and the flow conservation equations (2) for vehicles and (7) for

[*7] Currently under discussion to be modified in the UNECE World Forum for Harmonization of Vehicle Regulations (WP29).
[*8] The Cabinet Office Ordinance specifies the speed of walking assist vehicles to be at most 6 km/h in Japan.

relocation are replaced with (25) and (26), respectively. Note that the suffix s indicating the staff shift is omitted from variables $r', r, \tilde{r}$.

$$n_{jt+1} = n_{jt} - \sum_{\substack{i:\text{origin}_i=j\\ \text{start}_i=t}} z_i - r'_{jt} + \sum_{\substack{i:\text{dest}_i=j\\ \text{end}_i=t}} z_i + r_{jt} \qquad \forall j \in \boldsymbol{J}, t \in \{0, ..., t_{\text{last}-1}\} \tag{25}$$

$$\sum_{d\in\boldsymbol{B}} \tilde{r}_{bdt} = \sum_{j\in\boldsymbol{J}_b} r'_{jt}$$

$$\sum_{d\in\boldsymbol{B}} \sum_{t'\in \text{t.end}_{tdb}} \tilde{r}_{dbt'} = \sum_{j\in\boldsymbol{J}_b} r_{jt} \qquad \forall b \in \boldsymbol{B}, t \in \boldsymbol{T} \tag{26}$$

4.3.2 Results

Table 3 and Fig. 17 compare the results of optimization with staff-based relocation. The numbers are averages of 50 runs with different demand samples (satisfied demand and profit related numbers are shown as ratios of the results of the optimization to the ones of staff-based relocation).

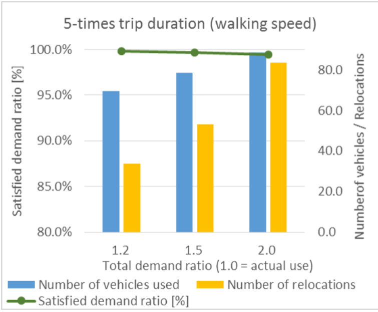
Table. 3: Autonomous drive relocation compared with base case

(a) 5-times trip duration (walking speed)

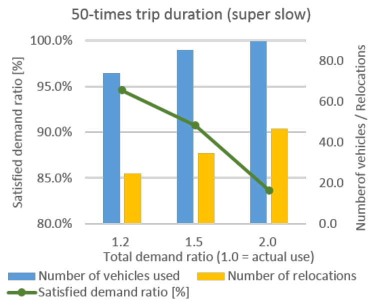
Total demand	×1.2	×1.5	×2.0
Satisfied demand ratio [%]	99.8%	99.7%	99.4%
Number of vehicles used	69.4	78.4	88.5
Number of relocations	33.9	53.1	83.6
Relocation / Vehicle	0.49	0.68	0.94
(Parking + 2×Relocation) / Vehicle	4.81	4.75	4.89
Profit ratio compared with base case	1.6	1.6	1.7
Revenue ratio	1.2	1.3	1.4
Relocation and moving costs ratio	6.0	9.1	13.8

(b) 50-times trip duration (super slow)

Total demand	×1.2	×1.5	×2.0
Satisfied demand ratio [%]	94.6%	90.7%	83.6%
Number of vehicles used	73.9	85.5	89.9
Number of relocations	24.5	34.8	46.6
Relocation / Vehicle	0.33	0.41	0.52
(Parking + 2×Relocation) / Vehicle	4.26	3.93	4.00
Profit ratio compared with base case	1.6	1.5	1.5
Revenue ratio	1.2	1.2	1.2
Relocation and moving costs ratio	3.9	5.2	6.5



(a) 5-times trip duration (walking speed)



(b) 50-times trip duration (super slow)

Fig. 17: Autonomous relocation with station clustering at different demand levels

In the walking speed case, the satisfied demand ratio is over 99% for all demand cases including 2.0 times higher than actual (Table 3a). The super slow case is a bit worse, 90% for 1.5 times demand and 84% for 2.0 times demand, but the number of satisfied demands is two times more compared with the no-relocation solution. The relocation cost increases in either case but its proportion remains a small fraction of the of revenue, and the profit improves to over 1.5 times that of the base case.

In the walking speed case, along with the increase in potential demand, the number of vehicles used increases and almost reaches the maximum number of vehicles available at 2.0 times demand. Not only the number of relocations but also the number of relocations per vehicle increases. The frequent vehicle relocation is considered to increase the per parking space operation ratio as well as the per vehicle one[*9] and avoids a situation where there is a lack of parking spaces caused by an increase in vehicles. We found the ratio of (parking + 2×relocation) to vehicle rate gives almost the same value (a little under 5) as the ratio of the parking to vehicle rate described in section 4.1.3. We conjecture that the additional term "2×relocation" for parking spaces comes from that one relocation has two effects on increasing parking capacity: one relocation obviously leaves open a parking space at a station the vehicle is relocated from and virtually adds a parking space at a station that the vehicle is to be relocated to during the period in which it is being dispatched.

In the super slow case, the number of relocations per vehicle does not increase as much as in the walking speed case. Relocation operations cannot meet the increase in demand, because super-slow driving takes too long to relocate vehicles (for example, driving a distance that usually takes 15 minutes takes 12.5 hours with super-slow driving, $15\text{minutes} \times 50 = 750\text{minutes}$). In addition, because of station clustering, relocation between adjacent stations in actual are replaced to the longest travels among the station clusters.

To check how clustering the stations affects the results, we developed another low-speed autonomous relocation model without station clustering. Because of the absence of relocation staff variables, the without-clustering model would be not as computationally expensive as the original model.

We replaced the variable $\tilde{r}$ representing relocation between clusters with $\bar{r}$ between stations and modified the parameter $\tilde{RC}$ for the relocating costs between clusters to $\bar{RC}$ between stations.

[*9] (The number of satisfied demands) / (the number of vehicles used + the number of relocations) stays at almost the same level, which indicates that the relocating vehicles works efficiently in high demand situation.

– $\bar{\mathrm{RC}}_{jl}$: relocating costs from station j to station l
– $\tilde{r}_{jlt} \in \mathbb{Z}^+,\ j,l \in \boldsymbol{J}, j \neq l,\ t \in \boldsymbol{T}$: number of vehicle(s) relocating from station j to station l departing during time interval t

The new formulation is as follows.

$$\text{maximize} \quad \sum_{i \in \boldsymbol{I}} \mathrm{P}_i z_i - \sum_{\substack{j,l \in \boldsymbol{J} \\ j \neq l}} \sum_{t \in \boldsymbol{T}} \bar{\mathrm{RC}}_{jl} \bar{r}_{jlt} \tag{27}$$

$$\begin{aligned} n_{jt+1} = n_{jt} &- \sum_{\substack{i:\mathrm{origin}_i = j \\ \mathrm{start}_i = t}} z_i - \sum_{l \in \boldsymbol{J} \setminus \{j\}} r_{jlt} \\ &+ \sum_{\substack{i:\mathrm{dest}_i = j \\ \mathrm{end}_i = t}} z_i + \sum_{l \in \boldsymbol{J} \setminus \{j\}} r_{ljt} \\ & \qquad \forall j \in \boldsymbol{J}, t \in \{0, ..., t_{\mathrm{last}-1}\} \end{aligned} \tag{28}$$

$$n_{jt} \geq \sum_{\substack{i:\mathrm{origin}_i = j \\ \mathrm{start}_i = t}} z_i + \sum_{l \in \boldsymbol{J} \setminus \{j\}} \bar{r}_{jlt} \qquad \forall j \in \boldsymbol{J}, t \in \boldsymbol{T} \tag{29}$$

The objective function (27) maximizes the profit, i.e., the revenue from satisfied trips minus the cost of autonomous relocation. The flow conservation equations for vehicles (28) replace (25), and constraints for vehicle status (29) replace (3). Constraints (26) are removed because relocation is represented by $\bar{r}$ without clusters.

Table 4 and Fig. 18 show the optimization results for with- and without- clustering models at 1.5-times potential demand. The numbers are averages of 50 runs with different demand samples. Each calculation took less than 10 seconds per 50 runs, as fast as expected.

For super slow relocation, the without-clustering cases have more relocations than the with-clustering cases, which supports the assumption that the station clustering prevents relocation between adjacent stations. Even in this case, removing station clustering increases the satisfied demand ratio to almost 99% by exploiting vehicles as much as in the walking speed case. In addition, the ratio of (parking + 2×relocation) to vehicle rate increased to the same level as in the walking speed case, which suggests that autonomous relocation sufficiently expands the parking capacity. This feature, virtual parking increase, especially suits the last-mile mobility scenario with many small stations scattered all over a city. Finally, Fig. 19 shows the results for one-day operation of autonomous relocation at 50 times the trip duration (super low) without station clustering.

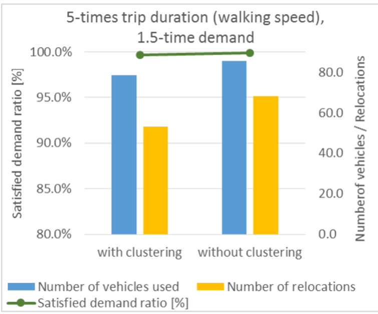
Table. 4: Autonomous relocation with and without clustering at 1.5-times demand

(a) Five-times trip duration (walking speed)

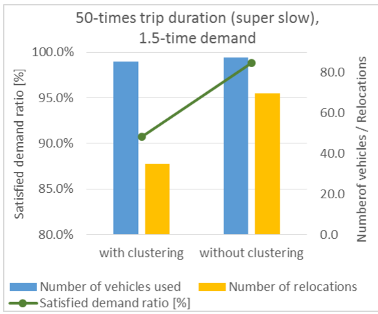
clustering	Yes	No
Satisfied demand ratio [%]	99.7%	99.9%
Number of vehicles used	78.4	85.5
Number of relocations	53.1	68.3
Relocation / Vehicle	0.68	0.80
(Parking + 2×Relocation) / Vehicle	4.75	4.71
Computation time [sec]	0.66	7.44

(b) Trip duration 50 times (super slow)

clustering	Yes	No
Satisfied demand ratio [%]	90.7%	98.8%
Number of vehicles used	85.5	87.4
Number of relocations	34.8	69.7
Relocation / Vehicle	0.41	0.80
(Parking + 2×Relocation) / Vehicle	3.93	4.64
Computation time [sec]	0.35	6.32



(a) 5-times trip duration (walking speed)



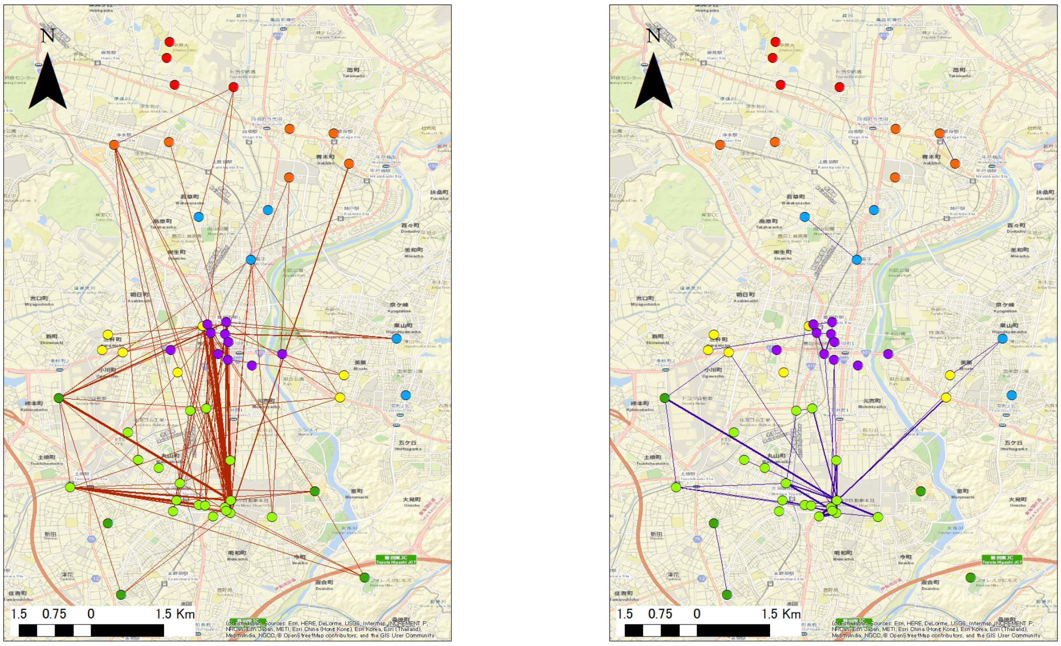
(b) 50-times trip duration (super slow)

Fig. 18: Autonomous relocation with and without station clustering at 1.5-times demand

5 Conclusion

We developed an optimization model specialized for last-mile mobility, short-trip one-way carsharing. The model reflected characteristics of last-mile mobility that requires both vehicles and parking spaces to be reserved and vehicles to be relocated by using a team in staff vehicles. In addition, we improved the station clustering algorithm by reducing its computational complexity.

We applied the developed model to three different cases, in combination with the improved station clustering. The first case, Ha:mo RIDE, revealed that the characteristic short trips did not generate enough



(a) Served trips **(b)** Autonomous relocation

Fig. 19: Diagram of one-day operation in 50-times trip duration (super low) autonomous relocation without station clustering at 1.5-times demand

additional revenue to cover relocation costs. The optimal solution to maximize the operator's profit was cutting relocation personnel costs, which would result in dissatisfaction and long-term customer defection. The operator must relocate vehicles not for profit, but for customer retention. The second case "Premium Service", which guarantees rides in return for price premiums, could not resolve the issue; low demand for the new service led to not enough premiums to cover relocation costs, while high demand was infeasible due to the shortage of resources even assuming whole-day demand given in advance. The third case, autonomous relocation by low-speed self-driving, showed the prospect for low-cost operation together with high demand satisfaction. Even driving at 1/10th of walking speed satisfied almost 1.5 times the demand that can be expected in actual use.

Though we assumed that autonomous relocation was available among all stations, this assumption would face challenges, such as regulations and different road environments, in an actual implementation. To demonstrate the effect of autonomous relocation, the first step should be a small-scale field test in a limited area, like a small valet parking. The 5th Science and Technology Basic Plan of the Japanese government [2] calls for implementations in specific areas such as the Tsukuba Mobility Robot Experimental Zone.

We assumed the demand for the whole day was given at the beginning of the day, but actual demand is at earliest 30 minutes before a ride. In addition, we generated demand samples from the 1-year average demand and ignored the deviations. Incorporating a mechanism for dealing with short-notice demand with deviations will be one of the biggest challenges facing an actual implementation. One idea to deal with it is combining long-term estimation and short-term modification. We observed substantial deviations in seasonal demand. Perhaps previous uses such as commuting in the morning can explain the trend. Or perhaps weather, day of the week, or other events can be explanatory variables.

Another research idea is adaptive pricing, which has been studied in Ha:mo RIDE but is not implemented yet. Appropriate pricing in combination with autonomous relocation can balance demand for vehicles and parking space supply and increase profit.

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