# The use of FLAC for the seismic evaluation of a concrete gravity dam including dam-watersediments-foundation rock interaction

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ABSTRACT: Seismic safety of 65m-high Licodia Eubea gravity dam located in Southeastern Sicily (Italy) has been assessed by means of advanced dynamic analyses of the tallest cross-section of the dam. The analyses were performed with two-dimensional, plane strain, finite difference FLAC code, taking into consideration simultaneously the dam-water-sediments-foundation interaction. The analyses have been carried out for an earthquake scenario corresponding to the Collapse Limit State (CLS), using a set of seven natural accelerograms for the simulations (both vertical and horizontal components). The physical and mechanical parameters of the concrete and foundation rock were obtained from in situ and laboratory tests campaigns.

A validation of the dynamic model in terms of fundamental mode vibration periods of the dam was carried out first. Then, linear analyses allowed to understand whether nonlinear analyses were necessary. From nonlinear analyses results, Licodia Eubea dam has been found to have an acceptable margin of safety for CLS. The implementation of nonlinearity at the dam-foundation interface reduced the tensile stress within the structure. The dam might experience limited sliding along its base, but its structural integrity would be preserved.

### 1 Introduction

Starting from the early 1980s, problems related to the seismic assessment of existing concrete dams have received considerable attention [1,2]. The literature has highlighted the importance of considering dam-water interaction, with water compressibility and reservoir bottom absorption, and dam-foundation interaction taking into account rock deformability. The use of advanced analyses as compared to standard methods based on simplifying assumptions (massless foundation rock and incompressibility of water) has demonstrated that these latter can lead to unreliable results, either on the conservative or unconservative side [3].

The FLAC code [4] is primarily used for static and dynamic analysis of soil and rock media. It employs a 2-D finite difference plane strain formulation and solves the dynamic stress-strain problem with explicit time-stepping procedure. The use of FLAC for the seismic analysis of concrete gravity dams has been rather limited [5,6]. Further, in these studies water is usually modelled as nodal masses by Westergaard approach and rarely [7] as a continuum.

This paper discusses the seismic safety assessment of an existing concrete gravity dam located in Southern Italy using FLAC and illustrates the features of the numerical model developed that take into account simultaneously the interaction the dam-reservoir-sediment-foundation rock interaction. Particular attention is devoted to the validation of the dynamic model.

# 2 Licodia Eubea dam, regional seismicity and seismic action

Licodia Eubea dam is a straight gravity dam (Figure 1a) located on the Dirillo River in the province of Catania (South-Eastern Sicily), owned and operated by the Raffineria di Gela (ENI). The construction of the dam started in 1960 and was completed in 1961. The dam impounds a reservoir with a capacity of 20.4·10<sup>6</sup> m<sup>3</sup>. The dam has a maximum height of 65 m

and a crest length of 327 m. It consists of 19 monoliths that are separated through joints along vertical planes, each monolith being about 15 m length. Figure 1b shows the cross-section of the tallest monolith of the dam. The maximum operating level is 328 m a.s.l. The dam is founded on calcareous rocks intercalated with marly-clayey layers. Geological surveys revealed the presence of bedding planes, slightly dipping upstream (inclination of about 8°) and along with sliding could occur. Prior to construction of the dam, a grout curtain was installed mainly to control seepage.

Two structural and geotechnical campaigns were carried out in 2001 and 2015 which allowed to thoroughly characterize concrete and foundation rock materials by means of in situ investigation and laboratory testing. Selected cores of concrete and rock were tested to determine physical and mechanical properties of interest (e.g., unconfined static compressive strength, modulus of elasticity, compressive wave velocity). Geophysical tests were also conducted both within the dam body (down-hole from the crest to the contact dam-foundation and a sonic tomography between upstream and downstream faces) and at the bedrock (down-holes, MASW, seismic refractions surveys) to obtain shear and compressional wave velocities.

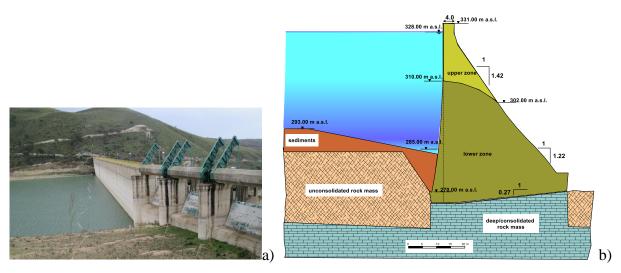


Figure 1: Picture of Licodia Eubea gravity dam (a) and maximum cross-section (b).

Southeastern Sicily is one of the most seismically active areas of Italy. The entire region was struck by high intensity (MCS I0=X-XI) historical earthquakes (e.g., 1169, 1542 and 1693 A.D.). Occasionally the area has experienced infrequent and moderate earthquake (e.g. 1991 EQ). A seismotectonic framework was defined by consulting the Database of Seismogenic Sources [8], supplemented by recent seismotectonic studies. Two tectonic features were concluded to be the most critical for the seismic hazard at the site, both located near the dam site. The maximum magnitude of these two faults was estimated to be 6.4 and 6.8, and the distance 13 and 5 km respectively. Seismic action was developed as 5% damping deterministic response spectrum by averaging the predictions of five well-accepted GMPEs (Ground Motion Prediction Equations) for rock outcropping condition and flat topography. The 84th percentile (mean+σ) spectrum estimate was compared with the spectrum derived from National code (NTC08) under the same conditions, for a return period of 1460 years (Collapse Limit State, CLS). The NTC08 spectrum was assumed as target spectrum as it was higher than response spectrum from GMPEs. The target PGAs on rock outcropping were 0.433g and 0.385g for the horizontal and vertical components, respectively. Seven records were then selected from web databases, characterized by magnitude and distance similar to those of the earthquake scenarios. The accelerograms were scaled such that their average horizontal spectrum reasonably matched the corresponding target response spectrum.

# 3 Numerical model and assumptions

The 2-D numerical model of the highest monolith of the dam with the foundation rock, the impounding water and the bottom sediments is shown in Figure 2. The model consists of about 24.000 quadrilateral zones. The reservoir length is assumed about three times the reservoir depth (i.e. 150 m from the crest dam) in the upstream direction; in the downstream direction the model was extended 50 m from the toe of the dam. For the full reservoir conditions, the maximum operating level of 328 m a.s.l. is considered for the simulations. The depth of the foundation rock is taken as 93 m, slightly higher than the dam height. The maximum thickness of the sediments at the reservoir bottom is assumed 8 m, according to available measurements. The quadrilateral zones are sized to transmit all frequencies of significance to the dam response, up to 20 Hz. A coarser mesh has been used for rock foundation and the dam (minimum size of zones equal to 3 m) whereas a very finely discretized grid has been employed for the sediment material of much lower stiffness (minimum size of zones 0.8 m), so as to ensure realistic representation of the propagating wavelengths. For water zones, the minimum size was assumed to about 3.0 m.

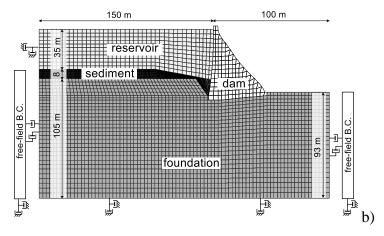


Figure 2: Finite difference model of the dam-water-sediments-foundation rock system for Licodia Eubea concrete gravity dam.

The numerical procedure consists of two steps: 1) initial static analysis of the dam-foundation rock system; 2) dynamic analysis of the dam-water-sediments-foundation rock system performed by linear and nonlinear analyses; this step was preceded by a validation of the dynamic response under elastic conditions.

The static analysis (step 1) aimed at computing the response of the dam-foundation rock system to the self-weight of the dam and to hydrostatic forces. Unit weight of concrete, rock mass and sediment material are assumed equal to 23.4, 23.0 and 15.7 kN/m³, respectively. Static linear elastic properties have been selected, within the range of values obtained from test results, such as the computed displacements matched measured ones during variations of the water levels; the measured displacements were appropriately adjusted for the displacements induced by thermal variations. Specifically  $E_{st}$ =21 GPa and 23 GPa have been assumed in the upper and lower parts of the dam body, respectively and Poisson's ratio  $\nu$ =0.3. The elastic properties of the rock base have been differentiated between the consolidated/deeper ( $E_{st}$ =3500 MPa) and surficial rock mass ( $E_{st}$ =1600 MPa) while the same Poisson's ratio is assumed ( $\nu$ =0.15). For the foundation rock mass an ubiquitous-joint elasto-plastic model was employed in the nonlinear analyses to take into account the presence of weak planes (bedding joints). The strength parameters of the rock mass are assumed 8 kPa and 42° for cohesion c' and friction angle  $\phi$ ', respectively, along a generic plane; according to direct shear tests on the clay bedding layers, it was assumed c'=0.08 MPa and  $\phi$ '=27.7° along the bedding planes. The average

compressive strength of the concrete has been determined as  $f_c$ =16.5 MPa; the tensile strength has been estimated based on empirical correlations resulting  $f_t$  =1.25 MPa. Interface elements were introduced along the dam base and the Mohr-Coulomb failure criterion was used to specify the shear strength at the contact interface. Specifically, the concrete-rock interface was assigned a friction angle of 42° and 0.8 MPa cohesion.

For the dynamic analyses, the boundary conditions are as follows: (i) at the bottom boundary, viscous dashpots are placed in order to take into account radiation damping [11]; (ii) "free-field" conditions are ensured at the two lateral boundaries through appropriate kinematic constraints, thus reproducing the "shear beam" type of motion produced by in-plane vertically incident SV waves. The input motion is applied at the bottom boundary in terms of shear and normal stresses time-histories, in order to simulate a transmitting base.

The model for dynamic simulations was set up according to available geological, geophysical and geotechnical data. The dam body was divided into two zones (Figure 1b), an upper one extending approximately from the crest to mid height of the dam and a lower one going down to the bottom of the dam, this latter part being slightly stiffer than the former. Analogously, for the foundation rock two zones (Figure 2) have been considered, i.e. a deeper rock mass stiffer than the surficial one, affected by weathering and jointing condition; however, the surficial rock mass immediately below the foundation was assumed as stiff as the deep rock due to grouting works carried out during construction. The bottom sediments, that essentially consist of silty-clayey soils, have been assigned physical and mechanical values from literature studies.

For linear analyses, a visco-elastic medium was assumed for both the body of the dam and the foundation rock. A perfect adhesion at concrete-rock interface was considered. Interface elements simulated the dam-water and the foundation-water contacts; these elements are characterized by null shear resistance and normal tensile strength equal to cavitation threshold. The water was treated as a compressible fluid ( $K_w$ = $2\cdot10^9$  Pa), with a negligible shear modulus (G=10 Pa) to avoid numerical instabilities, that produces hydrodynamic pressures that are dependent on the excitation frequency. Dynamic elastic modulus ( $E_{dyn}$ ) and Poisson's ratio ( $\nu$ ) were estimated using compressional and shear wave velocities from geophysical surveys. Damping ratio (D) assumed for concrete is 8%, which is in the range suggested for dynamic analyses of dams for high seismic loadings. For rock damping ratio was assumed equal to 1.0% and 0.5% for surficial and deep/consolidated rock, respectively. Damping ratio was taken according to the one-control frequency Rayleigh formulation, calculated as the mean value between the fundamental frequency of the dynamic system and the predominant frequency of the input motion. The main dynamic properties are listed in Table 1.

Material	$\gamma (kN/m^3)$	Vs (m/s)	Vp (m/s)	Edyn (GPa)	ν	D (%)
Dam – upper zone	23.4	2020	3800	25.6	0.3	8
Dam – lower zone	23.4	2140	4000	28.4	0.3	8
Foundation – sup.	23	600	1500	2.4	0.4	1
Foundation - deep	23	900	2200	5.3	0.4	0.5
Sediments	15.7	150	1440	0.1	0.495	2.5

Table 1: Mean material properties assumed for dynamic analyses.

In step 2, the verification of the dynamic model in the linear range was carried out first. The step 2 procedure then envisaged linear and nonlinear analyses and the safety evaluation of Licodia Eubea dam has been performed according to [9] guidelines.

In dynamic nonlinear analyses, the nonlinearity was restricted to sliding along the damfoundation contact and in the foundation rock. Mohr-Coulomb failure envelopes were used for the rock mass, both for the generic planes and for the weak orientation, as well at the concretefoundation rock interface, described by the strength parameters mentioned for the static model.

# 4 Validation of the dynamic model

The capability of FLAC to reproduce hydrodynamic forces on the dam was first investigated under assumptions similar to Westergaard procedure. To this aim, an increasing acceleration up to a constant value (Figure 3a) was applied as rigid input motion to the dam-rock mass system; no bottom sediments have been assumed. The maximum hydrodynamic pressure on upstream dam face calculated by FLAC is compared with that corresponding from Westergaard method, along with the hydrostatic pressure, in Figure 3b.

Prior to conducting the seismic analyses, the fundamental vibration periods of the dam calculated by FLAC were also verified to be consistent with those estimated by using the simplified procedure developed by [10]. To this aim, the model has been excited by a high-frequency Ricker pulse having central frequency  $f_E=6$  Hz ( $f_E=0.166$  s) and acceleration amplitude of 0.5g. Acceleration time-history is shown in Figure 3 along with the corresponding Fourier spectrum. The predominant frequency of the wavelet was chosen so as to maximize the response of the system. Due to its narrow–band nature the Ricker pulse is considered as particularly appropriate for bringing the frequency response trends to light. All the results refer to elastic analyses with a damping ratio of 0.5, 1 and 2.5% for the deep/consolidated rock mass, the dam and superficial rock mass, and bottom sediments, respectively.

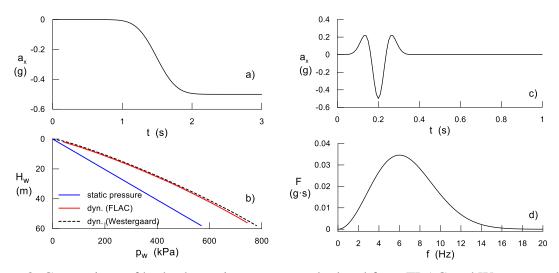


Figure 3: Comparison of hydrodynamic pressures calculated from FLAC and Westergaard method: input motion acceleration time-history (a); water pressures profiles on the upstream face of the dam (b); acceleration time-history (c) and Fourier spectrum (d) of the excitation Ricker wavelet with central frequency  $f_E = 6$  Hz.

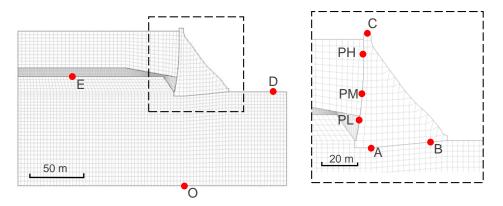


Figure 4: Mesh of the numerical model with location of control points

The response of the system is presented in terms of time histories of acceleration in representative points of the system as illustrated in Figure 4. Selected points are: O (base of the model), D (outcropping of the rock formation at a certain distance from the dam), E (sediments-rock interface), C (crest of the dam), A and B (dam-foundation rock interface). Fourier spectral ratio, that is the ratio of the amplitudes of the acceleration Fourier spectrum at the crest of the dam with respect to the outcrop rock motion or other control point, are also presented. In the following are examined separately the cases of dam on rigid/deformable foundation rock

In the following are examined separately the cases of dam on rigid/deformable foundation rock with empty/full reservoir and bottom sediments. The rigid rock condition has been simulated through an increase of the rock stiffness by a factor of 10 with respect to the real stiffness.

### 4.1 Dam on rigid foundation rock and empty reservoir

In case of rigid foundation, there is no interaction between the dam and the rock mass. This fact is evident by comparing the results of acceleration time histories at points A, B, D and E (Figure 5a) which are very similar irrespective of their position, at the ground surface (points D and E) or at the base of the dam (points A and B). On the contrary, the motion at the crest of the dam is significantly amplified as the ratio between the maximum crest acceleration and the maximum outcrop acceleration is around 4.5. The motion at the crest of the dam also shows sustained slightly-damped free oscillations after the first peak, which confirms the tendency of the vibration energy to remain within the dam. The crest-to-outcrop Fourier spectral ratio (Figure 5b) shows a peak at  $f_0$ =6.3 Hz ( $T_0$ =0.16 s) in correspondence of the first vibration mode; the spectral ratio at the fundamental frequency is  $r_{\rm C}/v_{\rm outcrop} \approx 14$ . Control points on the upstream face of the dam show lower spectral amplification but same fundamental frequency. In Figure 5a (bottom plot) the comparison between the time histories at the nodes D and E and the outcropping motion, which is twice the motion incident at the base of the model. The calculated motion at D and E reproduces very satisfactorily the outcropping motion. This is more evident for point E, that is farther away from the dam base than point D, and therefore free-field conditions are better simulated.

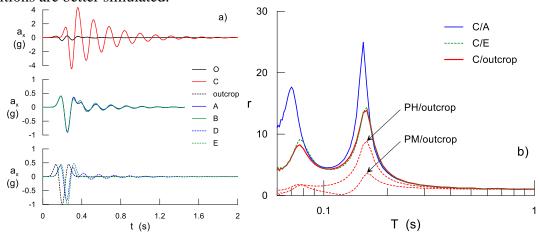


Figure 5: Rigid foundation and empty reservoir: acceleration time-histories at selected points (a) and Fourier spectrum ratios (b)

#### 4.2 Dam on rigid foundation rock, full reservoir and bottom sediments

The presence of the reservoir leads to a faster decay of the amplitude of free oscillations due the increasing damping because of the dam-water-sediments interaction (Figure 6a). This interaction leads to a lengthening of the fundamental period of the system ( $T_0$ =0.20 s  $f_0$ =5.0 Hz,) as illustrated in Figure 6b. This behavior is consistent with the increase of the water mass participating to the vibration of the dam.

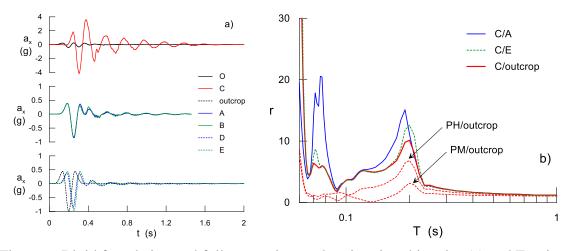


Figure 6: Rigid foundation and full reservoir: acceleration time-histories (a) and Fourier spectral ratios (b) at control points.

### 4.3 Dam on elastic foundation rock and empty reservoir

The presence of a flexible substratum drastically modify the seismic motion at the dam-foundation level, i.e. points A and B (Figure 7a). Overall the motion is rapidly attenuated, as compared with the motion calculated for rigid rock foundation and full or empty reservoir. The most evident effect is the low amplification of the maximum acceleration at the crest of the dam. As expected, the introduction of rock deformability led to an increase of  $T_0$ =0.29 s ( $f_0$ =3.4 Hz). The crest-to-outcrop spectral ratio has a peak of about  $r_{\text{C}/\text{outcrop}} \approx 4.5$ .

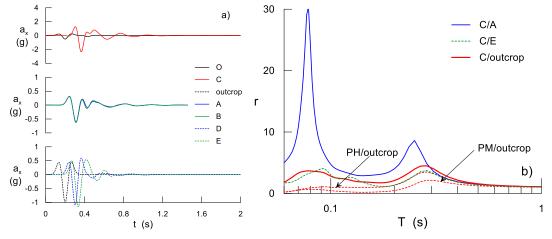


Figure 7: Elastic foundation and empty reservoir: acceleration time-histories (a) and Fourier spectrum ratios (b) at control points.

#### 4.4 Dam on elastic foundation rock, full reservoir and bottom sediments

Taking into account both the deformability of the rock foundation and the water compressibility determines a significant and rapid reduction of free oscillations and a farther lengthening of the period of the system ( $T_0$ =0.33 s,  $f_0$ =3.0 Hz). Unlike the previous cases, it can be noted a clear peak on the crest-to-control point E ratio which can be related to the fundamental frequency of the bottom sediments layer.

In Table 2 the fundamental periods calculated from FLAC are compared with those estimated by [10], considering separately the conditions of flexible and rigid bedrock, full and empty reservoir along with the effect of bottom sediments. A satisfactory agreement can be recognized in all cases.

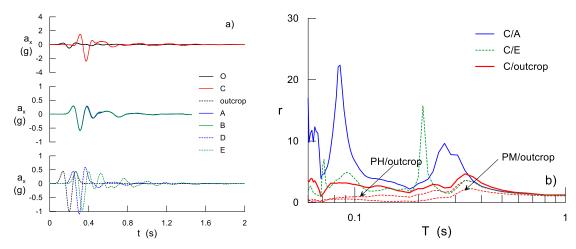


Figure 8: Flexible rock foundation and full reservoir: acceleration time-histories (a) and Fourier spectrum ratios (b) at control points

Table 2: Calculated (from FLAC) and estimated (from [10]) fundamental mode properties.

foundation	reservoir	T <sub>0</sub> (s) FLAC	f <sub>0</sub> (Hz) FLAC	T <sub>0</sub> (s) [10]	f <sub>0</sub> (Hz) [10]
rigid	empty	0.16	6.3	0.15	6.62
rigid	full	0.20	5.0	0.18	5.50
elastic	empty	0.29	3.4	0.25	3.98
elastic	full	0.33	3.0	0.30	3.29

## 5 Linear and nonlinear analyses

The methodology for seismic performance assessment and qualitative damage estimation was employed using results of linear and nonlinear analyses, according to the USACE (2007) guidelines. Linear analyses were conducted first and performance indices were calculated, such as the stress-demand capacity ratio DCR (i.e. the ratio of the calculated maximum principal stress to tensile strength of the concrete), the cumulative inelastic duration CD (defined as the total duration of stress excursions that exceeds a certain level of demand-capacity ratio) and the spatial extent of overstressed regions. These performance indices were used to establish a range of validity for linear analyses and to discriminate if nonlinear analyses were to be conducted.

#### **5.1** Linear analyses

Figure 9 show the contour lines of the maximum temporal value (upper envelope) of the principal tensile stress for two acceleration time histories for which most critical results are obtained. These plots indicate that higher tensile stresses generally develop at the heel and at the toe of the dam as well as near the changes of slopes of upstream and downstream faces. The largest values usually occur at the heel of the dam, where the allowable tensile strength is exceeded in localized areas. Compressive stresses in the concrete remained well below allowable values. Figure 10a shows the dam section overstressed areas as a function of DCR. The percentage of overstressed area falls well below the acceptance limit for the whole set of accelerograms. Figure 10b compare cumulative duration of stress cycles with the acceptance curve for zone A located at the heel of the dam (see Figure 9). It is evident that the cumulative duration curves are higher than 0.3 at DCR=1 for all time-histories while for DCR>1.2 most curves are generally within the zone of acceptable performance. It therefore results that the tensile strength is exceeded but only in very localized areas and the global consequences of the resulting damage are expected to be minor. Anyway, it was decided to conduct nonlinear analyses.

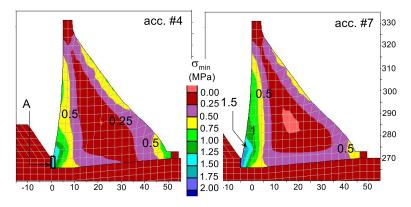


Figure 9: Contours of principal tensile stress envelopes in the dam body for the two most severe input motions

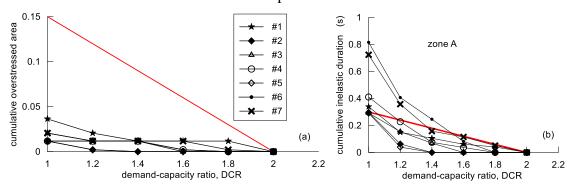


Figure 10: Performance curves for concrete gravity dams and results of linear analyses.

#### 5.2 Nonlinear analyses

The contours of principal tensile stress envelope obtained from the most severe nonlinear analysis are plotted in Figure 11a. It is evident that the values of principal tensile stresses decrease as compared to those from linear analyses. Further, no value exceeds the tensile strength of the concrete as the maximum tensile stress is 0.5 MPa, about one third of the maximum value calculated from linear analyses. The nonlinear analysis demonstrate therefore that no cracking will be forming in the body of the dam during strong ground shaking. Maximum shear strain contours are plotted in Figure 11b; this figure clearly indicates that sliding occurs towards downstream along a sliding band parallel at the contact dam-foundation, with maximum shear strains values of about 1.5%. The extension of the sliding band few meters inside the rock mass is consistent with a failure not concentrated on the foundation plane but developed on the weakest planes along the bedding orientation.

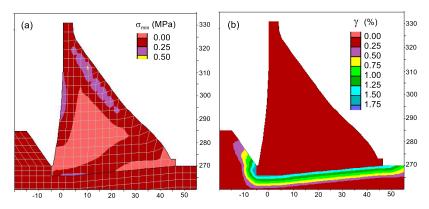


Figure 11: Nonlinear analyses results for the most severe input motion: contours of (a) principal tensile stress envelopes and (b) maximum shear strains.

### 6 Conclusions

The seismic performance of 65-m high Licodia Eubea concrete gravity dam under Collapse Limit State (CLS) earthquake scenario was investigated by advanced numerical analyses taking into account simultaneously the dam-water-sediments-foundation rock interaction. Overall seven spectrum-compatible acceleration time-histories were selected to represent the CLS scenario earthquake. The finite-difference code FLAC resulted to be effective in modelling the frequency-dependent response of the entire system, as evident from the comparison between the fundamental mode periods calculated under different hypothesis and those estimated by simplified relationships. It's worth noting that few examples do exist in the literature of seismic analyses of concrete dams in which also the reservoir domain is modelled as a continuum and thus also the frequency response of the water thrust on the dam is taken into account.

Numerical modeling comprised linear and nonlinear analyses. The results of linear analyses were used to assess the severity and extent of overstressed regions and to compute the cumulative duration of stress excursions over the strength. Dynamic tensile stresses locally exceeded the allowable tensile strength, mostly close the heel of the dam. The nonlinear analyses were conducted considering nonlinearity at the contact dam-foundation and in the rock mass. This introduced sliding especially along bedding planes in the rock mass that significantly reduced the tensile principal stresses within the dam body. It is concluded that an acceptable margin of safety for the CLS scenario can be considered.

# 7 Acknowledgements

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### 8 References

- [1] Hall J.F., Chopra A.K. (1982). Two-dimensional dynamic analysis of concrete gravity and embankment dams including hydrodynamic effects. Earth. Eng Struct Dynam, 10, 305–32.
- [2] Fenves G., Chopra A.K. (1985). Simplified Earthquake Analysis of Concrete Gravity Dams: Combined Hydrodynamic and Foundation Interaction Effects. Journal of Engineering Mechanics. ASCE, Vol. 111, 6, 736-756.
- [3] Chopra A. (2014). Earthquake analysis of concrete dams: factors to be considered. 10th U.S. Nat. Conf. on Earth. Eng., July 21-25, 2014, Anchorage.
- [4] Itasca (2011). FLAC-Fast Lagrangian Analysis of Continua, Ver. 7.0. User's Guide. Itasca Cons. Group, Minneapolis, USA
- [5] Bureau G. (2006). Use of FLAC for seismic analyses of concrete gravity dams. 4th Int. FLAC Symp. on Numerical Modelling in Geomechanics. Paper 04-01.
- [6] Bureau G., Zhou J., Keller T.O. (2007). Nonlinear structural analysis for San Vicente dam raise. 27th Annual USSD Conference, Philadelphia, March 5-9, 2007, 405-419
- [7] Yildiz E., Gurdil A.F. (2012). Review on seismic design of concrete gravity or RCC dams with design examples. 15 WCEE.
- [8] Basili R., G. Valensise, P. Vannoli, P. Burrato, U. Fracassi, S. Mariano, M.M. Tiberti, E. Boschi (2008). The Database of Individual Seismogenic Sources (DISS), Tecton., 4.
- [9] USACE (2007). Earthquake Design and Evaluation of Concrete Hydraulic Structures. EM 1110-2-6053.
- [10] Løkke A., Chopra A.K. (2013). Response spectrum analysis of concrete gravity dams including da-water-foundation interaction. Pac. Earth. Eng. Res. Center, PEER 2013/17.
- [11] Lysmer J., Kuhlemeyer R.L. (1969). Finite dynamic model for infinite media. J. Eng. Mech. Div. ASCE, 95, EM4: 859-877.