

A cyclostationarity-based classifier for digital linear modulations

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Résumé – Cet article traite de la classification des modulations numériques linéaires. L’algorithme de classification proposé est basé sur les statistiques cycliques du second ordre et d’ordre supérieur du processus d’observation. Des exemples de simulations sont présentées pour montrer l’efficacité de la méthode proposée.

Abstract – The paper deals with the classification of digital linear modulations. The classification algorithm is based on second- and higher-order statistics of the observed process. Simulation examples are presented to show the effectiveness of the proposed method.

1 Introduction

The problem of estimating the modulation format of an incoming signal corrupted by noise and interference plays an important role in several applications for both civil and military purposes, such as monitoring, surveillance, adoption of an appropriate warfare strategy, etc.

The modulation classification problem becomes very complicated in the presence of a large number of alternative modulations. In such a case, it can be appropriate to design the classifier by resorting to a decision-tree structure that is based on sub-classifiers that operate on subsets of possible alternatives. Such an approach, allows one to achieve computational savings with respect to the one-step classifier. Moreover, the design of each elementary classifier is often very simple and the modularity of the overall structure provides flexibility and simplifies its updating aimed at improving the performances as well as introducing new modulation formats among the possible alternatives.

The classification problem under consideration is frequently solved by extracting from the received signal some discriminating features. Among the features considered in the open literature, an outstanding position is held by those based on the cyclostationarity properties of the received signal, whose utilization in modulation format recognition was first proposed in [1] and, subsequently, exploited in several papers [2]-[5]. The main advantage of these features is that they are intrinsically tolerant to the presence of noise and spectrally and temporally overlapping interfering signals, provided that the observation time is sufficiently long and, moreover, at least a cycle frequency of the signal to be classified is not shared with the undesired signals.

In the present paper, we consider as features the cumulants of the symbol sequence, which were already recog-

nized [6] to be effective for modulation classification. In the paper, however, the considered features are estimated by exploiting the second- and higher-order cyclostationarity of the received signal. The proposed classifier operates on linear memoryless modulations and turns out to be effective in severe noise and interference conditions; moreover, it is flexible and adequate to work in a decision-tree structure. Therefore, it can be utilized as a preliminary classifier for partitioning various digital modulation formats into subclasses (e.g., for discriminating between one- and two-dimensional signal constellations), but may also be adopted within each subclass to determine the exact modulation format.

2 The proposed classifier

The down-converted received signal can be written as:

$$\begin{aligned} r(t) &= As(t) + i(t) \\ &= A \exp[j(\phi + 2\pi f_0 t)] \sum_{\ell=-\infty}^{+\infty} a_\ell p(t - \ell T + t_0) + i(t) \quad (1) \end{aligned}$$

where $A \geq 0$ represents the unknown (but nonrandom) channel attenuation, $p(t)$ is the known baseband pulse, T is the known symbol period, f_0 is the known residual carrier frequency or frequency offset, t_0 and ϕ are unknown (but nonrandom) and $\{a_\ell\}$ is a complex-valued zero-mean and i.i.d. symbol sequence normalized so that its variance is unity. The zero-mean additive disturbance $i(t)$ includes thermal noise and possible interfering signals and is independent of the signal $s(t)$, which is the signal to be classified.

Let us define $s(n) \triangleq s(nT_s)$ with T_s the sampling period. Under the above-mentioned assumptions, it can be shown [7] that the N th-order temporal cumulant function

of $s(n)$ is given by

$$\begin{aligned} C_s(n; \mathbf{m})_N &\triangleq \text{cum}[s^{(*)1}(n+m_1), s^{(*)2}(n+m_2), \dots, \\ &\quad s^{(*)N-1}(n+m_{N-1}), s^{(*)N}(n)] \\ &= c_{N,I} \exp[j(\phi_0 + 2\pi\nu_0(N-2I)n)] \sum_{\ell=-\infty}^{\infty} g(nT_s - \ell T; \mathbf{m}) \end{aligned} \quad (2)$$

where the superscript $(*)_h$ ($h = 1, \dots, N$) denotes optional conjugation, $c_{N,I} \triangleq \text{cum}[a_\ell^{(*)1}, a_\ell^{(*)2}, \dots, a_\ell^{(*)N}]$ with I the total number of conjugations, $\mathbf{m} \triangleq (m_1, m_2, \dots, m_{N-1})$, $\nu_0 = f_0 T_s$,

$$g(t; \mathbf{m}) \triangleq p^{(*)N}(t+t_0) \prod_{i=1}^{N-1} p^{(*)i}(t+t_0+m_i T_s) \quad (3)$$

$$\begin{aligned} \phi_0 &\triangleq (N-2I)\phi + 2\pi\nu_0[(\pm)_1 m_1 + (\pm)_2 m_2 + \dots \\ &\quad + (\pm)_{N-1} m_{N-1}] \end{aligned} \quad (4)$$

where $(\pm)_h$ is 1 in the absence and -1 in the presence of conjugation on the h th term.

By expanding in Fourier series the term $\sum_{\ell=-\infty}^{\infty} g(t - \ell T; \mathbf{m})$, it can be shown that the reduced-dimension cyclic temporal cumulant function of $s(n)$ is given by

$$\begin{aligned} C_s^\alpha(\mathbf{m})_N &\triangleq \langle C_s(n; \mathbf{m})_N \exp(-j2\pi\alpha n) \rangle = c_{N,I} \exp(j\phi_0) \\ &\times \sum_{\ell=-\infty}^{\infty} c_p^{(N)}(\ell; \mathbf{m}) \exp(j2\pi\ell t_0/T) \delta[\frac{\ell T_s}{T} + \nu_0(N-2I) - \alpha] \end{aligned} \quad (5)$$

where $\langle \cdot \rangle$ denotes infinite-time average, α is the cycle frequency,

$$\begin{aligned} c_p^{(N)}(\ell; \mathbf{m}) &\triangleq \\ \frac{1}{T} \int_{-\infty}^{+\infty} p^{(*)N}(t) \prod_{h=1}^{N-1} p^{(*)h}(t+m_h T_s) \exp\left(-j2\pi \frac{\ell}{T} t\right) dt \end{aligned} \quad (6)$$

and $\tilde{\delta}(t) = 1$ for t integer and $\tilde{\delta}(t) = 0$ for t noninteger.

Let us assume that the sampling period T_s is chosen so that it exists at least one discrete-time cycle frequency $\frac{kT_s}{T} + \nu_0(N-2I)$ corresponding to an unique continuous-time cycle frequency. Then, from (5) it follows that

$$|C_s^{\frac{kT_s}{T} + \nu_0(N-2I)}(\mathbf{m})_N| = |c_{N,I}| |c_p^{(N)}(k, \mathbf{m})| \quad \forall k \in K \quad (7)$$

where K denotes the nonempty set

$$K \triangleq \left\{ k : \frac{\ell T_s}{T} \neq \left(\frac{kT_s}{T} \right)_{\text{mod } 1} \quad \forall \ell \in L, \ell \neq k \right\} \quad (8)$$

with

$$L \triangleq \left\{ \ell : c_p^{(N)}(\ell, \mathbf{m}) \neq 0 \right\}. \quad (9)$$

With reference to the received signal $r(n)$, the additivity property of the cumulants leads to

$$\begin{aligned} &C_r^{\frac{kT_s}{T} + \nu_0(N-2I)}(\mathbf{m})_N \\ &= A^N C_s^{\frac{kT_s}{T} + \nu_0(N-2I)}(\mathbf{m})_N + C_i^{\frac{kT_s}{T} + \nu_0(N-2I)}(\mathbf{m})_N \\ &= A^N C_s^{\frac{kT_s}{T} + \nu_0(N-2I)}(\mathbf{m})_N \quad \forall k \in K - K_i \end{aligned} \quad (10)$$

where the last equality follows from the definition of K_i as the set of values of k such that kT_s/T is an N th-order cycle frequency of the sampled version of $i(t)$. Therefore, from (7) and (10) it follows that

$$|c_{N,I}| = \frac{|C_r^{\frac{kT_s}{T} + \nu_0(N-2I)}(\mathbf{m})_N|}{A^N |c_p^{(N)}(k, \mathbf{m})|} \quad \forall k \in K - K_i. \quad (11)$$

Let us note that, since $|c_{2,1}| = 1$, from (11), specialized for $N = 2$ and $I = 1$, it follows that

$$A^2 = \frac{|C_r^{\frac{kT_s}{T}}(\mathbf{m})_2|}{|c_p^{(2)}(k, \mathbf{m})|} \quad \forall k \in K - K_i. \quad (12)$$

Such an equation allows one to estimate the unknown channel attenuation A by exploiting only second-order cyclic statistics.

Equations (11) and (12) lead to consistent estimators of $|c_{N,I}|$. Different estimators can be obtained according to different choices for k , \mathbf{m} , I , and N and different estimators for $C_r^{\frac{kT_s}{T} + \nu_0(N-2I)}(\mathbf{m})_N$. Moreover, although not explicitly indicated, both $|C_r^{\frac{kT_s}{T} + \nu_0(N-2I)}(\mathbf{m})_N|$ and $|c_p^{(N)}(k, \mathbf{m})|$ also depend on the position of each optional conjugation but for $\mathbf{m} = \mathbf{0}$. Finally, note that for $N \geq 6$ estimators of the cyclic cumulant $C_r^{\frac{kT_s}{T} + \nu_0(N-2I)}(\mathbf{m})_N$ can be realized only on the basis of knowledge of the lower-order cycle frequencies of the overall disturbance.

Equations (11) and (12) constitute a useful classification method for digital linear modulations, in that different signal constellations exhibit different theoretical values of $|c_{N,I}|$. For example, the proposed algorithm, utilized assuming $N = 2$ and $I = 0$, that is, by exploiting only the second-order cyclostationarity, is able to discriminate one-dimensional from two-dimensional signal constellations with equally likely symbols. Moreover, in the case of $N = 2$ and $I = 0$, if the pulse $p(t)$ is real and the same pair of values (k, \mathbf{m}) is assumed in both (11) and (12), it results that knowledge of $p(t)$ is not required.

The discrimination between one- and two-dimensional constellations can be utilized as a first step in a decision-tree structure for the overall classification. In fact, the discrimination inside the set of one-dimensional constellations and inside the set of two-dimensional constellations can be obtained by utilizing $|c_{N,I}|$ for $N \geq 4$. Note that often $N = 4$ is an adequate choice. However, at least $N = J$ is required to discriminate J -PSK from M -PSK with $M > J$ (theoretically, $|c_{J,0}| = 1$ for J -PSK and $c_{J,0} = 0$ for M -PSK).

The implementation of the proposed classifier requires the comparison of the estimated feature (that is, an estimate of $|c_{N,I}|$) with a set of $N_c - 1$ thresholds, where N_c is the number of classes in which the digital modulated signals have to be partitioned.

Since the estimate of $C_r^{\frac{kT_s}{T} + \nu_0(N-2I)}(\mathbf{m})_N$ can be modeled as a Gaussian-distributed random variable [8], by further assuming that the channel attenuation estimate is practically error-free, the estimate of the discriminating feature provided by (11) can be reasonably modeled as a Rice-distributed random variable, whose variance σ^2 is

unknown, but is largely independent of the considered signal constellation [6]. The mean value, vice versa, depends on the considered constellation.

The threshold $\lambda(\sigma^2)$ and the probability of correct classification $P(\sigma^2)$ are both functions of σ^2 that can be numerically evaluated. Consequently, the threshold setting can be made by fixing a desired value P_d for the probability of correct classification, and determining the value $\sigma_d^2 = P^{-1}(P_d)$, and, finally, the threshold $\lambda_d = \lambda(\sigma_d^2)$. Note that the selected threshold is certainly satisfactory for values of probability of correct classification quite close to P_d . Finally, it is useful to emphasize that both the probability of correct classification and the threshold setting depend on the known *a priori* probability distribution of the candidate signal constellations.

3 Simulation results

To substantiate the effectiveness of the proposed classification algorithm, simulation experiments were carried out. All experiments consider a scenario with three independent signals, in which the signal to be classified is contaminated by a single interfering signal and thermal noise.

In all experiments, the signal to be classified is linearly modulated with $\nu_0 = 0$, $T/T_s = 4$ and a raised-cosine-shaped pulse $p(t)$ using a 50 % excess bandwidth. The interfering signal is a BPSK signal with the same pulse $p(t)$ and the same carrier frequency, but with a symbol period 1.4 times larger. Therefore, in (11) and (12) it is chosen $k = 1$ that leads to work with a cycle frequency not shared with the interfering signal. The selected lag vector is $\mathbf{m} = \mathbf{0}$. The inband signal-to-noise ratio (SNR) is set to 0 dB. The input parameter P_d for the threshold setting procedure (see Section 2) is set to 0.99 and the candidate signal constellations are assumed to be equally likely. All the results are based on 200 Monte Carlo trials for each candidate signal constellation.

In the first experiment, we consider the set { BPSK, 8-PSK } of alternative constellations. In such a case, the second-order cyclostationarity is sufficient to discriminate, in that $|c_{2,0}| = 1$ for BPSK and $c_{2,0} = 0$ for 8-PSK. The threshold resulting from the setting procedure is 0.58.

In Fig. 1, the probability of correct classification is reported as a function of the signal-to-interference ratio (SIR) and for different values of the number of received symbols. The interference-rejection capability of the method is evident. Satisfactory performances are already obtained with 100 symbols also in severe conditions of noise and interference (SIR=SNR=0 dB). A number of 1000 symbols allows one to well discriminate also in very severe interference situations (e.g., SIR=-8dB).

In the second experiment, we consider the case in which the set of constellations to be classified is { BPSK, 4-PSK, 8-PSK }. In such a case, to discriminate 4-PSK from 8-PSK it is not appropriate the second-order statistics c_{20} which is zero for both constellations. Then, the proposed strategy considers a first step in which the estimate of $|c_{2,0}|$ leads to the two sets { BPSK } and { 4-PSK, 8-

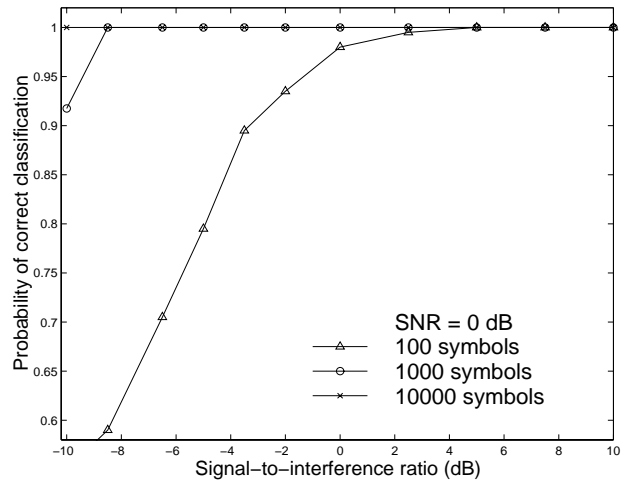


FIG. 1: The performance of the proposed classifier operating on the set { BPSK, 8-PSK }.

PSK } and a second step where the estimate of $|c_{4,0}|$ is used to discriminate between 4-PSK and 8-PSK. The selected thresholds are 0.61 in the first step and 0.58 in the second step. Let us note that the more expensive estimation of the fourth-order cyclic cumulant is performed only after the separation (by the first decision step) of BPSK from { 4-PSK, 8-PSK }. Both the computational complexity reduction and the probability of correct classification increase as the probability of occurrence of BPSK increases.

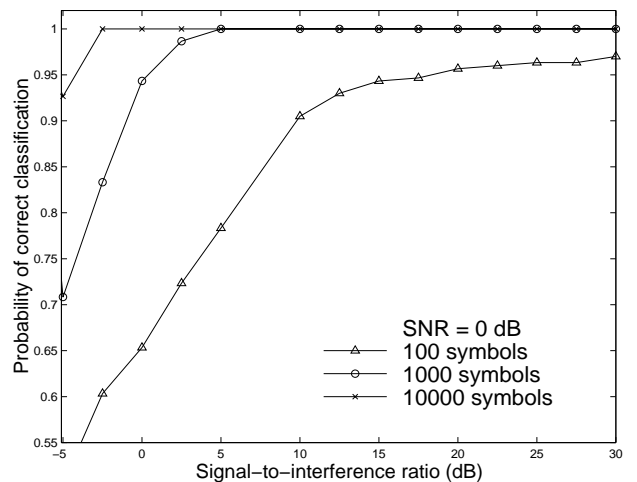


FIG. 2: The performance of the proposed classifier operating on the set { BPSK, 4-PSK, 8-PSK }.

Figure 2 presents the results of the second experiment. In such a case, 100 symbols are sufficient to work well in a severe noise environment (SNR=0dB) only for high values of SIR. However, 1000 symbols provide a very good performance in severe conditions of both noise and interference.

In the last experiment, we consider the case in which the set of constellations to be classified is { BPSK, 4-PSK, 8-PSK, 16-QAM }. In such a situation, at first the BPSK is discriminated from { 4-PSK, 8-PSK, 16-QAM } by the estimate of $|c_{2,0}|$ (theoretically, $|c_{2,0}| = 1$ for BPSK and

$c_{2,0} = 0$ for all others). Then, in a second step, we utilize the estimate of $|c_{4,0}|$ to discriminate inside the set $\{4\text{-PSK}, 8\text{-PSK}, 16\text{-QAM}\}$, in that theoretically $|c_{4,0}| = 1$ for 4-PSK, $c_{4,0} = 0$ for 8-PSK, and, finally, $|c_{4,0}| = 0.68$ for 16-QAM. The selected thresholds are 0.63 in the first step and 0.32 and 0.84 in the second step.

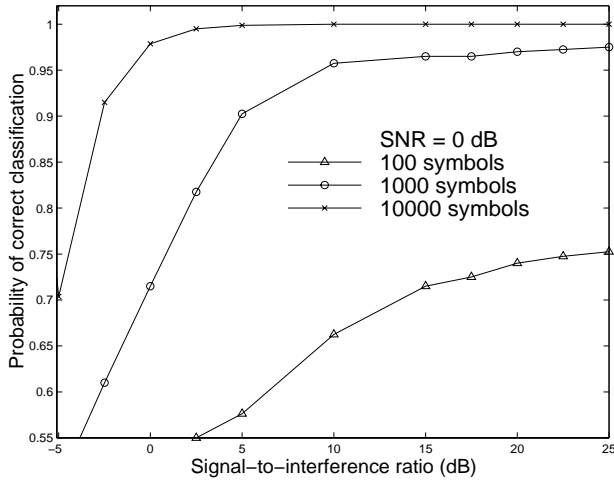


FIG. 3: The performance of the proposed classifier operating on the set $\{ \text{BPSK}, 4\text{-PSK}, 8\text{-PSK}, 16\text{-QAM} \}$.

Figure 3 shows the results of the third experiment. They confirm that also in severe conditions of noise and interference a few thousand symbols are required to obtain very good performances.

4 Conclusions

A new method for classifying linear memoryless digital modulations has been presented. The considered discriminating features are the cumulants of the symbol sequence. The utilization of these features leads to a simple and flexible design of the overall classifier for the considered set of alternative signal constellations. The main contribution of the paper lies in the method for estimating the considered features that exploits the second- and higher-order cyclostationarity properties of the received signals. As a consequence, the proposed classifier turns out to be effective

in severe noise and interference environments, as is confirmed by the results of the reported simulation experiments.

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