Orientation Based Second-Order Statistics for Texture Description

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Résumé – Nous nous intéressons dans cet article à la description de textures directionnelles et nous proposons une nouvelle approche, fondée sur les statistiques d'ordre 2 d'un champ d'orientations. Deux des principales propriétés haut-niveau conditionnant la perception des textures sont abordées : la directionnalité et la périodicité. Notre méthode se fonde sur la mesure de ressemblance entre deux orientations distantes d'un déplacement donné. Deux implantations de la mesure de ressemblance sont présentées. La première est la *Carte bidimensionnelle de Ressemblance d'Orientations* ; elle peut être décrite en termes de lignes et de points qui reflètent la périodicité ou la symétrie de la texture. La seconde implantation consiste en une *Fonction Curviligne de Ressemblance d'Orientations* qui fournit une mesure de la ressemblance moyenne entre deux orientations le long d'une courbe paramétrée et révèle une périodicité éventuelle. Les résultats expérimentaux sur textures synthétiques et naturelles montrent la pertinence de notre approche pour une description haut-niveau de la texture.

Abstract – This paper focuses on directional textures and proposes a new statistical approach for the second order description of an orientation vector field. We tackle two of the main high level properties that drive the perceptual grouping of texture: directionality and periodicity. Our method is based on the measure of similarity between two orientation vectors separated by a given displacement. We propose two implementations of the similarity measure. The first one is a 2-D *Orientation Similarity Map* which can be described in terms of blobs and lines that reflect texture periodicity and symmetry. The second implementation consists in a *Curvilinear Orientation Similarity Function* which provides a measure of the average resemblance between orientation vectors along a parametrized curve and brings out periodicity. We also provide experiments on both synthetic and natural images which show the relevance of our approach for the high level description of textures.

1 Introduction

Texture is undoubtedly one of the most important features used in image analysis. It arises, either randomly or deterministically, from the repetition of local patterns.

Statistical approaches and more particularly second order statistics have been widely reported in literature and have proved through years to be efficient for texture classification or segmentation. Textural features such as those based on cooccurrence matrices [8] or grey level difference histograms [3] provide measures of semantic notions as contrast, coarseness, randomness or directionality.

Among the high level features which drive the perceptual grouping of textures, three of them have been identified as fundamental [11]: directionality, periodicity and structural complexity. Unfortunately, the extraction of such features often rely on the distribution of pixel intensity values [1][3]. Several authors [6][7] brought out the fact that grey level based statistical methods are not suited to structural textures. These textures arise from an arrangement of non-elementary local patterns which can be described in terms of shape, size or orientation. For instance, cellular textures are better characterized by means of shape features than they would be by grey level based approaches.

This paper focuses on directional textures i.e. made up of elongated patterns. On such textures, directionality is obviously one of the most important features to be studied. That is the reason why we propose a new approach based on the texture orientation vector field and its second order statistical properties.

As we will see in section 2, the orientation vector field can be obtained by any gradient-based approach and consists of an orientation map and its associated coherence map. An interpolation technique is also given.

In section 3, an orientation resemblance function is proposed. This resemblance function is then used to draw up an *Orientation Similarity Map* or *OSM* which gives a second order description of texture. The *OSM* is exercized on both synthetic and Brodatz textures and is compared with grey level based interaction maps.

Finally, section 4 will provide an alternate derivation of the similarity measure: the *Curvilinear Orientation Similarity Function* or *COSF*. The *COSF* is suited for the description of elongated patterns and gives a measure of orientation correlation along a parametrized curve. This method is used to characterize composite material images taken from electronic microscopy and more particularly to retrieve the period and magnitude of textural ripple.

2 The Orientation Vector Field

2.1 Orientation estimation methods

Various techniques for orientation estimation have been reported in litterature. For instance, Bigün [2] set the problem of orientation detection in the least square sense, fitting an axis to the local Fourier Tranform. Kass and Witkin [9] grounded their algorithms on the difference of two Gaussians and on the statistical theory of directional data [10]. Such statistical methods raise the problem of the scale of analysis: one has to keep in mind the need for a local orientation estimate which is fitted to the size of the patterns involved in the texture. For this reason, statistics must be handled with care so that the orientation estimates remain local enough.
Very local operators (e.g. Prewitt's or Sobel's gradients) or size-adaptive operators(e.g. Deriche's gradient or Gradient of a Gaussian) are more likely to give good estimates of local orientations than larger scale statistical approaches. In recent works, Da Costa and al. [5] have also proposed a near optimal approach for fitting an orientation operator to the size of the textural patterns.

2.2 A discrete orientation map

Let $I : \mathbb{Z}^2 \to \mathbb{R}$ denote the image. On every pixel (i, j) of I, we can define an orientation vector $v_\theta^d(i, j)$ which consists of an argument $\theta_d(i, j)$ and a modulus $\eta_d(x, y)$.
θ_d is the discrete two-dimensional orientation map. Orientation being π-periodic, we restrict its values to $[0, \pi[$:

$$\begin{array}{lclcl} \theta_d & : & \mathbb{Z}^2 & \to & [0,\pi[\\ & & (x,y) & \to & \theta_d(x,y). \end{array} \quad (1)$$

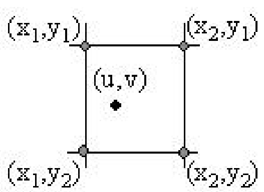
η_d is the corresponding coherence index map:

$$\begin{array}{lclcl} \eta_d & : & \mathbb{Z}^2 & \to & [0,1] \\ & & (x,y) & \to & \eta_d(x,y). \end{array} \quad (2)$$

The coherence or *confidence* index measures the degree to which a texture can be considered locally oriented. For example, this confidence index can be based on the computation of a directional variance upon a small neighborhood of the current pixel [9][10].

2.3 Interpolating orientations

Let interpolate the orientation and the coherence discrete maps on any point of real coordinates: $(u, v) \in \mathbb{R}^2$.



$$x_2 = x_1 + 1$$
$$y_2 = y_1 + 1$$

FIG. 1: Interpolating neighborhood

Let $\{(\theta_{kl}, \eta_{kl})\}_{k,l\in\{1,2\}}$ denote the orientations and coherence indices of the pixels surrounding (u, v). Then, using the Statistics of Directional Data [10], the orientation vector field can be interpolated from the neighboring pixels:

$$\begin{array}{lclcl} \theta(u,v) & = \theta_{u,v} & = & \frac{1}{2}\arg V \\ \eta(u,v) & = \eta_{u,v} & = & |V|^{\frac{1}{2}}. \end{array} \quad (3)$$

where $V = \sum_{kl} w_{kl}\eta_{kl}^2 e^{2j\theta_{kl}}$.
The w_{kl} are the weights associated to each neighbor and are based on the distance to the point (u, v):

$$w_{kl} = (1 - |u - x_{kl}|) \cdot (1 - |v - y_{kl}|). \quad (4)$$

3 The Orientation Similarity Map

3.1 The Resemblance Function

In order to construct an orientation similarity map, we have to define an orientation resemblance function R.

$$\begin{array}{lclcl} R & : & [0,\pi[\times[0,\pi[& \to & [0,1] \\ & & (\theta_1,\theta_2) & \to & R(\theta_1,\theta_2). \end{array} \quad (5)$$

R must be a decreasing function of the orientation difference Δ defined by:

$$\Delta(\theta_1, \theta_2) = \inf(|\theta_1 - \theta_2|, \pi - |\theta_1 - \theta_2|). \quad (6)$$

The first resemblance function we propose is the cosine function:

$$R_1(\theta_1, \theta_2) = \cos(\Delta(\theta_1, \theta_2)), \quad (7)$$

where Δ keeps the value of the difference between 0 and $\frac{\pi}{2}$. On textures where orientation variations are hardly discernable, the cosine function would not be suited because its first derivative is null at zero. In order to stress small orientation differences, let define R_2 by:

$$R_2(\theta_1, \theta_2) = \kappa \cdot e^{-\lambda\cdot\Delta(\theta_1,\theta_2)} + \gamma, \quad (8)$$

where κ, λ and γ allow to adapt the sensitivity to small differences and ensure:

$$\left\{ \begin{array}{lcl} R_2(\theta,\theta) & = & 1 \\ R_2(\theta,\theta+\frac{\pi}{2}) & = & 0. \end{array} \right. \quad (9)$$

3.2 The Orientation Similarity Map

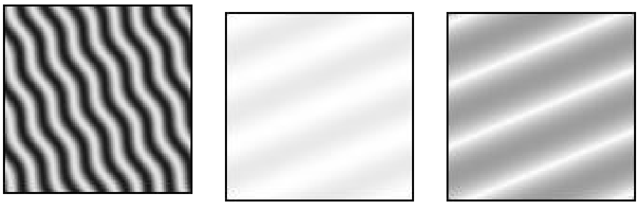
Let define the *Orientation Similarity Map* or *OSM* by:

$$M_S \; : \; \mathbb{Z}^2 \; \to \; [0,1] \quad (10)$$

and

$$M_S(\alpha, \beta) = \frac{\iint\limits_I \eta_{x,y}\eta_{x+\alpha,y+\beta} R(\theta_{x,y}, \theta_{x+\alpha,y+\beta}) dxdy}{\iint\limits_I \eta_{x,y}\eta_{x+\alpha,y+\beta} dxdy}. \quad (11)$$

$M_S(\alpha, \beta)$ measures the average resemblance over the image I between two orientation vectors separated by a displacement (α, β). Low values of $M_S(\alpha, \beta)$ reveal a possible periodicity on the orientation vector field.
Figure 2a represents a rippled texture. Figures 2b and 2c show the corresponding $OSMs$ computed using the resemblance functions R_1 and R_2 respectively. The origin is located at the center of the map.
OSMs can be interpreted in the same way as interaction maps [3]: white blobs and lines reveal the existence of periodicities or symmetries in the orientation vector field.

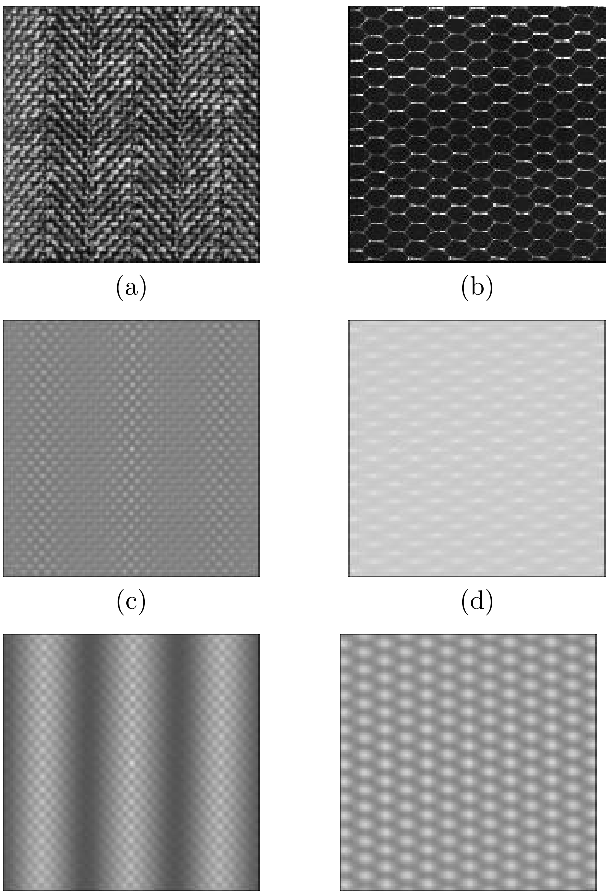


(a) (b) (c)

FIG. 2: Experiments on synthetic textures.

3.3 Results on Brodatz textures

The *OSM* method has been exercized on two natural textures from Brodatz's album. Textures d17 and d34 are presented on figure 3. For comparison, we provide the interaction maps based on the mean of the grey level difference histogram: *MGLDH-based* interaction maps [3].
Figure 3 shows that the interpretation is much more easier for *OSMs* than for interaction maps. For instance, on texture d34, the grey level information is contained in the edges between black cells. These edges are very thin and their grey level is strongly scattered. Thus, as grey level carries very few information, grey level based approaches are inadequate. On the contrary, such textures can be characterized meaningfully through their orientation vector field. That is why the *OSMs* reveal more information.



(a) (b) (c) (d) (e) (f)

FIG. 3: Experiments on Brodatz textures: (a) and (b) textures d17 and d34, (c) and (d) *MGLDH-based* interaction maps, (e) and (f) *OSMs* based on R_2.

4 Curvilinear 2$^{\text{nd}}$ order statistics

In this section, we deal with directional textures which consist in an arrangement of elongated patterns. On such textures, it is useless to extract 2$^{\text{nd}}$ order statistics through a 2-D approach. Indeed, elongated patterns can be described by crest lines or level curves running along them. If we can extract such level curves [4], then it is sufficient to study the second order statistics of the orientation vector field along these curves. Moreover, the curvilinear implementation will appreciably reduce the computational cost. So, let introduce the curvilinear likeness function, which is defined on any parametrized curve.

4.1 The Curvilinear Orientation Similarity Function

Let $\mathcal{C}$ denote a parametrized curve:

$$\begin{array}{cccc} \mathcal{C} & : & [0,L] & \rightarrow & \mathbb{R}^2 \\ & & s & \rightarrow & (x(s),y(s)), \end{array} \tag{12}$$

where s is the arc length and L the total length of the curve.
Let define the *Curvilinear Orientation Similarity Function* or *COSF* by:

$$f_S^{\mathcal{C}} \;:\; \mathbb{R} \;\rightarrow\; \mathbb{R}^+ \tag{13}$$

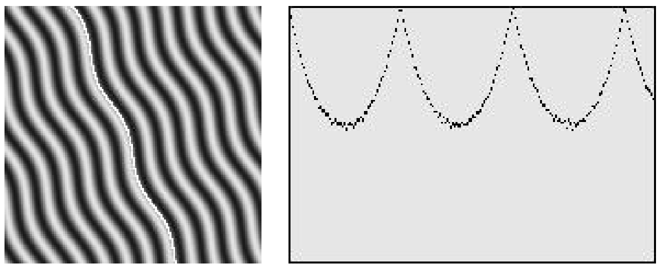
and

$$f_S^{\mathcal{C}}(t) = \frac{\int_{\mathcal{C}} \eta_s \cdot \eta_{s+t} \cdot R(\theta_s, \theta_{s+t}) \cdot ds}{\int_{\mathcal{C}} \eta_s \cdot \eta_{s+t} \cdot ds}. \tag{14}$$

$\theta_s = \theta(x(s), y(s))$ and $\eta_s = \eta(x(s), y(s))$ denote respectively the orientation and the coherence at arc length s. The definitions of orientation and coherence at any arc length rely on the interpolation rules defined in section 2. $f_S^{\mathcal{C}}(t)$ measures the average resemblance along the curve $\mathcal{C}$ between two points which are separated by a displacement t. Low values of $f_S^{\mathcal{C}}(t)$ reveal a periodicity on the image.

4.2 Results on synthetic textures

Figure 4a shows a level curve on a directional texture. In figure 4b, we give a representation of $f_S^{\mathcal{C}}(t)$ as a function of curvilinear distance.



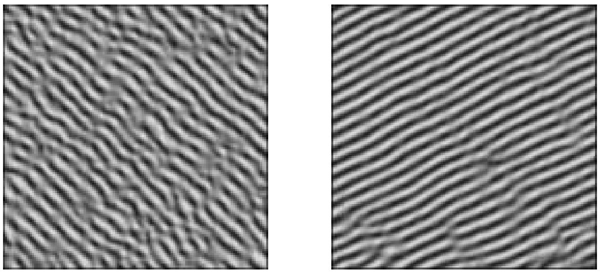
(a) (b)

FIG. 4: COSF (b) computed on a rippled texture (a).

The similarity function appears to be periodic: it reflects the presence of ripple along the pattern. The period of the curve is the curvilinear period of the ripple phenomenon whereas the depth of the minima is directly related to the ripple magnitude.

4.3 Application to material images

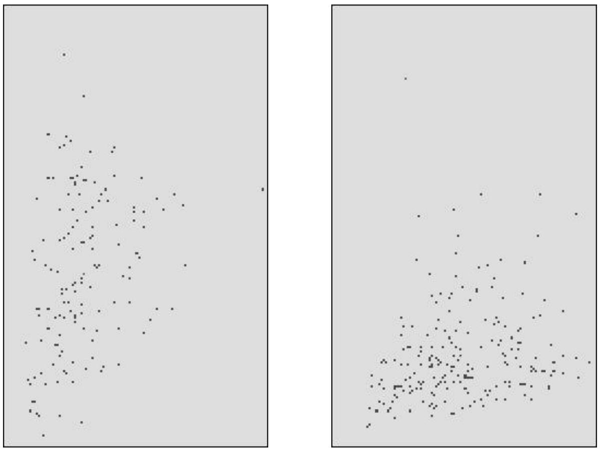
The *COSF* has been exercized on natural textures taken from transmission electronic microscopy. The images of figure 5 show atomic layers of composite materials observed at a microscopic scale. The extraction of the period and the magnitude of the undulation of layers is of great importance for the physical interpretation of those textures.



Material 1 Material 2

Fig. 5: Composite material images.

We choose to characterize the textural patterns by using the level curves of the image. For the level curve extraction, we implemented the algorithm presented in [4]. For each extracted level curve, we drew the Curvilinear Orientation Similarity Function. By a regression algorithm, the period and the magnitude of the undulation have been estimated. The occurrences of the *period-magnitude* couples on two different materials are reported on the bidimensional histograms of figure 6.
The distributions of the occurrences for the two images are quite different. They reflect the results obtained for a great number of images and turn out to confirm the hypotheses made by physicists.



Material 1 Material 2

Fig. 6: Tables of Period-magnitude occurrences: period on the *x-axis*, magnitude on the *y-axis*.

5 Conclusion

In this paper we have proposed a new and efficient approach for the characterization of directional textures. This approach gives a second order description of the texture orientation vector field and is based on the computation of an orientation resemblance function. It has been derived into two different implementations.
The first one is the *Orientation Similarity Map* which measures the average resemblance between two orientation vectors separated by a given displacement. Experiments on synthetic and natural images have shown the relevance of the *OSM* for the characterization of two-dimensional periodicity and directionality.
The second implementation consists in the *Curvilinear Orientation Similarity Function.* The *COSF* computes second order statistics of the orientation vectors along a parametrized curve. This method was exercized on composite material images. Associated to a level curve extraction algorithm, it proved to be efficient and suited for the extraction of the period and magnitude of texture ripple.

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