**Random discrete groups of Möbius transformations:**
**Probabilities and limit set dimensions.**

A Thesis presented in partial fulfillment of the requirements for the degree of
Doctor of Philosophy
in
Mathematics
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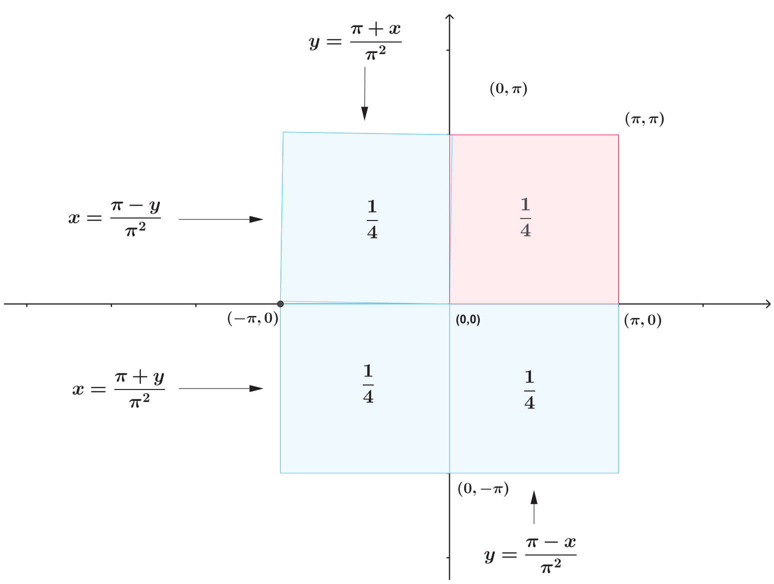


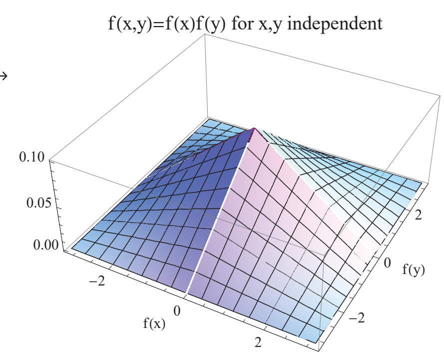


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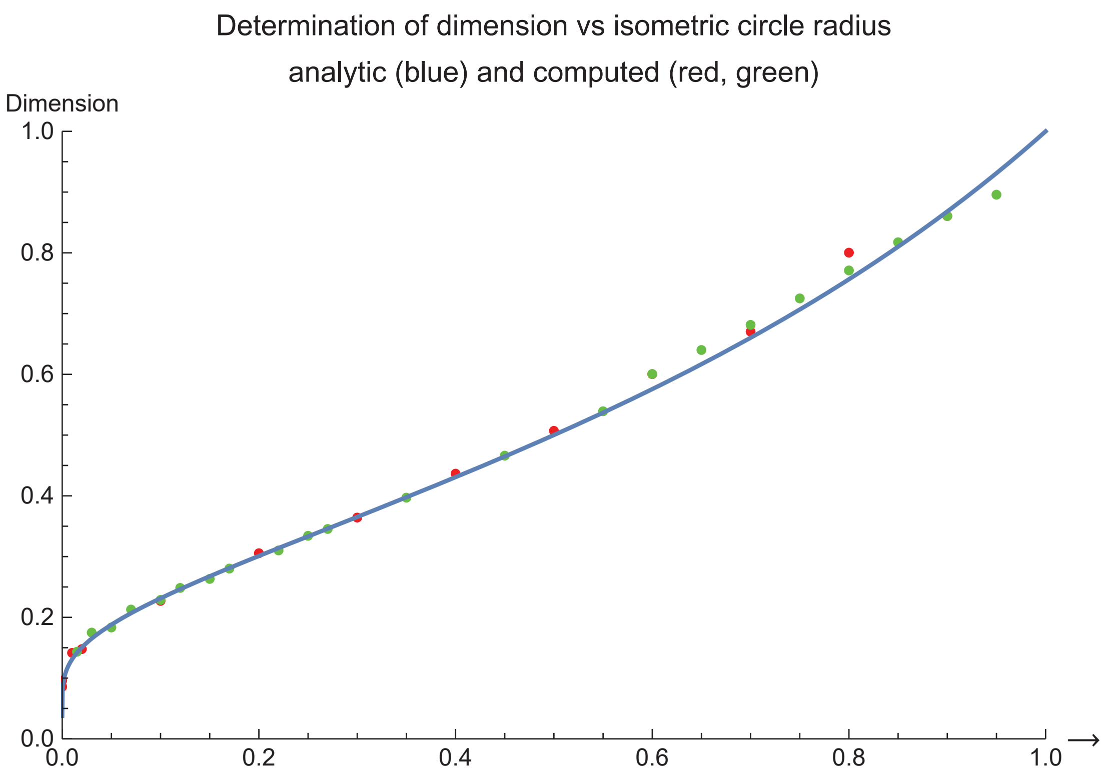




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It was my good fortune to have a great mathematician for my supervisor, but he made me work for the privilege. With his professional scepticism he made me fight for every claim, standard challenges were "I don't believe it", or maybe "It's either well known or it's wrong, I don't know which". Thanks Gaven.

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Thanks Shaun for your encouragement and tenacious insistence that the proof of a particular theorem be unassailable.

Lynette O'Brien, BBS (Hons), MSc (Mathematics)

Well, who could have a more loving, patient and supportive mathematical wife?

## ABSTRACT

This thesis addresses two areas related to the quantification of discrete groups. We study "random" groups of Möbius transformations and in particular random two-generator groups; that is, groups where the generators are selected randomly. Our intention is to estimate the likelihood that such groups are discrete and to calculate the expectation of their associated geometric and topological parameters. Computational results of the author [55] that indicate a low probability of a random group being discrete are extended and we also assess the expected Hausdorff dimension of the limit set of a discrete group. In both areas of research analytic determinations are correlated with computational results. Our results depend on the precise notion of randomness and we introduce geometrically natural probability measures on the groups of all Möbius transformations of the circle and the Riemann sphere.

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