Error Statistics Computation for Cellular CDMA Systems

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Abstract – The performance of the reverse link of a CDMA wireless system taking into account the effect of correlated Rician fading is investigated. An ideal open-loop power control which compensates for the long-term variations due to path loss and shadowing is considered. Conversely, no mechanism able to compensate for the small-scale multipath fading is supposed. Small scale multipath fading is still mitigated by ideal maximal ratio combining mechanisms. The outage probability is derived by assuming a constant multipath intensity profile (MIP). Under the same approximation, the average fade duration is evaluated by considering the expected number of times the received signal crosses the fade threshold. Simulations are carried out which validate the analytical results.

1. Introduction

Within the field of modern telecommunications, Code Division Multiple Access (CDMA) is one of the techniques which is gaining consideration for the development of personal communications [1]. The outage probability, i.e., the marginal probability that the Signal-to-Interference Ratio (SIR) is below a threshold, is one of the most important QoS metric for CDMA wireless systems. The goal of this contribution is to give an analytical model for the computation of the outage events statistics on the reverse link of a wireless Direct Sequence CDMA system, which is commonly believed to limit the system capacity. Indeed, due to the multipath access nature of the reverse link, one may expect that the forward link performance would be at least good as that of the reverse link.

Wideband high bit-rate systems, with non-negligible round trip delay are taken into consideration. An ideal open-loop power control which compensates for the long-term variations due to path loss and shadowing experienced by the mobile user, is assumed. Conversely, no sophisticated close-loop mechanism able to compensate for the small-scale multipath fading, is supposed. The propagation model considered in this paper accounts for frequency selectivity. Indeed, for high bit-rates wireless systems, the r.m.s. of the delay spread introduced by the channel is often significantly greater than the chip time of the spreading sequence. In an attempt to be thorough, we will assume that the multipath fading is mitigated at the base station through an ideal maximal ratio combiner (RAKE receiver). In particular, a constant multipath intensity profile (MIP) is considered. Finally, the general case of Rician fading channel is taken into consideration in the computation of the statistical properties of the channel.

This paper is organized as follows. Sections 2.1 and 2.2 present the analytical framework for the computation of the outage probability and the average fade duration respectively. Section 3 presents the simulations results which validate the theoretical approach. Conclusions are made in Section 4.

2. System model

2.1 Outage probability

Let us denote by L the number of the resolvable path of the frequency selective channel, by $\alpha_{i,p}(t)$ the *p-th* tap weight coefficient (relative to the *p-th* multipath contribution) of the *i-th* (i =0,N-1) user channel and by λ_i the global multipath power of the *i-th* user. It results:

$$\lambda_i = \sum_{p=1}^{L} \left|\alpha_{i,p}\right|^2 \tag{1}$$

Under the hypothesis of perfect long-term power control it is reasonable to assume $E(\lambda_0)= E(\lambda_1)=\ldots= E(\lambda_{N-1})=\eta$. The average received SNR evaluated in absence of interfering users may be expressed as:

$$\gamma = \frac{1}{2N_0} PG\eta \tag{2}$$

where PG is the process gain of the spreading procedure and N_0 is the spectral power density of the background noise. By referring to the well known Standard Gaussian Approximation (SGA) [2], the SNR conditioned to the set of random variables λ_i, i=0,N-1, is derived as:

$$SNR\left|\{\lambda_i\}_{i=0,N-1}\right. = \frac{\lambda_0\gamma}{\eta + \frac{2\gamma}{3PG}\sum_{i=1}^{N-1}\lambda_i} \tag{3}$$

Where the term 2/(3PG) refers to the cross-correlation properties of the spreading sequences [2].

In packet switching data transmission, the success/failure transmission process is often modelled as the outcome of a comparison of the instantaneous SNR to a threshold value, *SNRt* [3]. The packet is successfully decoded with probability one if the instantaneous SNR is above the threshold for all the packet duration. Otherwise, the packet is lost with probability one. Such threshold models implicitly rely on two assumptions. First, the fading process is slow enough so that the channel can be considered constant throughout a packet time. Second, the relationship between the instantaneous SNR and the bit error probability is close to a step function, i.e., it has a threshold behaviour. This model will be assumed later on in order to characterise the performance of a CDMA wireless system. The goal is to compute the outage probability P_o, defined as the probability that the SNR falls below the prescribed level, SNR_t. If F is the value of the fading margin, i.e. $\gamma = F \times SNR_t$, the outage probability is given by

$$P_o = \Pr(\lambda_0 < T) \quad (4)$$

where

$$T = \frac{1}{F}\left(\eta + \frac{2F \times SNR_t}{3PG}\sum_{i=1}^{N-1}\lambda_i\right) \quad (5)$$

is the SNR threshold. In presence of Rician fading, the received signal from a user may be thought as the superimposition of a direct component with many other resolvable scattered components. In the frequency selective fading channel considered here, the first multipath component of the *i-th* user, referred to as $\alpha_{i,1}$, encompasses the direct and a portion of the scattered components, while the remaining scattered components are temporary dispersed over $\alpha_{i,p}$, $p=2,L$, according to the multipath intensity profile of the channel. Invoking the central-limit theorem and under the assumption of uncorrelated scattering, the random variables $\alpha_{i,p}$ may be modelled as independent complex Gaussian random variables with means

$$\begin{cases} E(\alpha_{i,p}) = \nu & \text{for } i = 0, N-1 \text{ and } p = 1 \\ E(\alpha_{i,p}) = 0 & \text{for } i = 0, N-1 \text{ and } p \neq 1 \end{cases}$$

and variances

$m_p = E\left(\left|\alpha_{i,p} - E(\alpha_{i,p})\right|^2\right)$, *for* $i = 0,N\text{-}1$ *and* $p = 1,L$.

The Rice factor of the direct component, is defined as $K_R = \frac{\nu^2}{m_p}$. This parameter takes into account the power of the line of sight (LOS) component with respect to the scattered ones. The Rayleigh case is obtained by setting K_R=0.

The evaluation of the outage probability in (4) requires a statistical characterisation of the threshold T. We observe that the randomness of T derives from the summation of the N-1 independent random variables λ_i with mean η and variance $\left(\sum_{p=1}^{L} m_p{}^2 + 2m_1\nu^2\right)$.

Thus, provided that the number of users N is high, the central limit theorem allows us to characterise the threshold as a Gaussian random variable $f_T(T)$ with mean and variance given by

$$E(T) = \frac{\eta}{F}\left(1 + \frac{2F \times SNR_t(N-1)}{3PG}\right) \quad (6)$$

$$\sigma^2(T) = \frac{(N-1)}{F^2}\left(\frac{2F \times SNR_t}{3PG}\right)^2\left(\sum_{p=1}^{L} m_p{}^2 + 2m_1\nu^2\right) \quad (7)$$

respectively. The evaluation of (4) may now be performed as

$$P_0 = \int_0^\infty f_T(T)dT\int_0^T f_{\lambda_0}(l)dl = \int_0^\infty F_{\lambda_0}(T)\, f_T(T)dT \quad (8)$$

where $f_T(T)$ is the pdf of the Gaussian random variable T and $F_{\lambda_0}(T)$ is the cumulative distribution function (cdf) of the random variable λ_0. Under the hypothesis of constant MIP, $F_{\lambda_0}(T)$ results the cdf of a non-central chi-square distribution with 2L degrees of freedom and non-centrality parameter $\nu^2 = K_R m_p$ which can be numerically evaluated by means of reasonable computational effort [3].

2.2 Average fade duration

In [4] and [5] it is shown that a two-state Markov chain approximation for the block error process over correlated Rayleigh fading channels is a good model for a broad range of situations. In presence of a Markov error process, the steady state error rate and the average length of burst errors are sufficient to completely characterise the channel. One should note that in a radiomobile CDMA system, as discussed in previous Section, the interference level can be modelled as a slowly varying Gaussian random process. Thus, the statistical behaviour of the channel, expressed in terms of outage probability and average fade duration, depends on the threshold level.

The probability $F_{\lambda_0}(T)$ introduced in Section 2.1, represents the error-rate of a radiomobile CDMA system for a given value of the threshold T. We will then derive the average length of a burst error conditioned to the threshold T, assuming that the interference level does not dramatically change over a burst error time. The burst error length is defined as the average time the decision variable λ_0 remains below the threshold level T. In order to facilitate the notations, by indicating by $r_p = \left|\alpha_{0,p}\right|$, we get $\lambda_0 = \sum_{p=1}^{L} r_p^2$. Let us focus on the random variable r_p.

The tap weights coefficients $\alpha_{0,p}$ may be expressed as

$$\alpha_{0,p} = (X_p + \mu_p) + jY_p \quad (9)$$

where X_p and Y_p are Gaussian random processes with zero mean and variance $m_p/2$, μ_p is the average of the *p-th* path, i.e. $\mu_p=\nu$, for p=1 and μ_p=0, otherwise. The processes X_p and Y_p can be assumed to be stationary at least on the time scale of the fading variations. Thus, it is possible to define the correlation function:

$$R_p(t_0) = E\left(X_p(t)X_p(t+t_0)\right) = E\left(Y_p(t)Y_p(t+t_0)\right) \quad (10)$$

In a widely accepted model, the Gaussian processes are assumed to have a band-limited non-rational spectrum given by:

$$S_p(f) = \frac{m_p}{2}\left[1 - \left(\frac{f}{f_D}\right)^2\right]^{-\frac{1}{2}} \quad \text{for} |f| < f_D \quad (11)$$

and zero otherwise, where $f_D = V/\lambda$ is the Doppler bandwidth, V is the mobile speed, and λ is the carrier wavelength. This spectrum corresponds to the correlation function

$$R_p(t_0) = \frac{m_p}{2} J_0\left(2\pi f_D |t_0|\right) \quad (12)$$

where $J_0(x)$ is the Bessel function of the first kind and of zeroth order. The joint cdf of the processes r_p and of their derivatives $\dot{r}_p$ may be computed as in [5]:

$$f_{r_p \dot{r}_p}\left(r_p, \dot{r}_p\right) = \frac{2r_p}{m_p} I_0\left(\frac{2r_p \mu_p}{m_p}\right) e^{\frac{-\left(r_p^2 + \mu_p^2\right)}{m_p}} \frac{1}{\sqrt{-2\pi \ddot{R}_p(0)}} e^{\frac{\dot{r}_p^2}{2\ddot{R}_p(0)}} \tag{13}$$

Note that r_p and its derivative are independent thus resulting:

$$f_{\dot{r}_p}(\dot{r}_p) = \frac{e^{\frac{\dot{r}_p^2}{2\ddot{R}(0)}}}{\sqrt{-2\pi \ddot{R}(0)}} \tag{14}$$

The derivative $\ddot{R}_p(0)$ may be computed from (12), i.e., $\ddot{R}_p(0) = -m_p \pi^2 f_d^2$. The level crossing rate, $\Re(T)$, defined as the expected number of times per second the random process λ_0 crosses the threshold T in the positive direction is given by:

$$\Re(T) = \int_0^{+\infty} y f_{\lambda_0 \dot{\lambda}_0}(T, y) dy \tag{15}$$

where $f_{\lambda_0 \dot{\lambda}_0}\left(\lambda_0, \dot{\lambda}_0\right)$ is the joint cdf of the process λ_0 and of its derivative $\dot{\lambda}_0 = 2\sum_{p=1}^{L} r_p \dot{r}_p$. The random variable $(\dot{\lambda}_0 \mid \lambda_0 = z)$ is Gaussian distributed with zero mean and variance equal to $4m_p \pi^2 f_D^2 z$. Let us now consider the Gaussian random variable $q = \left((\dot{\lambda}_0 \mid \lambda_0 = z)/\sqrt{z}\right)$; this variable has zero mean and variance equal to $\delta^2 = 4m_p \pi^2 f_D^2$. Let us denote by $C_q(q_0)$ and by $c_q(q_0)$ the cdf and the pdf of q respectively.

The joint cdf of λ_0 and its derivative can be written as:

$$F_{\lambda_0, \dot{\lambda}_0}(x, y) = \int_0^x C_q\left(\frac{y}{\sqrt{z}}\right) f_{\lambda_0}(z) dz \tag{16}$$

Deriving expression (16) with respect to x and y we have:

$$f_{\lambda, \dot{\lambda}}(x, y) = \frac{1}{\sqrt{x}} c_q\left(\frac{y}{\sqrt{x}}\right) f_{\lambda_0}(x) \tag{17}$$

It is now possible to evaluate the integral (15) as:

$$\Re(T) = \int_0^{+\infty} y f_{\lambda_0, \dot{\lambda}_0}(T, y) dy = \int_0^{+\infty} \frac{y}{\sqrt{T}} c_q\left(\frac{y}{\sqrt{T}}\right) f_{\lambda_0}(T) dy \tag{18}$$

making the substitution $\omega = \frac{y}{\sqrt{T}}$ it results:

$$\Re(T) = f_{\lambda_0}(T)\sqrt{T} \int_0^{+\infty} \omega\, c_q(\omega) d\omega = f_{\lambda_0}(T)\sqrt{T}\sqrt{\frac{\delta^2}{2\pi}} = = f_{\lambda_0}(T) f_D \sqrt{2T\pi\, m_p} \tag{19}$$

The average fade duration $G(T)$ conditioned to the threshold level T can now be expressed as the ratio between the outage probability and the level crossing rate:

$$G(T) = \frac{F_{\lambda_0}(T)}{\Re(T)} \tag{20}$$

Finally, an estimation of the average fade duration G is given by

$$G = \int_0^{\infty} G(T) f_T(T) dT \tag{21}$$

3. Simulation results

In order to validate the analytical results derived in the previous Sections the proposed system model was simulated. The block diagram of the simulated signal received from the *i-th* user is shown in Figure 1(a). The mobile radio channel is supposed to be temporally divided into slot periods, referred to as H. We assume that the signal variation within the slot duration H is negligible, i.e., $f_D H << 1$. Accordingly, the tap weights coefficients of the *i-th* user's channel $\alpha_{i,p}(t)$ may be adequately generated from a white noise signal filtered by a Doppler digital filter with normalised cut-frequency equal to $f_D H$. The frequency response of the Doppler filter is settled according to (11). The variance of the *p-th* tap weight coefficient must be properly set following the multipath delay profile pattern of the channel; this is obtained by multiplying the white noise term by a scaling term referred to as $\sigma(\alpha_{i,p})$. The presence of a direct component in the multipath delay profile is modelled by the introduction of a constant term ν added to the first path. Finally, the decision variables λ_i are evaluated by a maximal ratio combining procedure.

The success/failure of data packets may be simulated as shown in Figure 1(b). The decision variables λ_0 must be compared with the threshold T in order to evaluate the outage events.

Simulation results have been collected for many different configurations of the parameters but for the sake of brevity and in order to fix some of the huge amount of variables which characterise the system, let us make the following assumptions which will hold in the rest of the paper: m_i=1 (i=1,L), SNR_t =7 dB, F=10, N=10 and PG = 100.

A comparison between simulation results and the analytical approach is shown in Figures 2 and 3. Figure 2 presents the comparison of the outage probability P_0 versus the number L of multiple paths, for different values of the Rice factor K_R. Figure 3 presents the graphics relative to the average fade duration G expressed in slot length H, versus the number L of multiple paths and for different values of K_R. The results shown in previous Figures assesses the validity of the analytical study discussed in section 2. Moreover, as one could have expected, the outage probability decreases as the Rice factor K_R increases and, for high values of K_R the advantages derived from the maximal ratio combining technique are less noticeable. As far as the average fade duration is concerned, curves shows that the analysis made in section 2.2 offers a good degree of approximation. As it can be noted in Figure 3, in the case of only one path, both theoretical approach and simulations present lower fade duration for K_R = 5 than for K_R = 10. This behavior can been explicated by considering that when there is a very strong direct path, the variability of the channel is reduced; in this case, outage events are less probable but longer.

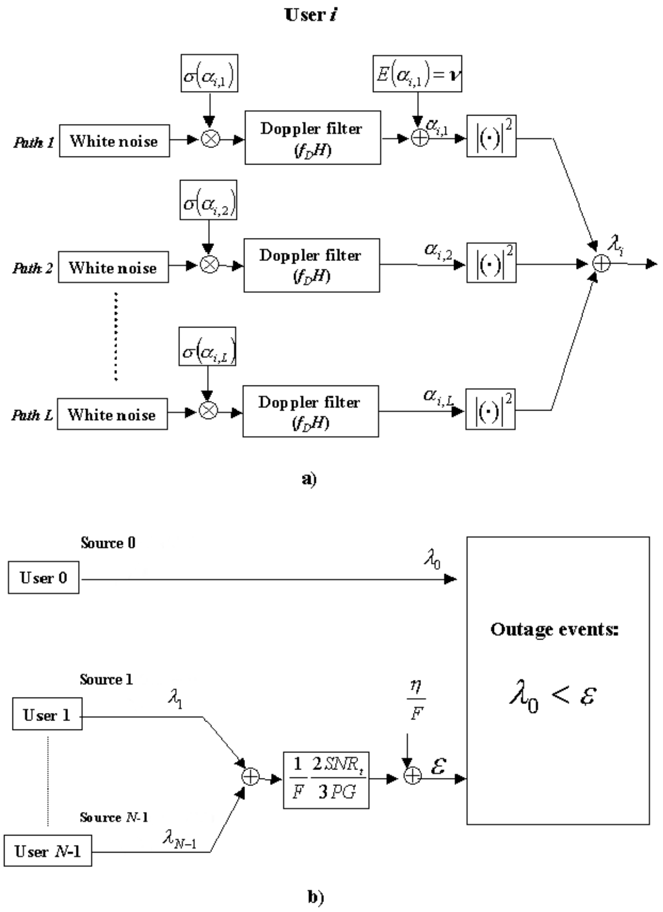

FIG. 1 : Block diagrams of the simulation procedure.(a).Generation of the simulated signal received from the *i*-th user. (b). Evaluation of the success/failure events

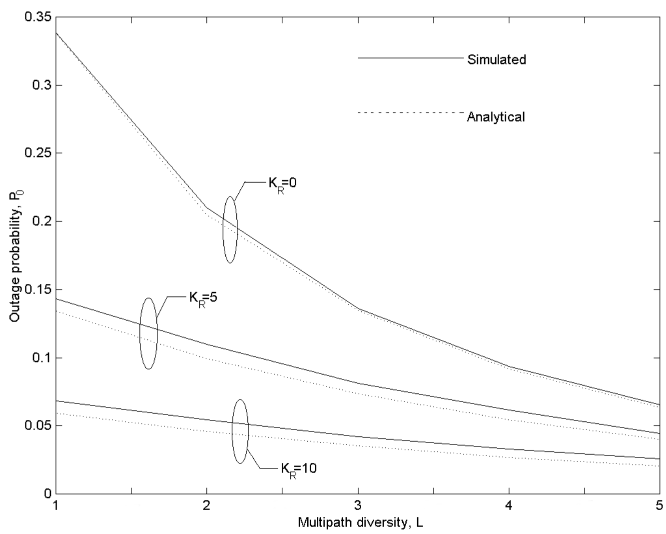

FIG. 2 : Comparison between the simulated and analytical values of the Outage Probability P_0 versus the number of multiple path per user L

4. Conclusions

In this work the performance of the reverse link of a DS/CDMA wireless system has been investigated in presence of correlated Rician fading. Under the hypothesis of constant MIP and supposing the presence of a RAKE receiver, the outage probability has been derived and a theoretical approach as been proposed for deriving the average fade duration. The analytical results have been validated through simulations for different environment conditions.

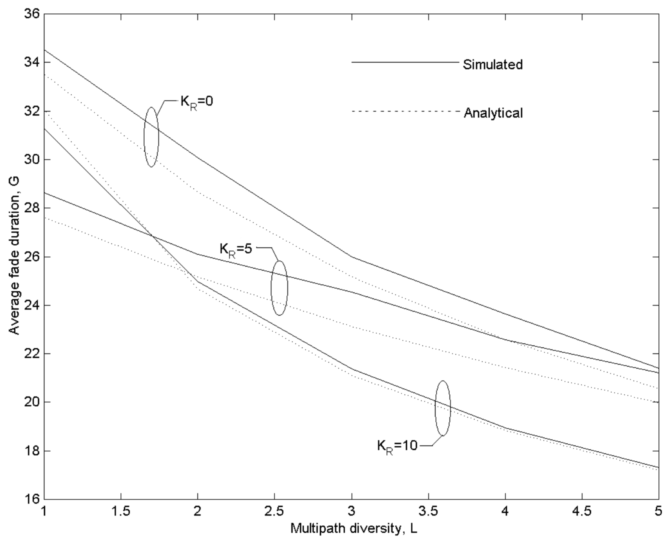

FIG. 3 : Comparison between the simulated and analytical values of the average fade duration G versus the number of multiple path per user L.

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