Diversity techniques for blind channel equalization in mobile communications

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RÉSUMÉ

Cet article traite de l'égalisation aveugle de canaux mobiles qui sont soit sélectifs en fréquence soit multiplicatifs. L'algorithme proposé peut-être utilisé pour compenser la distorsion de canaux á changements rapides où un ensemble restreint de données est disponible pour estimer les coefficients de l'égaliseur.

ABSTRACT

This paper deals with blind equalization of mobile channels, which are either frequency-selective or multiplicative. The proposed algorithm can be used for channel distortion compensation in diversity systems subject to rapidly varying channels, where a short set of data is available to estimate the equalizer coefficients.

1 Introduction

Mobile communications operate in a very hostile environment due to multipath propagation and vehicle displacement. Depending on the transmission rate and vehicle speed, either frequency-selectivity or Doppler spectrum spreading becomes the major concern ([1]). In this paper, a blind equalization technique is proposed which can be applied to compensate the distortion introduced by the multiplicative and the frequency-selective mobile channels.

The proposed approach relies on the availability of space or time diversity which enables the use of single-input multiple-output formulation (SIMO) of the transmission system. It is based on a criteria which allows for linear equalization of the received data. In fact, the proposed formulation is more general than the application suggested here and could also be applied in other environments. Thus, here it will be shown to be useful for defining a deterministic criteria for blind equalization, but it could also be applied to the problem of channel estimation by means of cyclostationary statistics-based methods (e.g. [2]).

The suggested algorithm has a low computational load and exhibits performance similar to that one of other deterministic criteria proposed in the literature: it obtains relatively good results for short data sets, it assumes the channel is FIR with known length (this constraint will be relaxed further on) and its original derivation does not take into account the additive noise, although it is of course considered when defining the method final formulation.

As opposed to methods which have appeared earlier in the literature, the proposed algorithm is based on the assumption that the receiver can observe the complete convolution of the transmitted data and the channel response. In the case of convolutive channels, the full channel output is available if a block transmission scheme is employed and a guard interval longer than channel response duration is inserted between consecutive transmitted frames. This is not a major constraint, given that this guard interval is inserted in most of burst transmission schemes anyway. In the case of a multiplicative channel, the benefits of OFDM (Orthogonal Frequency Division Multiplexing) modulation ([3]) have been shown in [4]. Since its application will turn the frequency flat fading into a convolutive distortion, the blind algorithm proposed here can also be applied in that case.

This paper extends the method proposed in [5], improving very significantly its robustness in front of the noise. The relationship of this algorithm with other methods proposed earlier in the literature is also explored.

2 Problem statement

Figure (1a) shows the discrete-time model for a diversity receiver: the same information signal $T[k]$ is transmitted through B diversity branches, it is distorted by different channel responses $C^i[k]$ and it is degraded by different additive white Gaussian noise terms $W^i[k]$. Using the z-transforms associated to these sequences, the received signal can be written as:

$$Y^i(z) = T(z) \cdot C^i(z) + W^i(z) \qquad i = 1, ..., B \qquad (1)$$

If the noise term is negligible it follows that:

$$Y^i(z) = T(z) \cdot C^i(z) \qquad i = 1, ..., B$$

and therefore:

$$T(z) = g.c.d.\{Y^i(z)\} \qquad i = 1, ..., B \qquad (2)$$

where $g.c.d.$ stands for the greatest common divisor. The algorithm presented here is based on the estimation of the transmitted data using equation (2). Of course, in order to apply this equation the complete z-transform $Y^i(z)$ must be

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available and, therefore, a block transmission scheme with a guard interval is needed. In the case of OFDM operating, no efficiency is lost because no guard interval is needed. Indeed, when working in the transformed domain, the OFDM received signal could be regarded as the result of a circular convolution with the DFT (Discrete Fourier Transform) of the channel distortion ([6]):

$$Y^i[k] = T[k] \circledast C^i[k] \qquad i = 1, ..., B$$

where $\circledast$ stands for circular convolution. Furthermore, oversampling the OFDM received signal is equivalent to zero padding the transformed domain sequences as long as there was no aliasing introduced when they were sampled at the symbol rate ([5]). Thus, if the received signal is oversampled, the circular convolution can be converted into a linear one:

$$Y^i[k] = X_{ZP}[k] \circledast C^i_{ZP}[k] = X[k] * C^i[k]$$

where ZP stands for 'zero-padding'. This way, the transmission of an OFDM signal through a multiplicative channel would fit also the model of equation (1).

Figure (1b) shows the linear equalization architecture employed in this paper. The multiple diversity branches are combined by means of FIR filters $E^i[k]$ to generate an output $R[k]$:

$$R(z) = \sum_{i=1}^{B} Y^i(z) \cdot E^i(z) = T(z) \cdot \sum_{i=1}^{B} C^i(z) \cdot E^i(z) \quad (3)$$

Thus, our problem can be stated as that one of designing the filters $E^i[k]$ in order to retrieve the transmitted data: $R[k] = T[k]$. Notice that if $B = 2$ (dual diversity) and the optimization criteria is based on forcing $R[k] = 0$ then the algorithm in [7] is obtained. In the present case, the perfect equalization (zero forcing) criteria requires $R(z) = T(z)$ and therefore

$$\sum_{i=1}^{B} C^i(z) \cdot E^i(z) = 1 \quad (4)$$

In the next section a blind algorithm is summarized which provides the equalizer coefficients $E^i[k]$.

3 Blind algorithm design

The proposed algorithm is based on the following property (Bezout equation):

Given B polynomials $\{A^i(z)\}$ the equation

$$\sum_{i=1}^{B} A^i(z) \cdot a^i(z) = 1$$

has a iff the B polynomials are coprime. Furthermore, the solution is unique (up to a multiplicative constant) iff

$$\deg\left\{a^i(x)\right\} + 1 = \frac{\deg\left\{A^i(x)\right\}}{B-1} \qquad i = 1, ..., B \quad (5)$$

If the polynomials $a^i(x)$ have a greater degree infinite solutions can be found for this equation. When this property is applied to equations (3)-(4), it turns out that perfect channel equalization can be obtained only when channel responses have no factor in common, a result well known in the literature ([8]). In case this condition is satisfied, the zero-forcing equalizer coefficients will be achieved by solving equation (4). Furthermore, from the previous property also follows that the zero-forcing equalizer is unique when the equalizer lengths are selected according to (5) and are non-unique if their filters are longer, being the difference among the possible solutions their performance in front of the additive noise ([11]). Thus, designing longer equalizers allowed for performance improvements in the BER.

Besides, notice that equation (3) says that the equalizer output $R(z)$ will always be a multiple of the transmitted data $T(z)$, and that

$$\deg\{R(z)\} = \deg\{T(z)\} + \deg\left\{\sum_{i=1}^{B} C^i(z) \cdot E^i(z)\right\}$$

Therefore, asking for an output of minimum length ($R(z)$ of minimum degree) is equivalent to asking for perfect channel equalization: $R[k] = T[k]$. This is the design criteria in which the proposed method is based: design $E^i[k]$ so that $R[k]$ has minimum length, then $R[k] = \alpha T[k]$, being α an unknown complex constant. The matrix formulation for the method can be found in [5] and is briefly summarized here in order to introduce the new method.

As shown in [5], equation (3) can be written using matrix notation as

$$\underline{Z} = \underline{\underline{Y}}\,\underline{E}$$

where $\underline{\underline{Y}}$ is a generalized Sylvester matrix. Besides, the perfect equalization case in (4) can be written as

$$\underline{\underline{Y}} = \begin{bmatrix} \underline{\underline{Y}}_t \\ \underline{\underline{Y}}_o \end{bmatrix} \qquad \begin{matrix} T\alpha = \underline{\underline{Y}}_t \underline{E} \\ \underline{0} = \underline{\underline{Y}}_o \underline{E} \end{matrix} \quad (6)$$

where the received data matrix $\underline{\underline{Y}}$ has been split in two parts. The minimum length criteria can be described then as finding those equalizer coefficients $\underline{\hat{E}}$ such that

$$\underline{0} = \underline{\underline{Y}}_o \underline{\hat{E}} \quad (7)$$

Thus the method proposed in [5] can be considered as a one based on the noise subspace of matrix $\underline{\underline{Y}}_o$. Once the equalizer has been estimated, the received data can be filtered to yield an estimation of the transmitted data:

$$\underline{\hat{T}} = \underline{\underline{Y}}_t \underline{\hat{E}} \quad (8)$$

4 Proposed algorithm formulation

The previous method has two main drawbacks which are solved by the new approach proposed here:

- The previous algorithm can only be applied when equalizer lengths satisfy (5), for if they were overdimensioned the algorithm might converge to a non-useful solution where constant α=0 and, therefore, $R[k] = 0$. Hence, the advantages of long equalizers in terms of noise cannot be exploited.
- The previous algorithm does not fully exploit the available data. Both $\underline{\underline{Y}}_t$ and $\underline{\underline{Y}}_o$ contain information on the channel and the transmitted data, but the algorithm described in [5] designs the equalizer taps based on $\underline{\underline{Y}}_o$ only.

According to these considerations, the new algorithm formulation tries to maximize the Signal-to-ISI-plus-noise-ratio (SINR) at the equalizer output. This SINR can be approximately estimated (see eq. (6)) as

$$\widehat{SINR} = \frac{\underline{E}^H \underline{\underline{Y}}_t^H \underline{\underline{Y}}_t \underline{E}}{\underline{E}^H \underline{\underline{Y}}_o^H \underline{\underline{Y}}_o \underline{E}} \tag{9}$$

This is the new cost function to be optimized. Notice that this new criteria is coherent with the algorithm in [5], given that it aims to find the solution which maximizes the mean power of detected symbols under the constraint of minimum length equalizer output and noise level reduction.

The covariance matrices associated to $\underline{\underline{Y}}_t$ and $\underline{\underline{Y}}_o$ are non-negative defined and thus the quotient in equation (9) corresponds to a typical Rayleigh quotient form ([10]). Therefore, it satisfies:

$$\lambda_{\min}\left[\underline{\underline{Y}}_t^H \underline{\underline{Y}}_t ; \underline{\underline{Y}}_o^H \underline{\underline{Y}}_o\right] \leqslant \frac{\underline{E}^H \underline{\underline{Y}}_t^H \underline{\underline{Y}}_t \underline{E}}{\underline{E}^H \underline{\underline{Y}}_o^H \underline{\underline{Y}}_o \underline{E}} \leqslant \lambda_{\max}\left[\underline{\underline{Y}}_t^H \underline{\underline{Y}}_t ; \underline{\underline{Y}}_o^H \underline{\underline{Y}}_o\right]$$

That is, the equalizer output SINR is bounded by the minimum and maximum eigenvalues of the data matrix $\underline{\underline{Y}}_t$ in the norm of $\underline{\underline{Y}}_o$. Thus, the equalizer that maximizes (9) can be obtained as the maximum generalized eigenvector:

$$\underline{\underline{Y}}_t^H \underline{\underline{Y}}_t \underline{E} = \lambda_{\max} \underline{\underline{Y}}_o^H \underline{\underline{Y}}_o \underline{E} \qquad \widehat{SINR} = \lambda_{\max} \tag{10}$$

Notice that this new cost function integrates the information contained in $\underline{\underline{Y}}_t$ and $\underline{\underline{Y}}_o$. Furthermore, the solution $R[k] = 0$ would yield a very poor SINR compared to the other solutions and, therefore, it can be rejected as a solution of the new cost function. Once the possibility of converging to this solution has been discarded, the length constraint in (5) can be released and longer equalizers can be employed. Simulations will show the performance obtained by increasing the equalizer length.

The equalizer performance can be further improved if a delay is allowed in the equalized signal. Many sets of equalizers can be obtained for different delays:

$$\sum_{i=1}^{B} C^i(z) \cdot E^i(z) = z^{-d} \quad 0 \leqslant d < \deg\left\{C^i(z) \cdot E^i(z)\right\} \tag{11}$$

providing different delayed estimates $R(z) = z^{-d}\hat{T}(z)$. Although in average terms some delays will provide better estimates than others ([9]), the simulations performed showed that all delays are useful for noise impairment reduction due to the reduced set of data available. Notice that, if the eigenvalue is taken as an estimate of the SINR, in order to decide which of the delays yields the better estimate of $\hat{T}(z)$ only the largest eigenvalue must be computed for the different values of the delay d. Unfortunately, this estimate is only reliable in high SNR scenarios, otherwise the full computation of the equalizer output must be carried out to find out which delay value is preferred.

5 Relation with other algorithms

In this section the algorithm in [5] (equations (7)-(8)) is compared with the extension of the deterministic method proposed in [12] to the block transmission case, rather than the continuous transmission case analyzed in the original paper.

Equation (6) can be written using matrix notation as

$$\underline{Z} = \underline{\underline{Y}}\,\underline{E} = \underline{\underline{T}}\,\underline{\underline{C}}\,\underline{E} + \underline{\underline{W}}\,\underline{E}$$

where the vectors and matrices are associated to the polynomials with the same letters. In order to compare both methods a singular value decomposition (SVD) must be performed to the generalized Sylvester matrix $\underline{\underline{Y}}$:

$$\underline{\underline{Y}} = \underline{\underline{U}}\,\underline{\underline{\Sigma}}\,\underline{\underline{V}}^H$$

Then, it can be seen that the proposed method is based on the signal subspace column vectors of matrix $\underline{\underline{U}}$, whereas the method proposed in [12] was based on its noise ones.

Furthermore, the algorithm proposed here has a computational load much lower that one of [12], even if several values of the delay d in equation (11) are used to reduce variance. Both methods have in common that they require SVD computation. However, the method proposed in this paper only one SVD must be computed and the matrix involved in it has the same size as the channel length, whereas the algorithm in [12] requires two SVD of matrices about the same size as the transmitted signal length. Since the frame duration must be chosen so that

$$deg\{T(z)\} >> deg\{C^i(z)\}$$

in order to keep efficiency high, the computational load of the proposed algorithm is much lower than that one of [12]. The dimension of the matrix involved, as well as the fact of working with the noise subspace singular vectors, has a second consequence: the algorithm in [12] is more sensitive to noise than the one proposed here.

The advantage of the method [12] in front of the one proposed here relies in the fact that the estimate provided by the former one doesn't need to be the result of a linear equalization of the received data, whereas the one proposed here does. This means that, in principle, better results can be obtained in ill-conditioned channels where the linear equalization can have noise enhancement problems (even though in the SIMO case they are not as bad as in the single channel case ([11])).

6 Simulations

Fig.2 and 3 illustrate the performance of the algorithm proposed in this paper. Both plots display the percentage of realizations (500 and 1000 were averaged) for which the equalizer output EbNo was higher than the value indicated in the x-axis. In all cases the transmitted data consisted of 128 QPSK symbols. Notice that the output EbNo depends changes on each run due to the algorithm sensitivity to the channel, data and noise realizations caused by the limited amount of data available for estimation.

Figure 2 shows the performance of the algorithm in its application to an OFDM transmission in a frequency-flat fading channel corresponding to a 25Kb/s transmission at 1GHz with a mobile moving at 100Km/h. In that case two antennas were used ($B = 2$) and EbNo=20dB. This figure shows the improvement obtained when the equalizer design criteria in (7) (I) is replaced by that one of equation (10) (II). Notice the algorithm performs correctly even though the multiplicative Rayleigh channel is a very difficult environment for the blind algorithm, for the multiplicative channel does not fulfill perfectly the finite length channel hypothesis.

Figure 3 shows the performance obtained when the algorithm is applied to a TDMA block transmission in a frequency selective channel. In this case, four antennas were simulated ($B = 4$) and channel responses:

$$C^1(z)=(1+j)+(-0.1-0.2j)z^{-1}+0.4z^{-2}+z^{-3}+0.5z^{-4}$$
$$C^2(z)=0.1j+z^{-1}-0.4jz^{-2}+0.2z^{-3}-0.5z^{-4}$$
$$C^3(z)=0.1+2z^{-1}-4jz^{-2}+0.2z^{-3}+z^{-4}$$
$$C^4(z)=(1+0.8j)-2jz^{-1}-0.4jz^{-2}+0.2z^{-3}+(1-0.5j)z^{-4}$$

This figure illustrates the improvement obtained by increasing the equalizer length. In this case, EbNo=15dB and the equalizers of length 2 (I) and 4 (II) were designed using equation (10).

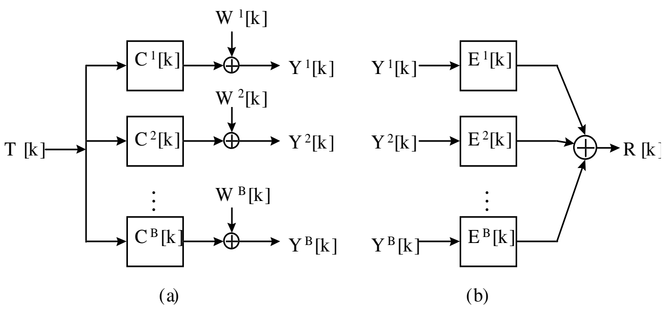

Figure 1 — Block diagram of the multichannel system. (a) Transmission; (b) Equalization

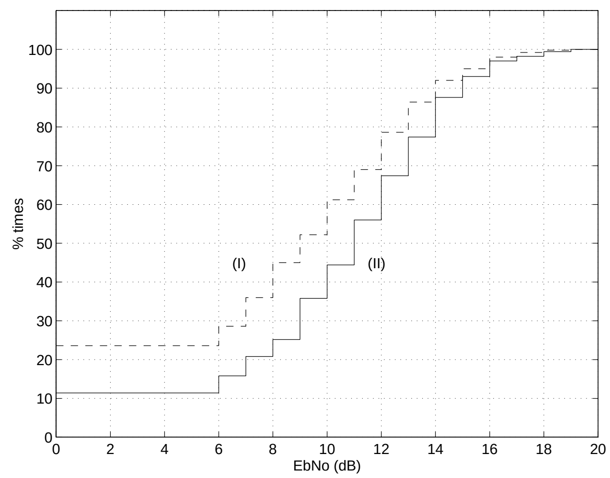

Figure 2 — Algorithm performance in the transmission of a OFDM signal in a Rayleigh frequency-flat fading scenario.

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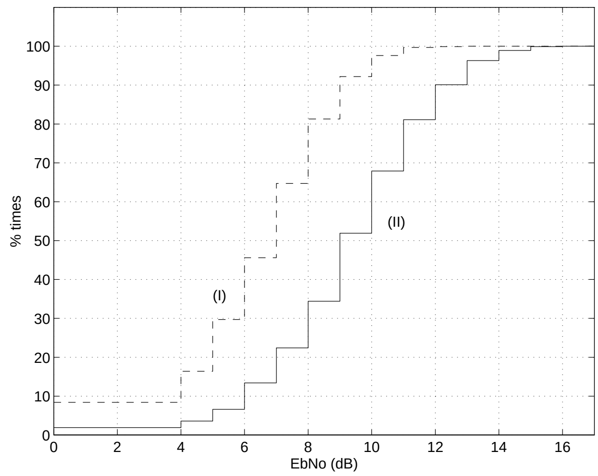

Figure 3 — Algorithm performance in the transmission of a TDMA signal in a Rayleigh frequency-selective channel

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