Using Stationary Data Model for Localization of Pulse Signal Sources

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RÉSUMÉ

Cet article traite de l'estimation des paramètres d'un ensemble de pulses par des méthodes de traitement d'antenne. L'approche proposée est basée sur la division de l'intervalle d'observation en sous-intervalles contenant des groupes de pulses assimilables à des données stationnaires. La condition sur la position des capteurs garantissant la présence de silences sur l'ensemble des capteurs est formulée en fonction des caractéristiques des pulses et de leurs directions d'arrivées. Un exemple d'application de cette condition au choix de la géométrie de l'antenne est présenté. Le domaine d'application de l'approche proposée est détaillé dans le cas d'un modèle Poissonien du flot de pulses.

ABSTRACT

The solved problem is the estimation of parameters of a stream of pulse signals from spatially distributed sources by array processing. The proposed approach is based on the division of the observation time interval for processing of subintervals which contain isolated group of pulses adequate to the stationary data model. The density condition of the antenna array depending on the characteristics of pulses and their direction-of-arrival is formulated when the processing intervals can be found by the simple algorithm of detecting the simultaneous pauses of signals in all sensors. An example of using this condition for the choice of the antenna array geometry is presented. The area of applicability of the proposed approach is detailed for Poisson pulse stream model.

Introduction

The problem of pulse signal parameters estimation appears in different areas namely radio communications, geophysics, sound ranging systems [1..3]. The nonstationarity of such signals demands to use special methods for their processing. The algorithms based on the nonstationary data model in the moving processing interval are studied in [2]. In some situations exploiting the stationary data model (SDM) and standard processing techniques is more attractive. The conditional maximum likelihood (CML) algorithm [4] is used in [5] for the processing of overlapped seismic pulses with known number of sources. However, the direct usage of SDM based methods in arbitrary time interval for pulse processing causes additional errors studied in [6].

The approach based on the procedure of searching intervals with data adequate to SDM is proposed below. It is shown that the condition of applicability of SDM is that the processing interval must include an isolated group of pulses (IGP). The demands for the antenna array geometry and for the pulse stream characteristics are established when IGP intervals can be found by means of detecting the simultaneous signal pauses in all spatial channels. The area of applicability of this approach is defined for Poisson pulse stream model. The processing algorithm for IGP signals based on CML and minimum description length (MDL) techniques with noise variance estimation during signal pauses is proposed. The improved detection performance of this algorithm compare with the known algorithm [7] is shown by simulations.

Problem formulation

Assume that M_s sources of pulse signals with unknown shape $s_i(t), i = 1...M_s$ and duration $T_i \in [T_{min}, T_{max}]$ are received by an antenna array with N sensors of known coordinates. The l-th sensor signal is expressed as following

$$x_l(t) = \sum_{i=1}^{M_s} s_i(t - T_{Bi} - \tau_l(\Theta_i)) + v_l(t),\ l = 1...N,\ t \in T_s, \tag{1}$$

where T_s : observation interval ; T_{Bi} : time of arrival of $s_i(t)$ to the first sensor ; $\tau_l(\Theta_i)$: propagation delay between the first and the l-th sensors for a signal from the direction-of-arrival (DOA) Θ_i ; $v_l(t)$: noise in l-th sensor assumed to be Gaussian and spatially and temporally white with unknown variance σ^2. All signals assumed zero mean.

The problem is to estimate of M_s and Θ_i, $i = 1...M_s$ by processing $x_l(t)$, $l = 1...N$, $t \in T_s$.

The direct using the whole observation interval T_s for estimating M_s and Θ_i is impossible because the inequality $M_s > N$ or even $M_s \gg N$ can hold for an arbitrary T_s. This situation causes the problem of dividing T_s into some processing subintervals $T_0^m < T_s$ which contain signals from some number of sources $M^m < M_s$.

The proposed approach is to detect the processing intervals T_0^m which contain data adequate to SDM and to estimate M^m and Θ_i^m by ML techniques. The following problems have to be solved in accordance with this approach :

- define the conditions of SDM applicability ;

- find the algorithm for detecting intervals with SDM signals and define its area of application ;

- find the algorithm for estimating number of sources and their parameters on each detected interval, which takes into account the features of pulse signals processing.

Condition of Stationary Data Model applicability

The standard SDM can be expressed as follows

$$\mathbf{X}(\omega_k) = \mathbf{A}(\omega_k, \Theta)\mathbf{S}(\omega_k) + \mathbf{V}(\omega_k),\ k = J_1 ... J_2, \qquad (2)$$

where $\Theta^T = \{\Theta_1, ..., \Theta_M\}$: DOA vector ; J_1, J_2 : bounds of significant frequency band ; $\mathbf{X}(\omega_k), \mathbf{S}(\omega_k), \mathbf{V}(\omega_k)$: vectors of DFT coefficients of array output signals, emitted signals and noise ; $\mathbf{A}(\omega_k, \Theta)$: $(N \times M)$ matrix of steering vectors i.e. the signal propagation model.

This model can be got by sampling (1) with sampling frequency f_t and transforming it into the frequency domain by DFT of length $L_f = T_0 f_t$ when each signal in each sensor is the delayed version of this signal in all other sensors on the processing interval T_0. The usual formulation of this demand for continuous signals is $T_A \ll T_0$, i.e. the signal propagation time along the array aperture T_A must be much less then the duration of a processing interval.

The SDM applicability condition for pulse signals can be formulated explicitly : the processing interval T_0 must include an isolated group of pulses, i.e. a set of pulses completely in all spatial channels containing in T_0.

So, the search of intervals which contain IGP is of decisive significance for the model (2) applicability in the problem considered.

Detection of Isolated Group of Pulses

Let us illustrate that this problem is not trivial in the general case. An example of spatial-temporal scenario for the array geometry and the pulse sources is presented in Fig.1,2. Let us consider the array of sensors 1, 2, 3. The pointed out intervals T_0 contain IGP from sources 1,2 and 1,3 for this array. The intervals T_F include some groups of pulses but they do not contain IGP and the model (2) is not adequate to the data on T_F . Classification of these intervals is a very complicated problem in the general case which requires an enumeration of these intervals at least.

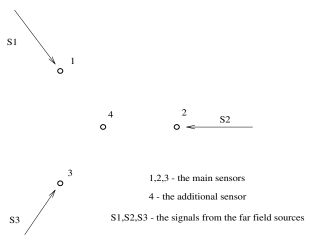

FIG. 1 — Spatial scenario for sensors and sources

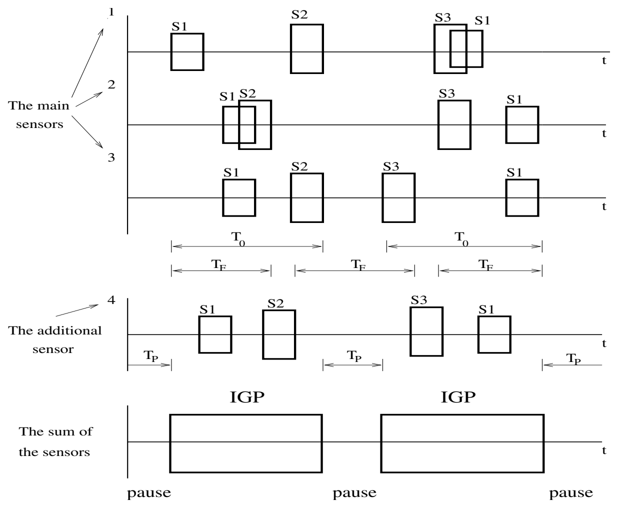

FIG. 2 — Temporal scenario for pulses

The following statement is formulated to overcome this difficulty : if the parameters of the array and the durations of pulses satisfy the following inequality

$$\max_{i=1,..,N;\ \Theta\in\tilde{\Theta}} \{\tau_{ij(i,\Theta)}(\Theta)\} < T_{\min}, \qquad (3)$$

where $\tau_{ij}(\Theta)$: propagation delay between the i-th and the j-th sensors for a signal from the DOA Θ ; $j(i,\Theta) = \arg\min_l\{\tau_{il}(\Theta) > 0\}$, $i = 1...N$: the number of the sensor which receives a signal from the DOA Θ after the i-th sensor ; $\tilde{\Theta}$: space observation area,

then the simultaneous pause intervals T_p satisfy the following condition

$$x_l(t) = v_l(t),\ l = 1...N;\ t \in T_p, \qquad (4)$$

separate the intervals T_0 with IGP.

The proof of this statement is straightforward.

The condition (3) can be simplified for the given array configuration, for example, for the linear array

$$j(i,\Theta) = \begin{cases} i+1 & \text{for } \Theta < 0,\ i = 1..N-1 \\ i-1 & \text{for } \Theta > 0,\ i = 2..N, \end{cases}$$

$$d_{\max} \sin\Theta_B / C < T_{\min}, \qquad (5)$$

where $d_{\max}$: maximal distance between sensors, C : signal propagation velocity ; $\tilde{\Theta} = [-\Theta_B, \Theta_B]$.

The condition (3) demands that the specific spatial density of the antenna array depends on the expected characteristics of pulses and their DOA. It has be taken into account together with the other usual conditions (sufficient size and number of sensors), for choosing the geometry of the antenna array for pulse stream processing.

The example of the antenna array of the sensors 1, 2, 3 in Fig.1, 2 does not satisfy this condition. The seismic antenna array in [5] is in fact dense in the sense (5) as one can see in Fig. 2 in [5].

It is worth emphasizing that the condition (3) or (5) can be used for changing the array configuration. For example, the additional sensor 4 in Fig.1 makes the array dense in the sense (3) and allows to detect IGP intervals T_0 by finding simultaneous pause intervals T_p (4) without any classifications.

Area of application

It is needed to assume some pulse stream model for the definition of the application area of the presented approach. Let us assume the Poisson model of a pulse stream [1]. The natural condition of effective using of the procedure of detecting the simultaneous pause intervals (4) is a low probability of the following event : the number of sources in some IGP interval is more then or equal to the number of sensors, i.e. $M \geqslant N$. This event is equivalent to the following one : $N-1$ pauses of Poisson stream are less then $T+T_A$ after arrival of one pulse on the array, where T is the average duration of pulses and T_A is the average time of signal propagation along the array aperture. This probability can be found by the general theory of pulse streams [1]

$$\left[1-e^{-\gamma(T+T_A)}\right]^{N-1} < \delta \quad \text{or}$$

$$\gamma(T+T_A) < \sqrt[N-1]{\delta}, \tag{6}$$

where $\delta \ll 1$ is the permissible probability of the pointed out event and γ is the average rate of pulse appearance (average number of pulses arrived on the array during one second).

This relation connects the geometry of the antenna array (the average time of signal propagation along the aperture), the average duration of pulses and the average rate of their appearance. It shows, for example, that increasing the size of the array or T_A causes decreasing of the rate of the stream which can be processed by the proposed way. This limitation is the price to pay for the simple procedure of the IGP intervals detection.

Processing algorithm

Let us use CML technique [4] for estimating the vector Θ for the given number of sources M_v which is not obligatory equal to the actual number of sources M. Taking into account that time averaging inside the processing interval T_0 is impossible, the expression of the CML estimator is the following

$$\hat{\Theta}(M_v) = \arg\min_{\Theta(M_v)} \sum_{k=J_1}^{J_2} tr\{\mathbf{P}^{\perp}(\omega_k, \Theta(M_v))\mathbf{X}(\omega_k)\mathbf{X}^*(\omega_k)\}, \tag{7}$$

where $\mathbf{P}^{\perp}(\omega_k,\Theta) = \mathbf{I} - \mathbf{A}(\omega_k,\Theta)[\mathbf{A}^*(\omega_k,\Theta)\mathbf{A}(\omega_k,\Theta)]^{-1}$ $\mathbf{A}^*(\omega_k,\Theta)$ is the projection matrix to the noise subspace.

MDL criterion [7] can be used for estimating the number of sources in T_0. This criterion has the following form in the case of averaging $J = J_2 - J_1$ frequency bins

$$\hat{M} = \arg\min_{M_v=0\ldots N-1} MDL(M_v), \tag{8}$$

$$MDL(M_v) = \ln f(\mathbf{X} \mid \hat{\mu}) + 0.5\nu \ln J, \tag{9}$$

where $f(\mathbf{X} \mid \hat{\mu})$ is the probability density function of the input vectors $\mathbf{X}(\omega_k)$; $\hat{\mu}$ is the ML parameter estimation ; ν is the number of real independent unknowns in μ.

The base of the considered pulse processing is a detection of the pause intervals T_p. It is natural to use them for the noise variance estimation

$$\hat{\sigma}^2 = \frac{1}{JNL_p}\sum_{l=1}^{L_p}\sum_{k=J_1}^{J_2}\|\mathbf{X}_p^l(\omega_k)\|^2, \tag{10}$$

where L_p is the number snapshots of the length L_f inside T_p ; $\mathbf{X}_p^l(\omega_k)$ is the vector of DFT coefficients of the l-th noise snapshot.

Substituting the least squares signal estimation $\hat{\mathbf{S}}(\omega_k) = (\mathbf{I}-\mathbf{P}^{\perp}(\omega_k,\Theta))\mathbf{X}(\omega_k)$ and (10) into (9), after transformations we get

$$MDL(M_v) = \frac{\sum_{k=J_1}^{J_2} tr\{\mathbf{P}^{\perp}(\omega_k, \hat{\Theta}(\hat{M}_v))\mathbf{X}(\omega_k)\mathbf{X}^*(\omega_k)\}}{\frac{1}{JNL_p}\sum_{l=1}^{L_p}\sum_{k=J_1}^{J_2}\|\mathbf{X}_p^l(\omega_k)\|^2} + 0.5M_v(2J+1)\ln J. \tag{11}$$

Summary of the proposed approach

1. Guarantee the fulfillment of the condition (3) for the given antenna array and characteristics of pulses, for example, by using additional sensors as shown in Fig.2. The average time of signal propagation along the aperture, the average rate and duration are connected by the inequality (6) for Poisson pulse stream model.

2. Processing the signals on the observation interval T_s by the following algorithm :

- detect the simultaneous pause intervals T_p^m (4) on the observation interval T_s by using of abrupt changes detection techniques [8]. Let us $m = 1\ldots I_p$.
- form the processing intervals $T_0^m = (t_e^m, t_b^{m+1}), m = 1\ldots I_p - 1$, where t_b^m and t_e^m are the beginning and the end of T_p^m.
- define the processing parameters : L_f^m, J_1^m, J_2^m.
- estimate $\hat{M}^m$ and $\hat{\Theta}(\hat{M}^m)$ in accordance with (8), (7), (11) on T_0^m, $m = 1\ldots I_p - 1$. If $\hat{M}^m < N$ then $\hat{\Theta}(\hat{M}^m)$ is the DOA estimate for $\hat{M}^m$ sources.

Simulation results

Let us compare performance of the key step of the presented algorithm : the detection of the number of sources in the given processing interval, with the MDL type algorithm from [7]. This algorithm estimates σ^2 by the input data $\mathbf{X}(\omega_k)$ and differs from the presented algorithm by the expression for MDL.

We simulate a uniform linear array of $N = 5$ sensors, spaced 20m apart, when $f_t = 256$Hz and $C = 340$m/s. The shape of pulses from all sources are shown in Fig.3. The following parameters are used in all experiments : $M = 3$, $\sigma^2 = 0.02$, $L_f = 128$, $J_1 = 4$, $J_2 = 24$, $L_p = 1$. The relative delay between pulses from different sources in the first sensor is $12 f_t^{-1}$. Calculations of $\hat{\Theta}(\hat{M}_v)$ in (7) are made by the alternating projection algorithm [4] with 10 iterations. The total number of trials with independent noise realizations is 100.

The signal parameters : pulse amplitude U_i and DOA Θ_i for i=1...M together with the estimated standard deviation of DOA $\sigma(\Theta_i)$ are presented in Table 1. The bias of the estimations $\hat{\Theta}(\hat{M}_v))$ are insignificant in all experiments. Simulation results for MDL (11), [7] and for their AIC versions which differ from the MDL algorithms by the absence of the coefficient $0.5 \ln J$ in the second parts of the expressions, are shown in Table 2. (It is pointed out in [7] that the AIC version of the algorithm can give better performance for a low volume of data.)

It is important to remark that for both the simple environment (experiment 1) and the complicated one (experiment 2, 3) with closely spaced, different power sources are chosen for simulations. Moreover, the comparison is made for coherent signals from different sources, i.e. in the case considered in [7].

The analysis of the presented simulation results shows that in all the considered situations the best detection performance is obtained by the MDL version of the proposed algorithm (11). Moreover, the improvement is higher in more complicated environment. For example, in situation 3 the incorrect detection of the number of sources ($M = 3$) is got in 35% of realizations for MDL [7], in 18% of realizations for AIC [7] and in 2% of realizations for MDL (11).

Conclusion

The approach based on the the division of the observation time interval for processing of subintervals which contain IGP is proposed for estimating of pulse stream parameters. The density condition of the antenna array depending on the characteristics of pulses and their DOA is formulated when IGP intervals can be found by the simple algorithm of detecting the simultaneous pauses of signals in all sensors. The algorithm for IGP processing is proposed. Its improved performance is demonstrated by simulations.

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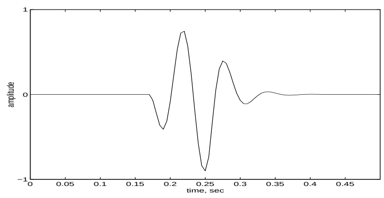

FIG. 3 — Signal shape

Nř	U_i	Θ_i	$\sigma(\Theta_i)$
1	1	-30	0.5
	1	10	0.4
	1	25	0.5
2	1	-30	0.6
	0.5	10	0.8
	1	25	0.6
3	1	0	0.7
	1	5	1.5
	0.5	15	1.6

Table 1. Signal parameters and estimated standard deviation of DOA

Nř	$\hat{M}$	MDL (11)	AIC (11)	MDL [7]	AIC [7]
1	0	0	0	0	0
	1	0	0	0	0
	2	0	0	9	0
	3	100	99	91	92
	4	0	1	0	8
2	0	0	0	0	0
	1	0	0	51	0
	2	0	0	0	0
	3	100	99	49	90
	4	0	1	0	10
3	0	0	0	0	0
	1	0	0	0	0
	2	2	0	35	12
	3	98	96	65	82
	4	0	4	0	6

Table 2. Number of realizations when the estimated number of sources equals $\hat{M}$