# Wavelet and Fractal Transforms for Image Compression

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RÉSUMÉ

Les algorithmes multirésolutionels de codage fractal exposés dans cet article combinent des transformées fractales et en ondelettes. Ils permettent d'améliorer les performances des algorithmes classiques de codage fractal dont ils réduisent les distortions caractéristiques (effets de bloc et images floues) par un meilleur codage des hautes fréquences.

ABSTRACT

The proposed multiresolution fractal coders are image compression schemes that combine wavelet and fractal transforms. They improve the performance of conventional fractal compression algorithms. They reduce the characteristic distortions of fractal algorithms: blocking artifacts and image blurring, by a better coding of high frequencies.

## 1 Introduction

The main idea behind all fractal coding algorithms is to exploit the similarities present within many natural images: one block of an image is represented by an affine transform of another larger block taken from the image itself [1, 2, 3]. The characteristic property of fractal coders is to exploit similarities between different scales. Wavelet transforms perform multiresolution decompositions of images, i.e, decompositions of the original images into subimages at different scales. The translation of the fractal property in the wavelet transform domain is straightforward: multiresolution decompositions through wavelet transforms of fractal coded images reveal strong relationships between subimages at different scales. These relationships limit the frequency content. Multiresolution fractal coders introduce degrees of freedom on these constraints.

## 2 The Fractal Algorithm in the Wavelet Transform Domain

### 2.1 Relationships

Fractal coded images are divided into non overlapping range blocks. Each of the $N_R^2$ range blocks, $j$, referred to in the whole image by a vector, $\mathbf{b}_j \in \{0, 1/N_R, \ldots, (N_R-1)/N_R\}^2$, is associated with an affine transform with scaling factor $\alpha_j$ and offset $\beta_j$, and with a domain block whose coordinates are denoted $\mathbf{d}_j$, with $\mathbf{d}_j \in \{0, 1/N_D, \ldots, (N_D-1)/N_D\}^2$. With these notations, the image $f$ is expressed as the sum of affine transformed copies of restrictions of it:

$$f(\mathbf{k}) = \sum_{j=1}^{N_R^2} \Big[ \alpha_j \cdot f(2(\mathbf{k}-\mathbf{b}_j)+\mathbf{d}_j) + \beta_j \Big] \cdot \chi(N_R(\mathbf{k}-\mathbf{b}_j)). \quad (1)$$

where $\chi(N_R(\mathbf{k}-\mathbf{b}_j))$ is the characteristic function of the $j^{th}$ range block and restricts the $j^{th}$ affine transformation to the $j^{th}$ range block. Explicit derivation of the relationships between scales of fractal coded images is obtained by translation of Equation (1) in the Haar wavelet transform domain. The obtained relationships show that when multiresolution decompositions are performed on fractal coded images, wavelet coefficients at a given resolution $m$ are scaled and shifted versions of wavelet coefficients at the previous resolution $(m-1)$, with a scaling factor proportional to $2^{-m}$. These relationships explain the resolution independence property of fractal schemes: if images are decoded at a larger size than the original one, extra details are added that mimic the real ones. Because the scaling factors between coefficients of successive scales are smaller than one, energy in octave subbands regularly decreases from the low resolutions to the high ones. The information in the high frequency components of fractal coded images is restricted to be low. When original images contain large information in high frequencies, fractal coding results in a poor rendering of high frequency components. Blur, lack of details, edge and texture degradation become noticeable.

### 2.2 Frequency Interpretation

By further exploiting the fractal property and its interpretation in the Haar wavelet transform domain, the conventional fractal algorithm is decomposed into a low-pass and a high-pass components, both using the same fractal transform.

The first part reconstructs a low resolution version of the original image, based on the conventional fractal transform

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followed by a low-pass filter. The low-pass filter reconstructs the low-resolution image obtained after one iteration of the wavelet transform. The low-pass part is a contractive transformation and, thus, possesses a unique fixed point: the low-resolution version of the fixed point of the fractal transform.

In the second part of the fractal transform, the high-pass information of the image is processed through the combination of the same fractal transform and a high-pass filter. This transformation has also a constant term called a condensation set. The high-pass part is a contractive transformation. With a careful choice of the condensation set, its fixed point is the high-pass version of the fixed point of the fractal transform.

The transformation resulting from this decomposition is indeed contractive and thus has a unique fixed point. With an appropriate choice of the condensation set, the attractor may be identical to the one obtained with the conventional fractal algorithm. The fixed point is decomposed into the sum of a low resolution image and a detail image.

# 3 Multiresolution Fractal Coders

## 3.1 First Multiresolution Fractal Coder

The constraints imposed on images by fractal coders do not exist within original images. The formulation of the fractal property in the Haar wavelet transform domain is thus modified to improve the conventional fractal algorithm. A new fractal coder is implemented: high frequencies are better rendered if two *different* fractal transforms are used inside the algorithm. The wavelet transform is applied once to the original image, yielding to a low resolution image and three detail images. These three detail images are recombined to obtain a high-pass image. The low-resolution image one fourth the original size, is coded with a first fractal code. The image obtained after iteration of this fractal code from any original image is used to compute a second fractal code to represent the high-pass original image. At the decoder, the reconstructed image is the combination of the low-resolution image obtained with the first fractal code and the detail image obtained with the second fractal code.

The preliminary results are very promising. A gain of about 4 dB is obtained with the original $128 \times 128$ "Lena" image. For larger images ($512 \times 512$ pixels) and when quantization is introduced, the average gain is of 0.7 dB, for PSNR around 25 dB. The bit rate is slightly increased, from 0.07 bit per pixel for the conventional coder to 0.11 bit per pixel for the new one. The visual quality is also clearly improved. Edges and textures are sharper. More details remain in the coded images [4, 5].

## 3.2 Extensions

The modification of the fractal coding algorithm may be generalized to any kind of frequency decomposition.

The generalized multiresolution fractal coder is decomposed into two steps. First, a wavelet transform is applied to the original image. Two subimages are obtained, a low-resolution one and a detail one. For the low-resolution image, a first fractal code is computed. For the detail subimage, a second fractal code is derived using information from the image obtained with the first fractal code. The block diagram of the generalized multiresolution fractal coder is depicted in Figure 1.

The decoding part iterates the first fractal code to construct an approximation of the low-resolution subimage. From this image and the second fractal code, an approximation of the detail subimage is reconstructed. The coded image is obtained by a combination of these subimages using an inverse wavelet transform. The block diagram of the generalized multiresolution fractal decoder is depicted in Figure 2.

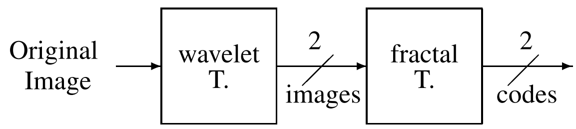



Figure 1 — General block diagram of multiresolution fractal coders. Each block represents one transform (T.).

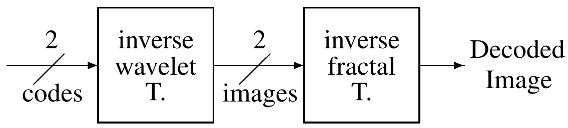



Figure 2 — General block diagram of multiresolution fractal decoders. Each block represents one inverse transform (T.).

## 3.3 Results

In the first multiresolution fractal coder described in the previous paragraph, the low-resolution image is obtained with one iteration of the Haar wavelet transform. Different coding schemes exist, according to the number of iterations of the wavelet transform and the wavelet basis used to perform them.

A second coder uses two iterations of the wavelet decomposition. Thus, the size of the low resolution image for the first part of the algorithm is one sixteenth the original size. Both PSNR and visual quality are improved. For $512 \times 512$ images, the average gain over the conventional fractal coder is about 1 dB with an increase in bit rate of only 0.04 bit per pixel. Edges and textures are better coded. The overall images are less blurred. When more than two iterations of the wavelet transform are performed, results in terms of PSNR and visual quality are improved.

Both new algorithms are easily generalized to any kind of wavelet basis. All wavelet bases are smoother than the Haar basis. The resulting coded images present less annoying blocking artifacts. However, the best wavelet basis in terms of numerical and visual quality depends on the original image. For a given image, it also depends on the range block sizes or the wavelet decomposition.

The results obtained with the multiresolution fractal algorithms are compared to those obtained with the JPEG compression standard. For very low bit rates, i.e., less than 0.1 bit per pixel, PSNRs for images coded with the proposed coders are larger than those obtained with the JPEG algorithm. Moreover, the visual differences are clearly significant. Images coded with the new fractal coders are more natural-looking. JPEG coded images are really blocky. Some edges are very straight but others completely disappear, as well as many details and most of the texture information.

The example depicted in Figure 3 summarizes all these results. The original $512 \times 512$ "baboon" image is represented in (a). It is coded with the conventional fractal coder (b). For this image, the highest PSNRs are obtained when the Daubechies wavelet with 20 coefficients is used in the wavelet transform. One, two, three and four iterations of the wavelet transform are performed to obtain the images depicted in Figure 3 (c), (d), (e), and (f), respectively.

# 4 Conclusion

Multiresolution fractal coders present all the advantages of conventional fractal coders and propose solutions to some of their drawbacks.

- Image quality of reconstructed images is good, even for very low bit rates.
- The characteristic distortions of fractal coders are reduced: blocking artifacts are less annoying, images are less blurred.
- For the implemented conventional fractal coder, the achievable bit rates are very limited. This range is drastically increased with the multiresolution coders.
- Since successive steps of the multiresolution fractal coders correspond to more and more details, these algorithms may be incorporated in a hierarchical scheme and progressive transmission to adapt to time-varying channel or display resources.
- Fractal decompression is fast. Decoding time is even reduced with the proposed schemes.
- For conventional coders, the computational for coding is very high. It is also high for the multiresolution fractal coders but is reduced with a proper chioce of range block sizes and number of iterations of the wavelet transform.

Future work includes other extensions of the multiresolution fractal coders. The two basis blocks of multiresolution fractal coders are wavelet transform and fractal transform. Only separable wavelets have been considered, either orthonormal or bi-orthogonal. Other multiresolution decomposition schemes may considered non-separable wavelets. The implemented fractal algorithm may also be improved to take into account domain block isometries or recursive splitting of range blocks.

Multiresolution fractal schemes are defined by many parameters: wavelet basis, number of bands and cut-off frequencies in the frequency decomposition, range block sizes. A thorough study still has to be performed to determine the parameters values yielding to the smallest image distortions for a target bit rate.

Good reconstructed image quality are obtained with multiresolution fractal coders at very low bit rates where they outperformed the JPEG standard algorithm, both in terms of PSNR and direct visual evaluation. Images obtained with the multiresolution fractal schemes are more natural-looking than those coded with JPEG. However, coding of the low-pass components of image blocks with the JPEG standard is very efficient. The high-pass component of the multiresolution coders may be incorporated in the JPEG scheme for a better coding of high frequency components of JPEG images. Alternatively, the low-pass component of the multiresolution fractal coders may be replaced by any other compression schemes that perform well on low resolution images.

# References

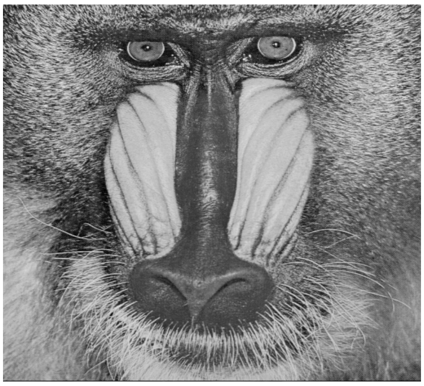
[1] M. Barnsley, *Fractals everywhere*. New York: Academic Press, 1988.

[2] A. E. Jacquin, *A fractal theory of iterated Markov operators with applications to digital image coding*. PhD thesis, Georgia Institute of Technology, 1989.

[3] E. Yuval Fisher, *Fractal Image Compression: Theory and Application*. Springer-Verlag, 1994.

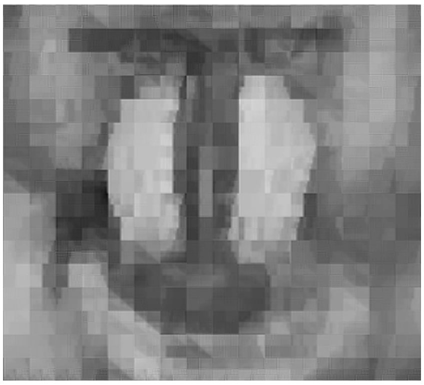
[4] F. C. Cesbron and F. J. Malassenet, "Multiresolution fractal coding of still images," in *International Conference on Signal Processing Applications and Technology, ICSPAT, Boston, USA*, Oct. 1996.

[5] F. C. Cesbron and F. J. Malassenet, "Wavelet and fractal transforms for image compression," in *Fractals in Engineering. From Theory to Industrial Applications* (J. L. Vehel, E. Lutton, and C. Tricot, eds.), pp. 329–38, Springer Verlag, 1997.

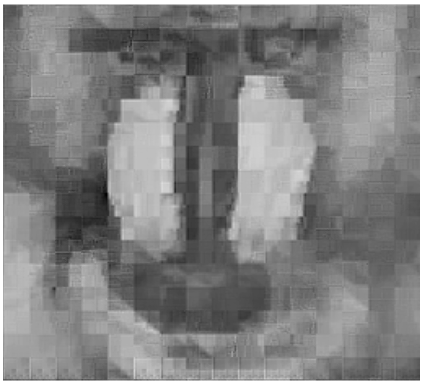


(a)

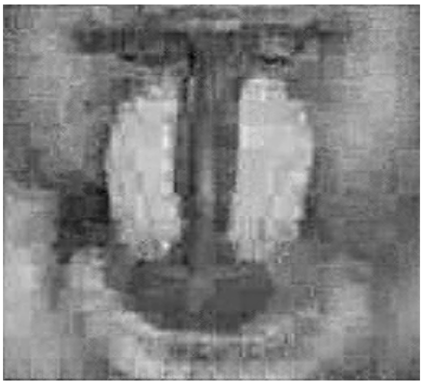
(b)



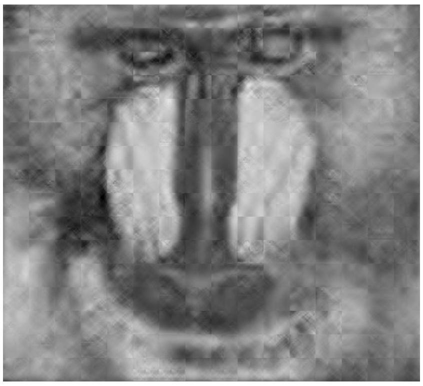
(c)



(d)



(e)



(f)

Figure 3 — "Baboon" image (512 × 512): (a) original image, (b) conventional fractal algorithm (PSNR = 17.87 dB, 0.016 bpp), multiresolution fractal coder with (c) one iteration (PSNR = 18.08 dB, 0.029 bpp), (d) two iterations (PSNR = 18.20 dB, 0.029 bpp), (e) three iterations (PSNR = 18.39 dB, 0.029 bpp), and (f) four iterations (PSNR = 18.64 dB, 0.029 bpp) of the wavelet transform.