Stability and Symmetry Breaking in the Two-Higgs-Doublet Model

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Motivations

- Standard Model (SM) contains one Higgs doublet $\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$
  - potential V$_{SM} = -\mu^2 \varphi^\dagger \varphi + \lambda(\varphi^\dagger \varphi)^2$.
  - after symmetry breaking : 4$-$3 $=$ 1 real d.o.f $\hat{=}$ 1 Higgs boson.
- motivations for extended Higgs sector:

  supersymmetry, baryogenesis,. . .
- Two-Higgs-Doublet Model(THDM) as "simplest ext.": $\varphi_1 = \begin{pmatrix} \varphi_1^+ \\ \varphi_1^0 \end{pmatrix}$, $\varphi_2 = \begin{pmatrix} \varphi_2^+ \\ \varphi_2^0 \end{pmatrix}$
  - potential more involved
  - after symmetry breaking: $8 - 3 = 5$ real d.o.f $\hat{=}$1 charged pair, 3 neutral Higgs boson.
- literatura on THDMs: huge amount (specific models, recently basis independent methods)
- here: stability and symmetry breaking in *most general* THDM at tree level (constructions for arbitrary basis)

Outline of the talk

1. THDM Potential
    - orbit variables

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2. Stability
    - Criteria for Stability
    - Example

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3. Symmetry Breaking
   - Stationary Points
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THDM Higgs Potential

How can we describe the most general THDM?

- two complex Higgs-doublet fields with hypercharge $y = +1/2$:

$$\varphi_1(x) = \begin{pmatrix} \varphi_1^+(x) \\ \varphi_1^0(x) \end{pmatrix}, \qquad \varphi_2(x) = \begin{pmatrix} \varphi_2^+(x) \\ \varphi_2^0(x) \end{pmatrix}.$$

- renormalisable, gauge invariant potential contains only

$$\varphi_i^\dagger \varphi_j, \qquad (\varphi_i^\dagger \varphi_j)(\varphi_k^\dagger \varphi_l), \quad i,j,k,l \in \{1,2\}$$

Definition: orbit variables K_0, K_1, K_2, K_3:

$$\begin{pmatrix} \varphi_1^\dagger\varphi_1 & \varphi_2^\dagger\varphi_1 \\ \varphi_1^\dagger\varphi_2 & \varphi_2^\dagger\varphi_2 \end{pmatrix} \equiv \frac{1}{2}(K_0 \mathbb{1} + K_a \sigma^a) \Leftrightarrow \begin{cases} K_0 = \varphi_1^\dagger\varphi_1 + \varphi_2^\dagger\varphi_2, & K_1 = 2\,\mathrm{Re}\,\varphi_1^\dagger\varphi_2, \\ K_3 = \varphi_1^\dagger\varphi_1 - \varphi_2^\dagger\varphi_2, & K_2 = 2\,\mathrm{Im}\,\varphi_1^\dagger\varphi_2 \end{cases}$$

- general THDM Higgs potential :

$$V(\varphi_1, \varphi_2) = V_2 + V_4 \quad \text{with} \quad \begin{cases} V_2 = \xi_0 K_0 + \xi_a K_a \\ V_4 = \eta_{00} K_0^2 + 2 K_0 \eta_a K_a + K_a \eta_{ab} K_b \end{cases}$$

- no gauge d.o.f. in this scheme, reduced powers

Orbit Variables

- domain($\mathbf{K} \equiv (K_1\ K_2\ K_3)^T$):

$$K_0 = ||\varphi_1||^2 + ||\varphi_2||^2 \quad \geq 0$$
$$K_0^2 - K^2 = 4\left(||\varphi_1||^2||\varphi_2||^2 - |\varphi_1^\dagger\varphi_2|^2\right) \quad \geq 0$$

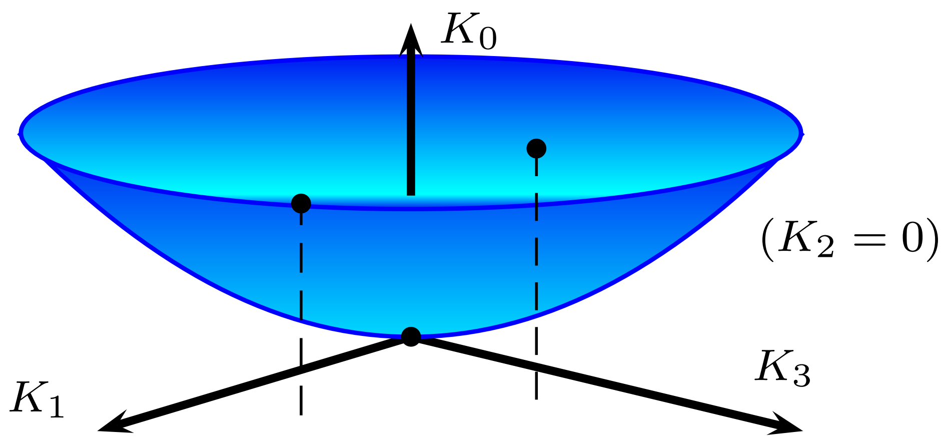
- change of doublet basis by $U \in U(2)$

$$\begin{pmatrix}\varphi_1' \\ \varphi_2'\end{pmatrix} = U \begin{pmatrix}\varphi_1 \\ \varphi_2\end{pmatrix},$$

  means for orbit variables

  $K_0' = K_0, \quad \mathbf{K}' = R(U)\mathbf{K}, \quad R(U) \in SO(3), \quad U^\dagger\sigma^a U = R_{ab}(U)\sigma^b.$

- Minkowski type structure:$(K_0, \mathbf{K})$ on and inside "forward light cone".



Stability

- stable potential: bounded from below
- stability determined by V in limit $(|\varphi_1|^2 + |\varphi_2|^2 =)\ K_0 \to \infty$,
- consider V_4 for $K_0 > 0$, define:

$$\mathbf{k} \equiv \mathbf{K}/K_0, \quad \text{with} \quad |\mathbf{k}| \leq 1.$$

$$V_4 = K_0^2 J_4(\mathbf{k}), \quad J_4(\mathbf{k}) \equiv \eta_{00} + 2\eta^T \mathbf{k} + \mathbf{k}^T E \mathbf{k}$$

- stability guaranteed by $V_4 \Leftrightarrow J_4(\mathbf{k}) > 0$ for all $|\mathbf{k}| \leq 1$
- domain of J_4 is unit ball: compact

  $J_4 > 0 \Leftrightarrow J_4|_{stat} > 0$ for all its stationary points
- stationary points of $J_4(\mathbf{k}) \equiv \eta_{00} + 2\eta^T k + k^T E k$ on domain $\mathbf{k}^2 \leq 1$

$|\mathbf{k}| < 1$: solve $\nabla_k J_4(\mathbf{k}) = 0 \quad \Leftrightarrow Ek = -\eta \quad$ with $\quad 1 - \mathbf{k}^2 > 0$

$|\mathbf{k}| = 1$: define $F_4(\mathbf{k}, u) \equiv J_4(\mathbf{k}) + u(1 - \mathbf{k}^2)$ with Lagrange multiplier u,

solve $\nabla_k F_4(\mathbf{k}, u) = 0 \quad \Leftrightarrow (E - u)k = -\eta \quad$ with $\quad 1 - \mathbf{k}^2 = 0$

Stability

Stability criteria via one function

- unified description with

$$f(u) \equiv F_4(\mathbf{k}(u), u) = u + \eta_{00} - \eta^T (E - u)^{-1} \eta$$

- for regular stationary points (set $u = 0$ for $|\mathbf{k}| < 1$):

$$f(u) = J_4(\mathbf{k}) = V_4(k)/K_0^2,$$

$$f'(u) = 1 - \mathbf{k}^2$$

- define $I =$ "set of all u values belonging to stat. pnts. of $J_4(\mathbf{k})$":

$$I := \{u | f'(u) = 0 \quad \vee \qquad \leftarrow \text{on sphere}$$

$$u = 0 \quad \wedge \quad f'(0) > 0\} \qquad \leftarrow \text{inside ball}$$

(exceptional solutions omitted here)

Theorem

Stability of potential guaranteed by $V_4 \;\Leftrightarrow\; f(u_i) > 0$ for all $u_i \in I$

Stability

Illustration of Stability Determining Function

- stability determined by $f(u) \equiv F_4(\mathbf{k}(u), u)$
- explicitely

$$f(u) = u + \eta_{00} - \eta^T (E - u)^{-1} \eta,$$
$$f'(u) = 1 - \eta^T (E - u)^{-2} \eta,$$

- in a basis where $E = \mathtt{diag}(\mu_1, \mu_2, \mu_3)$:

$$f(u) = u + \eta_{00} - \sum_{a=1}^{3} \frac{\eta_a^2}{\mu_a - u},$$
$$f'(u) = 1 - \sum_{a=1}^{3} \frac{\eta_a^2}{(\mu_a - u)^2}.$$

- $f'(u)$ has at most 6 zeros
- (exceptional solutions only possible if corresponding $\eta_a = 0$)

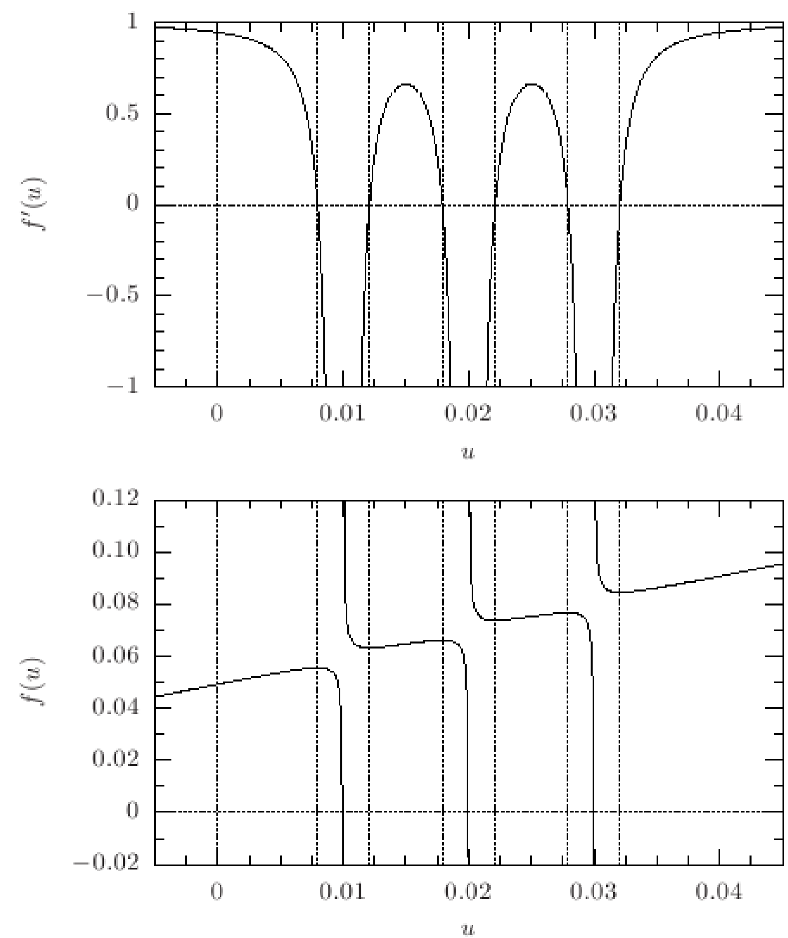


figure: $f'(u), f(u)$ for $\eta_{00} = 5 \cdot 10^{-2}$, $\eta_a = 2 \cdot 10^{-3}$, $(\mu_1, \mu_2, \mu_3) = (1, 2, 3) \cdot 10^{-2}$

Example: THDM of Gunion et al.

Potential

- Higgs potential:

$$
\begin{aligned}
V = {} & \lambda_1(\varphi_1^\dagger\,\varphi_1 - v_1^2)^2 + \lambda_2(\varphi_2^\dagger\,\varphi_2 - v_2^2)^2 + \lambda_3(\varphi_1^\dagger\,\varphi_1 - v_1^2 + \varphi_2^\dagger\,\varphi_2^\dagger - v_2^2)^2 \\
& + \lambda_4((\varphi_1^\dagger\,\varphi_1)(\varphi_2^\dagger\,\varphi_2) - (\varphi_1^\dagger\,\varphi_2)(\varphi_2^\dagger\,\varphi_1)) \\
& + \lambda_5(\mathrm{Re}(\varphi_1^\dagger\,\varphi_2) - v_1 v_2\cos\xi)^2 + \lambda_6(\mathrm{Im}(\varphi_1^\dagger\,\varphi_2) - v_1 v_2 \sin\xi)^2 \\
& + \lambda_7(\mathrm{Re}(\varphi_1^\dagger\,\varphi_2) - v_1 v_2\cos\xi)(\mathrm{Im}(\varphi_1^\dagger\,\varphi_2) - v_1 v_2\sin\xi)
\end{aligned}
$$

- V breaks $(\varphi_1, \varphi_2) \leftarrow (-\varphi_1, \varphi_2)$ only softly
- V_4 parameters:

$$\eta_{00} = \frac{1}{4}(\lambda_1 + \lambda_2 + 4\lambda_3 + \lambda_4),$$

$$\eta = \frac{1}{4}\begin{pmatrix} 0 \\ 0 \\ \lambda_1 - \lambda_2 \end{pmatrix}, \quad E = \frac{1}{8}\begin{pmatrix} 2(\lambda_5 - \lambda_4) & \lambda_7 & 0 \\ \lambda_7 & 2(\lambda_6 - \lambda_4) & 0 \\ 0 & 0 & 2(\lambda_1 + \lambda_2 - \lambda_4) \end{pmatrix}.$$

Example: THDM of Gunion et al.

Stability

- stability guaranteed by $V_4 \overset{\text{theorem}}{\Longleftrightarrow}$

$$\lambda_1+\lambda_3 > 0, \quad \lambda_2+\lambda_3 > 0, \quad \lambda_4, \kappa > -2\lambda_3-2\sqrt{(\lambda_1+\lambda_3)(\lambda_2+\lambda_3)}$$

where $\kappa := \frac{1}{2}(\lambda_5 + \lambda_6 - \sqrt{(\lambda_5 - \lambda_6)^2 + \lambda_7^2}.$

Stationary Points

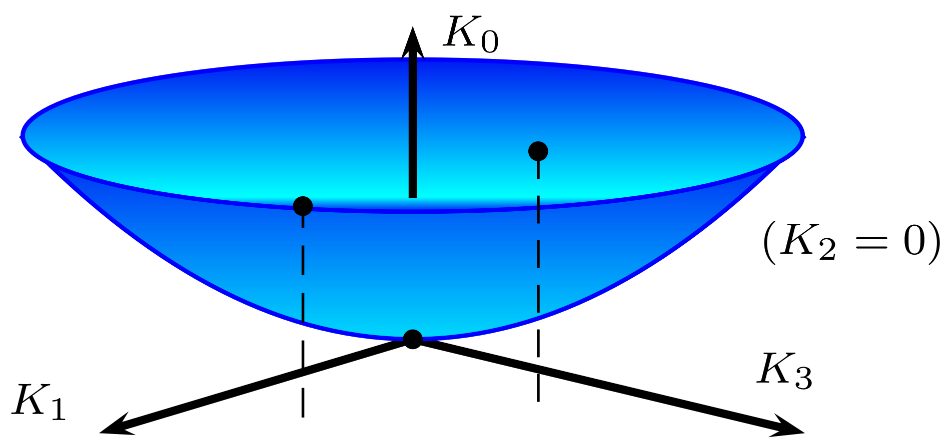
- 4-vector notation: $\tilde{\mathbf{K}} = \begin{pmatrix} K_0 \\ \mathbf{K} \end{pmatrix}$ $\quad \tilde{\xi} = \begin{pmatrix} \xi_0 \\ \xi \end{pmatrix}$ $\quad \tilde{E} = \begin{pmatrix} \eta_{00} & \eta^T \\ \eta & E \end{pmatrix}$
- potential $V = \tilde{\mathbf{K}}^T \tilde{\xi} + \tilde{\mathbf{K}}^T \tilde{E} \tilde{\mathbf{K}}$
- three classes of stationary points:

  $K_0 = \mathbf{K} = 0:$

  Trivial solution ($\varphi_1 = \varphi_2 = 0$)

  $K_0 > \mathbf{K}:$ solve ($\nabla_{\tilde{\mathbf{K}}} V = 0$)

  $K_0 = \mathbf{K} > 0:$ solve $\left(\nabla_{\tilde{\mathbf{K}},u} [V - u(K_0^2 - \mathbf{K}^2)] = 0 \right)$



Stationary Points

What are the implications for EWSB ?

- consider symmetry breaking behaviour of $\langle\varphi_1\rangle \equiv \begin{pmatrix} v_1^+ \\ v_1^0 \end{pmatrix}$ $\quad \langle\varphi_2\rangle \equiv \begin{pmatrix} v_2^+ \\ v_1^0 \end{pmatrix}$ for three global minimum $K_0, \mathbf{K}$ cases

$$K_0 = ||\varphi_1||^2 + ||\varphi_2||^2 \geq 0$$

$$K_0^2 - \mathbf{K}^2 = 4(||\varphi_1||^2||\varphi_2||^2 - |\varphi_1^\dagger\varphi_2|^2 \geq 0$$

$$(\langle\varphi_1\rangle \quad \langle\varphi_2\rangle) = \begin{pmatrix} v_1^+ & v_2^+ \\ v_1^0 & v_2^0 \end{pmatrix}$$

$K_0 = \mathbf{K} = 0$	$\Rightarrow$	$\langle\varphi_1\rangle = \langle\varphi_2\rangle = 0$	$\Rightarrow$	unbroken $SU(2)_L \otimes U(1)_Y$
$K_0 > \lvert\mathbf{K}\rvert$	$\Rightarrow$	$\langle\varphi_1\rangle, \langle\varphi_2\rangle$ lin. indep.	$\Rightarrow$	fully broken $SU(2)_L \otimes U(1)_Y$
$K_0 = \lvert\mathbf{K}\rvert > 0$	$\Rightarrow$	$\langle\varphi_1\rangle, \langle\varphi_2\rangle$ lin. dep.	$\Rightarrow$	$SU(2)_L \otimes U(1)_Y \longrightarrow U(1)_{em}$

Stationary Points

- 4-vector notation: $\tilde{\mathbf{K}} = \begin{pmatrix} K_0 \\ \mathbf{K} \end{pmatrix} \quad \tilde{\xi} = \begin{pmatrix} \xi_0 \\ \xi \end{pmatrix} \quad \tilde{E} = \begin{pmatrix} \eta_{00} & \eta^T \\ \eta & E \end{pmatrix}$
- potential $V = \tilde{\mathbf{K}}^T \tilde{\xi} + \tilde{\mathbf{K}}^T \tilde{E} \tilde{\mathbf{K}}$
- three classes of stationary points:

$K_0 = \mathbf{K} = 0$:

trivial solution $(\varphi_1 = \varphi_2 = 0)$

unbroken $SU(2)_L \otimes U(1)_Y$

$K_0 > \mathbf{K}$:

solve $\nabla_{\tilde{\mathbf{K}}} V = 0$

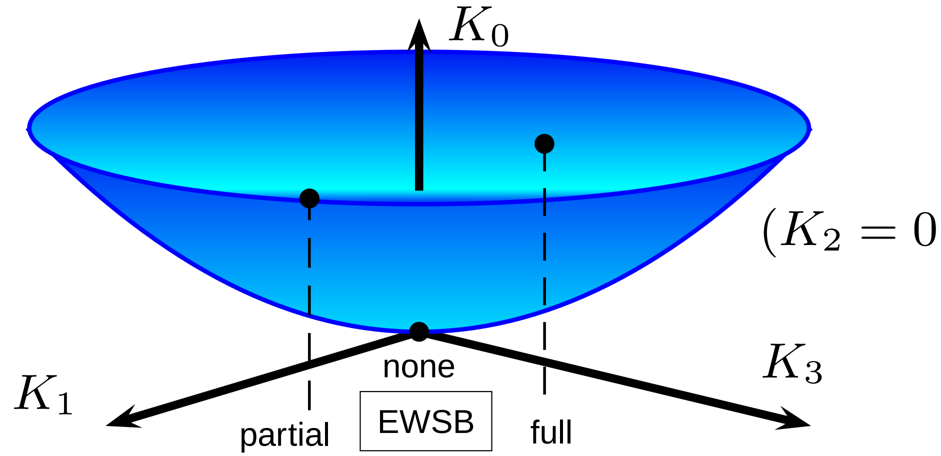
fully broken $SU(2)_L \otimes U(1)_Y$

$K_0 = \mathbf{K} > 0$:

solve

$\nabla_{\tilde{\mathbf{K}},u} \left[V - u(K_0^2 - \mathbf{K}^2) \right] = 0$

$SU(2)_L \otimes U(1)_Y$ broken to $U(1)_{em}$



Criteria for Symmetry Breaking

stationary points considerations easily give:

- $V = \frac{1}{2}\tilde{\mathbf{K}}^T\tilde{\xi} = -\tilde{\mathbf{K}}^T\tilde{E}\tilde{\mathbf{K}}$ at stationary points
  $\overset{\text{stab.}}{\Rightarrow} V < 0$ for all non-trivial stationary points
- mutually exclusive possibilities for local minima:
  - one or multiple min. with required EWSB $(K_0 = |\mathbf{K}|)$
  - one charge breaking minimum $(K_0 > |\mathbf{K}|)$
  - (degenerate set of solutions $(K_0 \geq |\mathbf{K}|)$)
  - trivial minimum $(\tilde{\mathbf{K}} = 0)$
- nontrivial minimum $\Leftrightarrow \xi_0 < |\xi|$,

Theorem

Global minimum with spont. symmetry breaking $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{em}$

is given and guaranteed by stat. pnt. of type $K_0 = |\mathbf{K}| > 0$
with largest Lagrange multiplier $u_0 > 0$.

Potential after Symmetry Breaking

- vacuum

$$\langle\varphi_1\rangle \equiv \begin{pmatrix} 0 \\ v_1^0 \end{pmatrix} \quad \langle\varphi_2\rangle \equiv \begin{pmatrix} 0 \\ v_2^0 \mathrm{e}^{i\xi} \end{pmatrix}$$

must lie on the "forward light cone"

- change of base with $\tan\beta = v_2^0/v_1^0$, unitary gauge:

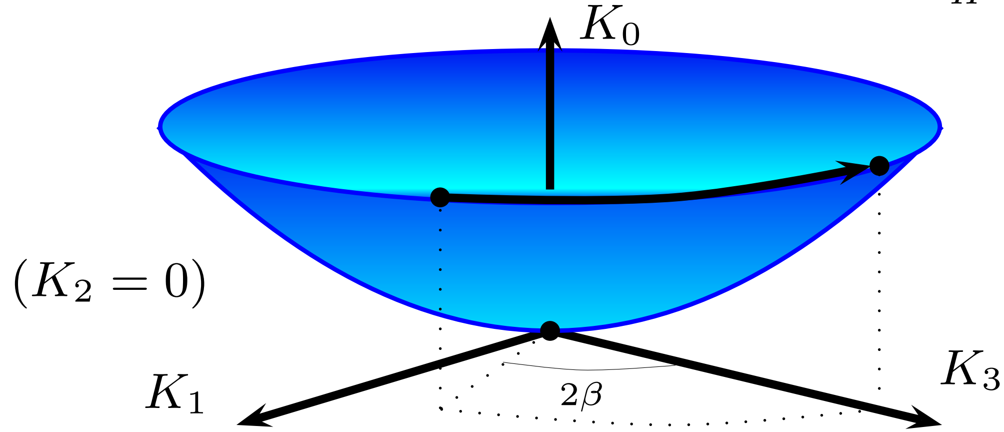
$$\langle\varphi_1'\rangle \equiv \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}v_0, \end{pmatrix}, \quad \langle\varphi_2'\rangle \equiv \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Leftrightarrow \tilde{\mathbf{K}}'^T = \begin{pmatrix} v_0^2/2 & 0 & 0 & v_0^2/2 \end{pmatrix}$$

separates Goldstone modes

- charged mass squared

$$m_{H^\pm}^2 = 2u_0 v_0^2$$



Example: THDM of Gunion et al.

Potential

- Higgs potential:

$$V = \lambda_1(\varphi_1^\dagger\,\varphi_1 - v_1^2)^2 + \lambda_2(\varphi_2^\dagger\,\varphi_2 - v_2^2)^2 + \lambda_3(\varphi_1^\dagger\,\varphi_1 - v_1^2 + \varphi_2^\dagger\,\varphi_2^\dagger - v_2^2)^2$$
$$+ \lambda_4((\varphi_1^\dagger\,\varphi_1)(\varphi_2^\dagger\,\varphi_2) - (\varphi_1^\dagger\,\varphi_2)(\varphi_2^\dagger\,\varphi_1))$$
$$+ \lambda_5(\mathrm{Re}(\varphi_1^\dagger\,\varphi_2) - v_1 v_2 \cos\xi)^2 + \lambda_6(\mathrm{Im}(\varphi_1^\dagger\,\varphi_2) - v_1 v_2 \sin\xi)^2$$
$$+ \lambda_7(\mathrm{Re}(\varphi_1^\dagger\,\varphi_2) - v_1 v_2 \cos\xi)(\mathrm{Im}(\varphi_1^\dagger\,\varphi_2) - v_1 v_2 \sin\xi)$$

- for all $\lambda_i > 0$ global minimum obvious:

$$\langle\varphi_1\rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle\varphi_2\rangle = \begin{pmatrix} 0 \\ v_2 \mathrm{e}^{i\xi} \end{pmatrix}$$

- stability conditions:

$$\lambda_1 + \lambda_3 > 0, \quad \lambda_2 + \lambda_3 > 0, \quad \lambda_4, \kappa > -2\lambda_3 - 2\sqrt{(\lambda_1+\lambda_3)(\lambda_2+\lambda_3)}$$

where $\kappa := \frac{1}{2}(\lambda_5 + \lambda_6 - \sqrt{(\lambda_5-\lambda_6)^2 + \lambda_7^2}.$

Example: THDM of Gunion et al.

Stationary Points

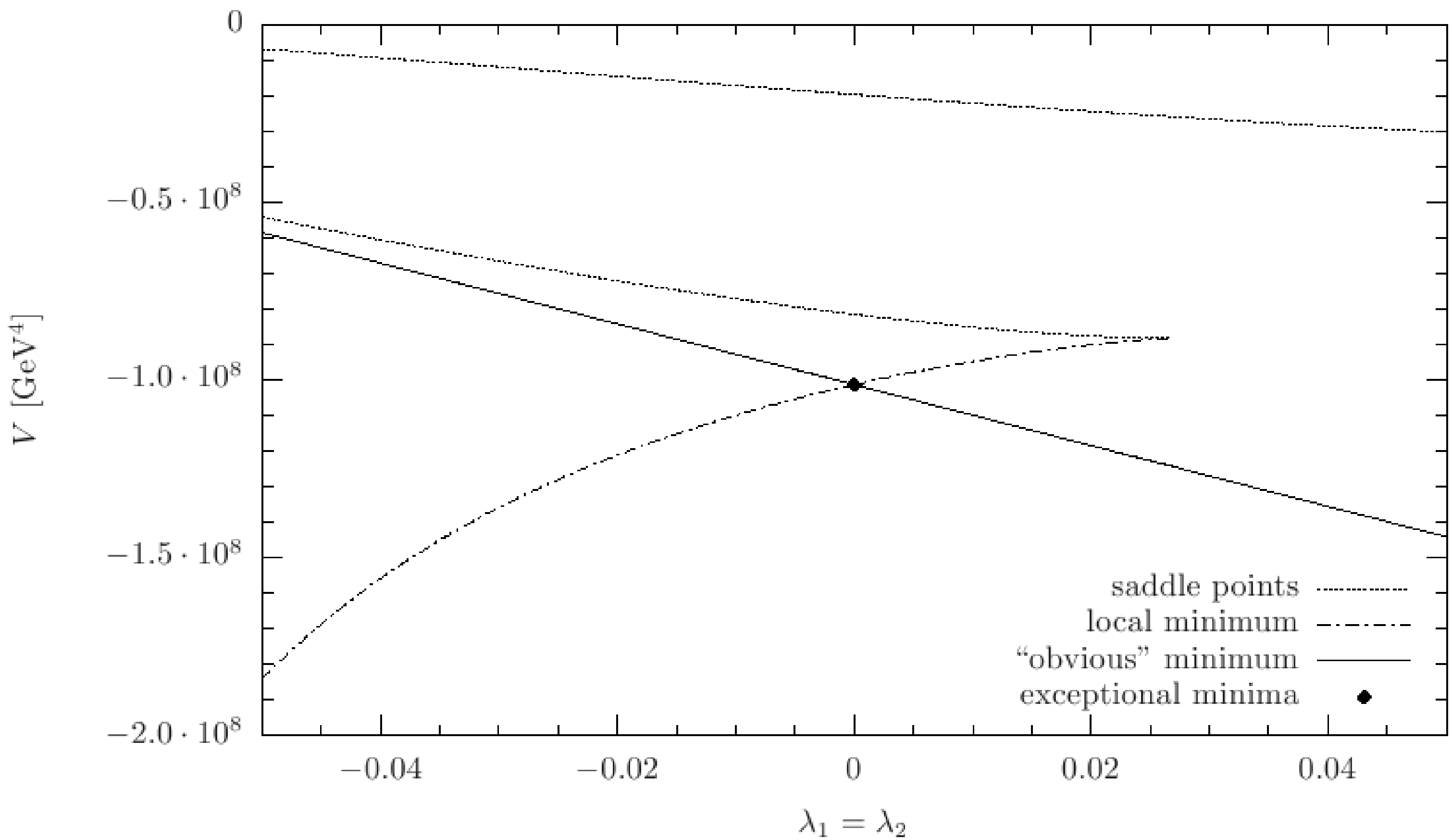


Figure: V (shifted $V(\tilde{\mathbf{K}} = 0)$ at all stationary points with $K_0 = |\mathbf{K}| > 0$ for $\lambda_3 = 0.1, \lambda_4 = 0.2, \lambda_5 = \lambda_6 = 0.4, v_1 = 30$ GeV, $v_2 =$171 GeV, $\lambda_7 = 0, \xi = 0$.

Summary

General THDM Higgs potential:

- orbit variables simplify access to structure:
  - no gauge d.o.f.
  - reduction of powers
- results for stability and for symmetry breaking
  - general conditions via univ. function
    (explicit solutions for stat. pnts., arbitrary basis)
  - structural statements:
    existence of minima, . . .
- specific application: clarifications for THDM of Gunion et al.
  - explicite general stability conditions
  - global minimum implications