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Letters to the Editors

Threshold energy of impact ionization by electrons and holes in germanium

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A number of workers (Miller 1955, Chynoweth *et al* 1960, Ivakhno 1972, Tyagi 1973) have determined threshold energy of impact ionization in germanium by considering it as a parameter and adjusting its value to fit the values of ionization coefficient (α_i) to the theoretical curves of α by Shockley (1961) Wolf (1954) and Baraff (1952). Hauser (1966) has theoretically calculated the threshold energy but confined his attention to (111) valley of the conduction band and heavy hole only. Anderson and Crowell (1972) and Ballinger *et al* (1973) have used detailed band structure but their methods are highly involved and require large computer time. We have attempted to calculate threshold energy using detailed band structure by analytical method of Shekhar and Sharma (1974, 1975).

The following parameters are used in the calculations. $<111>$ *valleys* : Location $k_0 = \frac{2\pi}{a}$ $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$, longitudinal mass $m_l = 1.64m_0$, density of state mass $m_d = .22m_0$. $E_g = .665$ eV. (000) *valley* . Location $k_0(000)$, electrons spherical mass $0.04m_0$, $E_g = .705$ eV.

$<100>$ *valleys* : Location $k_0 = .86\times\frac{2\pi}{a}$ (1, 0, 0), electrons' conductivity mass $= .20m_0$, $E_g = .865$ eV.

Valence band : Loation (0, 0, 0); heavy hole mass $m_h = .319m_0$, light hole mass $m = .043m_0$ and split off hole mass $m_s = 075\, m_0$, split off band is .29 eV below the coincident tops of heavy hole and light hole bands, lattice constant $a = 5.6575$ Å.

For the calculation of threshold energies for impact ionization both normal and Umklapp processes have been considered for both electrons and holes. The momentum conservation equation for ionization by electrons for normal and Umklapp processes respectively are

$$k_1 = k_1'+k_2'+k_h \qquad \ldots \quad (1a)$$

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$$k_1 = k_1' + k_2' + k_h + G \qquad \ldots \quad (1b)$$

where k_1 is the wave vector of primary electron before impact, k_1', k_2' and k_h are wave vectors of primary electron, secondary electron and hole after the impact process; G is reciprocal lattice vector. Assuming the bottom of the conduction band as energy reference, the energy of the electron is given by

$$E = \frac{h^2}{2m_e}(k-k_0)^2$$

where k_0 is the wave vector of the corresponding conduction band minimum. We expect that the minimum energy for impact ionization will occur when the motions of all the particles involved in the impact process are confined in the direction of valley minimum In this case the vector momentum eq (1) reduces to one dimensional scalar equztion. The energy conservation equation for ionization process for typical combination of location of electrons before impact in (111) valley and after impact in (111) and ($\bar{1}\bar{1}\bar{1}$) valleys is

$$\frac{h^2}{2m_e}(k-{}_1k_0)^2 = \frac{h^2}{2m_e}(k_1'-k_0)^2 + \frac{h^2}{2m_e}(k_2'+k_0)^2 + E_g + \frac{h^2k_h^2}{2m_h} \qquad \ldots \quad (3)$$

After eliminating k_h from eq. (3) with the help of eq. (1)a and then minimizing it with respect to k_1' and k_2'; eq. (3) becomes

$$k_1^2\left(\frac{m_e+m}{2m_e+m}\right) - 2k_0k_1 + \left(k_0^2 - \frac{2m_eE_g}{h^2}\right) = 0 \qquad \ldots \quad (4)$$

Eq. (4) is quadratic in k_1 and is solved numerically to give two values. The energy is calculated by using eq (2) for both vazlues of k_1. The lower value of energy is taken as threshold energy provided that k_1 is real and within the first Brillouin zone; corresponding k_1', k_2' and k_h are all real and within first Brillouin zone and the wave vectors of the electrons must correspond to their valleys assumed in a particular process. Such physical restrictions are applied in the calculation of threshold energies for all combinations of locations of electrons and holes. The lowest set of values of threshold energies for electrons in each valley and holes in each branch of valence band are respectively given in tables 1 and 2.
Column (a) is for longitudinal mass and column (b) for density of state mass for electrons in <111> and conductivity mass in <100> valleys (as other masses in <100> valley are not known). N stands for normal and U stands for Umklapp process.

From table 1 it can be seen that for electrons in (111) valley threshold energy ranges from 0.814 eV to 1.053 eV. Assuming the concentration of holes in the valence band is proportional to $mh^{3/2}$, the weighted average of threshold energy for the electrons in (111) valey is 0.837 eV.

Table 1 Threshold energy of electrons in germanium

Position of primary electron before impact	Positions of electrons after impact	Hole kind	Threshold energy in eV (a)	Threshold energy in eV (b)	Pro
(000)	$(\bar{1}11)$ (111)	*hh*	0·553	0·556	
		h	0·558	0·574	
		sh	0·827	0·886	
(111)	(111) $(\bar{1}\bar{1}1)$	*hh*	0·814		
		h	0·823	3·029	
		sh	1·053	3·112	
(100)	(100) $(\bar{1}00)$	*hh*		1·489	
				1·728	
				1·893	

Table 2. Threshold energy of holes in germanium

Primary hole before impact	Holes after impact	Electron position after impact	Threshold energy in eV (a)	Threshold energy in eV (b)	Process
hh	*hh hh*	(000)		1·51	*N*
	hh h	(000)		3·28	*N*
	hh h	(111)	1·32	2·01	*N*
h	*hh hh*	(000)		0·85	*N*
	hh h	(000)		0·89	*N*
	hh hh	(111)	1·79	3·29	*N*
sh	*hh hh*	(000)		0·57	*N*
	h h	(000)		0·62	*N*
	hh hh	(111)	1·43	2·74	*N*

The threshold energy for electrons in (000) valley ranges from 0.553 eV to 0.827 eV with weighted average at 578 eV and that for electrons in (100) valley it varies from 1.489 eV to 1.893 eV having weighted average at 1.495 eV The transition probability for impact ionization and fraction of electron in each of these valleys will give the contribution to impact ionization by these valleys It is established by these calculations that impact ionization in germanium will be initiated by electrons in (000) valley. It is expected that at high fields involved in impact ionization, appreciable fraction of electrons may exist in (000) valley hence the threshold energy of impact ionization will range from 0.578 eV to 0.837 eV

Table 2 shows that the threshold energy for heavy hole, light hole and split off hole are 1.32 eV, 0.85 eV and 0.57 eV respectively The fraction of split off holes even at high fields is expected to be small, because of small mass and large gap between split off branch and other two branches The threshold energy of holes will thus range from 0.85 eV to 1.32 eV. These calculations clearly establish that the impact ionization in semiconductor with multivalley conduction band and degenerate valence band can not be sharp but gradually increasing process as more and more impact processes come into play with increase of electric fields

Our values are in excellent agreement with experimentally determined values by Mackay and McAfee (1953) ($0.7 \text{ eV} \leqslant E_{te}$, $E_{th} \geqslant 2.9 \text{ eV}$); Tyagi (1973) ($E_{te} = 1.8 \text{ eV}$, $E_{th} = 2.6 \text{ eV}$), Miller (1955) ($E_{th} = E_{th} = 1.50 \text{ eV}$) and Ivankho and Novak (1972) ($0.66 \text{ eV} \leqslant E_{te}$, $E_{th} \geqslant 0.74 \text{ eV}$). These values are also in excellent agreement with the theoretical values of Hauser ($E_{te} = 0.91 \text{ eV}$, $E_{th} = 1.3 \text{ eV}$) Ballinger *et al* ($E_{te} = .74 \text{ eV}$) and Anderson & Crowell (1972) ($E_{ti} = .76 \text{ eV}$) Hauser's method is similar to ours except that he has considered only (111) valley of conduction band and heavy hole. He has not considered other conduction band minima and other holes. His calculations indicate a sharp ionization Methods of Ballingers *et al* and Crowell are modified Franz's graphical construction involving heavy computation giving only one threshold energy and hence indicating a sharp ionization. Important feature of our calculations is that impact ionization in germanium will be started by electrons in central (000) valley at $E_t = 0.578 \text{ eV}$ which is less than the energy gap of semiconductor and enhanced by electrons in (111) valley of $E_t = 0.837 \text{ eV}$ and further enhanced by electrons in (100) valley at $E_t = 1.495 \text{ eV}$ indicating an increasing impact ionization with the increase of energy of electrons

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