Indian J. Phys 52A 286–288 (1978)

Letters to the Editors

Off-shell K matrix for the Morse function*

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(*Received* 3 *June* 1977)

In a recent paper (Talukdar *et al*, 1977) (hereafter cited as paper I) we have derived a wave function method for computing the off-shell K matrix elements for potentials regular at the origin. The method has been found to have certain computational advantages More specifically, we could relate the K matrix elements for the exponential potential to Bessel functions. The object of the present communication is to demonstrate explicitly that basic results of paper I can also be used to obtain the K matrix for the Morse function in terms of tabulated transcendental functions. The Morse function represents a static soft core potential which can account for the behaviour of s-wave nucleon-nucleon phase shifts at high energies, in particular that these phase shifts become negative (Darewych & Green, 1967) The Morse function has been used to approximate the velocity dependent nucleon-nucleon potential (Green *et al*, 1967; Lodhi, 1969, 1970)

The s-wave part of the off-shell K matrix is given by (Talukdar *et al*, 1977)

$$<p\,|\,K\,|\,q> = -\frac{2}{\pi pq}\int_0^\infty \sin pr\; V(r)\phi(k,q,r)dr, \qquad \ldots\ (1)$$

where $\phi(k,q,r)$ is an off-shell wave function regular at the origin It satisfies a standing wave boundary condition and can be written as

$$\phi(k,q,r) = -\frac{1}{4}\pi q <k\,|\,K\,|\,q> [f(k,r)+f(-k,r)]$$

$$+\frac{1}{2i}[f(k,q,r)-f(k,-q,r)] \qquad \ldots\ (2)$$

The functions $f(k,r)$ and $f(k,q\,r)$ represent the on and off-shell Jost solutions (Fuda & Whiting 1973) The object $<k\,|\,K\,|\,q>$ stands for the half-off-shell K matrix given by

$$<k\,|\,K\,|\,q> = \frac{2\,\mathrm{Im}\,f(k,q)}{\pi q\,|f(k)|\,\cos\delta(k)} \qquad \ldots\ (3)$$

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* Work supported in part by the Department of Atomic Energy, Government of India.

** Based in part on a thesis to be submitted by one of the authors (NM) to the University of Burdwan, West Bengal, India, for the Ph.D. degree.

In eq. (3) $f(k)$ and $f(k, q)$ represent the on and off-shell Jost functions and $\delta(k)$, the phase of the Jost function, which is obviously the phase shift. The on and off-shell Jost solutions for the Morse potential

$$V(r) = V_0(e^{-2(r-d)/b} - 2e^{-(r-d)/b}) \qquad \dots \quad (4)$$

can be written in the forms (Talukdar *et al*, 1975).

$$f(k, r) = \alpha^{ikb}\, e^{ikd}\, z^{-ikb}\, e^{-\frac{1}{2}z}\; {}_1F_1(a, c, z) \qquad \dots \quad (5)$$

and

$$f(k, q, r) = \sum_{n'=0}^{\infty} G_{n'}(V_0, b, d, q, \rho)\, \frac{\sigma(\sigma+c-1)}{(\sigma+n')(\sigma+n'+c-1)}$$

$$\times e^{-\frac{1}{2}Z}\, Z^{n'+\alpha-ikb}\; {}_2F_2\begin{pmatrix} 1 \quad \sigma+n'+a \\ \sigma+n'+1,\; \sigma+n'+c \end{pmatrix} Z \Bigg). \qquad (6)$$

where $\quad Z = \xi\, e^{-r/b}$

and $\quad G_{n'}(V_0, b, d, q, \rho) = (2V_0^{\frac{1}{2}}b)^{iqb}\, e^{iqd}\, \frac{\rho^{n'}}{n'!}$

with

$$\rho = \tfrac{1}{2},\; \xi = \alpha\nu,\quad \alpha = 2V_0^{\frac{1}{2}}b,\quad \nu = e^{d/b},$$

$$\sigma = ikb - iqb,$$

$$a = \tfrac{1}{2} - ikb - V_0^{\frac{1}{2}}b,$$

and $\quad c = 1 - 2ibk.$

Combinings eq (1), (2), (5) and (6) we get the off-shell K matrix in the form

$$<p|K|q> = \frac{2V_0\nu}{\pi pq} \sum_{m,\, n\geq 0}^{\infty} \frac{(-1)^m\, \xi^{m+n}}{2^m\, m!} \Bigg[B\,(k,q)A(k)\, \frac{(a)_n}{(c)_n n!}$$

$$\times\{\xi^{-ikb}\,(\nu I(2, -k, 0) - 2I(1, -k, 0) + \xi^{ikb}(\nu I(2, k, 0) - 2I(1, k, 0))\}$$

$$+ \frac{1}{2i}\sum_{n'=0}^{\infty} \Bigg[D_{n'}(q, \sigma)\, \frac{(\sigma+n'+a_n)}{(\sigma+n'+1)_n(\sigma+n'+c)_n}\, \xi^{(n'-iqb)}\{\nu I(2, -q, n') - 2I(1, -q, n')\}$$

$$- D_{n'}(-q, \sigma')\, \frac{(\sigma'+n'+a)_n}{(\sigma'+n'+1)_n(\sigma'+n'+c)_n}\, \xi^{n'+iqb}\{\nu I(2, q, n') - 2I(1, q, n')\}\Bigg]\Bigg] \qquad (7)$$

where

$$B(k, q) = -\frac{1}{4} \pi q \langle k | K | q \rangle,$$

$$A(k) = \xi^{ikb} = \alpha^{ikb} e^{ikd},$$

$$(\alpha)_n = \frac{\Gamma(\alpha+n)}{\Gamma(\alpha)},$$

$$Dn'(\pm q, x) = G_n'(V_0, b, d, \pm q, \rho) \frac{x(x+c-1)}{(x+n')(x+n'+c-1)}$$

and

$$I(j, g, n') = \frac{pb^2}{p^2b^2+(m+n+j+n'+igb)^2}.$$

The Pochhammer symbol $(\alpha)_n$ as well as the integral $I(j, g, n')$ characterising the off-shell K matrix in eq. (7) converge uniformly (Luke, 1969) as n and $n' \to \infty$. It is therefore possible to sum the series on a computer and use it to check on programmes which attempt to compute the K matrix by using any iterative procedure.

References

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