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Comments on the correlation function in deep inelastic scattering

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Two-particle and three-particle correlation functions in the central region have been worked out in order to study the behaviour of multiparticle productions at very high energy The two-particle correlation function is expressed in terms of relative distance between the two particles in rapidity space. The three-particle correlation function is also a function of relative distance in rapidity space apart from containing individual rapidity terms

1. INTRODUCTION

The structure of hadrons and their interactions are very little known even to-day Both theoretical and experimental efforts have been going on to interpret the constituents of hadrons in terms of a few fundamental units (Gell-Mann 1964 and Zweig 1964) without much success. Recently, high energy electron has been used to probe the structure of the hadron

In deep inelastic scattering, the coupling constant involving the virtual photon is model dependent The virtual photon nucleon scattering is similar to that of the hadron-hadron scattering when the mass of the virtual photon is fixed. The observed scaling phenomena suggests that the coupling constant (Cahn & Colglazier 1973) of the virtual photon should be independent on Q^2 when Q^2 tends to infinity In deep inelastic scattering, physical quantities are dependent on Bjorken variable only (Bjorken & Paschos 1969). Our earlier work (Pal 1975) shows that the two particle correlation function is independent on Q^2 and ω when the particles are not produced in the central plateau. When both of them are produced in the central region, their sub-region is much smaller than other sub-regions. This, of course, does not follow from the hadron-hadron scattering. We note that for large ν and Q^2 the sub-region of the virtual photon and the hadron produced nearer to it is larger than other sub-regions. Using the technique of Abarbonel (Abarbonel 1970), the correlational function is proportional to $\exp[-(y_1-y_2)/L]$ where y_1 and y_2 are the repidities of the produced particles We note that it is independent of the virtual mass and the target. When three hadrons are produced in the central region, the correlation function is proportional to sum of the terms containing factors $\exp[-(y_1-y_3)/L)$ and $\exp(-y_i/L)$, $i=1, 2, 3$ if $y_1 > y_2 > y_3$ The first term is expected, for it contains the relative distance between the end particles in rapidity space.

To new feature of the three particle correlation function is that it damps exponentially with a characteristic range for rapidity $y_i\alpha[1-\alpha_R(0)]$, $i = 1, 2, 3$

2. Kinematics and Correlation Functions

Let the momenta of the virtual photon, the target and the observed hadrons be $q_1 P, p_1, p_2$ and p_3 respectively. The rapidity variable y_i which is defined by $y_i = \frac{1}{2}\ln(\omega_i+p_{Li})/(\omega_i-p_{Li})$ is convenient for our considerations at high energy. Here ω_i and p_{Li} are the energy and longitudinal momentum of the i-th particle. At high energy, the transverse momentum of any particle is finite and does not play any role for various physical quantities of interest. At laboratory frame, the square of the total energy of the virtual photon and the target s is given by $s = M^2-Q^2+2M\nu$. Let $S_{\gamma i}$ and S_{Ai} be the square of the sub-energies of the virtual photon and i-th particle, the virtual photon and the target and the particles 1 and 2 respectively. Therefore, we have

$$s_{\gamma_i} \simeq -2m_{\perp i}[\nu\exp(-y_i)-M/\omega\sinh y_i]-Q^2$$

$$S_{Ai} \simeq -Mm_{\perp i}\exp(y_i) \qquad (1)$$

and

$$S_{12} \simeq m_{\perp 1}m_{\perp 2}\exp[-(y_2-y_1)]$$

where

$$m_{\perp i}^2 = m_i^2+p_{\perp i}^2$$

$$Q^2 = -q^2 \quad \text{and}$$

$$\nu = \text{energy of the virtual photon,}$$

provided

$$y_1 > y_2 \text{ and } \ln m_{\perp i}/M \leqslant y_i \leqslant \ln s/Mm_{\perp i}$$

To start with, we have got the virtual projectile γ (the photon) and the target proton P. The one particle distribution is defined as the invariant quantity of distribution of one particular type of particles due the collision between the virtual particle and the target. The particle distribution can be defined similarly when we are interested in distribution of two particular types of particles due to collision between the virtual projectile and the target. σ_{tot} is the total cross-section of the virtual projectile and the target. With the help of generalized optical theorem of Mueller (Mueller 1970), the total cross-section σ_{tot}, particle 1, particle 2 and particle 1 and 2 produced together distributions are given by the following equations :

$$\sigma_{tot} \simeq \beta_P(Q^2)\beta_A{}^P + \beta_R(Q^2)\beta_A{}^R s^{-(1-\alpha_R(0))} \qquad \dots \quad (2)$$

This is obtained in (figure 1a). The first terms in eq. (3) is due to Pomeronchon exchange. Here $\beta_P(Q^2)$ is the coupling constant of the virtual photon with the Pomeronchon trajectory and $\beta_A{}^P$ is that of the target with the Pomeronchon trajectory. The second term in eq. (2) is due to other trajectories R $\beta_R(Q^2)$ and $\beta_A{}^R$ are the corresponding coupling constants due to exchange of trajectories R.

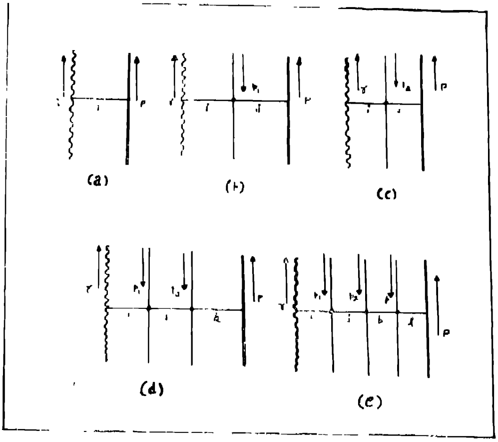

Fig (1a). Total Cross-section of virtual projectile represented by wavy line and the target P by thick line. The thin line i represents Pomeronchon and Regge trajectory exchange between the projectile and the target

Fig. (1b) One particle inclusive cross-section $\rho_{gA}{}^1$ The thin line i represents Pomeronchon and Regge trajectory exchange between the virtual projectile and the observed hadron p_1. The thin line j represents Pomeronchon and Regge trajectory exchange between the target P and the observed hadron p_1

Fig (1c). Similar interpretation holds good as in Fig (1b). Only one has to replace the observed hadron p_1 by p_2.

Fig (1d) Two-particle inclusive cross-section $\rho_{gA}{}^{1,2}$ due to collision between the virtual projectile γ and the target P. The interpretation of thin lines i and k is similar to that of i and j in Fig, (1b). The thin line j here represents Pomeronchon and Regge Trajectory exchange between the observed hadrons p_{1g} and p_2.

Fig (1e) The three-particle inclusive cross-section $\rho_{gA}{}^{1,2,3}$ due to collision between the virtual projectile γ and the target P. The thin line i represents Pomeronchon and Regge trajectory exchange between virtual projectile γ and observed hadron p_1. The lines j and k represent Pomeronchon and Regge trajectory exchange between observed hadrons p_1 and p_2 and observed hadrons p_2 and p_3 respectively. The thin line l represents Pomeronchon and Regge trajectory exchange between observed hadron p_3 and the target P.

$$\rho_{gH}{}^1 \simeq \beta_P(Q^2)\beta_A{}^P f_{PP}(p_{\perp 1})+\beta_P(Q^2)\beta_A{}^R f_{PR}(p_{\perp 1})\,|s_{A_1}|^{-(1-\alpha_R(0))} \quad .. \quad (3)$$

(This is given in figure (1b))

$$\rho_{gA}{}^2 \simeq \beta_P(Q^2)\beta_A{}^P f_{PP}(p_{\perp 2})+\beta_P(Q^2)\beta_A{}^R f_{PR}(p_{\perp 2})\,|s_{A_2}|^{-(1-\alpha_R(0))} \quad ... \quad (4)$$

(See figure (1c))

$$\rho_{gA}{}^{1,2} \simeq \beta_P(Q^2)\beta_A{}^P f_{PP}(p_{\perp 1}) f_{PP}(p_{\perp 2}) +$$

$$+\beta_P(Q^2)\beta_A{}^R f_{PR}(p_{\perp 1}) f_{RP}(p_{\perp 2}) \mid s_{12} \mid^{-(1-\alpha_R(0))} \quad (5)$$

(See figure (1d))

In statistical mechanics (Mueller 1971), correlation functions are defined to measure the difference between the particle distributions actually observed in an interaction and the particle distributions expected if there are no correlations present. The two particle correlation function $G_2(y_1, p_{\perp 1}, y_2, p_{\perp 2})$ is defined by the following equation

$$G_2(y_1, p_{\perp 1}, y_2, p_{\perp 2}) = \rho_{gA}{}^{1,2}/\sigma_{tot} - (\rho_{gA}{}^1/\sigma_{tot})(\rho_{gA}{}^2/\sigma_{tot}) \quad .. \quad (6)$$

This is zero when the total energy including all sub-energies are allowed to tend to infinity. The sub-energy $|s_{12}|^{\frac{1}{2}}$ is the smallest of all. We note, therefore, how the correlation function tends to zero with tre separation between observed hadrons in rapidity is given by

$$G_2(y_1, p_{\perp 1}; y_2, p_{\perp 2}) \simeq f_{PR}(p_{\perp 1}) f_{RP}(p_{\perp 2}) \mid s_{12} \mid^{-(1-\alpha_R(0))}$$

$$= f_{PR}(p_{\perp 1}) f_{RP}(p_{\perp 2}) (m_{\perp 1} m_{\perp 2})^{-\frac{1}{2}} \exp[-(y_1 - y_2)/L] \quad .. \quad (7)$$

where

$$L = [1-\alpha_R(0)]^{-1} \simeq 2$$

and $\rho_{gA}{}^1$, $\rho_{gA}{}^2$ and $\rho_{gA}{}^{1,2}$ are invariant distributions of particle 1, particle 2 and particles 1 and 2 together respectively. The three-particle correlation function $G_3(y_1, p_{\perp 1}; y_2, p_{\perp 2}, y_3\ p_{\perp 3})$ is defined by the following equation .

$$G_3(y_1, p_{\perp 1}; y_2, p_{\perp 2}; y_3, p_{\perp 3}) = \rho_{gA}{}^{1,2,3} -$$

$$1/\sigma_{tot}{}^2[\rho_{gA}{}^{2,3}\rho_{gA}{}^1 + \rho_{gA}{}^{3,1}\rho_{gA}{}^2 + \rho_{gA}{}^{1,2}\rho_{gA}{}^3] + 2/\sigma_{tot}{}^3 \rho_{gA}{}^1 \rho_{gA}{}^2 \rho_{gA}{}^3 \quad . \quad (8)$$

where $\rho_{gA}{}^{1,2,3}$ is the invariant distribution of particles 1, 2 and 3 together (see figure 1e). This is zero when the total energy including all sub-energies are allowed to tend to infinity. The sub-energies $s_{12}{}^{\frac{1}{2}}$ and $s_{23}{}^{\frac{1}{2}}$ are the smallest of all. The measure of the rate of approach to the scaling limit is given by the next leading trajectory R. The three-particle correlation function expressed in terms of the separation between observed hadrons in rapidity is given by

$$G_3(y_1, p_{\perp 1}; y_2, p_{\perp 2}; y_3, p_{\perp 3}) \simeq$$

$$-f_{PP}(p_{\perp 2}) f_{RP}(p_{\perp 3}) f_{PR}(p_{\perp 1}) (m_{\perp 1} m_{\perp 3})^{-\frac{1}{2}} \exp[-(y_1 - y_3)/L]$$

$$-3 f_{PP}(p_{\perp 2}) f_{RP}(p_{\perp 3}) f_{PR}(p_{\perp 1}) \beta_R/\beta_P (2\pi m_{\perp 3})^{-\frac{1}{2}} \exp(-y_1/L)$$

$$-3 f_{PP}(p_{\perp 1}) f_{PP}(p_{\perp 2}) f_{PR}(p_{\perp 3}) \beta_R/\beta_P (2\pi m_{\perp 3})^{-\frac{1}{2}} \exp(-y_3/L)$$

$$-3 f_{PP}(p_{\perp 1}) f_{PP}(p_{\perp 3}) f_{PR}(p_{\perp 2}) \beta_R/\beta_P (2M m_{\perp 2})^{-\frac{1}{2}} \exp(-y_2/L). \quad ... \quad (9)$$

Thus we observe that the characteristic distance over which the correlation function is effective is given by $1/[1-\alpha_R(0)]$, where R is the next to Pomeronchon trajectory which controls the approach to sacling, for ρ, ω, A_2 and f trajectories, we have $\alpha_R(0) \simeq \frac{1}{2}$ and $L \simeq 2$ We have used our earlier work (Pal 1975). The notations used here are properly explained there. Expressions for various physical quantities e.g., total cross-section, correlation functions etc. have been properly interpreted on the basis of the assumed model there. For the sake of completeness, we are reproducing some of our earlier works. We have used smooth functions $f_{PR}(p_{\perp 1})$ and $f_{RP}(p_{\perp 2})$ in the correlation function $G_2(y_1, p_{\perp 1}, y_2, p_{\perp 2})$ in eq. (7). Here $p_{\perp 1}$ and $p_{\perp 2}$ are the transverse momentum of the observed hadrons of energy-momentum p_1 and p_2 respectively The lower suffix P and R denote pomeronchon exchange from one side and Regge trajectory exchange from the other side The f's are the coupling constants of the observed hadrons with Pomeronchon and Regge trajectory. Other smooth functions in eq. (9) have similar interpretation.

3. Conclusion

When hadrons are produced in the central retion, the correlation function becomes independent of the beam and the target. The two-particle correlation function depends only on the relative distance of the two particles in rapidity space. The three-particle correlation function behaves similarly but it also contains terms as function of each individual rapidity However, when any of the hadrons is produced in the current fragmentation region, the correlation function highly model-dependent and depends on the mass of the virtual photon (Pal 1975). We hope that it will be easier to visualize the many-body system at high energies with the help of higher correlation function

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