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# Spectral shape of sea level muons derived from the model of Bull et al. using ISR results on kaon-pion ratio

D. P. BHATTACHARYYA, BIVA CHOUDHURY AND D. BASU

*Indian Association for the Cultivation of Science, Jadavpur, Calcutta* 700032.

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The pion production spectrum derived by different authors from the sea level muon spectrum has been fairly reviewed in our earlier works (Bhattacharyya 1972, 1974). In these calculations the contribution of kaons in the muon flux was neglected. The Intersecting Storage Ring experiments of CERN Group (Antinucci *et al* 1973) have found that the production of kaons in pp collision is not negligible. The average charged pion and kaon multiplicities produced in a pp collision obey the following relations

$$<n_\pi \pm> = 1.71 \ln s + 3.7 s^{-0.5} - 4.3 \quad \ldots \quad (1)$$

$$<n_k \pm> = 0.24 \ln s + 1.5 s^{-0.5} - 1.02 \quad \ldots \quad (2)$$

where $s$ is the square of the total center of mass energy. The kaon-pion ratio, $<n_k \pm>/<n_\pi \pm>$ calculated from relations (1) and (2) has been plotted in the figure 1 as a function of the square of center of mass energy It is found that the increase of $K/\pi$ ratio is very slow.

Recently Thompson & Whalley (1975) have studied the data of Ayre *et al* (1975) on $2 \times 10^5$ particles above 20 GeV muon energy which have provided impetus to us to consider $K/\pi$ ratio from the ISR data for the production of muons due to decay of kaons as well as pions. We have fitted the ISR data on the $K/\pi$ ratio in the converntional diffusion model of Bull *et al* (1965) and have calculated the spectral shape of the sea level muons in the energy range 5-650 GeV.

*Theoretical model after Bull et al* (1965)

The diffusion equation for pions in the atmosphere has been studied by Bull *et al* (1965) For equal absorption length of protons and interaction length of pions the sea level muon spectrum can be calculated from the following expression

$$N(E_\mu)dE_\mu = A.P_\mu (E_\mu + \Delta E_\mu)^{-\gamma}. \left\{ \frac{r_\pi{}^{\gamma-1} B_\pi}{E_\mu + \Delta E_\mu + B_\mu} + \frac{K}{\pi} \frac{r_k{}^{\gamma-1} B_k}{E_\mu + \Delta E_\mu + B_k} \right. dE_\mu \quad \ldots \quad (3)$$

where the energy degradation factors are $r_\pi = 0.757$, $r_k = 0.52$; the critical energies for pion and kaon decays are $B_\pi = 121$ GeV, $B_k = 450$ GeV; $\Delta E_\mu$ is the muon energy loss from production to ground level and is taken from the following relation

$$\Delta E_\mu = 2.333\text{GeV} + 0.0026 E_\mu \quad \dots (4)$$

The muon survival probability is calculated from the expression

$$P_\mu = [0.097 E_\mu/(2.33 + 1.0026 E_\mu)]^{1.08/(2.583+1.003\, E_\mu)} \quad \dots (5)$$

and the kaon-pion ratio at production has been taken from the figure 1. It is

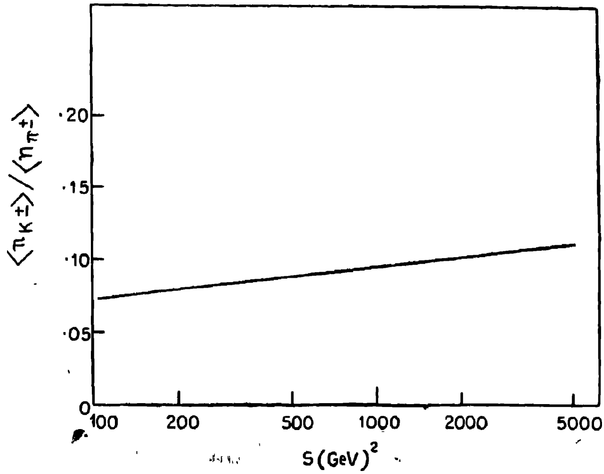


Fig. 1. Kaon pion ratio estimated from the CERN Interesting Storage Ring data has been plotted as a function of the square of total center of mass energy.

assumed that the exponent of the pion and kaon spectra are similar to that of the measured primary proton spectrum of Ryan *et al* (1972) which is $\gamma = 2.75$.

The calculated sea level muon spectrum has been fitted to the measured data of Allkofer *et al* (1971) and Ayre *et al* (1975) in the spectral range 5-650 GeV. Figure 2 shows the derived sea level muon spectrum with the fitting parameters $A = 0.202$ and $\gamma = 2.75$ along with the experimental data of Kiel and Durham Groups. The theoretical results and experimental data of Allkofer *et al* (1971) and Ayre *et al* (1975) have been used in a $\chi^2$ test to find the best values of $\gamma$ and $A$. The analysis show that for $A = 0.202$ and $\gamma = 2.75$ the calculated spectrum is in accord with the measured values

The $\chi^2$ value for 25 degrees of freedom has been calculated and was found to be 16.63 which corresponds to significance levels of 0.88 for the muon energy range 5-650 GeV.

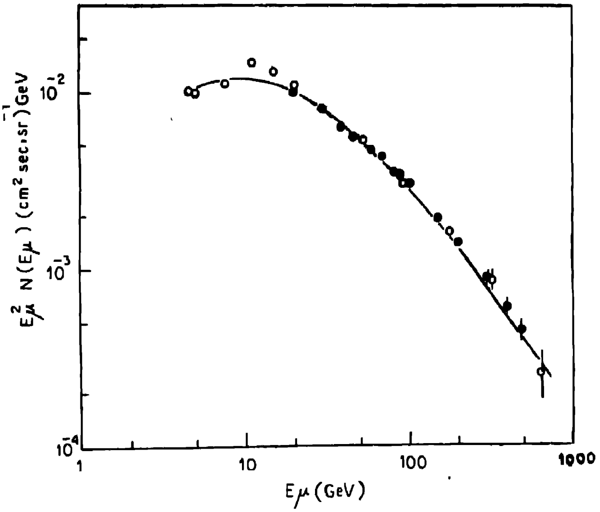


Fig. 2. The calculated sea level muon spectrum for $A = 0{\cdot}202$ and $\gamma = 2{\cdot}75$. Experimental data : O Allkofer *et al* (1971); ● Ayre *et al* (1975).

The present study indicates that a single energy exponent ($\gamma = 2.75$) can explain the entire spectral shape of the energy spectrum of muons derived from the diffusion model of Bull *et al* (1965).

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