PROPERTIES OF 35-PLET OF ONE BARYON AND TWO-BARYON SYSTEMS

S. T. H. ABIDI

DEPARTMENT OF PHYSICS, INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR INDIA

(Received October 30, 1967, Resubmitted June 17, 1968)

ABSTRACT. The states bolonging to (4,1) irreducible representations of SU_3 have been classified by their charge and U-spin. This classification has then been used to derive mass relations. The probable value of spin J of a baryon-baryon system belonging to (4, 1) representation has been shown to be 1. It has also been found that $(JP = 5/2^+, B = 1)$ baryon states can only belong to (4, 0, 0, 0, 1) irreducible representation of SU_6 . Some branching ratios in the decays $35 \rightarrow 8\bigotimes 27$ are calculated.

INTRODUCTION

Recently Dashen and Sharp (1965) have used the static model of Abers' et al (1964) in conjunction with S-matrix perturbation theory of Dashen and Frautschi (1964) to discuss the masses and decay of 35-plet of baryons $J^P = 5/2^+$ into (i) an octet of baryons and an octet of mesons and (ii) an octet of mesons and a deciment of baryons. Zimerman and Fagundes (1966) have calculated certain sum-rules among the coupling constants in the decay of 35-plet into octet of 0mesons and deciment of $3/2^+$ baryons. In the present paper the states belonging to (4, 1) irreducible representations of SU_3 have been classified according to their charge and U-spin (introduced by Lipkin, Mesbkov and Levinson 1963). This classification has been found useful in deriving mass relations among the different members of the multiplet. The relations obtained are in perfect agreement with the masses calculated by Dashen and Sharp. The possible irriducible representation of SU_6 into which this (4,1) multiplet can be accomodated has been shown to be (4, 0, 0, 0, 1).

The results obtained by this analysis may be utilized with advantage to study the properties of the 35-plet of dibaryons. Some attempts (Oakes 1963, Gerstein 1964 and Lipkin 1965) have already been made to classify the multi-baryonic states according to irreducible representations of SU_3 with limited success. Oakes (1963), for example, has shown that deuteron is a member of a $\overline{10}$ -multiplet. The other members of this multiplet may, possibly, be produced in baryonbaryon interactions. The possible multiplets that may arise in baryon-baryon interactions are an octet, a decimet, a 27-plet and a 35-plet. Some properties of this 35-plet have been studied by the method presented in this paper. The possible spin values are specified and some interesting branching ratios in the decay of the 35-plet into an octet and a 27-plet have been calculated.

U-SPIN EIGEN-STATES OF 35-P LET

In SU₃ multiplets like $(3\mu, 0)$ and $(0, 3\mu)$ where μ is any integer, since there is never more than one state at a given point on the $I_3 - Y$ diagram the transformation from I-spin classification to U-spin classification of states obtained simply by a rotation of $I_3 - Y$ diagram by an angle of 120°. In the multiplet (4,1), however, one finds that there are twenty points on $I_3 - Y$ diagram where two states occur at the same point. Consequently, in 35-plet the eigenfunctions of I^2 are not the eigen-functions of U^2 . The U-spin eigen-states can, however, be expressed as linear combinations of isor pin eigen-states. In table 1 the U-spin eigen-states, along with their charges, as linear combinations of isospin states are listed. The coefficients in these combinations have been determined with the help of commutation relations between raising and lowering operators U_+ , I_+ , $U_$ and I_- . For the isospin eigen-states notations of Lipkin and Harari (1964) have been used.

Charge Q	U	U ₃	Notation
3	1/2	+1/2	<i>I</i> ₂ ⁺⁺⁺
3	1/2	-1/2	N_{5}^{*+++}
2	1	+1	<i>I</i> ₂ ⁺⁺
2	1	U	$\tilde{N}_5^{*++} = \left(\frac{3}{5}\right)^{\frac{1}{2}} N_3^{*++} + \left(\frac{2}{5}\right)^{\frac{1}{2}} N_5^{*++}$
2	1	-1	Y2 ^{*++}
2	0	0	$\tilde{N}_{3}^{*++} = \left(\frac{3}{5}\right)^{\frac{1}{2}} N_{5}^{*++} - \left(\frac{2}{5}\right)^{\frac{1}{2}} N_{3}^{*+}$
1	3/2	+3/2	<i>I</i> ₂ +
1	3/2	+1/2	$\tilde{N}_{5}^{*+} = \left(\frac{1}{5}\right)^{\frac{1}{2}} N_{5}^{*+} + \left(\frac{4}{5}\right)^{\frac{1}{2}} N_{5}^{*+}$
1	3/2	- 1/2	$\tilde{Y}_{2}^{*+} = \left(\frac{1}{2}\right)^{\frac{1}{2}} Y_{2}^{*+} + \left(\frac{1}{2}\right)^{\frac{1}{2}} Y_{1}^{*+}$
1	3/2	-3/2	Ξ ₃*+

Table 1 U-spin eigen-states

Charge Q	U	υ	s Notation
1	1/2	+1/2	$\hat{N_{3}}^{*+} = \left(\frac{4}{5}\right)^{\frac{1}{2}} N_{5}^{*+} = \left(\frac{1}{5}\right)^{\frac{1}{2}} N_{3}^{*+}$
1	1/2	- 1/2	$\tilde{Y}_{1}^{*+} = \left(\frac{1}{2}\right)^{\frac{1}{2}} Y_{2}^{*+} - \left(\frac{1}{2}\right)^{\frac{1}{2}} Y_{1}^{*+}$
0	2	+2	1 ₂ 0
0	2	+1	$\tilde{N}_5^{*0} = \left(\frac{1}{10}\right)^{\frac{1}{2}} N_5^{*0} + \left(\frac{9}{10}\right)^{\frac{1}{2}} N_3^{*0}$
0	2	0	$\tilde{Y}_{2}^{*0} = \left(\frac{1}{4} \right)^{\frac{1}{2}} Y_{2}^{*0} + \left(\frac{3}{4} \right)^{\frac{1}{2}} Y_{1}^{*0}$
0	2	- 1	$\overline{\Xi}_{3}^{*0} = \left(\frac{1}{2}\right)^{\frac{1}{2}} \Xi_{3}^{*0} + \left(\frac{1}{2}\right)^{\frac{1}{2}} \Xi_{1}^{*0}$
0	2	- 2	Ω_1^{*0}
0	1	- 1	$ ilde{N}_3^{*0} = \left(egin{array}{c} 9 \\ 10 \end{array} ight)^{rac{1}{2}} N_5^{*0} = \left(egin{array}{c} 1 \\ 10 \end{array} ight)^{rac{1}{2}} N_3^{*0}$
U	1	0	$\tilde{Y}_1^{*0} = \left(\frac{3}{4}\right)^{\frac{1}{2}} Y_2^{*0} - \left(\frac{1}{4}\right)^{\frac{1}{2}} Y_1^{*0}$
0	1	- 1	$\mathbf{\tilde{\Xi}}_{1}^{*0} = \left(\frac{1}{2}\right)^{\frac{1}{2}} \mathbf{\Xi}_{3}^{*0} - \left(\frac{1}{2}\right)^{\frac{1}{2}} \mathbf{\Xi}_{1}^{*0}$
-1	5/2	-+ 5/2	<i>I</i> ₂ -
- 1	5/2	+3/2	$\tilde{N}_5^{*-} = \left(\frac{1}{25}\right)^{\frac{1}{2}} N_5^{*-} + \left(\frac{24}{25}\right)^{\frac{1}{2}} N_3^{*-}$
1	5/2	-+ 1/2	$\tilde{Y}_{2}^{*-} = \left(\frac{1}{10}\right)^{\frac{1}{2}} Y_{2}^{*-} + \left(\frac{9}{10}\right)^{\frac{1}{2}} Y_{1}^{*-}$
- 1	5/2	- 1/2	$\tilde{\Xi}_{3}^{*-} = \left(\frac{1}{5}\right)^{\frac{1}{2}} \Xi_{3}^{*-} + \left(\frac{4}{5}\right)^{\frac{1}{2}} \Xi_{1}^{*-}$
- 1	5/2	- 3/2	$\tilde{\Omega}_1^{*-} = \left(\frac{2}{5}\right)^{\frac{1}{2}} \Omega_1^{*-} + \left(\frac{3}{5}\right)^{\frac{1}{2}} \Omega_0^{*-}$
- 1	5/2	- 5/2	X1-
- 1	3/2	+3/2	$\tilde{N}_{3}^{*-} = \left(\frac{24}{2\kappa}\right)^{\frac{1}{2}} N_{8}^{*-} - \left(\frac{1}{2\kappa}\right)^{\frac{1}{2}} N_{3}^{*-}$

Table 1 (contd.)

Change Q	υ	Ua	Notation
-1	3/2	-+-1/2	$\tilde{Y_1}^{*-} = \begin{pmatrix} 9\\10 \end{pmatrix}^{\frac{1}{2}} Y_2^{*-} - \begin{pmatrix} 1\\10 \end{pmatrix}^{\frac{1}{2}} Y_1^{*-}$
1	3/2	-1/2	$\tilde{\Xi}_1^{*-} = \left(\begin{array}{c}4\\5\end{array}\right)^{\frac{1}{2}} \Xi_3^{*-} - \left(\begin{array}{c}1\\\overline{5}\end{array}\right)^{\frac{1}{2}} \Xi_1^{*-}$
-1	3/2	3/2	$\tilde{\Omega}_0^{*-} = \left(\begin{array}{c}3\\-5\end{array}\right)^{\frac{1}{2}} \Omega_1^{*-} - \left(\begin{array}{c}2\\5\end{array}\right)^{\frac{1}{2}} \Omega_0^{*-}$
2	2	+ 2	N5*
-2	2	+1	Y ₂ *
-2	2	0	Ξ 3 [*]
2	2	1	Ω1 [*]
<u> </u>	2	-2	X1

Table 1 (contd.)

MASS-RELATIONS

The above U-spin classification may be used to find relations among the masses of (4, 1) multiplet. If the strong interactions were fully invariant under SU_3 all members of a given multiplet would have the same mass provided that the small mass splitting due to electromagnetic interaction be neglected. Since the masses of different particles classified in the same multiplet are different, the strong interaction Hamiltonian operator may be written as

$$H = H_0 + H_1 \qquad \dots \qquad (1)$$

where H_0 is SU₃ invariant and H_1 is not. H_1 , novertheless, being a strong interaction Hamiltonian must not violate hypercharge and isospin conservation. The simplest assumption, usually made, is that mass breaking operator H_1 transforms a $\cdot \eta$ -meson i.e.,

where U^1 is a U-spin vector operator and U^0 is scalar operator. From eqs. (1) and (2) one obtains the following matrix elements of mass breaking operator :

$$= \frac{\sqrt{3}}{4} < \frac{1}{2} || U^{1} || \frac{1}{2} > -\frac{1}{2} < \frac{1}{2} || U^{0} || \frac{1}{2} >$$
$$< N_{5}^{*+++} | H_{1} | N_{5}^{*+++} > = -\frac{\sqrt{3}}{4} < \frac{1}{2} || U^{1} || \frac{1}{2} > -\frac{1}{2} < \frac{1}{2} || U^{0} || \frac{1}{2} >$$

$<\!I_2^{++} H_1 I_2^{++} > = rac{\sqrt{3}}{2} <\!\! 1 \parallel U^1 \parallel 1 \!> - rac{1}{2} <\!\! 1 \parallel U^0 \parallel 1 \!>$
$<\! ilde{N}_{5}^{*++} H_1 ilde{N}_{5}^{*++} > = - <\! 1 \parallel U^0 \parallel \! 1 >$
$ = -\frac{\sqrt{3}}{2} <1 \parallel U^1 \parallel 1 > -\frac{1}{2} <1 \parallel U^0 \parallel 1 >$
$<\!I_2^+ H_1 I_2^+\!> = \frac{3\sqrt{3}}{4} < \tfrac{3}{2} \parallel U^1 \parallel \tfrac{3}{2} > - \tfrac{1}{2} < \tfrac{3}{2} \parallel U^0 \parallel \tfrac{3}{2} >$
$<\!\tilde{N}_5^{*+} H_1 \tilde{N}_5^{*+}\!> = \frac{\sqrt{3}}{4} < \tfrac{3}{2} \ U^1 \ \tfrac{3}{2} > - \tfrac{1}{2} < \tfrac{3}{2} \ U^0 \ \tfrac{3}{2} >$
$<\! ilde{Y}_2^{st+}\!\mid\! H_1\!\mid\! ilde{Y}_2^{st+}\!\!>=-rac{\sqrt{3}}{4}<\!rac{3}{2}\!\parallel U^1\!\parallel\!rac{3}{2}\!>-rac{1}{2}<\!rac{3}{2}\!\parallel U^0\!\parallel\!rac{3}{2}>$
$<\!\Xi_3^{*+}\!\mid\!H_1\!\mid\!\Xi_3^{*+}\!>=-rac{3\sqrt{3}}{4}<rac{3}{2}\parallel U^1\parallelrac{3}{2}\!>-rac{1}{2}<rac{3}{2}\parallel U^0\parallelrac{3}{2}>$
$<\!I_2^0 H_1^{} I_2^{0}\!> = \sqrt{3} <\!2 \ U^1^{} \ \!2\!> - rac{1}{2} <\!2 \ U^0^{} \ 2\!>$
$< ilde{N}_5{}^{*0} H_1 ilde{N}_5{}^{*0} > = rac{\sqrt{3}}{2} < 2 \ U^{\prime 1} \ 2 > - rac{1}{2} < 2 \ U^0 \ 2 >$
$< ilde{Y}_2^{*0} H_1 ilde{Y}_2^{*0} > = - rac{1}{2} < 2 \ U^0 \ 2>$
$< ilde{\Xi}_3^{*0} \mid H_1 \mid ilde{\Xi}_3^{*0} > = - \; rac{\sqrt{3}}{2} < 2 \parallel U^1 \parallel 2 > \; - \; rac{1}{2} < 2 \parallel U^0 \parallel 2 >$
$<\!\Omega_1^{*0} H_1 \Omega_1^{*0}\!> = -\sqrt{3} < 2 \ U^1 \ 2\!> \!- \frac{1}{2} < 2 \ U^0 \ 2>$
$<\!I_2^-\!\mid\!H_1\!\mid\!I_2^->=rac{5\sqrt{3}}{4}<rac{5}{2}\parallel U^1\parallelrac{5}{2}>-rac{1}{2}<rac{5}{2}\parallel U^0\parallelrac{5}{2}>$
$< \! ilde{N}_{5}^{st-} \! \left H_{1} \! \left ilde{N}_{5}^{st-} \! > = rac{3\sqrt{3}}{4} \! < rac{5}{2} \ U^{1} \ rac{5}{2} > - rac{1}{2} \! < rac{5}{3} \ U^{0} \ rac{5}{2} \! >$
$< \check{Y}_2^{*-} H_1 \: \check{Y}_2^{*-} > = rac{\sqrt{3}}{4} \: < rac{5}{2} \parallel U^1 \parallel rac{5}{2} > - rac{1}{2} < rac{5}{2} \parallel U^0 \parallel rac{5}{2} >$
$< ilde{\Xi}_3^{*-} H_1 ilde{\Xi}_3^{*-} > = - rac{\sqrt{3}}{4} < rac{5}{2} \ U^1 \ rac{5}{2} > - rac{1}{2} < rac{5}{2} \ U^0 \ rac{5}{2} > $

S. T. H. Abidi

where $\langle a \parallel U \parallel a \rangle$ is the reduced matrix element between states with U spin equal to a. These matrix elements lead to the following relations among the masses of different particles :

These relations are found to be in perfect agreement with the masses given by Dashen and Sharp (1965).

SU6 CLASSIFICATION AND PROBABLE VALUES OF SPIN

In this section we find out the irreducible representations of SU_6 to which the 35-plet of SU_8 may belong. Let us first consider the baryon states having baryon number B = 1. These states will appear as resonances in mesonbaryon interactions at high energies. They will, therefore, by contained in one, or more, of the irreducible representations of SU_6 that are obtained in the reduction of the direct product 56 \otimes 35. This reduction is given by Itzykson and Nauenberg (1966)

$$56 \otimes 35 = 700 \oplus 1134 \oplus 70 \oplus 56$$
 ... (5)

Now if we examine the (SU_3, SU_2) contents of all the irreducible representations occuring on the right side of equation (5) we find that there are (i) two 35-plets of SU_3 with spins J = 5/2 and J = 3/2 respectively in the 700-dimensional representation of SU_6 and (ii) two 35-plets with J = 3/2 and J = 1/2 in 1134 dimensional representation of SU_6 . No 35-plet occurs in 70-dimensional and 56-dimensional representation. We, therefore, find that $J^P = \frac{5}{2}^+$ baryons can only be accomodated in 70-dimensional representation of SU_6 .

The baryon-baryon 35-plets (B = 2) will, similarly, be found in some irreducible representations of SU_6 that are obtained in the reduction : (Itzykson and Nauenberg 1966)

$$56 \otimes 56 = 1050 \oplus 490 \oplus 1134 \oplus 462 \qquad \dots \qquad (6)$$

The generalized Pauli principle allows only the two anti-symmetric representations 490 and 1050 to be realized in nature. The 35-plets of SU_3 occur in both these representations. In 490-dimensional irreducible representation only those 35-plet of SU_3 occur which have spin J = 1 while in 1050-dimensional representation of SU_6 there are three 35-plets with spin values J = 1, 2 and 3. Thus the low-lying baryon-baryon states will have spin J = 1.

BRANCHING RATIOS IN THE DECAY OF 35-PLET

The decays of a 35-plet of single baryons $J^P = 5/2^+$ into (i) a meson octet and baryon octet and into (ii) a meson octet and a baryon dociment have been discussed by Dashen and Sharp (1965) and by Zimmerman and Fagundes (1966). Their results could be used for di-baryon states also if we replace the meson octet by baryon octet. We want to discuss the decay of 35 di-baryon states into an octet of meson and a 27-plet of di-baryons. We assume that for higher dimensional resonances the violation of unitary symmetry primarily affects the phase space and form factors in the decay widths. The branching ratios may then be predicted from the following wave-functions :

$$\begin{aligned} |I_{2}^{+++}\rangle &= -(\frac{1}{3})^{1/2} |K^{+}_{3/2}X^{1}_{3/2}\rangle + (\frac{2}{3})^{1/2} |\pi^{+}_{1}X_{1}^{2}\rangle \\ |I_{2}^{++}\rangle &= -(\frac{1}{4})^{1/2} |K^{+}_{3/2}X^{1}_{1}\rangle - (\frac{1}{12})^{1/2} |K^{0}_{3/2}X^{1}_{3/2}\rangle + (\frac{1}{3})^{1/2} |\pi^{+}_{1}X_{0}^{2}\rangle \\ &+ (\frac{1}{3})^{1/2} |\pi^{0}_{1}X_{1}^{2}\rangle \end{aligned}$$

$$|I_{2}^{+}\rangle = -(\frac{1}{6})^{1/2} |K_{3/2}^{+}X_{-1/2}^{+}\rangle - (\frac{1}{6})^{1/2} |K_{3/2}^{0}X_{1/2}^{+}\rangle + (\frac{1}{9})^{1/2} |\pi_{1}^{+}X_{-1}^{2}\rangle + (\frac{4}{5})^{1/2} |\pi_{0}^{0}X_{0}^{2}\rangle + (\frac{1}{9})^{1/2} |\pi_{-1}^{-}X_{1}^{2}\rangle |I_{2}^{0}\rangle = -(\frac{1}{12})^{1/2} |K_{3/2}^{+}X_{-3/2}^{1}\rangle - (\frac{1}{4})^{1/2} |K_{-3/2}^{-}X_{-1/2}^{1}\rangle + (\frac{1}{3})^{1/2} |\pi_{0}^{0}X_{-1}^{2}\rangle + (\frac{1}{3})^{1/2} |\pi_{-1}^{-}X_{0}^{2}\rangle \qquad (7)$$

where $_{I'}X_{I'_3}^{Y'}$ stand for a member of 27-plot having quantum numbers Y = Y', I = I' and $I_3 = I'_3$. The other thirty-one wave-functions can be similarly written down using the Clebsch-Gordon coefficients of SU₃. The branching ratios are then given by (Gasiorowics 1964):

$$R = \frac{\Gamma(I_2 \to M_{1+}{}^{27}B_1)}{\Gamma(I_2 \to M_2 + {}^{27}B_2)} \left(\frac{P_{M_2}}{P_{M_1}}\right)^{2l+1} \left(\frac{P^2 M_1 + \Lambda^2}{P' M_2^2 + \Lambda^2}\right)^{l}$$
(8)

Here M_1 and M_2 are the members of meson octet; ${}^{27}B_1$ and ${}^{27}B_2$ are members of 27-plet of baryon-baryon states; P_{M_1} and P_{M_2} are the momenta in the respective decays, l is the orbital angular momentum in the decay and Λ reflects the source size. Thus, using wave-functions (7) we obtain :

$$R_{1} = \frac{M(I_{2}^{+++} \rightarrow K^{+} + \frac{3}{2}X_{1}^{1})}{M(I_{2}^{+++} \rightarrow \pi^{+} + \frac{1}{2}X_{1}^{2})} = \frac{1}{2}$$

$$R_{2} = \frac{M(I_{2}^{++} \rightarrow K^{+} + \frac{3}{2}X_{1}^{1})}{M(I_{2}^{++} \rightarrow \pi^{+} + \frac{1}{2}X_{1}^{2})} = \frac{3}{4}$$

$$R_{3} = \frac{M(I_{2}^{++} \rightarrow K^{0} + \frac{3}{2}X_{1}^{3})}{M(I_{2}^{++} \rightarrow \pi^{0} + \frac{1}{2}X_{2}^{2})} = \frac{1}{4}$$

$$R_{4} = \frac{M(I_{2}^{+} \rightarrow K^{+} + \frac{3}{2}X_{1}^{1})}{M(I_{2}^{+} \rightarrow \pi^{+} + \frac{1}{2}X_{2}^{2})} = \frac{3}{2}$$

$$R_{5} = \frac{M(I_{2}^{+} \rightarrow K^{0} + \frac{3}{2}X_{1}^{1})}{M(I_{2}^{+} \rightarrow \pi^{0} + \frac{1}{2}X_{2}^{0})} = \frac{3}{8}$$

$$R_{6} = \frac{M(I_{2}^{0} \rightarrow K^{+} + \frac{3}{2}X_{1}^{1} - \frac{3}{2})}{M(I_{2}^{0} \rightarrow \pi^{0} + \frac{1}{2}X_{2}^{0})} = \frac{1}{4}$$

$$R_{7} = \frac{M(I_{2}^{0} \rightarrow K^{0} + \frac{3}{2}X_{1}^{1} - \frac{1}{2})}{M(I_{2}^{0} \rightarrow \pi^{-} + \frac{1}{2}X_{2}^{0} - \frac{1}{2})} = \frac{3}{4}$$

The branching ratios in the decays of other particles of this plet can als) be similarly found out.

ACKNOWLEDGMENT

I am greatly indebted to Dr. G Bandopadhyay for his suggestions and encouragement.

REFERENCES

Abers, E. S., Balazs, L. A. P. and Hara, Y., 1964, Phys. Rev., 186B, 1382.

Dashen, R. F. and Sharp, D. H., 1965, Phys. Rev., 188B, 223.

Dashen, R. F. and Frautschi, S. C., 1964, Phys. Rev., 185, B 1190.

Gasiorowez, S., 1963, Phys. Rev., 181, 2808.

Itzykson, C. and Nauenberg, M., 1966, Rev. Mod. Phys., 88, 95.

Lipkin, H. J., Meshkov, S. and Levinson, C. A., 1963, Phys. Rev. Lett., 10, 361.

Lipkin, H. J., and Harari, H., 1964, Phys. Rev. Lett., 18, 345.

Zimmerman, A. H. and Fagundes, H. V., 1966, Il Nuovo Cimento 48, 233.