Letters to the Editor

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A NOTE ON A RELATIVISTIC FORMULAE FOR IONIZATION BY PROTON IMPACT

A. ROY CHOWDHURY

PHYSICS DEPARTMENT, JADAVPUR UNIVERSITY, INDIA.

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We consider the collision of a proton with a hydrogen like atom, in which process the electron become ionized. (We neglect the translatory motion of the nucleus).

Let ϕ, ψ denote the proton and electron field operators. Then the initial bound state wave function is defined as (Roy, 1960 a).

$$\psi_s = \langle 0 | \psi_s^{op}(x) | \psi_1 \rangle \qquad .. \quad (1)$$

where

$$|\psi_1\rangle = \int \sum_{s=1}^{4} d^3k g_s(k) a^{s+}(k) |0\rangle \qquad .. \quad (2)$$

the bound state vector for the single electron. $a^{s+}(k)$ is the usual notation for creation operator of an electron, and A for proton. Also we have

$$\psi_\rho^{op}(x) = \int \sum_{i=1}^{4} a^i(p) u_\rho^i e^{ip.x} d^3p \qquad .. \quad (3)$$

u_ρ^i is the Dirac Spinor. Now (1), (2), (3) give :

$$\psi_\rho(x) = \sum_{s=1}^{4} d^3k g_s(k) u_\rho^s(k) e^{ik.x} \qquad .. \quad (4)$$

Let $\qquad V_\rho(k) = \int \psi_\rho(x) e^{-ik.x} d^3x.$ denote the

Fourier transform of bound state wave function; then (4) gives

$$V_\rho(k) = \sum_{s=1}^{4} u_\rho^s g_s(k) \qquad .. \quad (5)$$

which is the equation determining the four-vector $g_s(k)$. It is interesting to note that we need not determine g's explicitly, but can use equation (5) to convert them in terms of $V(k)$, which are determined easily.

The matrix element of the process is given by:

$$M_{fi} = <\psi_f \mid S^{(2)} \mid \psi_i> \tag{6}$$

where

$$\mid \psi_i> = \int \sum_{s=1}^{4} d^3k g_s(k) a^{s+}(k) A^+(P_i) \mid 0>$$

$$\mid \psi_f> = A^+(P_f) a^+(q_f) \mid 0> \tag{7}$$

$$S^{(2)} = \frac{e^2}{2!} \int \int P[\{\bar{\phi}(x')\gamma_\mu A_\mu(x')\phi(x')\}\{\bar{\psi}(x)\gamma_\nu A_\nu(x)\psi(x)\}]dxdx'$$

(6) with the help of (7) gives

$$M_{fi} = D. \frac{\bar{v}(P_f)\gamma^\mu v(P_i)\bar{u}(q_f)\gamma_\nu V(p)}{(P_i - P_f)^2} \delta(\epsilon_f - \epsilon_i + E_f - W) \tag{8}$$

with $p = P_f - P_i + q_f$, D contains some constant factors. In non-relativistic limits (8) becomes

$$M_{fi} = \frac{\phi_{1un}(P_f - P_i + q_f)}{(P_i - P_f)^2} = \int \int \frac{1}{|\bar{r} - \bar{R}|} . e^{i(p_f - p_i).R} \quad {}^{iq_f.r} \quad d\bar{r}d\bar{R} \tag{9}$$

which is the usual Born approximation (Mott and Massey 1949). For the 1s state the solution of Dirac's equation is (Akheiezer and Berettross Keii; 1953):

$$\psi(r) = N_i(1 + ik_1\gamma_0\gamma.\bar{n})u(0)e^{-\eta r}; \; \bar{n} = \bar{r}/r$$

$$V(p) = N_1(J_1 + ik_1\gamma_0\gamma.\bar{p}J_2)u(0) \qquad .. \tag{10}$$

where $J_1 = \int e^{-\eta r}e^{i\bar{p}.\bar{\gamma}}$ $J_2 = J_1/\eta$ and $k_1 = \frac{1}{2}\ \alpha z$. From (8) we see $\Sigma \mid M_{fi} \mid^2$ proportional to

$$\frac{A}{(P_i - P_f)} . \delta(\epsilon_f - \epsilon_i + E_f - W) \tag{11}$$

where A stands for:

$$T_r[\Lambda_+(P_f)\gamma^\mu\Lambda_+(P_i)\gamma^\gamma]T_r[\Lambda + (q_f)\bar{Q}\Lambda - (0)Q] \tag{12}$$

and

$$Q = \gamma_\mu(J_1 + ik_1\,\gamma_0\,\gamma.\bar{p}\,J_2)$$

Now the differential cross-section for the process is given by:

$$d\sigma = 2\pi\rho_F\Sigma \mid M_{fi} \mid^2/v. \tag{13}$$

where (Roy, 1960b)

$$\rho_F = \frac{P_f^2 q_f^2}{(2\pi)^6} \cdot \frac{dP_f}{dE} . dq_f d\Omega_P d\Omega \tag{14}$$

when (8), (11), (12) are used in (13) the expression for the differential cross section boils down to:

$$d\sigma = \frac{e^4}{4a_0^3} \cdot \frac{d^3P_f d^3q_f}{(2\pi)^4} \cdot \frac{\epsilon_i}{P_i} \cdot \frac{A}{(P_i - P_f)^4} \cdot \delta(\epsilon_f - \epsilon_i + E_f - W) \quad .. \quad (15)$$

where:

$$A = \frac{1}{M^2 m}\left[J_1^2 A_1 - \frac{1}{4} A_2 J_2^2 \right] \quad .. \quad (16)$$

$$A_1 = [2(P_f.P_i)m + 4(P_i.P_f - M^2)m + 2(P_f q_f)(P_i)_0 + 2(P_i.q_f)\times$$

$$\times (P_f)_0 - 2(P_i.P_f)(q_f)_0 - 2(q_f)_0(P_i.P_f - M^2)]$$

$$A_2 = \alpha^2 p^2[2(P_i)_0(P_f.q_f) + 2(P_f)_0(P_i.q_f) - 2(q_f)_0\times$$

$$(P_i.P_f - M^2) - 2(P_i.P_f)(q_f)_0]$$

$$J_1 = \frac{8\pi\eta}{(p^2+\eta^2)^2}, \quad J_2 = J_1/\eta$$

where (q) represents the zeroeth component of the four-vector q. The formula for cross-section contains two terms A_1 and A_2, of which the latter is proportional to For the physically interesting situation we may neglect 2nd part and the formula (15) then gives.

$$d\sigma = \frac{8r_0^2}{\pi^6\alpha^3} \cdot \frac{\epsilon_i}{mP_i} \cdot \frac{d^3P_f d^3q_f}{(P_i - P_f)^4} \cdot \frac{\delta(\epsilon_f + E_f - E)}{\{z^2 + a_0^2(P_f - P_i + q_f)^2\}^4}$$

$$\times \left\{ 2 + \frac{\vec{P_f}.\vec{P_i}}{M^2} - \frac{\bar{P}_f \cdot \bar{q}_f}{Mm} - \frac{\bar{P}_i \cdot \bar{q}_f}{Mm} \right\} \quad .. \quad (17)$$

(ϵ_i, P_i) being energy momentum of the incoming proton. Formula (17) is easily seen to be reduced to that of the usual Born approximation results in non-relativistic limit.

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