EQUATIONS OF ELECTRON TRAJECTORIES UNDER THE INFLUENCE OF ORTHOGONAL ELECTRIC AND MAGNETIC FIELDS IN A SEMI-CIRCULAR SPECTROMETER

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(*Received December* 26, 1967 ; *Resubmitted April* 27 *and July* 22, 1968)

ABSTRACT. Equations of electron trajectories under the influence of orthogonal electric and magnetic fields in a semi-circular spectrometer are derived. The influence of preacceleration on the resolving power and the transmission of the spectrometer are discussed.

INTRODUCTION

Resolution less than 1 in 1000 can be set in a semi-circular spectrometer. Using radiographic films, and nuclear emulsion plates, a large part of the conversion electron spectrum can be surveyed and important informations about nuclear structure can be obtained. However, photographic detecters have lower energy limit of detection around 7 keV. By using preacceleration technics, electrons down to zero energy can be recorded. In the present article, we derive equations of electron trajectories under the influence of orthogonal electric and magnetic fields, and show that preacceleration does not seriously affect the resolving power and transmission of the spectrometer.

EQUATION OF ELECTRON TRAJECTORIES

Figure 1 is a trihedral Oxyz showing the magnetic, electric and velocity vectors

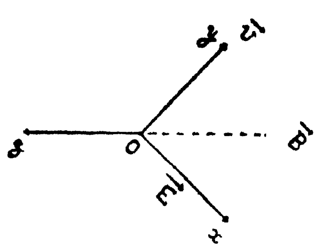

Figure 1. Trihedral Oxyz showing $\vec{B} \perp \vec{E} \perp \vec{v}$

$\vec{B}$, $\vec{E}$, and $\vec{v}$ respectively with 0 as the centre of the radioactive source emitting electrons. The Lorentz force for an electron under such conditions is given by $e[\vec{E}+\vec{v}\,\Lambda\,\vec{B}]$ with

$$v\,\Lambda\,\mathrm{B}\left|\begin{matrix} i & j & k \\ \dot{x} & \dot{y} & \dot{z} \\ 0 & 0 & -B \end{matrix}\right.$$

The equations of motion are :

$$m\ddot{x} = eE - e\dot{y}B \tag{1}$$

$$m\ddot{y} = e\dot{x}\mathrm{B} \tag{2}$$

$$m\ddot{z} = 0 \tag{3}$$

where m is the electron mass.

For the initial conditions

$$x_0 = y_0 = z_0. \quad v_0 = \quad v_{0y}$$

From equations (3),

$$z = v_{0z}t \text{ at time } t.$$

From equations (1) and (2), we get

$$\frac{d}{dt}(\dot{x}+i\dot{y}) - \frac{ieB}{m}(\dot{x}+i\dot{y}) = \frac{e}{m}E,$$

$$\dot{x}+i\dot{y} = Ae^{i\omega t}+i\frac{E}{B} \text{ (where } A = ae^{i\alpha}\text{, and } \omega \text{ is the angular velocity)}$$

$$= a(\cos\alpha+i\sin\alpha)(\cos wt+i\sin wt)+iE/B \tag{4}$$

a and α are defined by the initial conditions :

$$v_{0x}+iv_{0y} = a\cos\alpha+ia\sin\alpha+iE/B.$$

Hence, we have

$$v_{0x} = a\cos\alpha, \quad \text{and} \quad v_{0y} = a\sin\alpha+E/B.$$

From equation (4), we get :

$$\dot{x}(t) = v_{0x}\cos wt - \left(v_{0y} - \frac{E}{B}\right)\sin wt \tag{5}$$

$$\dot{y}(t) = \left(v_{0y} - \frac{E}{B} \right) \cos wt + v_{0x} \sin wt + \frac{E}{B} \quad \ldots \quad (6)$$

$$x(t) = \frac{v_{0x}}{w} \sin wt + \frac{1}{w} \left(v_{0y} - \frac{E}{B} \right) (\cos wt - 1) \quad \ldots \quad (7)$$

$$y(t) = \frac{1}{w} \left(v_{0y} - \frac{E}{B} \right) \sin wt - \frac{v_{0x}}{w} (\cos wt - 1) + \frac{E}{B} t \quad \ldots \quad (8)$$

$$z(t) = v_{0z} t \quad \ldots \quad (9)$$

Equations 5 to 9 define completely the trajectories of the electrons with $\vec{E}$ and $\vec{B}$.

PARTICULAR CASE OF PLANE TRAJECTORIES, ORTHOGONAL TO THE MAGNETIC INDUCTIONS B, WITHOUT PREACCELERATION

For $v_{0z} = 0$, $z(t) = 0$, the trajectory lies in the xy plane, and with $E = 0$, we have :

$$x = \frac{v_{0x}}{w} \sin wt + \frac{v_{0y}}{w} (\cos wt - 1)$$

$$y = \frac{v_{0y}}{w} \sin wt - \frac{v_{0x}}{w} (\cos wt - 1)$$

$$\left\{ x - \left(- \frac{v_{0y}}{w} \right) \right\}^2 + \left\{ \left(y - \left(+ \frac{v_{0x}}{w} \right) \right\}^2 = \frac{v_{0x}^2 + v_{0y}^2}{w^2} = \frac{v_0^2}{w^2}.$$

The trajectory is a circle with the centre :

$$(x_c, y_c) = \left(- \frac{v_{0y}}{w} , \frac{v_{0x}}{w} \right)$$

and radius

$$\rho = \left| \frac{v_0}{w} \right| = \left| \frac{mv_0}{eB} \right|.$$

In figure 2, the origin is displaced by a distance $2D$ from the plane of the detector, thus limiting the angle of emission in the xy plane.

Suppose that $X = -2D + x,$

and $Y = y.$

The trajectory is still a circle given by the equation :

$$\left\{ X+\left(2D+\frac{v_{0y}}{w}\right)\right\}^2+\left\{Y-\frac{v_{0x}}{w}\right\}^2=\frac{v_0^2}{w^2}.$$

a) Suppose that the electron trajectory makes an angle α_0 with the X—axis, and passes through the centre of the diaphragm as shown in figure 2. Let us precise the initial conditions :

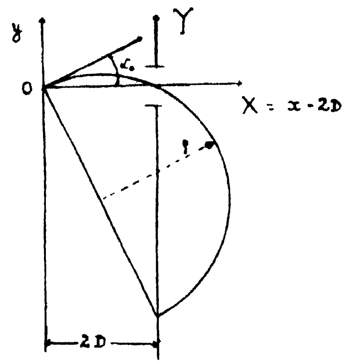

Figure 2. Showing the central trajectory, with initial velocity v_0 making an angle α_0 with OX ; the origin is displaced to X = x − 2D.

$$v_{0x}=v_0\cos\alpha_0$$

and $$v_{0y}=v_0\sin\alpha_0.$$

For $X=0$, the trajectory will cut the Y axis at two points Y_1, and Y_2 defined by :

$$\left(2D+\frac{v_0}{\omega}\sin\alpha_0\right)^2+\left(Y-\frac{v_0}{\omega}\cos\alpha_0\right)^2=\frac{v_0^2}{\omega^2}.$$

i.e.,

$$\left.\begin{matrix}Y_1\\Y_2\end{matrix}\right\}=\frac{v_0\cos\alpha_0}{\omega}\mp\sqrt{\frac{v_0^2}{\omega^2}\cos\alpha_0-4D\left(D+\frac{v_0}{\omega}\sin\alpha_0\right)}$$

The central trajectory is given by $Y_1=0$, thus defining the angle of emission by the condition $\left(\frac{v_0}{\omega}\right)\sin\alpha_0=-D$.

The point of impact on the plate is $Y_2=Y_0=2\frac{v_0}{\omega}\cos\alpha_0=\frac{2(B\rho)}{\omega}\cos\alpha_0$, where $B\rho$ is the magnetic rigidity of the electron.

b) Let us consider the case of a trajectory situated in a plane parellel to the median plane, but whose initial velocity makes an angle $\alpha = \alpha_0 \pm \phi$ with OX.

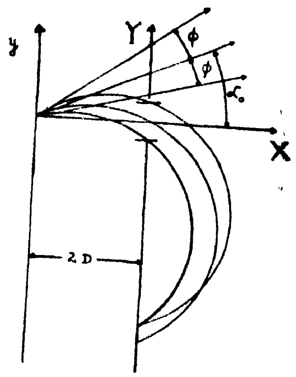

Figure 3. Showing the initial velocity making an angle $\alpha = \alpha_0 + \phi$ with the X axis.

The trajectory will cut OY at two points :

$$\left.\begin{matrix} Y_1 \\ Y_2 \end{matrix}\right\} = \frac{v_0}{\omega} \cos\alpha \left\{ 1 \mp \sqrt{1 - \frac{4D\omega(D\omega + v_0 \sin\alpha)}{v_0^2 \cos^2\alpha}} \right\}$$

Using the relation $v_0 \sin\alpha_0 = -D$, we get the point of impact on the plate :

$$Y_2 \approx \frac{2v_0 \cos\alpha}{\omega} \left[1 - \frac{\sin\alpha_0 (\sin\alpha_0 - \sin\alpha)}{\cos^2\alpha} \right]$$

$$= Y_0 \frac{\cos\alpha}{\cos\alpha_0} \left[1 - \frac{\sin\alpha_0 (\sin\alpha_0 - \sin\alpha)}{\cos^2\alpha} \right].$$

Let us recall that Y_0 is the point of impact for the central trajectory. From the above discussions, we conclude that a point source emitting monoenergetic electrons produces a ray of a finite width on the detector. The resolving power of the spectrometer is defined by the quantity $R_0 = \frac{Y_0 - Y_2}{Y_0} =$

$$1 - \frac{\cos\alpha}{\cos\alpha_0} \left[1 - \frac{\sin\alpha_0 (\sin\alpha_0 - \sin\alpha)}{\cos^2\alpha} \right]$$

The source has a finite width s, being placed in the plane parallel to that of the detecter. The total resolution is given by the expression :

$$R = \frac{s}{Y_0} + \frac{\phi_0^2}{2},$$ where ϕ_0 is half of the solid angle of emission.

INFLUENCE OF PREACCELERATION ON THE RESOLUTION AND TRANSMISSION OF THE SPECTROMETER

Figure 4 shows the case of a homogeneous accelerating space between the source fixed to a plane electrode, and a grid parallely situated at a distance d

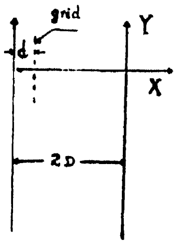

Figure 4. Preacceleration, showing the grid.

from the source. At this geometry, the motion of electrons is defined by the equations (5) to (9). To estimate the influence of preacceleration, we consider two cases :

1) an electron emitted an angle α_0 with the initial velocity V_0 and energy W_0.

2) an electron emitted an angle α with initial velocity v_0, energy ω_0, and accelerated under a potential of V Kilovolts such that on leaving the grid, its energy bears the relation $W_0 = (\omega_0 + V)$keV.

If T and T_E are the respective times of stay of the two electrons in the accelerating medium.

$$T = \frac{d}{V_0 \cos \alpha_0}, \quad \omega T = \frac{d}{\cos \alpha_0} \leqslant 1, \quad \omega T_E = \frac{d}{\cos \alpha},$$

$$\sin \omega T = \omega T, \quad \text{and} \quad \cos \omega T = 1 - \left(\frac{\omega^2 T^2}{2} \right).$$

At the exit of the grid, for an accelerated electron,

$$\dot{x}(T_E) = v_{0x} \left(1 - \frac{\omega^2 T_E^2}{2} \right) - v_0 \, \omega T_E + \gamma T_E,$$

$$\dot{y}(T_E) = v_{0y} \left(1 - \frac{\omega^2 T_E^2}{2} \right) + v_{0x} \omega T_E + \frac{\gamma \omega T_E^2}{2}.$$

$$y(T_E) = v_{0y}T_E - \frac{v_{0x}}{\omega} \cdot \frac{\omega^2 T_E^2}{2}, \text{ where } v_{0x} = v_0 \cos\alpha,\ v_{0y} = v_0 \sin\alpha,$$

and $\gamma = \frac{eE}{m}$ = acceleration due to E in the direction OX.

For an electron without preacceleration,

$$Y(T) = V_{0y}T - V_{0x}\frac{\omega^2 T^2}{2}, \text{ where } V_{0x} = V_0 \cos\alpha_0 \text{ and } V_{0y} = V_0 \sin\alpha_0$$

The relative displacement between the two images is

$$\Delta y = V_{0y}T - v_{0y}T_E.$$

In order to associate the two trajectories, we consider $\Delta y = 0$.

Then,
$$v_{0y} = V_{0y}\frac{T}{T_E}.$$

$$T_E = T.\ \frac{2d\ V_{0x}^2}{V_0^2 - v_0^2}\left[\sqrt{1+\frac{V_0^2 - v_0^2}{V_{0x}^2}} - 1\right]$$

The above equations show that we have to diminish the distance d between the source and the grid, subject to discharge conditions. The relative displacement is of the order $\frac{\omega^2 T^2}{2}$.

For a spectrometer, where $d = 5$ mm, radius of curvature $= 125$ mm,

$$\frac{\omega^2 T^2}{2} \approx \frac{1}{2}\left(\frac{d}{r}\right)^2 \approx 10^{-3}.$$

Thus, the position of a ray with preacceleration is displaced by a quantity equal to the geometrical resolution of the spectrometer in comparison to a ray without preacceleration. However, for electrons of low energy, the resolution depends largely on the dimensions of the source. The above discussion is comparable with experimental observation obtained by studying the 148,08keV ray F of ThB, using Kodirex films as detecters, and applying a potential of 5KV.

TRANSNISSION

For optimum conditions, the solid angle is given by the relation $\phi_0 = \sqrt{\frac{2s}{Y_0}}$ The transmission is a function ϕ_0. Δy due to the preacceleration is negligible. Thus ϕ_0, and hence transmission remains practically unaffected.

This work is a part of the thesis for the doctorate degree presented by the author at the University of Lyon in May 1967. We thank Professor R. Chery of the Institute of Nuclear Physics, LYON, for his valuable guidance.