A Performance Analysis of Multivariate Nonparametric Control Charts

by

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Abstract

Robust and efficient multivariate control charts are not common in literature. This report explores the versatility of the few distribution-free, nonparametric multivariate Statistical Process Control (MSPC) charts suitable for average run length (ARL) analysis. Current datasets are becoming increasingly complex, large, and less likely to follow distributional properties required for traditional parametric statistics, a fact especially true for a multivariate setting. The purpose of our study is to compare the newest available methods, not previously compared with one another in cases and data structures not yet explored. Due to the versatility and robustness of the types of data these methods can accommodate, finding real world applications is trivial. The five methods applied here are able to exploit different types of changes to the structure of a distribution, rather than simply detect a mean shift. These methods have similar features, able to avoid lengthy data-gathering steps, and applicable in short-run and start up situations. By establishing cut-off values simultaneously based on input observations, rather than beforehand, the methods are applying data-dependent control limits which shows their truly distribution-free property. Some of the current areas of improvement continue to be on creating more computationally efficient algorithms for these methods.

I would like to dedicate this to my son, Nathan, who may one day find inspiration in this work to create his own legacy in whichever path he chooses. All my family and friends. My mom who I love and miss everyday. My dad because he's my dad. My grandma for always looking out for me. My girlfriend because she loves, supports, encourages, and backs me every step of the way. My buddy, Jamal, the consummate professional, who makes me realize the importance of working hard, never being satisfied, and lifelong education. The pets in my life, the boys, Rocco, Marty, and Milo. My high school math teacher, Mr. Morrison, who always encouraged my brilliance. Finally, my poppy, who's eyes would light up every time I would mention becoming a scientist, professor, or another professional career associated with my degree, and using my mind rather than my body.

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Statement of contributions

One important contribution is creating a comprehensive review of the latest nonparametric multivariate control chart methods, as many authors noted there was no valid methods available for comparison at the time of their respective papers. Another point is that the magnitude of the distributional shifts applied here is smaller than was presented in the works of some of the authors. Finally, the application of all of these methods to real datasets, establishing control limits based on few phase I IC observations, and showing their flexibility and robustness in the types of real life data they accommodate.

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Chapter 1

Introduction

1.1 Overview

Quality is an important factor in the decision-making process of most consumers. The perception of quality, once reserved for the manufacturing sector, has recently spread to different areas, including the service sector. Before detailing current quality control (QC) developments, a brief history of QC is necessary. Researchers did not formally develop the science of QC until the beginning of the 20th century. The inspection department at the Western Electric Company is at least partially credited with pioneering the area in 1907, thanks to the work of Joseph M. Juran who later became a well-known QC consultant. By 1925, at the Bell Laboratories, Walter A. Shewart began to introduce the idea of control charts based upon hypothesis testing. He first developed his Shewart control charts in the 1930s to monitor the means of process variables. Some years later, coinciding with World War II, and the major advancements in the manufacturing industry, the area of QC saw significant progress. In 1947, while working at the Statistical Research Group at Columbia University, Harold Hotelling introduced his theory behind the famous Hotelling T^2 test statistics. Since its inception, the area of QC has seen much progress from the research and development departments worldwide. For a more thorough and comprehensive examination on the history of quality control, see Wadsworth, Stephens, and Godfrey (2002) or Qiu (2014).

Garvin (1987) recognized that as QC developed, it was necessary to formally define quality, and created the eight dimensions of quality. These eight dimensions determining the quality of a process are: performance, features, reliability, conformance, durability, serviceability, aesthetics, and perceived quality. Of these eight dimensions, the aspect that has been scrutinized the most from a statistical viewpoint, is conformance.

One approach favoured by many companies to achieve product conformance through QC involves establishing definitions for conformance by applying Six Sigma techniques. The original idea behind Six Sigma is a process resulting in 3.4 defective parts per million opportunities (DPMO) (Tennant 2001). Over time the Six Sigma approach has grown from a performance metric to an associated quality initiative. Its structure includes a methodology for implementation, and certifying individuals to develop plans, analyze data, and ultimately improve the process. Six Sigma owes its tools and techniques to the areas of total quality management and statistical process control (SPC).

1.2 Statistical Process Control

As an important tool for monitoring product conformance, SPC's goal is to monitor and control a process to conform to its required specifications, or perceived standard of quality from the consumers perspective. SPC is applied to any process with a measurable output. The person or machine responsible for the conformance of this product would want the manufacturing process to cease as quickly as possible in the event of the process failing to conform. The operator is only concerned whether a process is meeting conformance standards or not.

To decide whether a process is conforming or not, the analysis of data is divided into two stages: phase I and phase II. In phase I, the dataset from a conforming process is referred to as "in-control" (IC). In parametric methods, enough observations must be obtained to establish the IC distribution. In contrast, the target of self-starting, distribution-free methods is to not require a minimum number of phase I observations. If a process is not conforming, it is said to be "out-of-control" (OC). When establishing phase I conditions, if the process goes OC, it must be re-started until a sufficient number of data points are collected. In phase II analysis, an online sequential monitoring of the process is performed, which is believed to be IC at the start. The process will be monitored to ensure it remains IC, and a signal is to be triggered as quickly as possible by the presumably correct detection of the occurrence of OC conditions. The most common technique in SPC monitoring is done by control charts.

Control charts are used to determine whether or not a process is currently under control; that is, variation coming from chance causes alone (McNeese 2006). Common control charts such as $\bar{X}$ charts, for a shift in the mean, R charts and S charts, for a shift in the variability of a distribution are useful for normal, univariate data. However, generally in manufacturing, a process's quality tends to be judged upon many characteristics or process variables simultaneously; hence, the need for multivariate SPC (MSPC). One advantage of multivariate normal data is that the form of the marginal distributions can be found easily from the joint distribution.

1.3 Multivariate Process Monitoring

In MSPC, if the control chart is built under the assumption of normality, the Hotelling T^2 test statistic is considered a standard procedure. The Hotelling T^2 control chart monitors the mean vector of multivariate normal processes. These methods have been around longer than most and are more common in literature. Some variations of the Hotelling T^2 include: successive differences, minimum volume ellipsoid, and minimum covariance determinant based estimates of mean and covariance matrices. These methods were used to compare against the method developed by Cheng and Shiau (2015), known as the Multivariate Sign Shewart (MSS) chart of the selected methods for this practicum. Other methods, such as those developed by Liu(1995) including the R, Q, and S charts, were created for comparison with Shewart control charts. These methods based on data depth, simplicial and Mahalanobis distances, are useful in a multivariate setting.

Historically, multivariate control charts were predominantly used in the manufacturing industry. However, over the past two or three decades, they have begun to spread to non-manufacturing industries such as health care, banking, and insurance, among others. With respect to the statistical distribution of the process data, non-manufacturing applications tend to be non-normal in nature (Ning, Yeh, Wu, and Wang, 2014), and for many methods, deviation from the parametric assumption may lead to wrong conclusions.

Given the recent surge in data recording and analysis, in terms of both size and frequency, and the ever-increasing complexity of the data itself, the applicability of methods free of distributional constraints has become well-established and logical. Therefore, we introduce the theme of this practicum, nonparametric MSPC (NPMSPC). One advantage of nonparametric methods, especially in a non-manufacturing setting, is that nonparametric methods can be applied to cases with a randomly defined covariance structure. It is worth noting that in literature, the terms “distribution-free tests” and “nonparametric methods,” are often used interchangeably. In this practicum, the distinction is made that nonparametric methods may be applied to cases meeting certain distributional assumptions. As an example, the most recent control chart techniques monitoring phase I situations require data to be from the elliptical family of distributions, where as distribution-free methods avoid the notion of a probability distribution. Nonparametric methods are developed based on theories such as the empirical cumulative distribution function (ECDF), empirical likelihood ratio tests, spatial sign and rank, and rank correlation through Monte Carlo methodology.

1.4 Literature Review of NPMSPC

The following synopsis focuses on available methods for multivariate nonparametric control charts. The study of NPMSPC charts begins with a discussion of the simpler, univariate cases. Due to their ease and familiarity, univariate, nonparametric Shewart control charts have received more attention in literature, for both phase I and II process monitoring. Nonparametric Cumulative Sum (NPCUSUM) charts were first proposed by Bakir and Reynolds (1979), and for a more recent study on NP SPC, see the Wilcoxon signed rank test proposed by Bakir (2004). Another approach is nonparametric change-point detection (CPD) charts, based on the Mann-Whitney two-sample test statistic, wtih the first sample taken before the change-point occurs, and the second sample taken from the distribution after the change-point, for related discussion, see Hawkins and Deng (2010). For an overview of NP SPC from CUSUM theory and onto the development of nonparametric Exponentially Weighted Moving Average (EWMA) charts, see Graham, Chakraborti, and Human (2011). Unfortunately, many univariate methods prove unreliable when the normality assumption is invalid, and extending to a multivariate setting has proven tricky, in practice. Some recent works suggest future research on checking univariate methods carrying over to the multivariate setting.

One important paper connecting univariate and multivariate methods comes from Qiu and Li (2011). The authors discuss NP SPC of univariate processes with the goal of extending well-known nonparametric methods to a multivariate setting. The focus is phase II monitoring of univariate processes of continuous, or categorical data when the IC response CDF is unknown. The technique uses a CUSUM-based form derived from different test statistics. Despite promising results, the methods are not used in this comparative study due to the fact that they were created primarily for use in phase II, univariate analysis. A study by Jayathava and Pongpullponsak (2014), important in solving the economic design problem, furthers the NP SPC study into multivariate analysis. The technique utilizes control charts to compare a sign-test with the Mann-Whitney test, and the Hodges-Lehmann estimator, using simulated normal and Weibull data. The choice to not include this method is because another method based on the Mann-Whitney test statistic was selected. Another univariate method for possible future study is given by Liu, Zhang, and Zi (2015), investigated dual nonparametric CUSUM control charts based on ranks. Although each of these methods attempt to extend to the multivariate case, they fall short, and the concern of this practicum is studying methods developed for multivariate analysis.

In recent literature, much study has been dedicated to NPMSPC methods. Some of these approaches include theory such as cross-component or anti-ranks (Qiu and Hawkins 2001), the use of log-linear modeling to categorize data such as in the paper by Qiu (2008), and spatial sign or rank (Zou, Wang, and Tsung 2012; Cheng and Shiau 2015). A longitudinal ranking method was explored by Boone and Chakraborti (2012) where they used multivariate nonparametric Shewart charts based on component-wise signs and signed ranks effectively as pointed out by Bell, Jones-Farmer, and Billor (2014). Also, for methods based on the empirical likelihood ratio test (LRT) see Sun and Zi (2013, Ning et al. (2014), or Li (2015). More recently, Kakde, Peredriy, Chaudhuri, and McGuirk (2016) proposed a method using support vector data description (SVDD) for high frequency data. Qiu and Zhang (2015) attempted to apply a normal transformation to data, however their results did not indicate improved performance. These methods study shifts in either location or shape parameters or both simultaneously. Many scholars have studied methods detecting shifts in the location parameter: for example, see Qiu and Hawkins (2001), Chen, Zi, and Zou (2015), and Jayathava and Pongpullponsak (2014). For detecting shifts in location and scale parameters simultaneously, see Chowdury, Mukherjee, and Chakraborti (2015). Many papers that detect location parameter changes suggest that future studies should extend their proposed methodologies to monitor variation or scale parameters. Now that the different NPMSPC methods have been covered, further discussion of methods differentiating between phase I and phase II is necessary.

NPMSPC research is divided into phase I and phase II data analysis. Recent literature shows phase I analysis to be more difficult, because the IC distribution must be established for a IC data set, or a self-starting method must be implemented. According to Bell et al. (2014), "currently, there exists no distribution-free phase I multivariate techniques." For other recent examples on phase I data, see works by: Ning et al. (2014), Cheng and Shiau (2015), Paynabar, Zou, and Qiu (2015), Capizzi and Masarotto (2017) among others. For some recent works on phase II analysis, see Chen et al. (2015), Chowdury et al. (2015), Liu et al. (2015), Qiu and Zhang (2015), and Zhang, Chen, and Zou (2016). Control charts that do not apply self-starting methods require certain distributional assumptions, such as being from the elliptical family of distributions, as in Cheng and Shiau (2015).

Self-starting methods are implemented at the start-up of a process, and require no distributional assumptions. For some current works on self-starting methods, see Zou and Tsung (2011), Sun and Zi (2013), Chen et al. (2015), and Zhou, Zhou, Liu, and Geng (2015). The proposed method by Chen et al. (2015) uses the Wilcoxon rank sum statistic, and has been shown to be efficient in detecting shifts of heavy tailed or skewed process distributions. Another advantage is that it determines control limits online, rather than beforehand. One disadvantage is that it may not completely factor in correlation among process variables. The method by Zhang et al. (2016) is an extension, hence, comparable to the Chen et al. (2015) method. The latter method is based on bivariate goodness-of-fit tests, and attempts to account for correlation structures. Other methods that may be applied to this situation and included in future comparisons are discussed next.

Li, Dai, and Wang (2014) proposed a multivariate change-point control chart for phase I analysis based upon data depth. Since this method is developed for the purpose of phase I analysis, it requires little prior knowledge of the underlying distribution. As a self-starting methodology, it does not restrict the minimum sample size of phase I observations, and has shown to be robust against normality. The authors proposed a generalized Mann-Whitney statistic, and it is similar to the method proposed by Chen et al. (2015), a distribution-free EWMA (DFEWMA). They indicate that no competing nonparametric multivariate detecting scheme for phase I analysis exists, and therefore, it is compared with the LRT of Sullivan and Woodall (2000), which requires the assumption of multivariate normality. The performance comparison is made with true signal probabilities. Since similarities exist with the DFEWMA method, we chose not to include it separately in this comparative study.

Balakrishnan, Paroissin, and Turlot (2014) proposed a one-sided control chart based on weighted precedence statistics and focuses on the distribution of smaller than expected lifelengths. Their method was found to be superior when compared against a Mann-Whitney, and CUSUM control chart. Given the comparison made with a Mann-Whitney chart, future study could be done with the methods included in our study. However, given the nature of their focus on lower limits, one-sided data, their method was not included in this comparative study.

Another method that could potentially be modified for future comparisons with our selected methods is by Mukherjee, Graham, and Chakraborti (2015). The proposed method is based on exceedence statistics using a nonparametric CUSUM (NPCUSUM) control chart for detecting smaller to moderate shifts in location parameters. The reason it was not included here is because they do not consider the null distribution case. However, the paper suggests that future work should compare the proposed NPCUSUM-EX method with existing self-starting charts, and extend it to develop EWMA charts. One important contribution to the nonparametric field is the use of median run length, as opposed to average run length (ARL). This is because performance for ARL can be skewed for CUSUM charts so it could carry over to EWMA charts. The usefulness of EWMA charts is in their robustness as noted by Stoumbos and Sullivan (2002).

Li, Liu, Zou, and Jiang (2014) developed a self-starting control chart for high dimensional and short run processes. Under the null hypothesis, the charting statistic of the change-point model asymptotically follows a multivariate normal distribution. This fact renders the method not distribution-free, so it was not included in the comparative study. Future work may be found in creating a hybrid method with more traditional nonparametric methods, and attempting to solve the high dimension problem.

The empirical likelihood ratio method proposed by Ning et al. (2014) is a control chart for individual or sub-grouped observations. Under the change-point model framework, testing the hypothesis $H_0 : \mu_0 = \mu_1$ versus $H_1 : \mu_0 \neq \mu_1$, the authors compared their method with existing nonparametric methods to accommodate data from non-normal distributions, commonly found in non-manufacturing industries. The methods the authors compared were control charts applying a two sample Mann-Whitney test statistic, Cramer-von-Mises statistic, and Kolmogorov-Smirnov statistic. Ning et al. (2014) mention that, until this point, nonparametric phase I control charts have not been based on the empirical likelihood theory, and future comparisons could be made with data depth methods. Two reasons for not including this method in our comparative study are that the method looks at sub-grouped data and that its purpose is to show the asymptotic distribution of the test statistics based on p-values associated with the IC signal probabilities, rather than to analyze ARL values. From the description of this and previous methods, finding truly nonparametric and distribution-free techniques, even in today's literature is difficult.

The lack of nonparametric methods is mentioned by Bell et al. (2014) who proposed a distribution-free multivariate phase I location control chart for sub-grouped data from elliptical distributions. This method is an extension of a univariate method, which requires the data to be ranked; therefore, the method introduces the idea of data depth, measuring how deep (or central) a data point is with respect to a certain probability distribution function (PDF). A data depth reduces multivariate data from a p-vector to a univariate depth value. The data is assumed to be unimodal, with a large depth value indicating centrality. The properties which classify a distribution as elliptical, required for a data depth function, are affine invariance, maximum at centre, monotonicity from deepest point, and vanishing at infinity indicating outliers. From the univariate case, using ranks render the control charts distribution-free, likewise, in the multivariate case using a multivariate mean rank chart to look at the data depth renders them distribution-free. The data depth function is applied to a continuous set of variables $\mathbf{X}_{n\times p}$, resulting in n corresponding depth values for D, with respect to x and $F_0(n)$, where $F_0(n)$ is the empirical distribution function (EDF) of the pooled reference sample. The method is compared against a Hotelling T^2 type chart. The paper uses the wine quality dataset, which is analyzed in this practicum. Despite the suitability of this method for our comparison study, it lacks the distribution-free property, and is designed for sub-grouped data, as opposed to individual observations, so was not included. However, potential exists for future comparison with the methods in this practicum if perhaps a hybrid method is developed to eliminate the distribution-free property.

The distribution-free phase II CUSUM control chart for joint monitoring location/scale by Chowdhury et al. (2015) requires an appropriate reference sample of size m_0 from an IC process dataset with a describable CDF. The test statistic is derived from data coming from an unknown univariate population, and does not cover the multivariate case. The test statistic requires continuous data to calculate a mean and a variance, and for performance analysis the ARL is studied. The explorative dataset used is the well-known piston ring data from Montgomery (2001). This dataset includes measurements of the inside diameter of the piston rings with an IC reference dataset of 125 observations, and looks at phase II samples of size $n = 5$. Due to the usefulness as a self-starting method if an extension to the multivariate case could be found, it would be meaningful to compare this method.

The multivariate Goodness-of-fit (GoF) method proposed by Zhang et al. (2016) is an important method for NPMSPC, but was not included in the full comparative study due to the computational effort required. Part of the importance of the GoF method is to compare to the DFEWMA method, and relevance to the theory and applications of the other methods. The GoF and other methods here require a meaningful representation of the multivariate EDF (MEDF). Zhang et al. (2016) report that, through theoretical and numerical analysis, it can be considered to be distribution-free, even with cases of small reference sample size m_0. The method checks for general distributional changes in the multivariate process variables, rather than just a mean (or location) vector, or covariance (or shape) matrix shift. The proposed method is a multivariate GoF based on the nonparametric likelihood ratio, supported by Zhang (2002). Instead of monitoring marginal distributions separately or the entire joint distribution, the charting statistic is constructed through a series of bivariate GoF tests, and is capable of detecting a much broader category of changes. The authors make a call for future research in finding more efficient computational algorithms.

The advantages of NPMSPC, although they may seem trivial, are worth mentioning. In literature, the so-called "curse of dimension" (Bickel 1969) is often encountered. Self-starting methods are able to overcome this problem by reducing the number of phase I observations required for that method to effectively detect IC behaviour.

Additionally, often the only distributional assumption is that the distribution is a continuous multivariate random variable. Another major advantage of these methods is that they avoid a lengthy data gathering step.

Some disadvantages of multivariate, distribution-free methods include the lack of research done on self-starting methods, especially those examining non-normal data structures. In general, nonparametric methods suffer in terms of performance when compared with parametric methods where the data meets the distributional requirements.

Our objective in this practicum is to include a detailed and comprehensive comparison of the most recent NPMSPC methods available in literature designed for individual observations in terms of phase II analysis. The five methods included for the comparative study are as follows: i) The spatial rank, MEWMA (SREWMA, for our notation as in the original paper) based method proposed by Zou et al. (2012). Of the five methods in this study, when introduced, two of the methods compared the SREWMA. Zou et al. (2012) mention it is the first method ever developed to be considered a self-starting nonparametric method, so it can be seen as a benchmark for recent methods. ii) The method directly motivates the DFEWMA and GoF methods; this being the case, the DFEWMA is included in the study. iii) Another ideal method, is proposed by Sun and Zi (2013), which is a self-starting method based on a weighted version of an adjusted empirical likelihood (AEL) ratio test incorporating a MEWMA control scheme, and denoted as EWAEL control scheme. iv) Zhou et al. (2015) developed the Multivariate Smirnov Test (MST) method for analysis of phase I data, which requires the MEDF, and taking the supremum of the difference of two or more distributions. Similar to the DFEWMA paper, the authors suggest that the only existing comparable method was the SREWMA method. The MST paper is a continuation of Bickel (1969), who had showed the MST is distribution-free and consistent against all alternatives. v) The fifth method in our comparison, Cheng and Shiau (2015) introduced a distribution-free multivariate control chart for phase I analysis, denoted as the MSS method. To summarize, the five methods to be compared in this practicum are known as: i) SREWMA, ii) DFEWMA, iii) EWAEL, iv) MST, and v) MSS.

This comparison of the methods is made in terms of carefully selected performance measures, described in detail later. We compare the latest available methods in cases not previously explored, covering many different distributions, and include smaller distributional shifts, δ, than previous works. The aim is to discover the most robust method in a multivariate setting. The distribution of each variable may be different, or unknown, with a covariance structure that is not well-defined. As will be shown, the reason for selecting the distributional cases, as that are in this report is to create consistency across each of the different methods and to aid future studies. In addition, we compare the methods using real data to show their versatility in real world applications.

The organization of the remaining part of this practicum is as follows. In Chapter 2, we describe all the five NPMSPC methods selected for performance comparison. In Chapter 3, we discuss using Monte Carlo simulations and the types of data most applicable for each of the selected methods. Also, we will discuss performance measures to use for analytic purposes and perform a thorough comparison of the five methods. In Chapter 4, we discuss the implementation of all five methods using two case examples. Conclusions and recommendations are given in Chapter 5.

Chapter 2

Methods

Most of the methods discussed in Chapter 1 are not included in this comparative study for the aforementioned reasons mentioned in Section 1.4. Hereafter, the methods discussed were scrutinized to meet the criteria to allow for meaningful comparison. The criteria for each method is that they be useful in self-starting situations, with a small IC phase I dataset size, m_0, and a large p dimension. In this chapter, a detailed description of the theory and methodology for each of the five methods will be provided.

To start our discussion, we define the change-point model used throughout this practicum to distinguish between changes in the distribution as it pertains to IC and OC conditions. We simulate data, where the observations are independent and identically distributed (*iid*) continuous variables in more than one dimension, such that $\mathbf{x}_1, ..., \mathbf{x}_{m_0}, \mathbf{x}_{m_0+1}, ..., \mathbf{x}_{m_0+t}, ..., \mathbf{x}_{m_0+n} \in \mathbb{R}^p$ for some integer, $p > 1$. The first m_0 phase I observations are considered IC, and the next $t = 1, ..., n$ phase II observations are OC. We shall define the multivariate change-point model as:

$$\mathbf{x}_i \sim \begin{cases} F_0(\mathbf{x}), & \text{for } i = 1, ..., m_0, \\ F_1(\mathbf{x}), & \text{for } i = m_0 + 1, ..., m_0 + n, \end{cases} \tag{2.1}$$

where F_0 and F_1 represent the IC and OC CDFs, respectively.

Before applying control charts, an appropriate set of observations of minimum size m_0 must be selected; here we set $m_0 = 50$ as explained in the sections for each method. For each of the five methods, once the m_0 IC phase I observations are collected, we then calculate each of the $t = 1, ..., n$ charting statistics of the respective methods based upon the phase II observations.

Exploring datasets is of practical importance in this study and all NPMSPC methods. Both the DFEWMA and MSS methods analyze the well-known wine quality dataset, and we investigated this data in Chapter 4 as an example of a case study. Once appropriate data has been selected, we applied our methodology to create control charts, and attain the ARL results for a measure of performance. In this chapter, we outline the procedure for calculating the charting statistic of each of the five methods. After the charting statistic has been found, we will obtain our performance measures as discussed in Chapter 3.

2.1 SREWMA method

Of the five methods in this study, when introduced, two of the methods were compared to the SREWMA method proposed by Zou et al. (2012). Motivated by the Zou and Tsung (2011) spatial sign and self-starting scheme, the newer of the two methods is a pioneering paper for self-starting, nonparametric methods. The SREWMA method monitors location parameters, requires few observations ($p+2 < m_0$), and no IC distributional parameters. One drawback of the method is its distribution-free property holds exactly over a limited class of multivariate, elliptical distributions. The authors outline the need for further research in developing a more robust method in detecting shifts of different magnitudes and checking the method's performance in high dimensional cases. They suggest integrating change-point detection and two sample spatial rank test theories to develop such a method.

The SREWMA method uses spatial sign and ranks, and the charting statistic, like the method by Zou and Tsung (2011), requires the affine invariance property. In general, a matrix is said to satisfy the affine invariant property if it does not depend on the coordinate system that was used to calculate it. Equivalently, multiplying $\mathbf{X}$ by a full matrix, $\mathbf{D}$, does not change the value of the charting or test statistic. An example here is the matrix $\mathbf{S_y} \geq 0$ satisfies the affine invariance property because $\mathbf{S_{Ax+b}} = \mathbf{AS_xA^T}$. Once the data is transformed to attain affine invariance, the spatial rank function is applied. In general, finding spatial ranks in the multivariate case is as follows. Define

$$U(\mathbf{X}) = \begin{cases} ||\mathbf{X}||^{-1}\mathbf{X}, & \text{for } \mathbf{X} \neq \mathbf{0} \\ \mathbf{0}, & \text{for } \mathbf{X} = \mathbf{0}, \end{cases} \tag{2.2}$$

where $||\mathbf{X}|| = (\mathbf{X}^T\mathbf{X})^{1/2}$ or simply the Euclidean length. Here, the empirical spatial rank function is given by:

$$\mathbf{r}_i = R_E(\mathbf{x}_i) = \frac{1}{n}\sum_{j=1}^{n} U(\mathbf{X}_i - \mathbf{X}_j), \text{ for } i = 1, ..., n, \text{where } i \neq j \tag{2.3}$$

The $\mathbf{r}_i$ values reflect the relative magnitudes of the data. The theoretical R with respect to F is $R_F(\mathbf{X}) = E_y[U(\mathbf{X} - \mathbf{Y})]$, where the distribution of $Y \sim F$. For a spherical distribution F, $R_F(\mathbf{X}) = q_F(r)\mathbf{U}$, with $r = ||\mathbf{X}||$, $\mathbf{U} = U(\mathbf{X})$, where $q_F(\cdot)$ is a scalar function. Despite the distributional assumption, the spatial rank has been shown to be robust (Zou et al. 2012).

Now, for the details of calculating the n charting statistics, for each of the $t = 1, ..., n$ phase II OC observations, start by finding the sample covariance $\hat{\mathbf{S}}_t$ based on the prior $m_0 + t - 1$ observations

$$\hat{\mathbf{S}}_t = \frac{1}{m_0 + t - 1}\sum_{j=1}^{m_0+t-1} (\mathbf{X}_j - \bar{\mathbf{X}}_t)(\mathbf{X}_j - \bar{\mathbf{X}}_t)^T \tag{2.4}$$

where $\bar{\mathbf{X}}_t$ is sample mean of $m_0 + t - 1$ observations. Next, we find the eigenvalues Λ and eigenvectors ν of the sample covariance, and using the Cholesky factorization (Johnson and Wichern 2007), we determine the transformation matrix $\hat{\mathbf{M}}_t$ such that:

$$\hat{\mathbf{M}}_t = \hat{\mathbf{S}}_t^{-1/2} = \nu\Lambda^{-1/2}\nu^T \tag{2.5}$$

With n of the $\hat{\mathbf{M}}_t$ values calculated, we apply the affine invariance transformation and the spatial rank function to our simulated data as follows:

$$R_E(\hat{\mathbf{M}}_t\mathbf{X}_{m_0+t}) = \frac{1}{m_0 + t - 1}\sum_{j=1}^{m_0+t-1} U\left(\hat{\mathbf{M}}_t(\mathbf{X}_t - \mathbf{X}_j)\right) \tag{2.6}$$

Once each of the values $R_E(\hat{\mathbf{M}}_{t-1}\mathbf{X}_{m_0+t})$ for $t = 1, ..., n$ have been found, to calculate the charting statistic for the SREWMA method the variability of the $R_E(\hat{\mathbf{M}}_{t-1}\mathbf{X}_{m_0+t})$ values is assessed by

$$CoV[R_F(\mathbf{MX})] = E[||R_F(\mathbf{MX}_t)||^2]\mathbb{I}_p/p, \text{note: CoV = covariance matrix.}$$

where $E[||R_F(\mathbf{MX}_t)||^2]$ is approximated by $\hat{E}[||R_F(\mathbf{MX}_t)||^2]$ such that,

$$\hat{E}[||R_F(\mathbf{MX}_t)||^2] = \frac{\sum_{j=1}^{m_0} ||\tilde{R}_E(\hat{\mathbf{M}}_0\mathbf{X}_j)||^2 + \sum_{j=m_0+1}^{m_0+t} ||R_E(\hat{\mathbf{M}}_{j-1}\mathbf{X}_j)||^2}{m_0 + t - 1}, \ t = 1, ..., n.$$

where the initial conditions for $t = 1$ is:

$$\tilde{R}_E(\hat{\mathbf{M}}_0\mathbf{X}_j) = \frac{1}{m_0}\sum_{k=1}^{m_0} U\left(\hat{\mathbf{M}}_0(\mathbf{X}_j - \mathbf{X}_k)\right) \text{ for } j = 1, ..., m_0, \text{where } j \neq k. \tag{2.7}$$

Making use of these calculated $E[||R_F(\mathbf{MX}_t)||^2]$ and $R_E(\hat{\mathbf{M}}_t\mathbf{X}_{m_0+t})$ values the author's proposed a charting statistic determined by the recursive EWMA scheme:

$$Q_t^{R_E} = \frac{(2-\lambda)p}{\lambda\eta_t}||\mathbf{v}_t||^2, \tag{2.8}$$

where $\mathbf{v}_t = (1-\lambda)\mathbf{v}_{t-1} + \lambda R_E(\hat{\mathbf{M}}_{t-1}\mathbf{X}_t)$, and λ is the smoothing parameter of any EWMA scheme. Also, $v_0 = 0$, $\eta_t = \hat{E}[||R_F(\mathbf{MX}_t)||^2]$.

This process is continued until $t = 1, ..., n$ values of $Q_t^{R_E}$ are obtained. Like each of the methods explored here, this is repeated for 1000 Monte Carlo simulations, and the appropriate empirical quantiles are found corresponding to the cut-off values. Many distribution-free methods, including those used here, mention that when obtaining charting statistics if from the m_0 IC phase I observations case an OC condition occurs, the process must be restarted until only IC observations are considered. Thus, start monitoring the process and obtain observations $\mathbf{X}_t$ sequentially, and for new observations find $R_E(\hat{\mathbf{M}}_{t-1}\mathbf{X}_t)$ and obtain the charting statistic $Q_t^{R_E}$. In general, once the $Q_t^{R_E}$ statistic exceeds some control limit, a signal shall be triggered otherwise using rank-one downdating Cholesky factorization obtain $\hat{\mathbf{M}}_t$, and also update $\hat{E}[||R_F(\mathbf{MX}_t)||^2]$ (Zou et al. 2012).

Some asymptotic properties (Zou et al. 2012) involved in these calculations are

$$CoV(R_E(\mathbf{MX}_t)R_E(\mathbf{MX}_k)) = 0, \ \forall\ k \neq t. \tag{2.9}$$

Furthermore, as $\lambda \to 0$, and $\lambda t \to \infty$, $Q_t^{R_E}$ converges to the χ^2 distribution, with p degrees of freedom, and the SREWMA sequence is considered a self-starting method, if and only if, $m_0 \geq p + 2$. However, as large a value of m_0 as possible is preferred. In order to stabilize the process from the phase I observations, Zou et al. (2012) suggest $m_0 + t \geq 50$. Note, in choosing λ (the smoothing parameter of any EWMA scheme), the smaller, the quicker the detection, and the more robust the method. A remark is the SREWMA method does not accommodate tied observations.

2.2 DFEWMA method

The method by Chen et al. (2015) produces a distribution-free control chart to study multivariate process data. It was motivated by the SREWMA method as a way to compare the effectiveness of both methods in high dimensional cases. The method of Chen et al. (2015) is preferred since it does not require the underlying distribution to have the elliptical class property. In both the SREWMA and DFEWMA papers, they use a minimum of $m_0 = 50$ phase I observations in their comparative studies to satisfy the requirements of other methods in their study, not directly required for the use of their proposed methods given their self-starting nature. Analogous to the SREWMA method, the DFEWMA method requires $m_0 \geq p + 2$. The data should be ordinal as the method requires ranking to be meaningful, noting that variables do not necessarily have to be continuous. However, in the DFEWMA method, the idea of continuity should be taken cautiously, since having many ties in the data may cause problems as with any Wilcoxon statistic.

The DFEWMA control chart establishes component-wise ranks for $\mathbf{X}$ to construct a two-sample Wilcoxon-Rank Sum statistic. From the change-point model described in equation (2.1) for individual observations, the charting statistic is

$$T_t(w, \lambda) = \sum_{j=1}^{p} T_{jt}^2(w, \lambda), \text{ for } t = 1, ..., n, \tag{2.10}$$

with the T_{jt} based on each $j = 1, ..., p$ dimension are

$$T_{jt}(w, \lambda) = \sum_{i=m_0+t-w+1}^{m_0+n} (1-\lambda)^{n-i} \frac{R_{jni} - w(m_0 + n + 1)/2}{\sqrt{w(m_0 + n + 1)(m_0 + n - w)/12}}, \tag{2.11}$$

where $w > 0$ is the window size, λ is a smoothing parameter, R_{jni} is the rank of X_{ji} among the sample for $i = 1, ..., m_0 + n$, for each of the j variables. Since the indexing on the summation covers $m_0+w+1, ..., m_0+n$, the corresponding R_{jni} values represent the rank of the OC observations based on all the $m_0 + n$ observations. We notice the window size w prevents us from checking the first w OC phase II observations, where w should be chosen so it is the smallest integer that satisfies $(1-\lambda)^w \leq 0.05$. For instance, if we set $\lambda = 0.05$, this would yield $w = 58$, however, Chen et al. (2015) mention that it has to be at least $w = 5$, so that was our selection. This is to avoid the false detection of an early OC condition and to help stabilize the process. For each new $t = 1, ..., n$ observation, the charting statistic T_t is calculated until all n of these charting statistics are found. This process is repeated for 1000 Monte Carlo simulations, with the appropriate control limits taken based on the appropriate level α quantiles. $T_{jn}(w, \lambda)$ is a weighted version of the two-sample Wilcoxon rank-sum statistic (Chen et al. 2015). In general, a location shift at an unknown change-point $m_0+t \leq m_0+n$ corresponds to a large $|T_{jn}(w, \lambda)|$ value, and the test statistic $T_n(w, \lambda)$ signals an alarm.

2.3 EWAEL method

Sun and Zi (2013) proposed a self-starting method, which is a weighted version of an adjusted empirical LRT incorporating a MEWMA control scheme. The method assigns probabilities according to the MEDF function exploring p dimensional observations within the real space, $\mathbf{X} = \{\mathbf{X_1}, ..., \mathbf{X_{m_0+n}}\}, \mathbf{X_i} \in \mathbb{R}^p, i = 1, ..., m_0+n$, iid, where the distribution function changes at an unknown time point $m_0+t \leq m_0+n$. For the observation in the j^{th} dimension $X_{i,j}$ define the marginal CDF as $F_j(t), t \in \mathbb{R}$. Now, consider the joint distribution of $X_{i,j}$, and $X_{i,k}$ with CDF $F_{jk}(\mathbf{t})$, $\mathbf{t} = [t_1, t_2] \in \mathbb{R}^2$, which represents a single bivariate point on the real plane. The following details in relation with the MEDF are also relevant to the MST method. Partition the set of samples of $\mathbf{X}$ into four disjoint regions according to whether $X_{i,j}$ ($X_{i,k}$) is smaller than t_1 (t_2) or not. Define $H_0(\mathbf{t}) : F_{jk}(\mathbf{t}) = F_{0,jk}(\mathbf{t}), \forall\ \mathbf{t} \in \mathbb{R}^2$ against $H_1(\mathbf{t}) : F_{jk}(\mathbf{t}) \neq F_{0,jk}(\mathbf{t})$ for at least one of j or k. The number of samples $[X_{i,j}, X_{i,k}]$ in each of the four regions of the multinomial distribution with probabilities, under H_0 such that:
$P^1_{0,jk}(\mathbf{t}) = P(X_{i,j} \leq t_1, X_{i,k} \leq t_2)$
$P^2_{0,jk}(\mathbf{t}) = P(X_{i,j} \leq t_1, X_{i,k} > t_2)$
$P^3_{0,jk}(\mathbf{t}) = P(X_{i,j} > t_1, X_{i,k} \leq t_2)$
$P^4_{0,jk}(\mathbf{t}) = P(X_{i,j} > t_1, X_{i,k} > t_2)$.
Testing the hypothesis where as $\hat{P}^r_{jk}(t)$ is based on the MEDF, and extending to more than two dimensions, $\mathbf{t} \in \mathbb{R}^p,\ p > 2$ is a useful interpretation for both the MST and

EWAEL methods.

The EWAEL method, unlike the SREWMA method, does not require any assumptions of the underlying process distribution. One of the main advantages of this method is that it avoids a lengthy data-gathering step, making it useful in phase I applications.

To begin, a general definition of the empirical likelihood (EL) function is required where $p_i = F(x_i) - F(x_i-)$, and $F_n(x) = n^{-1}\sum_{i=1}^{n} I(x_i \leq x)$ is the empirical CDF, note when maximized $p_1 = ... = p_n = n^{-1}$. From this, the regular profile log-likelihood of $\boldsymbol{\mu}$ becomes (Sun and Zi 2013):

$$l(\mu) = \sup\{\sum_{i}^{n} log(p_i) | \sum_{i}^{n} p_i = 1, \sum_{i}^{n} p_i(x_i - \mu) = 0, p_i \geq 0\} \qquad (2.12)$$

However, Chen et al. (2008) point out that a shortcoming of the EL, is that for a given $\boldsymbol{\mu}$ the log of the likelihood ratio function is well-defined only if the convex hull of $\{x_1, ...x_n\}$ contains the p-dimensional $\mathbf{0}$ vector. Also, the existence of the maximum EL solution depends on the selection of μ. Chen et al. (2008) proposed an adjusted EL, called the profile-adjusted empirical likelihood (AEL) to overcome this problem of existence. Let $\bar{x} = n^{-1}\sum_{i=1}^{n} x_i$ and $x_{n+1} = -\eta_n \bar{x}$ for some $\eta_n > 0$. Since our p dimensional vector of each observation $\bar{\mathbf{x}}$, $\mathbf{x_i} \in \mathbb{R}^p$, $i = 1, ..., m_0 + n$ and $\mathbf{x_{n+1}}$ are always on opposite sides of $\mathbf{0}$, the AEL is always well defined (Chen et al. 2008). Therefore, the observation vector $\mathbf{x_{n+1}}$ is added to the data.

In order to understand the development of the test statistic, consider the following: $\boldsymbol{\mu}_i = E(\mathbf{X}_i) - \boldsymbol{\mu}_0$,
where

$$\boldsymbol{\mu}_i = \begin{cases} 0 \text{ for } i = 1, ..., m_0, m_0 + 1, ..., m_0 + t \\ \boldsymbol{\mu} \text{ for } i = m_0 + t + 1, ..., m_0 + n, \end{cases} \qquad (2.13)$$

with t being the unknown change-point. From $\hat{\boldsymbol{\mu}}_i$, we establish the AEL framework such as:

$$L_t(\mathbf{0}) = \sup\{\prod_{i=0}^{m_0+t} p_i^{w_i} | \sum_{i=0}^{m_0+t} p_i = 1, \sum_{i=0}^{m_0+t} p_i y_i = 0, p_i \geq 0, \text{ for } i = 0, ..., m_0 + t\}$$

where $y_{i,t} = x_i - \mu_0$, $i = 1, ..., m_0 + t$. Note, since the true population mean μ is unknown, the IC phase I sample is used to estimate $\hat{\mu}$ and denoted as μ_0, where $\mu_0 = \Sigma_{i=1}^{m_0} x_i/m_0$. Also, $y_{0,t} = -\frac{\eta_t}{a_t}\sum_{i=1}^{m_0+t} w_i y_{i,t}$, $\quad a_t = \sum_{i=1}^{m_0+t} w_i$, $\eta_{m_0+t} > 0$, $\forall\ t =$

$1, ..., n$. The selection of w_i, is the EWMA scheme weights $w_i = (1-\lambda)^{m_0+n-i}$, with $0 \leq \lambda \leq 1$, the smoothing parameter. Based on $L_t(0)$, the weighted adjusted empirical log-likelihood ratio (WAELR) is defined as follows:

$$l_t(\mathbf{0}) = 2b_t \ \log[L_t(\mathbf{0})/L_t(\hat{\mu})]$$

where $L_t(\hat{\mu}) = \sup\{\prod_{i=0}^{m_0+t} q_i^{w_i} | \sum_{i=0}^{m_0+t} q_i = 1, q_i \geq 0\}$. Note that $b_t = {}^{a_t}/_{\sum_{i=1}^{m_0+t} w_i^2}$, is a scaling constant. Using Lagrange multipliers, the authors showed that:

$$p_i = \frac{1}{a_{m_0+t} + w_0} \frac{w_i}{\mathbf{1 + \boldsymbol{\theta}^T \boldsymbol{y_i}}}, \ q_i = \frac{w_i}{a_{m_0+t} + w_0}, \ \text{for } i = 1, ..., m_0 + t.$$

Therefore, the corresponding WAELR charting statistic becomes:

$$l_t(\mathbf{0}) = 2b_t \sum_{i=0}^{m_0+t} w_i \log(1 + \theta^T y_i), \ \text{for } t = 1, ..., n,$$

with θ satisfying the score equation

$$g(\theta) = \sum_{i=0}^{m_0+t} \frac{w_i y_{i,t}}{\mathbf{1 + \boldsymbol{\theta}^T \boldsymbol{y_{i,t}}}} = \mathbf{0}. \tag{2.14}$$

To solve the score equation and obtain solutions for the values of θ, the Newton-Raphson method was applied. In order to find an iterative update to θ, we solve:

$$\boldsymbol{\theta}_{k+1} = \boldsymbol{\theta}_k - J_k^{-1}(\boldsymbol{\theta}_k) g(\boldsymbol{\theta}_k). \tag{2.15}$$

To iteratively update the inverse of the Jacobian J^{-1} consider:

$$J_{k+1}^{-1}(\boldsymbol{\theta}_{k+1}) = J_k^{-1}(\boldsymbol{\theta}_k) + \frac{[(\boldsymbol{\theta}_{k+1} - \boldsymbol{\theta}_k) - J_k^{-1}(\boldsymbol{\theta}_k)\left(g(\boldsymbol{\theta}_{k+1}) - g(\boldsymbol{\theta}_k)\right)](\boldsymbol{\theta}_{k+1} - \boldsymbol{\theta}_k) J_k^{-1}(\boldsymbol{\theta}_k)}{\left(\boldsymbol{\theta}_{k+1} - \boldsymbol{\theta}_k\right)^T J_k^{-1}(\boldsymbol{\theta}_k)\left(g(\boldsymbol{\theta}_{k+1}) - g(\boldsymbol{\theta}_k)\right)} \tag{2.16}$$

When the step-size $J_k^{-1}(\boldsymbol{\theta}_k) g(\boldsymbol{\theta}_k)$ does not reduce the error in $|\boldsymbol{\theta}_{k+1} - \boldsymbol{\theta}_k|$ from the previous iteration, it is reduced by a factor of 0.5 repeatedly until the new θ value converges.

Our solution to the score equation for θ_k from (2.16), is plugged into our WAELR charting statistic in (2.14). This process is repeated for 1000 Monte Carlo simulations, and then taking the appropriate α quantile values. Based on (2.14), the proposed control chart triggers a signal if $l_t(\mathbf{0}) > C$, C is chosen based on the limits to achieve a specific IC run length distribution. As a remark, Chen et al. (2008) showed that if the w_i's are equal ($\lambda_i = 0$) the WAELR $l_t(\mathbf{0})$ reduces to the conventional AEL ratio function. At this stage we label the method as EWMA WAELR or (EWAEL) control scheme.

2.4 MST method

Zhou et al. (2015) developed a method based on the multivariate Smirnov test, and similar to the Chen et al. (2015) paper, they suggest the only existing comparable method was the SREWMA method by Zou et al. (2012). The MST paper analyzed the ARL values of various sizes of δ shifts to multivariate distributions, similar to the cases in the comparison made by Li et al. (2014). The MST method is robust, self-starting, good for small IC sample size, and is designed to detect various magnitudes of mean shifts.

The MST technique examines a multivariate $\mathbf{X}$ matrix of data observations. The data is assumed to be *iid*, and follow a distribution so as to be distinguishable between IC and OC, as defined in our original hypothesis model in equation (2.1). The data, $\mathbf{X} \in \mathbb{R}^{(m_0+n)\times p}$, with p variables, must be meaningfully represented with the MEDF. For the first m_0 phase I IC observations, define the MEDF as

$$F_{m_0}(\mathbf{y}) = m_0^{-1} \sum_{j=1}^{m_0} I_{A(\mathbf{y})}(\mathbf{x}_j), \tag{2.17}$$

for $\mathbf{x}_1, ..., \mathbf{x}_{m_0} \in \mathbb{R}^p$, where $A(\mathbf{y}) = \{\mathbf{s} : \mathbf{s} \leq \mathbf{y}\}$, with $\mathbf{s} \leq \mathbf{y}, (s_1 < y_1, ..., s_p < y_p)$ being defined component-wise. Part of the difficulty in finding F is in considering the possible orderings of each variable, such that

$$F_{m_0}(\mathbf{y})^* = 1 - F_{m_0}(\mathbf{y}). \tag{2.18}$$

From F similarly define $G_n(\mathbf{y})$ as the MEDF of n phase II OC observations $\mathbf{x}_{m_0+1}, ..., \mathbf{x}_{m_0+n}$.

Since $F(\mathbf{y})$ and $G(\mathbf{y})$ are defined component-wise, i.e.

$$F(\mathbf{y}) = \begin{pmatrix} F(y_1) \\ F(y_2) \\ \vdots \\ F(y_p) \end{pmatrix}, \tag{2.19}$$

to find the Smirnov distance, start by calculating

$$\sup_{x \in \mathbb{R}^j} |F(x_j) - G(x_j)|. \tag{2.20}$$

for each of the $j = 1, 2, ..., p$ dimensions, each time checking both cases with F replaced by F^*. Define the Smirnov distance, $D_t(F, G)$, for $t = 1, ..., n$ as the maximum of each p vector of supremum values. This is defined as:

$$D_t(F, G) = \max |F(\mathbf{x}) - G(\mathbf{x})|, \tag{2.21}$$

which, for each $t = 1, ..., n$, is simply the maximum within the $1 \times j$ vector for the $j = 1, 2, ..., p$ calculations from (2.21). This process is continued until all n of the D_t test statistics are calculated. We repeat this for 1000 Monte Carlo simulations, and compute the level α quantiles.

2.5 MSS method

Cheng and Shiau (2015) introduced a distribution-free multivariate control chart for phase I analysis based on the multivariate sign, or spatial sign. Given the common goal with the other methods of creating a nonparametric, self-starting method, and considering the fact that the method was applied to the wine quality data set, it was included in our comparative study. One drawback with this method is that it was originally designed for individual observations. However, the method produced favourable results in performance analysis so we transformed the method to accommodate individual observations by using subgroups of size one.

To apply the method, let $\{\mathbf{X}_1, ..., \mathbf{X}_{m_0}\}$ be an *iid* random sample from a continuous p-dimensional distribution, with a location parameter, such that $F(\mathbf{X} - \boldsymbol{\mu}_0)$ indicating shift to the mean vector. The null hypothesis is similar to (2.1) with the

OC observations having the same distribution function, but here having a different location parameter, such that $F(\mathbf{X} - \boldsymbol{\mu}_1)$, where $\boldsymbol{\mu}_1 \neq \boldsymbol{\mu}_0$.

This paper follows the authors' description of the process of calculating the test statistic, insofar as the central point of a distribution tends to be unknown in practice. That is, it must be estimated from the data empirically. The literature commonly recommends the use of the spatial median as the multivariate centre because of its robustness. The (sample) spatial median $\hat{\theta}$ shall be set to minimize the function:

$$\sum_{i=1}^{m_0} ||\mathbf{X}_i - \boldsymbol{\theta}||. \tag{2.22}$$

If we set the gradient of the function in (2.23) to zero, $\hat{\theta}$ will satisfy:

$$\sum_{i=1}^{m_0} \frac{\mathbf{X}_i - \boldsymbol{\theta}}{||\mathbf{X}_i - \boldsymbol{\theta}||} = \mathbf{0}. \tag{2.23}$$

Since there is no ordering in multivariate data, from (2.2), as in the SREWMA method, define the multivariate sign function, and transform the data $\mathbf{u}_i = U(\mathbf{M}\mathbf{X}_i)$ for $i = 1, ..., m_0$, with the matrix $\mathbf{M}$ as defined in Section 2.1. Randles (2000) constructed an affine invariant multivariate sign test using a transformation, retransformation, and a directional transformation. Applying the Tyler Transformation Matrix, and a procedure proposed by Hettmansperger and Randles (2002) to phase I data, the estimators for θ and M are solutions to the two equations:

$$\frac{1}{m_0} \sum_{i=1}^{m_0} U\left(\mathbf{M}(\mathbf{X}_i - \boldsymbol{\theta})\right) = \mathbf{0}, \tag{2.24}$$

and

$$\frac{1}{m_0} \sum_{i=1}^{m_0} U(\mathbf{M}(\mathbf{X}_i - \boldsymbol{\theta})) U(\mathbf{M}(\mathbf{X}_i - \boldsymbol{\theta}))^T = \frac{1}{p}\mathbb{I}_p. \tag{2.25}$$

Accordingly, the Hettmansperger-Randles estimators can be found by the following algorithm.

i.) First of all, set $(\theta, \mathbf{S})$ to $(\tilde{\theta}, \mathbb{I}_p)$ where $\tilde{\theta}$ is the p regular sample medians of the p components. $\mathbf{S}^{1/2}$ is any $p \times p$ matrix such that $\mathbf{S}^{1/2}\mathbf{S}^{1/2} = \mathbf{S}$.

ii.) Fix $\mathbf{S}$, and compute $\mathbf{S}^{-1/2}$, then we let $\mathbf{Z}_i = \mathbf{S}^{-1/2}(\mathbf{X}_i - \boldsymbol{\theta})$, for $i = 1, ..., m_0$, and iteratively update θ by:

$\theta + \frac{\sum_{i=1}^{m_0} U(\mathbf{Z}_i)}{\sum_{i=1}^{m_0} ||\mathbf{Z}_i||^{-1}}$, until convergence.

iii.) Fix θ, and let $\mathbf{Z}_i = \mathbf{S}^{-1/2}(\mathbf{X}_i - \boldsymbol{\theta})$, for $i = 1, ..., m_0$, then iteratively update $\mathbf{S}$ by: $p\mathbf{S}^{1/2}\left(\frac{1}{m_0}\sum_{i=1}^{m_0} U(\mathbf{Z}_i)U(\mathbf{Z}_i)^T\right)\mathbf{S}^{1/2}$, until
$||p\left(\frac{1}{m_0}\sum_{i=1}^{m_0} U(\mathbf{Z}_i)U(\mathbf{Z}_i)^T\right) - \mathbb{I}_p||$ is sufficiently small, then set $\mathbf{S} = \frac{p}{trace(\mathbf{S})}\mathbf{S}$.
iv.) Repeat steps ii) and iii) until θ and $\mathbf{S}$ converge. The estimate values $\hat{\theta}$ and $\mathbf{S}_t$ with $\mathbf{M}_t^T\mathbf{M}_t = \mathbf{S}_t^{-1}$ and the corresponding multivariate sign vector of each $\mathbf{X}_i$ is:
$u_i = U\left(\mathbf{M}(\mathbf{X}_i - \boldsymbol{\theta})\right)$ and the charting statistic is:
$Q = m_0\bar{\mathbf{u}}^T(\frac{1}{m_0}\sum_{i=1}^{m_0}\mathbf{u}_i\mathbf{u}_i^T)^{-1}\bar{\mathbf{u}}$.

Note that $\bar{\mathbf{u}}$ is the average of $\mathbf{u_i}$'s, $\bar{\mathbf{u}} = \frac{\sum_{i=1}^{m_0}\mathbf{u_i}}{m_0}$ and $\frac{1}{m_0}\sum_{i=1}^{m_0} U(\mathbf{M}_{m_0-1}\mathbf{x}_i)(\mathbf{M}_{m_0-1}\mathbf{x}_i)^T = \frac{1}{p}\mathbb{I}_p$.
For any $\mathbf{M}_i$ where $\mathbf{M}_i^T\mathbf{M}_i = S_i^{-1}$, S_i is the Tyler scatter matrix, and M_i is the corresponding Tyler transformation matrix. The test statistic becomes: $Q = m_0 p\bar{\mathbf{u}}^T\bar{\mathbf{u}}$, and H_0 is rejected when Q is larger than some critical value. As a remark, Q has the affine invariant property when $m_0 > p(p-1)$. This will hereafter be referred to as the multivariate sign Shewart (MSS) chart.

The following is a procedure to be used to determine control limits with the multivariate normal distribution.
i.) Specify the dimension p, and the desired false alarm rate α.
ii.) Generate iid sample $\{\mathbf{X}_i,\ i = 1, ..., m_0\}$ from, say, $N_p(0, \mathbb{I}_p)$ to form one group of size m_0. To find our cut-off values, add $t = 1, ..., n$ observations from the same $N_p(0, \mathbb{I}_p)$ distribution.
iii.) Calculate corresponding Q_t statistic for $t = m_0 + 1, ..., m_0 + n$
iv.) Take sample upper α quantile of $\{Q_t\}$ as the control limits based on 1000 Monte Carlo simulations

The test statistic created has a small sample distribution-free property when the underlying class of distributions is of elliptical directions. Therefore, in the paper's comparative simulation study, adaptations of the Hotelling multivariate T^2 charts are investigated. Like the other methods, the authors compare with the multivariate normal, t, and gamma distributions, here with n values of five and 10. The real life case study was on the wine quality data set, as explored in the MST paper.

Chapter 3

Performance Analysis

3.1 Cut-off Values

In order to complete the performance analysis of the five methods explained in Chapter 2, we discuss the development of the cut-off values or control limits for the charting statistics of the methods. We use a Monte Carlo simulation approach and begin by generating a sample of size $m_0 + n$ observations from a multivariate distribution, with a mean vector $\boldsymbol{\mu} = \mathbf{0}$, and a $p \times p$ covariance matrix of $\boldsymbol{\Sigma} = \mathbb{I}$. In order to create a comparison of the five methods as self-starting methods, different values of $m_0 = 50, 100$ observations are generated from the null distribution, representing the phase I IC data set. Based on 1000 Monte Carlo simulations, the appropriate $(1 - \alpha)100\%$ quantile values of the test statistics are used as cut-off values. For each of n phase II data points, the number of cut-off values calculated is based on the level of significance, if $\alpha = 0.05$, $n = 100$, and if $\alpha = 0.01$, $n = 500$. We restrict ourselves to 1000 Monte Carlo simulations due to the fact we arrive cut-off values for such a large number of data positions, for instance $\alpha = 0.01$, and $m_0 + n = 600$. Our limited simulations running 5000 simulations does not change the ARL values much, so we restrict our simulations to 1000.

In our simulation study for each of the five methods, we calculate test statistics for each of the following distributional cases:

i.) Multivariate normal, $N_p(\mathbf{0}, \mathbb{I})$, for $p = 2, 3, 4, 5$, $\alpha = 0.05, 0.01$, and $m_0 = 50, 100$.

ii.) For non-normal data with $p = 2$, $\alpha = 0.05, 0.01$, $m_0 = 100$, $X_1 \sim N(0, 1)$, $X_2 \sim$ Uniform$(-\sqrt{3}, \sqrt{3})$ and $X_1 \sim N(0, 1)$, $X_2 \sim t_3$, where t_3 is the student t distribution with 3 degrees of freedom.

iii.) For non-normal data with $p = 5$, $\alpha = 0.05, 0.01$, $m_0 = 100$, $X_1, X_2, X_3 \sim N(0, 1)$, $X_4, X_5 \sim$ Uniform$(-\sqrt{3}, \sqrt{3})$ and $X_1, X_2, X_3 \sim N(0, 1)$, $X_4, X_5 \sim t_3$.

The reason for using mostly cases involving the normal distribution is due to its ease, and because within the structure of the five methods, the $\delta = 0$ results would more consistently obtain the desired ARLs. The cases with the uniform and t_3 distributions analyze the robustness of the methods. The uniform distribution is a way to check the method's behaviour in heavy-tailed distributional cases and the t distribution checks the behaviour in skewed distributions. The selection of the p variables was designed to check a low dimension case ($p = 2$), and a high dimension case ($p = 5$). For a complete recap of the calculated cut-off values, see Appendix A, Tables A.1 through A.30. As an example, Table 3.1 represents the first ten lines of Table A.1, which is the multivariate normal distribution and covers the cases for $p = 2-5$, $\alpha = 0.05$, and $m_0 = 50$ of the DFEWMA method. Similar tables for the other α and m_0 values are created for each of the other four methods. For the non-normal cases, all five methods have two tables, one for each $\alpha = 0.05, 0.01$ value, and contained within them are the $p = 2, 5$ for uniform and t distribution cases.

Table 3.1: DFEWMA Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.05$, $m_0 = 50$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
6	52.69	77.93	102.99	129.06	54	219.91	328.61	435.65	543.65
7	64.99	96.31	127.78	159.03	55	220.50	328.35	436.37	543.78
8	76.97	114.06	151.21	188.13	56	220.33	327.97	436.19	544.40
9	88.52	131.75	174.50	217.58	57	219.98	328.07	436.07	544.86
10	99.66	148.20	197.15	244.86	58	219.88	328.54	435.78	544.10
11	110.11	163.79	217.51	271.69	59	220.26	327.99	435.95	543.25
12	120.25	178.58	237.59	295.82	60	219.65	328.05	436.20	543.91
13	129.15	192.31	255.59	318.53	61	220.43	328.02	436.03	543.67
14	138.00	205.60	272.67	340.02	62	219.75	328.22	435.31	544.07
15	145.80	217.32	288.59	360.06	63	219.84	327.94	436.33	544.01
16	153.00	227.53	302.75	377.86	64	220.13	327.98	435.86	542.92

3.2 Performance Measure Analysis

In this practicum, in order to conduct our comparative performance analysis, we study the run length (RL) distribution of the control charts for each of the five methods. The RL distribution is studied in terms of ARL, and the standard deviation of the run length sd(RL). The main reason for selecting the ARL and sd(RL) was that similar metrics were used to compare the performance of the methods in their respective papers. To acquire our ARL results, a similar approach to that of the DFEWMA and MST papers was applied. We start by finding RL results by calculating the test statistic for n observations to compare against the cut-off values as described in Section 3.1. The RL is found by counting $t = 1, 2, ..., n$ observations until the test statistic triggers an alarm by exceeding the cut-off values. The process is continued for 1000 Monte Carlo simulations, and the ARL and sd(RL) are calculated from the results. To calculate the ARL, within each simulation we find the RL and take the average of the 1000 Monte Carlo simulations. An additional note, within each simulation, for example the $\alpha = 0.05$ case, if an OC condition has not been detected by $t = 100$, the value of RL=100 is recorded. The ARL values we obtained were based upon shifts to the IC and OC distributions. For each simulation, m_0=50 or 100 phase I IC observations were generated, followed by n observations of the shifted phase II OC data. The shift in mean was made in terms of

$$\delta = (\boldsymbol{\mu_1} - \boldsymbol{\mu})^{\boldsymbol{T}} \boldsymbol{\Sigma}^{\boldsymbol{-1}} (\boldsymbol{\mu_1} - \boldsymbol{\mu}) \tag{3.1}$$

for values of $\delta =$ 0, 0.5, 1, 1.5, 2, 3, 4, and 5, similar to the cases explored in the literature (Zhou et al. 2015, Chen et al. 2015, Cheng and Shiau 2015, and many others). Now, we only applied the shift to the mean vector, with the covariance matrix, $\boldsymbol{\Sigma}$, remaining constant. As a remark, the value of $\delta = 0$ represents the null case, and should achieve the optimal ARL values, for example if $\alpha = 0.05$, ARL$\approx$ 20. These values can be used for discussion on the robustness of each of the methods. For the most part, each of the methods adequately achieve the desired ARL values for the null case, $\delta = 0$, which is located in the first row of Tables B.1 through B.24 in Appendix B.

We begin by discussing some of the results from the multivariate normal cases. As an example, for the MST method, when exploring the normal cases, with $\alpha = 0.05$, the farthest ARL from 20, is ARL= 18.987, in the $p = 3$, and $m_0 = 100$ case shown in Table B.7. Similar conclusions of nearly achieve the optimal ARL can be made for both the DFEWMA and EWAEL methods. The MSS results are interesting in that they seem to improve as p increases. For the $\alpha = 0.05$, $m_0 = 50$, $p = 2$ case, the ARL=18.695 in Table B.1, however, from Tables B.11 and B.15 when $\alpha = 0.05$, $m_0 = 100$, $p = 4$ and 5, the ARL = 20.012 and 20.025, respectively.

One interesting point is that most of the ARL values for $\delta = 0$ is less than the optimal level. The explanation for this could lie in the fact the RL was cut at a level of 5 × optimal ARL. However, the one method that does not follow this pattern is the SREWMA method, for example when $\alpha = 0.05$, $m_0 = 50$, $p = 2$, ARL=22.765.

Overall, for the $\delta = 0$, normal cases, the MSS method seems to be better than the other methods at achieving the desired ARL. One drawback of the MSS method is that it has high sd(RL) values, such as the case where $\alpha = 0.01$, $m_0 = 50$, $p = 4$, the sd(RL)=132.864. In terms of sd(RL) results, each of the DFEWMA, EWAEL, and MST methods tend to be lower, and thus they perform better; however, in general all five methods are subject to high variability in RL results.

For the other distributions, uniform and t when $\delta = 0$, the desired ARL is not always achieved, perhaps bringing into question the truly distribution-free nature of certain methods. The SREWMA method is often higher than desired, such as the uniform case, $\alpha = 0.05$, $p = 2$ with ARL=32.428 in Table B.17, or the t distribution $\alpha = 0.01$, $p = 2$ with ARL=114.899, from Table B.22. The other methods tend to fall below the desired ARL, as is the case for the t distribution, $\alpha = 0.01$, $p = 2$ which is evident in Figure 3.3, and seen in Table B.22, the ARL values are: ARL = 84.917, 85.076, 114.899, 88.404, and 91.044 for the DFEWMA, EWAEL, SREWMA, MST, and MSS methods respectively. It is interesting to note that, when p is increased from 2 to 5, the ARL values for the $\delta = 0$ seem to get closer to the optimal values. From Figure 3.4, specifically the $\alpha = 0.01$ plots, we see the ARL values are closer to 100 in the $p = 5$ than the $p = 2$ case. From the results for $\delta = 0$, we have seen that they nearly achieve optimal values.

Now, a more detailed comparison of the detection capabilities of the five methods is performed on simulated data with various size δ shifts to the distribution in an attempt to explore which methods may indeed be better. The quicker the detection of a shift of size δ to the distribution, indicates a superior method. The results are very consistent, with the detection of the MST method apparently the superior method having the smallest ARL values for small, moderate, and large magnitude δ shifts to the distribution.

For the case of small shifts to the distribution of $\delta = 0.5$, the MST outperforms the other methods, the DFEWMA and EWAEL methods both perform well, and the SREWMA and MSS method lack in detection capabilities. Take for example Figure 3.2, the bottom left graph in particular, or from Table B.15 with multivariate normal distribution $\alpha = 0.05$, $m_0 = 100$, $p = 5$. When $p = 2$, the DFEWMA results are closer to the EWAEL and MST method results when $m_0 = 50$ than for $m_0 = 100$, for example, see Figure 3.1, or in Tables B.1 and B.3 for the $\alpha = 0.05$ case, when $\delta = 0.5$, $m_0 = 50$. Compared to the MST and EWAEL ARL results, we see that the DFEWMA method is performing better for $m_0 = 50$ than when $m_0 = 100$, while the SREWMA and MSS methods perform poorly in both instances. The five methods DFEWMA, EWAEL, SREWMA, MST, and MSS produce the results for ARL = 8.628, 6.594, 17.797, 4.736, and 17.791, and for $m_0 = 100$: ARL = 10.658, 6.877, 15.669, 4.663, and 18.459.

The SREWMA and MSS method results portray some interesting behaviour throughout the simulation study. The better of the two methods seems contingent on the different values of α considered. The SREWMA method is better when $\alpha = 0.05$, and the MSS method is better when $\alpha = 0.01$. Take the case, for example, when $\alpha = 0.05$, $m_0 = 100$, $p = 2$, $\delta = 4$. The results for the DFEWMA, SREWMA, and MSS methods are ARL=7.653, 8.157, 12.001, compare against $\alpha = 0.01$, $m_0 = 100$, $p = 5$, ARL=12.948, 39.312, 21.542. We mention the DFEWMA as a benchmark to compare the ARL results relative to the other methods.

For many of the cases, the EWAEL outperforms the DFEWMA method, as when $\alpha = 0.05$, $m_0 = 100$, $p = 2$, from Figure 3.1, or Table B.3, for the methods DFEWMA and EWAEL when $\delta = 0.5$, ARL = 10.658 and 6.877 and when $\delta = 4$, ARL = 7.653 and 3.605. However, as p increases, the DFEWMA outperforms the EWAEL method as in the case $\alpha = 0.01$, $m_0 = 50$, $p = 5$, from Figure 3.2 or Table B.14, the two methods DFEWMA and EWAEL for the two cases of $\delta = 0.5$, achieve values of the ARL = 17.747, 20.465 and for $\delta = 4$, ARL = 8.218, 10.675, respectively. In general, as the δ shift increase, the MSS and SREWMA methods ‘catch up’ to the other methods, seen in Figure 3.2, or in Table B.16 such as $\alpha = 0.01$, $m_0 = 100$, $p = 5$, summarized by comparing the results from $\delta = 0.5$ against those of $\delta = 4$. We notice the ARL values get much closer together, between for instance, the DFEWMA and MSS or SREWMA, and as usual the MST method provides the best detection capability.

For the non-normal cases, the inconsistent results are a good indication which methods may lack in robustness. This was indicated earlier, as the SREWMA method fails to achieve the desired ARL values. For the other methods, the increase in the p dimension improves the DFEWMA results, but the EWAEL method does provide better detection in all but one of the cases, in t distribution $\alpha = 0.05$, $p = 5$, when $\delta = 0.5$ as shown in Table B.23, or in Figure 3.4, the bottom left plot. However, the EWAEL performs better than the DFEWMA method in the $\delta = 2$ and larger cases, a pattern not seen in the multivariate normal cases. If we look at the t distribution cases for $\alpha = 0.01$, $p = 5$, and $\alpha = 0.05$, $p = 2$, the fact that the EWAEL performs better is again evident in Figure 3.4, in particular the top left plot. For an overall ranking of the methods based on robust distribution results, from better to worst, would be MST, EWAEL, DFEWMA, SREWMA, and MSS. The best synopsis of this conclusion can be drawn from the uniform $\alpha = 0.05$, $p = 5$, $\delta = 1, 3$ cases, from Figure 3.3, the bottom left plot, and also the corresponding Table B.19. Clearly, the MST method performs better in different magnitudes of δ shifts, and the SREWMA and MSS lag behind in the smaller shifts, but do improve in the larger shift cases. Unlike the normal cases, the MSS method do not do as good of a job in the robust cases of improving over the SREWMA method, exemplified by the case of the uniform distribution with $\alpha = 0.01$, $p = 2$, $\delta = 2, 3, 4$ from Figure 3.3, the top right plot where as in the normal cases for $\alpha = 0.01$, the MSS method achieved smaller ARL values than did the SREWMA method.

The superiority of the MST method is also seen in terms of the sd(RL) values. In the multivariate normal case, $\alpha = 0.05$, $m_0 = 100$, $p = 2$, $\delta = 4$ case for the DFEWMA, EWAEL, SREWMA, MST, and MSS the sd(RL)s are 4.006, 1.289, 3.259, 0.486, and 2.869, respectively. Even in the null case, when the sd(RL) are relatively closer to one another, we see a big jump in the results from the $\delta = 0$ to the $\delta = 0.5$, as in the case of $\alpha = 0.01$, $m_0 = 100$, $p = 3$, from Table B.8. Also, consider the non-normal case of the t distribution $\alpha = 0.01$, $p = 2$, in Table B.22.

Figure 3.1: **ARL values of $N_2(\mathbf{0}, \mathbb{I})$ cases**

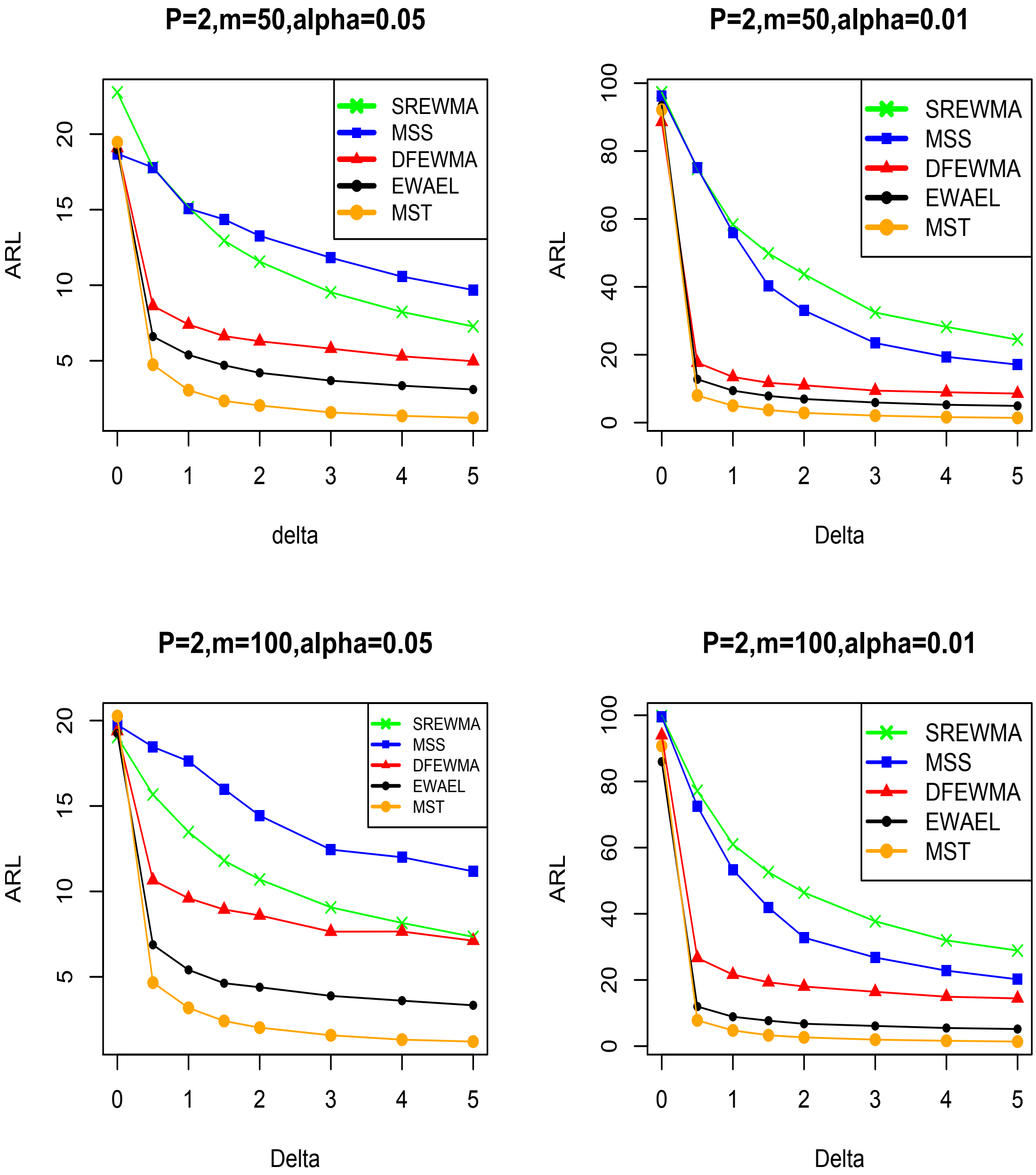


Figure 3.2: **ARL values of** $N_5(\mathbf{0}, \mathbb{I})$ **cases**

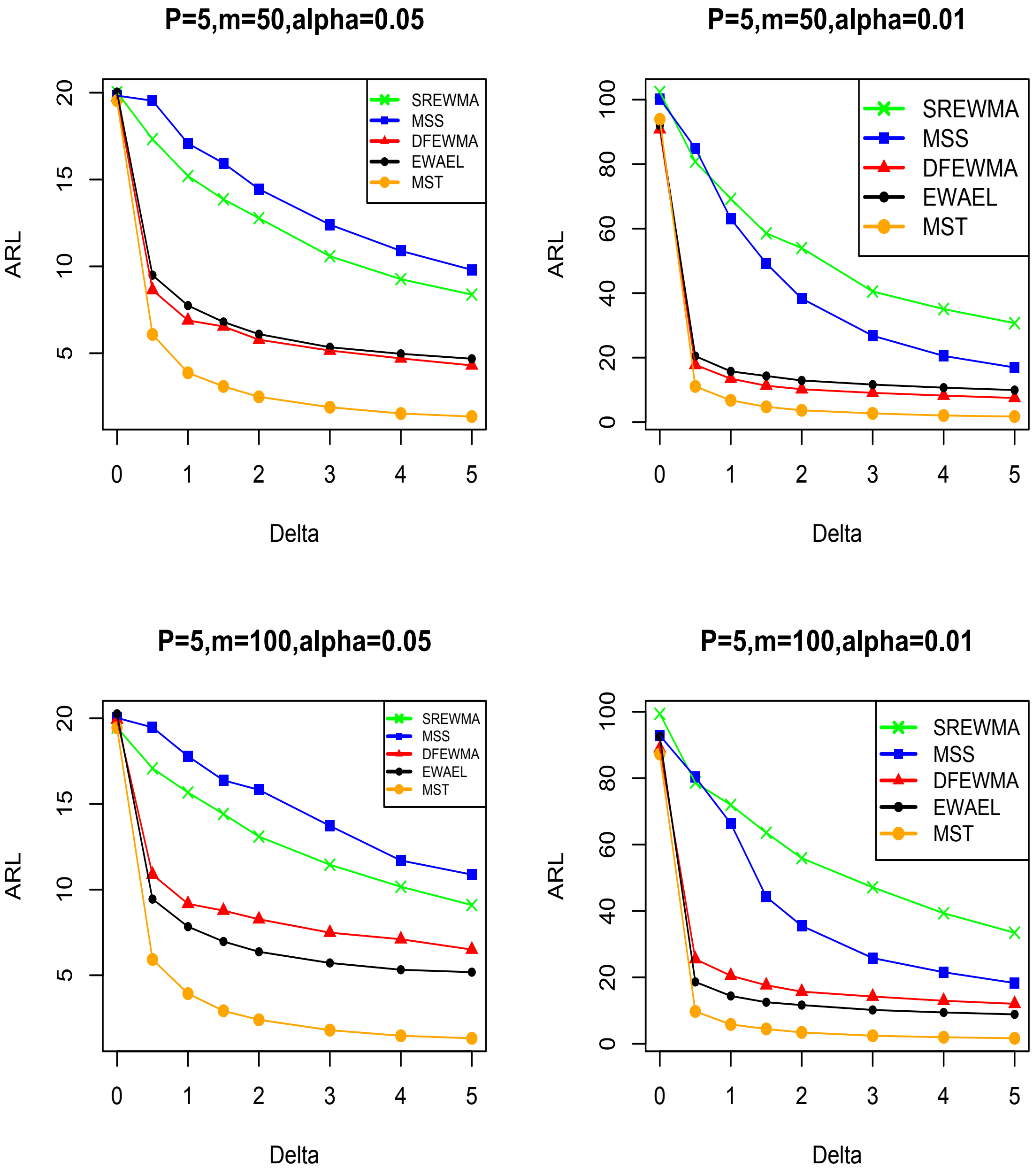


Figure 3.3: **ARL values for** $m_0 = 100$, **for p=2,** $X_1 \sim N(0,1)$, **and** $X_2 \sim$ **Uniform**$(-\sqrt{3},\sqrt{3})$, **and p=5,** $X_1, X_2, X_3 \sim N_3(\mathbf{0}, \mathbb{I})$**;** $X_4, X_5 \sim$ **Uniform**$(-\sqrt{3},\sqrt{3})$

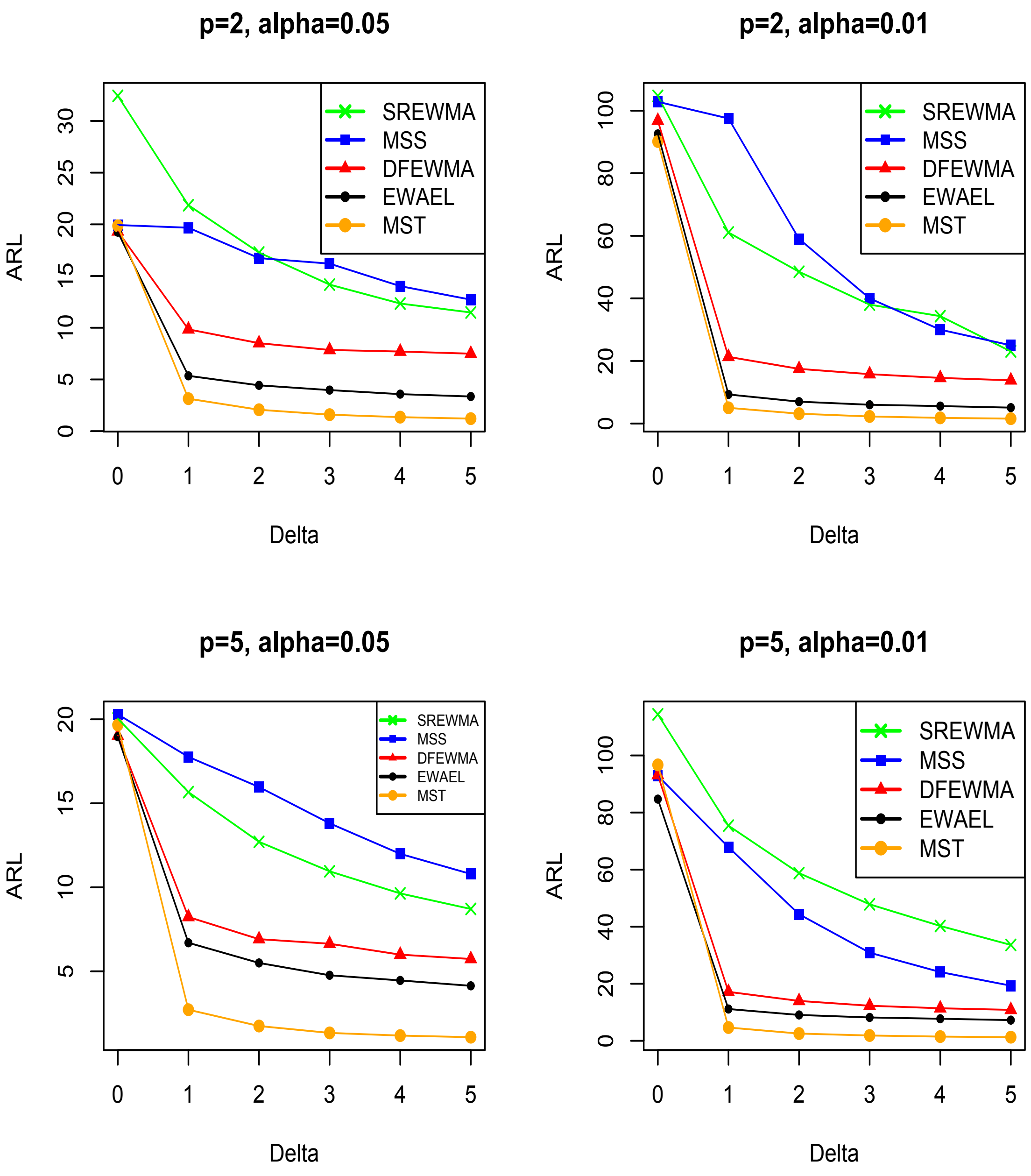
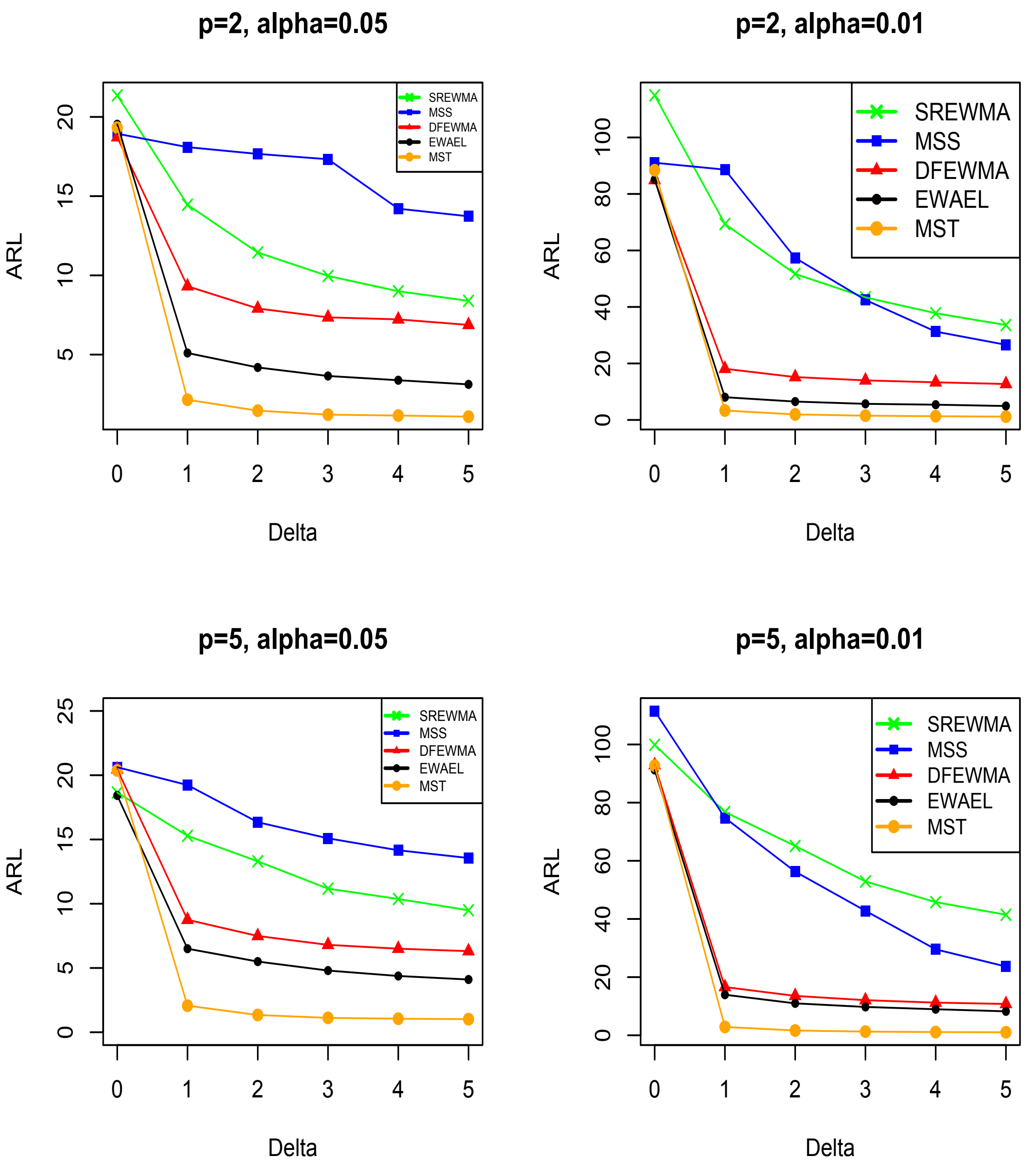


Figure 3.4: **ARL values for $m_0 = 100$, for p=2, $X_1 \sim N(0,1)$, and $X_2 \sim t_3$, and p=5, $X_1, X_2, X_3 \sim N_3(\mathbf{0}, \mathbb{I})$; $X_4, X_5 \sim t_3$**



3.3 Robustness

In this section, we analyze the robustness of the methods. In practice, cut-off values may be difficult to attain, so we check whether the cut-off values depend on the distribution of the data. If the cut-off values are not affected by the different distribution assumptions that they are calculated based on, this shows the method to be robust.

3.3.1 Cut-off value analysis

In this section, we analyze the robustness for two cases where the data is generated by $X_1, X_2, X_3 \sim N_3(\mathbf{0}, \mathbb{I})$; $X_4, X_5 \sim$ both t_3 and Uniform$(-\sqrt{3}, \sqrt{3})$ distribution cases for $m_0 = 100$ and $\alpha = 0.05$. First, we compare the $n = 100$ cut-off values of the test statistics that were calculated for each of the five methods with those calculated using the $N_5(\mathbf{0}, \mathbb{I})$ distribution.

From the cut-off value plots, Figures 3.5 and 3.6, the robust nature and lack of sensitivity to change of the distributions of the MST method is realized in that the cut-off values are very similar for both cases. This point is also true of the DFEWMA method. The EWAEL cut-off are close to being similar, however this method's cut-off values are more susceptible to fluctuations and subtle variability. The surprising plots are that of the SREWMA and MSS methods, where both seem to have a noticeable shift in the cut-off values, both seem to grow bigger for both non-normal distributions, perhaps due to the heavy-tailed nature of the distributions. This characteristic of different cut-off values of the SREWMA and MSS methods shows their possible lack of robustness and sensitivity to the distribution from which they are calculated.

Figure 3.5: **Cut-off value comparison for five methods** $m_0 = 100$**, for p=5,** $N_5(\mathbf{0}, \mathbb{I})$**, and** $X_1, X_2, X_3 \sim N_3(\mathbf{0}, \mathbb{I})$**;** $X_4, X_5 \sim t_3$

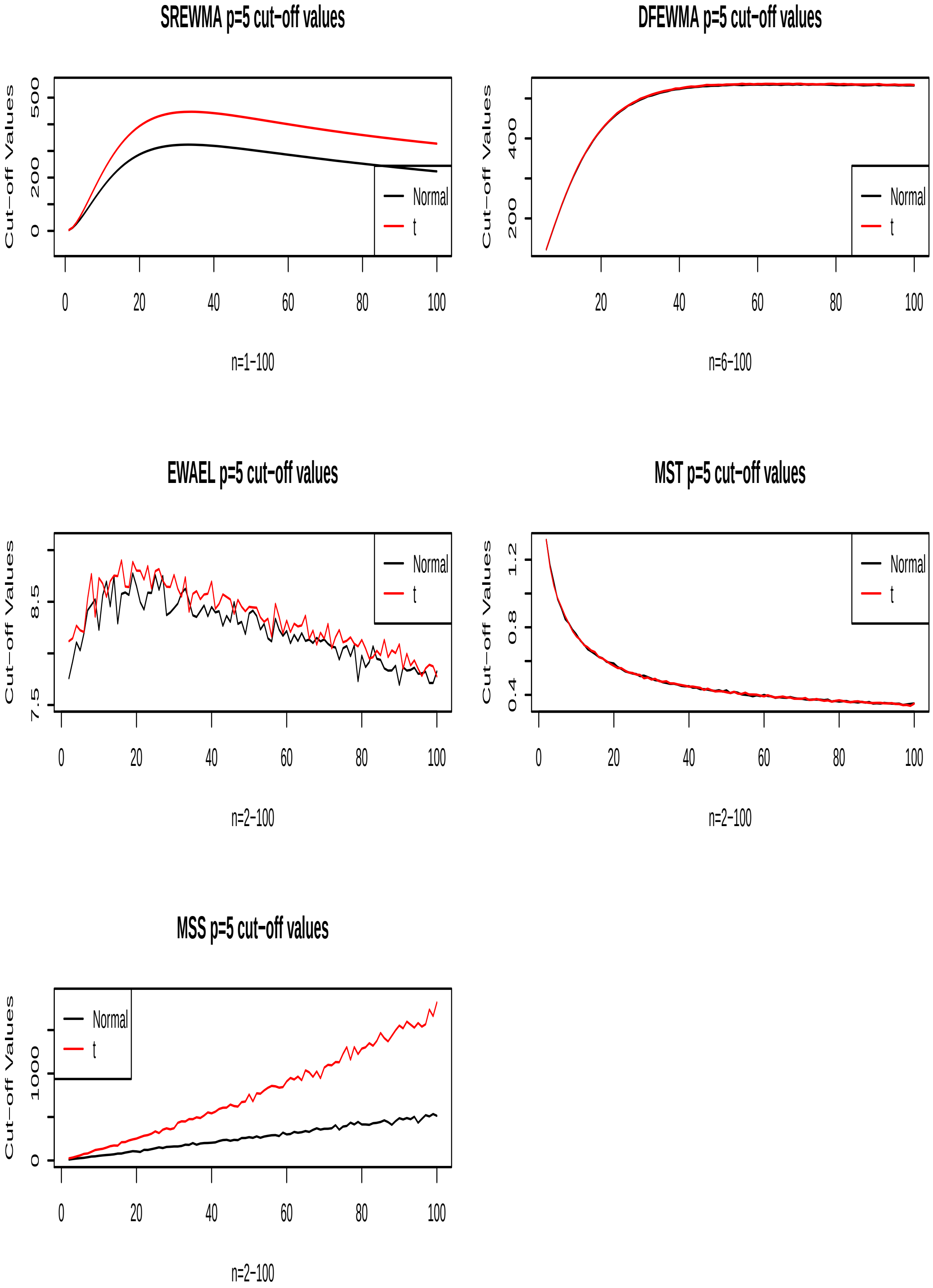
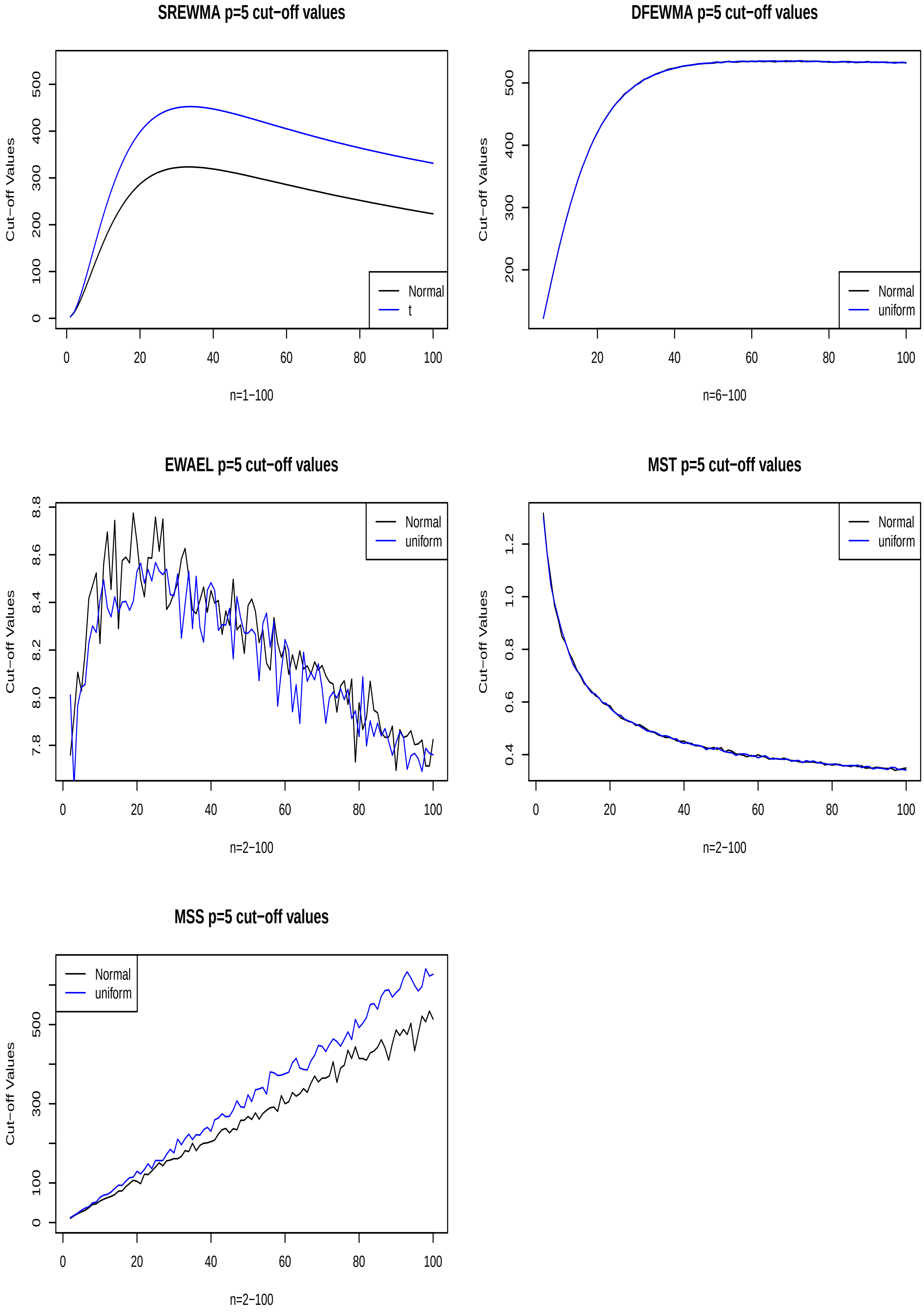


Figure 3.6: **Cut-off value comparison for five methods** $m_0 = 100$**, for p=5,** $N_5(\mathbf{0}, \mathbb{I})$**, and** $X_1, X_2, X_3 \sim N_3(\mathbf{0}, \mathbb{I})$**;** $X_4, X_5 \sim$ **Uniform**$(-\sqrt{3}, \sqrt{3})$

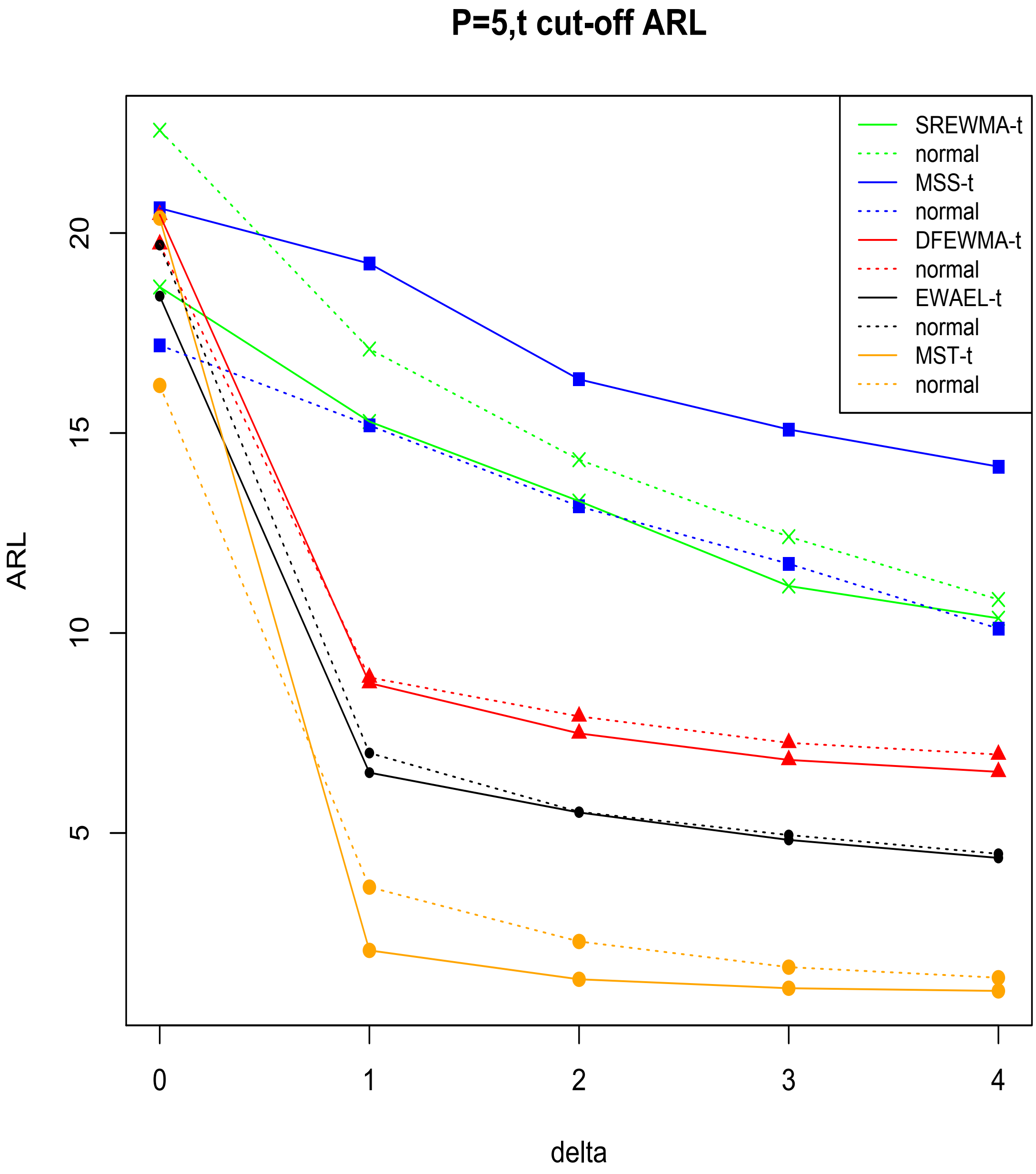


3.3.2 ARL results

Next, we consider the case $X_1, X_2, X_3 \sim N_3(\mathbf{0}, \mathbb{I})$; $X_4, X_5 \sim t_3$ to compare the ARL values for shifts of $\delta = 0, 1$, and 2, when using its own cut-off values, and when using the $N_5(\mathbf{0}, \mathbb{I})$ cut-off values. One point that makes them so surprising is the fact the ARL values do not reflect the different distributions. This is most likely a result of the fact when the data does detect an OC it is well beyond the 95% cut-off limit which we are studying in this analysis.

In terms of the ARL results, for the methods to be truly distribution-free, there should be little change in the ARL values for whichever cut-off values are chosen. As we see, both the DFEWMA and EWAEL achieve very similar results, regardless of the cut-off values used, suggesting that the test statistics that are calculated for these two methods may be robust with regards to the distribution. The MSS and MST methods are below the values. The fact that the ARL=16.258 for the MST method when $\delta = 0$ draws into question the robustness of the MST method in terms of its cut-off values, and also for the shifted data, the detection capabilities becomes poorer. In contrast, when calculating the relevant type I error $\alpha = 1/ARL = 1/16.258 = 0.0615$ is not an unreasonable value. The SREWMA method has ARL values consistently above that for when the data is run for the t distribution. These results are summarized in Table B.25, or Figure 3.7.

Figure 3.7: **ARL comparison for cut-off values of $m_0 = 100$, for p=5,** $X_1, X_2, X_3 \sim N_3(\mathbf{0}, \mathbb{I})$**;** $X_4, X_5 \sim t_3$



Chapter 4

Real Data Application

In this chapter, we present a comparison of the five methods using two real data examples, a manufacturing process of acceleration data, and data from a production process regarding wine quality.

4.1 Wine Quality Data

The dataset included of the white wine production process is studied in the MST and MSS papers. The white wine data set is publicly available in the UC Irvine Machine Learning Repository (ref. http://archive.ics.uci.edu/ml/datasets/Wine+Quality). The dataset includes 11 continuous variables; however, as in the papers that proposed the MST and MSS methods, the three most relevant variables for quality were analyzed: residual sugars, sulfates, and alcohol. One further variable in the dataset is a categorical one ranging from 0 to 10 quality levels, 0 as the worst, and 10 as the best. Cortez et al. (2009) note that as more quality ratings are recorded, online monitoring is required to maintain high standards of quality. As previous analyses of this data set established, a quality rating of level 7 is considered IC data, and a level 5 rating is OC. The original data included 4898 observations, including 880 observations as a level 7, and 1457 observations as a level 5.

Our approach for analysis was similar to that of attaining the results from the simulation study section. A bootstrap technique was applied, in that obtaining our $m_0 = 100$ phase I observations, we sampled with replacement from the 880 level 7 observations. To accommodate the SREWMA method, and create continuous type data, error terms on the order of 10^{-3} were added to avoid ties.

In the null case, $n = 100$ were similarly sampled with replacement from the level 7 observations, whereas for the shifted data $n = 100$ observations were sampled with replacement from the level 5 observations. As expected from the Table 4.1, each of the five methods achieve the desired $ARL \approx 20$, given the level of significance $\alpha = 0.05$. In terms of the analysis of the shifted distribution data, each of the methods do show a considerable leap in their respective results. The DFEWMA, SREWMA, and MST methods each show the expected behaviour of having $ARL < 20$, and the MST is the superior method in terms of quick detection, while the DFEWMA and SREWMA methods report similar ARL values to one another. As for the EWAEL and the MSS methods, each do detect a change in the distribution; however, their ARL values are larger than 20, which is possibly a result of the method's lack of robustness in non-normal cases, or caused by an inability to assign order in a multivariate dimensional setting. The fact that all the ARL results for the shifted results are different than 20 is a good indication that each of the five methods detect that a shift has occurred in the distribution of the three variables.

Table 4.1: Wine Data Set Performance Analysis

Methods	Null Case			
DFEWMA	EWAEL	SREWMA	MST	MSS
19.269(28.948)	19.848(42.272)	18.914(13.290)	20.080(34.510)	21.530(11.823)
		Shifted Data		
16.987(30.236)	22.237(45.859)	17.350(19.882)	4.204(5.352)	39.180(35.887)

4.2 Acceleration Data

The second data set we consider explores a manufacturing process from an electronic company in the city of St. John's, NL, Canada as studied by Variyath and Vattathoor (2013). Due to confidentiality, we cannot provide further details on the manufacturing process. The dataset includes 105 observations of 3 axial components of acceleration measured by an accelerometer on a e-compass unit fixed on the objects, and within the 105 observations, it is known that a shift has taken place.

The phase I IC size was set to $m_0 = 50$, and the remaining $n = 55$ represents the phase II OC observations. For this discussion, consider Figures 4.1 to 4.5, where each of the MST, EWAEL, and MSS charts indicate a change by an upward trend around the $n = 20$ to 25 range. In this range, the MST begins trending towards OC although remains IC, and the EWAEL method does detect an OC condition. The

MSS chart detects an OC condition earlier than the other methods, and remains OC for the duration of the data. These results may be deceiving as the chart does not indicate a noticeable mean shift until after the $n = 40$ to 45 observation. The strength of detection after this point is convincing for each of the MSS and MST methods, and even the the DFEWMA does show a definite change in its behaviour. However, it does not go OC until the end of the $n = 55$ OC observations. An argument could be made of the MST superiority given its behaviour in detecting a mean shift around $n = 30$ to 40, where the EWAEL went very much IC through these time points, the MST began to detect OC conditions. A comment on the SREWMA method, is although it does not ever indicate OC conditions, it does get close throughout observational time points $n = 40$ to 50.

Figure 4.1: **DFEWMA Control Chart For the Acceleration Data set**

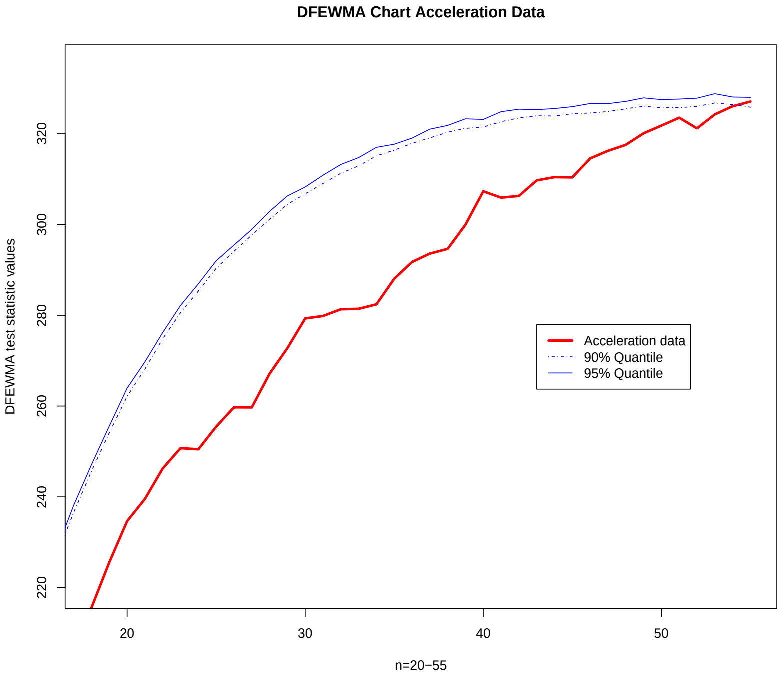


Figure 4.2: **EWAEL Control Chart For the Acceleration Data set**

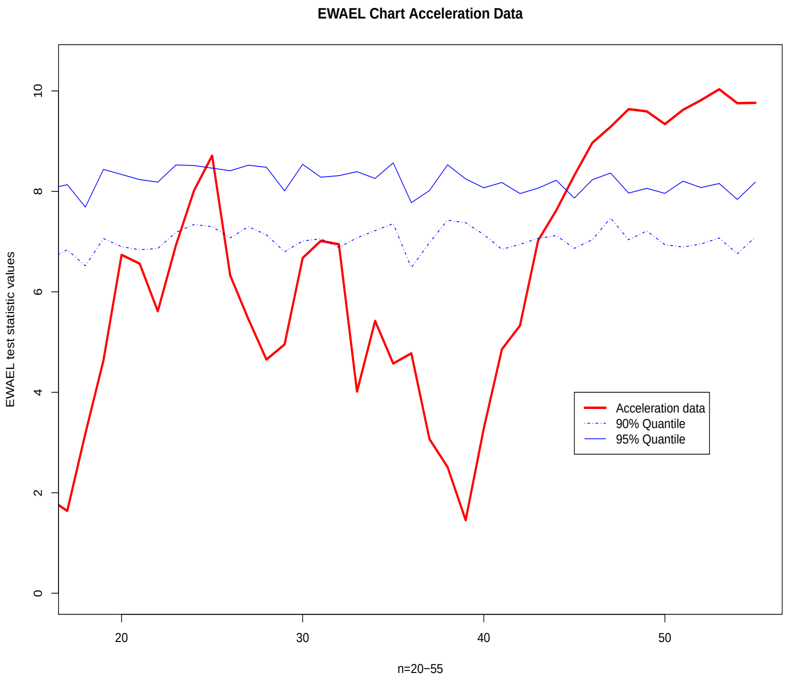


Figure 4.3: **SREWMA Control Chart For the Acceleration Data set**

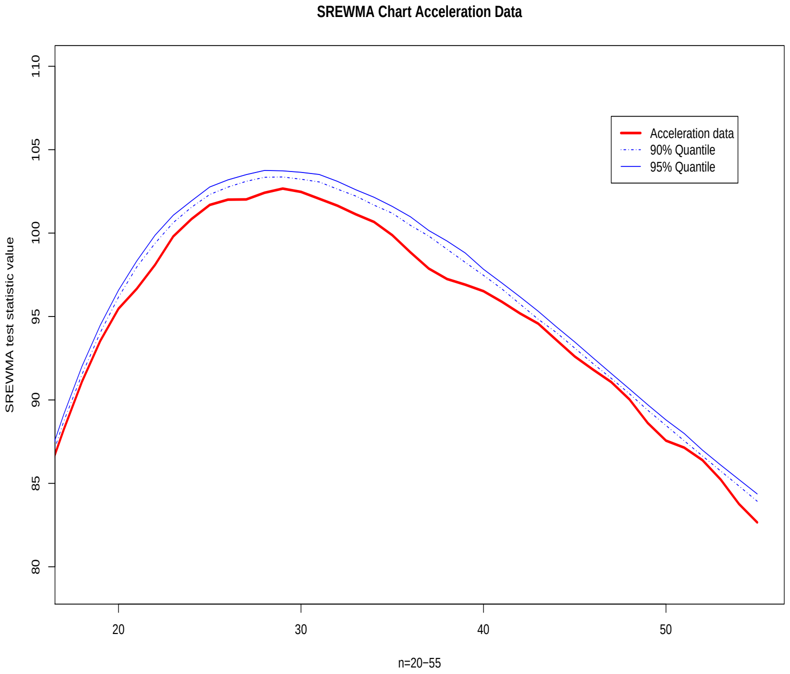


Figure 4.4: **MST Control Chart For the Acceleration Data set**

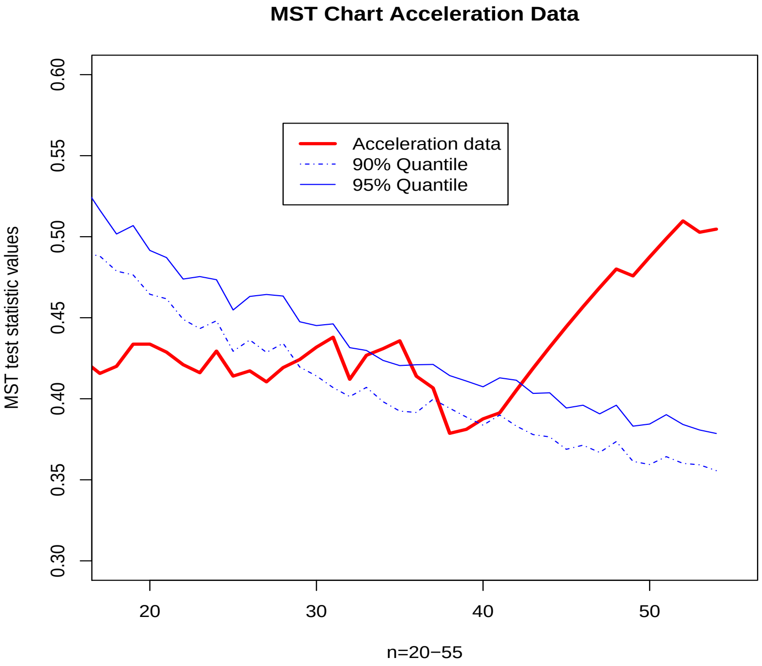
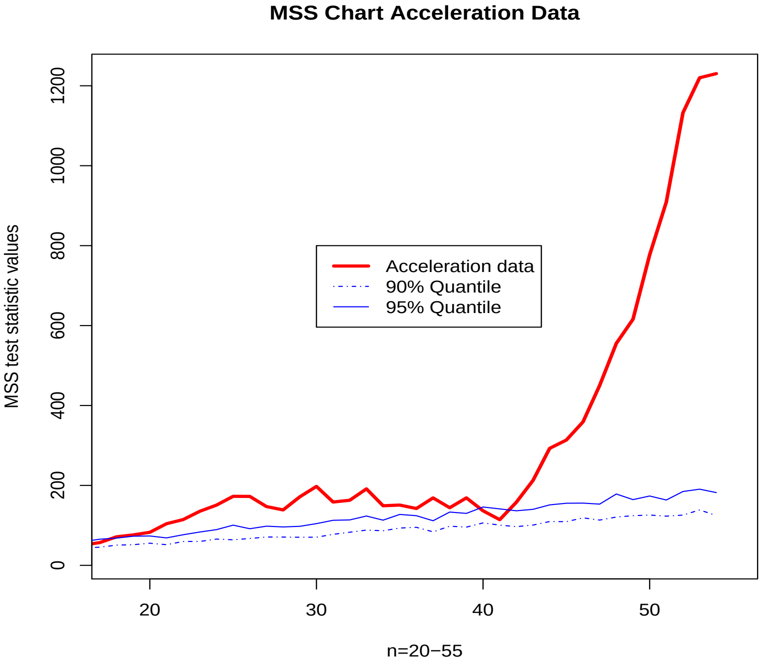


Figure 4.5: **MSS Control Chart For the Acceleration Data set**



Chapter 5

Summary and Conclusions

The science of QC has undergone much advancement over the past century, and the idea of QC developed into SPC, and the creation of control charts. Until recently, SPC's main area of application was in the manufacturing industry, and as is well-known, many manufacturing processes require that several process characteristics are monitored simultaneously. For the most part, when applied in a manufacturing setting, MSPC was acceptable under the multivariate normal assumption, such as with the Hotelling T^2 Control Chart. However, as MSPC expanded into more non-traditional areas, such as finance, insurance, and medicine, the idea that the data must be normally distributed was no longer reasonable. This fact has necessitated the search for an alternative, generally accepted approach, which in turn has led to the growth in recent literature of nonparametric techniques.

The importance of nonparametric methods in the study of multivariate control charts is in their robustness, as they are sensitive to small shifts of different sizes within skewed or heavy-tailed distributions. Nonparametric methods that are self-starting schemes can monitor a process online and check for OC conditions in real time. As a result they avoid lengthy data-gathering steps and can be applied at the start-up and in short-run processes where the reference sample size is small, and the dimension of the data is large.

Creating methods that are able to simultaneously monitor changes to the location and covariance structure continues to be a problem. The DFEWMA and MST methods do attempt to overcome these difficulties, in that they tend to analyze distributional changes rather than a single parameter, evidenced by their results showing sensitivity to small distributional shifts. Even in the MSS method, which measures a single parameter, it uses the median rather than the mean, as it is known to be more robust with respect to the RL distribution. From our results in the simulation study, the MST method seems to be the far superior method, in terms of ARL and sd(RL) performance analysis. The main evidence contradicting this is from the robust results, and the fact that the MST method falls off from the optimum $\delta = 0$ ARL ≈ 20 value. Furthermore, the DFEWMA and EWAEL both show favourable detection capabilities given the distributional cases studied. The MSS method, even though it performs poorly in small shifts to the distribution in the simulation study, seems to be a viable method when it comes to working with real world data.

Finally, one important area for current research effort is the search for more computationally efficient algorithms, whether through new statistical methods, or rather improving old ones. Further research could be devoted to perhaps combining certain theories from current methods studied here, such as working with the Smirnov distance, Wilcoxon rank-sum, spatial ranks and signs, empirical likelihood theory, and also goodness-of-fit methods. One potential approach could consider how the Spearman Rank Correlation theory can be applied to a multivariate setting, and investigate the relationship between the variables in terms of positive and negative correlation.

Chapter 6

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Appendix A

Cut-off values

Table A.1: DFEWMA Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.05$, $m_0 = 50$

	p variables					*p* variables			
n	2	3	4	5	n	2	3	4	5
6	52.69	77.93	102.99	129.06	54	219.91	328.61	435.65	543.65
7	64.99	96.31	127.78	159.03	55	220.50	328.35	436.37	543.78
8	76.97	114.06	151.21	188.13	56	220.33	327.97	436.19	544.40
9	88.52	131.75	174.50	217.58	57	219.98	328.07	436.07	544.86
10	99.66	148.20	197.15	244.86	58	219.88	328.54	435.78	544.10
11	110.11	163.79	217.51	271.69	59	220.26	327.99	435.95	543.25
12	120.25	178.58	237.59	295.82	60	219.65	328.05	436.20	543.91
13	129.15	192.31	255.59	318.53	61	220.43	328.02	436.03	543.67
14	138.00	205.60	272.67	340.02	62	219.75	328.22	435.31	544.07
15	145.80	217.32	288.59	360.06	63	219.84	327.94	436.33	544.01
16	153.00	227.53	302.75	377.86	64	220.13	327.98	435.86	542.92
17	159.69	238.50	316.69	394.27	65	219.99	327.53	435.81	543.30
18	166.08	247.01	328.15	409.78	66	219.69	327.54	435.58	542.82
19	171.60	255.66	338.86	423.51	67	219.75	327.83	434.63	543.00
20	176.09	263.14	349.76	436.07	68	219.98	327.91	434.76	542.65
21	181.53	270.28	358.96	447.71	69	219.40	327.70	434.85	542.33
22	185.25	276.73	367.12	457.55	70	219.65	327.45	434.69	542.64
23	189.14	282.18	374.70	468.06	71	219.75	326.88	435.32	542.11
24	192.51	287.14	381.03	475.03	72	219.82	327.03	434.49	542.45
25	195.74	292.01	387.70	483.09	73	219.93	326.32	434.63	542.47
26	198.74	295.78	392.37	490.58	74	219.50	326.83	434.24	542.25
27	200.68	299.34	398.19	496.56	75	219.17	326.85	434.52	541.94
28	202.77	302.75	402.96	501.95	76	218.99	326.93	433.80	541.37
29	204.92	305.73	406.08	506.26	77	219.43	327.24	434.01	542.34
30	207.08	308.77	410.49	511.01	78	218.89	326.99	434.07	540.79
31	208.24	311.37	412.85	515.33	79	219.08	326.90	433.38	540.80
32	209.67	313.67	416.31	519.28	80	219.00	326.40	433.84	541.04
33	211.42	315.44	419.32	522.13	81	218.63	326.32	433.12	541.49
34	212.48	317.15	420.90	525.44	82	218.66	326.54	433.21	540.17
35	213.28	317.97	422.45	527.61	83	218.65	326.25	433.48	540.26
36	214.14	319.68	424.92	530.09	84	218.31	326.37	432.90	540.25
37	215.15	320.29	426.36	531.19	85	218.38	326.10	433.40	540.41
38	215.81	321.89	427.65	532.66	86	218.20	326.31	432.77	540.99
39	217.10	322.34	428.98	534.38	87	218.49	325.88	432.94	539.60
40	216.95	323.56	430.58	536.44	88	218.07	325.45	432.36	540.40
41	217.33	324.19	430.83	537.29	89	218.16	325.19	432.59	539.76
42	217.51	324.64	432.05	538.41	90	218.31	325.40	432.70	539.98
43	218.41	325.34	432.99	538.68	91	218.49	325.75	432.20	539.71
44	218.55	326.25	432.92	540.23	92	218.16	325.60	432.01	539.69
45	218.52	326.20	433.04	541.18	93	218.24	325.15	431.83	538.56
46	218.81	326.24	433.77	542.40	94	217.88	325.39	431.68	538.24
47	219.52	326.92	435.42	542.21	95	218.36	324.76	431.30	538.20
48	219.38	327.26	435.33	542.76	96	218.12	324.76	432.25	538.46
49	219.39	327.12	434.79	542.93	97	217.66	325.59	431.51	538.34
50	219.39	327.72	435.80	543.04	98	217.56	324.68	431.82	538.30
51	219.61	327.45	435.79	543.66	99	217.82	325.50	431.20	537.68
52	220.19	328.13	435.55	543.66	100	217.88	324.68	431.74	537.90
53	220.42	328.27	436.11	544.15					

Table A.2: DFEWMA Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 50$

	p variables					*p* variables			
n	2	3	4	5	n	2	3	4	5
6	53.89	79.09	104.47	130.57	92	221.12	328.60	434.99	543.68
7	66.40	98.27	129.93	160.60	93	221.75	328.76	435.83	542.60
8	78.26	115.75	153.69	190.79	94	220.93	328.49	435.03	542.65
9	89.78	133.74	177.21	220.64	95	221.08	327.97	436.61	542.75
10	101.16	149.91	199.25	247.31	96	220.69	327.71	436.01	542.89
11	111.80	166.15	220.15	274.93	97	219.69	329.00	435.51	542.59
12	122.08	180.41	240.74	298.63	98	220.57	328.51	435.54	542.57
13	130.62	194.84	258.10	322.09	99	220.54	328.96	435.22	543.50
14	140.11	209.13	275.78	343.14	100	220.89	327.66	434.84	543.38
15	147.23	219.72	291.70	363.84	101	220.16	327.61	434.42	541.84
16	155.41	230.07	304.92	381.53	102	220.53	327.19	434.73	541.25
17	162.19	241.71	320.53	397.87	103	220.43	327.60	434.05	541.15
18	168.71	250.11	332.06	412.39	104	220.48	327.33	435.06	541.62
19	173.49	258.94	343.74	426.72	105	219.82	327.04	433.95	541.32
20	178.61	265.94	353.52	439.02	106	219.89	328.46	435.02	541.38
21	184.13	273.79	362.22	451.02	107	220.33	327.41	435.78	541.78
22	187.50	279.29	370.42	460.75	108	219.25	326.71	433.05	541.76
23	191.04	285.72	377.74	472.13	109	220.35	327.06	433.99	541.67
24	195.18	289.76	384.38	479.29	110	219.46	327.40	433.70	541.23
25	198.29	295.03	390.38	487.95	111	220.10	327.29	434.30	539.45
26	200.78	298.41	396.53	495.12	112	219.29	327.43	434.61	541.61
27	203.40	301.67	401.18	501.26	113	220.17	326.99	433.09	539.47
28	205.86	305.84	406.19	505.41	114	219.95	325.76	432.88	541.03
29	207.09	309.28	409.79	512.89	115	220.10	327.00	433.47	540.27
30	209.13	311.40	413.06	515.64	116	219.57	325.73	432.10	539.67
31	210.62	313.98	416.03	518.99	117	219.24	326.89	434.33	540.59
32	212.02	316.63	419.89	524.18	118	219.64	326.33	432.76	538.95
33	213.76	318.09	423.34	526.28	119	219.66	325.88	433.50	540.03
34	214.70	320.89	425.04	529.71	120	219.44	325.45	432.01	538.74
35	216.30	321.26	426.60	531.49	121	219.97	325.82	433.31	539.88
36	215.96	323.08	429.02	533.57	122	218.89	326.28	433.20	540.15
37	217.53	323.85	430.50	534.78	123	218.38	326.93	432.01	539.67
38	218.23	325.41	431.07	538.26	124	219.84	326.52	433.77	540.07
39	220.12	325.86	432.04	538.82	125	219.14	326.17	431.72	538.91
40	220.03	327.29	433.63	540.81	126	219.83	325.95	432.96	539.49
41	220.03	327.77	435.44	541.60	127	219.54	326.28	432.50	537.36
42	219.92	328.66	435.33	542.35	128	219.13	325.68	432.62	538.61
43	221.32	327.70	436.16	543.59	129	218.37	325.65	432.70	539.62
44	221.31	328.91	437.49	544.04	130	218.53	325.33	431.69	538.25
45	221.51	329.58	437.08	544.73	131	219.28	326.81	431.33	538.67
46	221.94	329.98	437.49	546.34	132	218.65	325.61	432.10	537.90
47	221.47	329.35	439.08	546.37	133	218.71	324.85	432.01	537.97
48	221.94	330.67	439.32	546.51	134	219.20	325.62	431.48	538.58
49	222.08	331.14	438.79	547.00	135	218.23	325.25	431.85	538.57
50	222.39	330.72	439.61	546.63	136	218.11	325.73	432.60	536.94
51	222.10	330.06	438.31	548.15	137	218.11	324.88	431.11	537.05
52	222.81	330.90	438.81	547.56	138	218.77	326.34	431.07	539.51
53	222.54	330.72	440.29	548.15	139	218.10	324.87	431.57	538.25
54	222.56	331.73	439.36	548.31	140	218.41	326.04	431.54	537.66
55	223.14	331.07	440.80	547.46	141	218.82	325.89	431.38	537.90
56	222.56	331.14	439.19	549.36	142	217.76	325.45	431.85	537.19
57	222.72	331.58	440.18	549.00	143	217.84	325.08	431.16	536.85
58	222.27	331.64	439.57	548.23	144	218.36	325.52	431.12	537.43
59	222.50	331.38	439.72	547.81	145	218.81	325.06	431.05	537.56
60	222.82	331.66	440.91	548.30	146	217.73	325.70	431.40	537.02
61	222.54	331.17	439.53	548.51	147	218.71	324.53	431.47	537.05
62	222.16	331.09	440.72	548.62	148	218.41	326.28	431.02	536.50
63	223.03	331.14	440.48	549.05	149	217.54	324.93	431.80	535.79
64	222.74	330.94	440.01	547.76	150	218.25	325.01	429.31	536.08
65	222.44	329.90	439.57	548.32	151	218.68	323.77	430.40	536.35
66	222.55	330.38	439.72	547.79	152	218.37	324.70	430.64	536.16
67	223.01	331.22	438.65	547.42	153	218.37	323.90	431.22	536.34
68	223.11	330.91	438.58	546.41	154	218.52	324.05	431.49	536.09
69	221.57	331.38	438.46	547.11	155	218.24	324.87	430.89	536.14
70	222.08	331.66	439.78	546.97	156	218.11	324.68	430.65	535.83
71	222.26	330.21	438.60	545.90	157	218.13	324.43	431.34	535.37
72	222.79	330.50	439.06	547.77	158	217.88	324.83	430.45	536.27
73	222.00	329.96	438.39	546.02	159	217.17	323.58	429.41	536.44
74	221.90	329.80	436.94	545.82	160	218.20	324.88	429.97	535.72
75	222.25	329.40	438.92	545.87	161	218.60	325.04	431.38	536.28
76	221.87	329.99	437.93	545.86	162	217.85	323.95	430.03	535.54
77	221.49	330.38	436.85	545.62	163	218.37	325.15	430.45	535.60
78	220.85	330.25	437.93	545.02	164	217.97	324.85	429.74	535.87
79	222.08	329.86	436.91	544.40	165	217.90	323.43	429.74	535.64
80	222.68	329.96	437.57	545.20	166	217.90	324.59	430.55	536.88
81	221.20	329.79	436.93	545.97	167	217.80	324.21	431.34	535.31
82	221.86	329.41	435.82	544.68	168	218.20	324.35	430.37	535.48
83	221.73	330.23	438.27	544.25	169	217.78	323.51	430.78	535.99
84	221.08	330.74	436.94	543.97	170	217.69	324.76	430.91	535.00
85	221.40	329.20	436.94	545.30	171	216.90	324.29	429.75	535.26
86	220.47	329.73	435.99	545.49	172	216.97	323.57	429.85	536.21
87	221.03	328.68	436.08	543.21	173	217.74	323.42	429.02	536.05
88	220.85	329.53	436.35	543.74	174	218.06	323.64	430.76	535.58
89	220.65	328.20	436.61	544.67	175	217.83	324.19	429.48	536.06
90	220.99	328.37	435.93	543.99	176	217.92	324.06	430.15	534.68
91	220.99	328.37	435.94	543.77	177	217.87	323.55	429.53	536.88

DFEWMA Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 50$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
178	217.42	322.89	428.98	533.78	264	216.19	321.06	425.14	531.40
179	217.84	323.88	429.49	534.73	265	216.49	321.95	426.06	530.82
180	217.55	323.63	428.70	535.21	266	215.21	321.62	426.49	531.18
181	217.69	324.20	428.62	535.12	267	215.66	320.75	425.61	531.04
182	217.21	323.35	429.66	535.44	268	215.83	321.47	426.91	531.78
183	217.12	323.21	429.37	534.88	269	215.19	321.71	427.07	531.24
184	217.83	323.63	428.25	536.53	270	216.09	320.70	426.44	530.64
185	218.26	324.32	429.24	535.21	271	215.49	322.09	426.80	530.90
186	216.15	322.92	428.96	534.98	272	216.09	321.48	425.80	530.31
187	218.56	323.76	429.12	534.72	273	215.74	321.60	425.48	531.51
188	217.09	322.50	428.24	534.93	274	215.83	321.10	425.70	531.68
189	217.04	322.83	429.01	534.18	275	215.89	321.97	426.81	530.69
190	218.79	323.07	429.25	533.41	276	216.03	322.39	426.79	531.81
191	217.21	323.82	430.25	534.66	277	215.11	320.01	426.10	530.73
192	217.24	323.13	429.43	533.53	278	216.16	320.78	426.52	530.91
193	216.73	323.44	427.87	534.27	279	216.07	321.66	426.73	531.84
194	217.69	323.10	428.20	533.28	280	215.56	321.28	425.06	530.54
195	216.44	324.19	429.23	534.77	281	215.67	321.04	426.09	530.30
196	216.96	323.52	429.14	534.56	282	216.40	320.95	427.08	531.53
197	216.39	323.56	427.87	534.86	283	215.69	321.37	425.73	529.92
198	217.78	322.40	428.34	534.39	284	215.48	320.69	426.21	530.41
199	217.24	323.76	428.90	534.31	285	216.41	319.54	425.31	529.86
200	217.81	322.37	429.46	534.75	286	215.87	320.33	425.63	529.86
201	216.86	322.30	428.55	532.35	287	216.22	322.12	426.37	530.51
202	217.46	323.12	428.24	533.59	288	215.71	321.22	424.56	530.37
203	216.80	321.48	427.78	533.74	289	216.29	320.88	425.84	531.79
204	216.34	323.41	429.02	533.87	290	215.29	321.48	425.30	531.34
205	217.23	323.23	427.47	535.36	291	215.81	320.30	427.07	531.29
206	216.92	323.32	428.91	534.08	292	216.27	321.12	426.32	530.76
207	217.28	323.95	429.78	533.06	293	215.58	320.81	426.19	530.36
208	217.47	322.96	428.58	533.28	294	215.95	320.67	425.84	531.15
209	216.63	321.92	428.51	533.65	295	216.04	320.53	425.28	530.40
210	216.41	322.51	426.85	533.08	296	215.15	321.12	426.72	529.76
211	216.99	322.03	428.51	533.58	297	215.75	320.54	425.44	530.08
212	216.48	323.33	427.93	533.36	298	215.36	320.37	425.11	530.58
213	217.02	323.17	428.93	532.54	299	215.47	321.24	426.04	531.18
214	217.34	322.13	427.94	532.15	300	215.97	320.44	426.45	530.42
215	217.22	322.53	426.51	533.17	301	215.12	321.06	426.78	529.46
216	216.72	322.54	427.27	533.06	302	215.25	321.30	426.90	530.43
217	217.47	322.27	427.62	533.23	303	215.36	320.23	425.06	530.05
218	216.83	322.00	427.72	532.34	304	216.05	321.02	425.42	532.07
219	216.37	322.31	427.86	532.46	305	214.97	321.48	426.08	530.25
220	217.20	322.18	427.64	532.94	306	215.92	321.53	424.69	529.59
221	216.83	322.13	428.28	534.32	307	216.09	320.68	427.02	529.99
222	217.06	322.61	430.14	532.34	308	214.76	319.78	425.32	529.71
223	216.79	322.45	427.46	532.86	309	215.56	321.22	424.88	529.82
224	215.94	321.81	427.11	533.04	310	215.62	321.74	425.31	529.47
225	216.09	321.68	427.54	533.64	311	214.92	320.78	425.47	530.93
226	216.06	321.92	426.40	532.55	312	215.60	321.77	425.85	530.68
227	216.37	323.44	427.58	532.48	313	215.22	320.92	426.04	530.73
228	216.54	322.37	428.68	531.48	314	215.99	320.35	425.63	531.67
229	216.28	322.99	427.62	533.80	315	215.84	320.22	425.68	530.01
230	216.58	322.04	427.03	533.94	316	215.81	321.40	425.07	528.80
231	215.92	322.49	427.26	532.13	317	215.08	319.85	426.15	530.15
232	216.22	321.24	426.99	531.91	318	215.60	320.76	425.88	528.91
233	216.43	321.44	428.24	532.00	319	215.27	320.47	426.56	529.43
234	215.98	321.60	427.90	531.52	320	216.72	319.84	424.42	528.73
235	216.63	321.75	426.98	533.11	321	214.96	320.31	425.71	528.53
236	216.37	322.21	426.97	532.68	322	215.18	321.49	426.01	530.32
237	216.36	323.28	427.64	532.16	323	215.88	319.77	424.28	529.73
238	216.08	321.94	427.22	531.70	324	214.79	321.69	424.32	530.41
239	216.09	322.48	427.72	531.87	325	216.06	320.30	425.03	530.39
240	216.34	320.48	426.55	531.50	326	216.46	320.18	424.43	529.65
241	216.06	321.36	427.52	533.67	327	215.41	319.65	424.24	529.92
242	216.45	322.17	426.84	532.23	328	215.23	320.92	424.46	528.90
243	216.18	321.48	427.63	531.45	329	215.99	320.50	426.35	530.40
244	216.24	321.22	428.02	532.84	330	215.26	320.58	424.37	528.69
245	216.20	322.99	428.13	530.93	331	216.00	320.67	425.12	529.48
246	215.97	321.54	426.26	531.78	332	214.87	320.76	424.70	528.44
247	216.31	321.76	427.26	530.76	333	215.16	320.39	425.49	529.88
248	216.05	321.42	427.14	533.30	334	215.59	320.33	424.23	528.73
249	215.99	321.15	427.12	531.43	335	214.85	320.04	424.94	529.96
250	217.01	321.60	426.94	532.29	336	215.28	320.59	423.74	528.95
251	215.76	321.26	425.57	532.27	337	215.40	321.42	424.95	529.28
252	215.79	321.89	428.42	531.73	338	215.14	319.80	426.36	529.10
253	216.75	322.37	427.68	532.71	339	215.25	320.54	425.13	529.08
254	216.07	322.17	425.48	530.98	340	215.16	320.41	425.80	530.37
255	216.78	321.77	426.57	531.69	341	215.40	320.88	424.17	529.39
256	215.98	321.78	426.42	532.18	342	215.93	320.54	424.51	529.78
257	216.13	321.77	426.43	531.04	343	216.32	320.80	424.34	529.43
258	216.44	322.51	426.83	531.36	344	215.60	320.40	423.36	529.59
259	216.03	322.23	427.07	533.13	345	214.66	319.83	425.06	528.75
260	217.08	321.84	426.71	531.55	346	214.58	320.84	425.78	530.67
261	216.95	321.13	427.06	531.97	347	215.40	319.73	424.01	529.31
262	215.80	321.30	425.80	532.36	348	214.62	320.58	424.74	529.39
263	216.92	322.10	426.26	530.49	349	215.41	320.73	424.17	530.13

DFEWMA Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 50$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
350	214.91	320.58	424.76	530.31	426	215.02	319.96	424.24	527.62
351	215.60	320.59	427.03	529.59	427	213.90	319.34	423.51	527.74
352	215.38	320.08	424.62	529.69	428	213.94	318.41	423.55	527.48
353	214.59	320.24	425.33	529.20	429	214.92	318.97	423.75	528.72
354	214.59	320.62	425.23	528.76	430	215.22	320.32	423.51	528.73
355	216.37	319.94	425.01	528.54	431	214.58	320.15	423.90	528.52
356	215.70	319.84	424.72	529.21	432	215.89	319.27	423.41	527.13
357	215.33	321.50	424.96	528.59	433	214.01	319.56	423.85	527.35
358	215.45	320.30	425.37	529.66	434	214.80	319.57	422.58	527.69
359	216.76	320.38	424.66	527.96	435	214.63	318.72	423.16	527.17
360	214.67	320.15	424.16	528.49	436	214.42	319.21	423.01	526.77
361	215.90	320.34	425.29	529.69	437	214.09	319.38	423.02	528.92
362	215.82	320.03	424.68	530.28	438	215.03	319.56	423.69	526.27
363	215.16	320.85	424.89	531.20	439	214.57	319.07	425.15	527.44
364	214.74	319.79	425.35	530.79	440	214.91	319.80	423.12	527.95
365	214.79	320.53	425.54	528.19	441	214.89	319.47	423.49	526.57
366	215.18	320.45	424.35	530.19	442	214.17	319.39	422.31	528.24
367	214.19	319.49	424.00	530.18	443	214.89	320.31	422.77	528.65
368	215.44	319.56	423.63	528.20	444	214.58	318.96	423.47	527.89
369	214.23	319.54	423.78	528.53	445	214.48	319.21	423.42	527.50
370	214.46	321.31	424.79	528.03	446	214.40	319.56	424.61	527.83
371	215.47	319.54	423.87	529.21	447	214.39	318.92	423.16	528.88
372	215.14	320.32	426.14	529.34	448	214.82	318.64	424.53	528.14
373	215.03	320.06	424.88	528.85	449	214.99	318.95	423.36	527.01
374	215.34	319.80	424.29	530.10	450	214.03	319.72	423.38	527.16
375	214.57	319.93	425.54	528.04	451	214.75	319.84	422.98	526.91
376	215.17	319.39	426.18	528.22	452	213.51	318.96	423.58	528.22
377	214.72	319.90	424.55	527.68	453	214.91	319.65	424.30	526.83
378	214.82	320.01	424.68	528.88	454	214.08	320.20	424.11	528.79
379	215.05	319.25	424.99	529.61	455	214.57	319.09	424.34	527.05
380	215.31	319.53	425.27	528.56	456	215.10	318.90	423.82	528.69
381	215.58	319.72	424.18	529.36	457	214.93	319.61	422.29	527.40
382	214.93	320.20	423.76	529.04	458	215.70	318.69	422.20	528.66
383	215.31	319.34	424.00	527.75	459	214.49	318.89	424.71	527.60
384	214.60	320.14	424.35	527.98	460	214.58	319.31	423.79	527.16
385	215.13	319.73	424.14	529.27	461	214.01	320.27	423.18	527.32
386	214.74	319.38	424.56	528.70	462	214.84	319.69	424.15	527.71
387	215.04	319.49	424.28	529.24	463	214.18	319.29	422.90	527.60
388	215.09	320.93	424.57	529.83	464	214.29	319.20	423.25	526.99
389	215.18	320.26	424.93	527.36	465	214.09	319.25	425.42	527.85
390	214.59	319.87	424.90	527.19	466	214.48	319.21	422.17	525.92
391	215.14	319.12	424.99	528.61	467	214.34	319.44	423.28	527.58
392	214.78	319.78	424.66	529.10	468	215.31	320.83	424.11	527.24
393	215.29	320.12	424.09	526.72	469	214.20	320.70	423.49	528.00
394	214.37	320.21	425.30	527.92	470	214.18	318.20	424.65	527.75
395	214.72	320.38	423.80	529.45	471	214.41	319.32	422.43	527.54
396	215.39	320.41	424.02	529.29	472	215.12	319.17	423.63	527.76
397	215.06	319.44	424.69	529.21	473	214.56	319.22	422.55	526.48
398	215.08	320.39	423.14	529.05	474	214.92	318.50	424.25	528.57
399	214.44	319.71	423.60	528.03	475	214.65	319.73	423.67	526.97
400	214.20	319.11	424.93	529.00	476	214.78	318.59	423.11	526.43
401	214.91	319.70	423.66	527.59	477	214.22	319.32	423.68	527.07
402	215.03	319.07	425.92	530.48	478	213.60	318.70	422.95	527.97
403	214.19	320.22	424.08	529.96	479	215.10	319.67	423.22	528.17
404	215.54	318.62	423.75	526.95	480	214.08	318.43	424.41	526.61
405	214.82	319.70	423.95	527.87	481	214.49	319.19	423.66	526.73
406	214.21	318.91	424.83	527.74	482	214.75	319.61	422.90	527.44
407	215.02	318.82	424.52	528.52	483	213.88	318.80	422.62	527.40
408	214.16	318.96	424.77	527.85	484	214.46	318.95	423.54	526.85
409	214.33	319.90	423.72	527.39	485	213.88	319.87	423.57	527.62
410	215.11	319.62	424.95	528.47	486	214.45	318.54	422.37	527.37
411	214.94	318.80	423.72	528.40	487	214.23	318.80	424.53	526.24
412	215.74	319.82	424.09	527.72	488	214.87	318.49	423.53	526.28
413	215.24	319.73	424.16	528.55	489	214.58	319.52	423.00	526.68
414	214.19	320.35	423.61	527.36	490	214.60	318.84	423.36	526.19
415	214.53	318.87	423.69	527.18	491	214.29	319.59	423.13	528.06
416	214.37	318.78	424.04	529.46	492	213.95	319.31	422.86	528.05
417	215.09	320.09	424.50	526.59	493	213.53	318.49	422.87	527.37
418	215.07	320.40	424.96	527.11	494	214.51	319.73	422.65	526.97
419	214.86	319.29	423.92	528.53	495	214.96	318.83	424.72	526.54
420	214.93	320.15	423.81	528.29	496	214.65	318.50	422.73	526.36
421	214.68	319.52	424.23	528.08	497	214.50	318.16	422.92	526.03
422	214.32	321.13	423.68	527.97	498	214.87	318.07	422.97	528.34
423	214.75	319.54	424.02	528.09	499	214.44	319.10	422.74	527.57
424	214.19	318.88	423.45	528.75	500	214.20	319.26	423.42	527.80
425	215.59	320.46	424.26	529.48					

Table A.3: DFEWMA Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.05$, $m_0 = 100$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
6	50.08	74.17	98.41	122.15	54	216.06	322.24	427.59	534.44
7	61.65	91.59	121.15	150.97	55	216.19	322.51	428.67	533.83
8	73.30	108.82	144.33	179.46	56	216.09	322.32	427.89	533.36
9	84.41	125.90	166.21	207.32	57	215.94	322.12	429.15	534.17
10	94.96	141.57	188.49	234.13	58	215.98	322.02	428.50	534.44
11	105.31	156.88	208.03	259.09	59	215.54	322.01	428.53	534.42
12	114.72	171.03	227.04	283.23	60	216.09	323.41	428.33	534.62
13	123.97	184.30	244.69	305.24	61	216.23	322.85	428.91	534.28
14	132.10	196.78	262.10	325.42	62	216.51	322.74	429.27	534.90
15	139.96	208.64	276.33	345.31	63	216.59	322.49	429.16	534.25
16	146.79	219.20	291.58	363.39	64	216.37	323.03	429.15	534.79
17	153.43	229.53	304.11	378.69	65	216.72	322.79	428.73	534.75
18	159.56	237.96	315.81	394.47	66	216.46	322.67	428.58	533.84
19	165.55	246.31	326.89	408.37	67	216.72	322.98	428.80	534.90
20	170.78	254.31	337.17	420.63	68	216.61	322.29	428.96	535.01
21	174.78	260.73	346.29	432.12	69	216.73	322.29	428.47	534.25
22	179.44	267.33	355.45	442.40	70	216.48	322.91	428.85	535.32
23	182.81	272.52	362.75	451.60	71	216.59	323.07	428.93	534.51
24	186.19	278.21	369.05	460.34	72	216.25	322.42	428.59	535.33
25	189.45	283.07	375.41	468.05	73	216.87	322.99	428.90	534.26
26	192.26	287.02	380.00	474.82	74	216.18	323.27	428.83	534.94
27	195.20	290.82	385.86	482.41	75	215.74	322.26	428.26	534.79
28	197.37	294.28	390.50	486.46	76	215.88	322.67	429.07	534.93
29	199.13	296.84	394.81	491.89	77	216.22	322.69	428.37	534.77
30	200.84	299.92	398.40	496.80	78	216.34	322.60	428.30	534.40
31	203.03	301.91	401.93	500.89	79	216.46	322.13	428.69	534.05
32	204.12	304.91	404.83	505.38	80	216.32	322.89	428.85	533.39
33	205.74	306.56	407.29	507.56	81	215.87	322.40	428.08	533.70
34	207.07	308.55	409.87	510.80	82	215.61	322.05	428.16	533.28
35	208.17	310.16	411.91	514.00	83	215.75	321.52	427.93	533.81
36	209.09	311.92	414.13	516.29	84	215.80	323.00	427.85	534.07
37	210.17	312.86	415.38	518.26	85	215.95	322.46	428.02	534.39
38	210.52	314.16	417.34	521.13	86	216.09	321.92	427.65	533.66
39	210.93	315.32	419.06	522.83	87	215.76	321.88	428.07	532.73
40	212.32	316.36	420.18	523.62	88	216.17	322.09	427.62	533.15
41	212.42	317.00	420.99	525.33	89	215.65	322.11	428.00	533.29
42	213.04	318.53	422.40	526.81	90	215.61	322.09	427.98	534.39
43	213.31	318.75	423.18	527.53	91	215.31	322.11	428.12	532.99
44	213.75	319.37	423.78	528.69	92	215.90	322.07	427.71	533.65
45	213.82	318.77	425.08	529.32	93	215.69	321.97	427.67	533.09
46	214.37	320.45	425.21	530.63	94	215.95	321.17	427.70	533.30
47	215.17	320.36	425.41	530.71	95	215.58	321.42	427.24	533.34
48	214.98	320.53	425.86	531.46	96	215.52	321.24	426.90	532.60
49	215.12	321.37	426.89	531.76	97	215.63	321.36	426.79	533.06
50	215.42	321.29	426.96	531.82	98	215.49	321.62	427.22	532.59
51	216.14	321.32	427.15	532.95	99	215.53	321.39	427.44	532.63
52	215.76	321.13	427.06	533.02	100	215.89	321.01	427.53	532.45
53	215.71	321.90	427.89	533.71					

Table A.4: DFEWMA Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 100$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
n=6	51.27	75.46	100.35	124.01	92	219.15	325.69	432.05	536.38
7	63.07	93.27	123.26	153.86	93	218.26	324.78	431.75	538.02
8	75.17	110.64	146.26	181.87	94	218.55	324.87	432.93	536.34
9	86.38	127.82	168.79	209.35	95	217.65	324.79	431.32	537.83
10	97.31	143.33	189.82	236.05	96	218.74	324.86	430.46	537.58
11	107.06	159.13	209.84	260.76	97	218.30	325.27	430.29	537.31
12	116.49	173.17	231.17	286.06	98	218.21	325.18	432.35	536.40
13	126.24	187.73	247.18	308.14	99	217.88	324.34	432.39	539.25
14	133.96	199.45	263.87	329.42	100	218.57	324.85	430.00	536.52
15	142.28	211.51	279.89	348.33	101	217.67	323.98	431.41	537.84
16	149.03	222.73	294.16	365.82	102	217.63	325.79	430.17	536.23
17	155.82	231.91	308.71	382.47	103	218.92	324.63	430.09	537.35
18	162.64	241.09	320.11	399.90	104	217.73	325.21	431.29	535.91
19	168.04	249.43	329.60	412.23	105	218.35	324.93	431.16	537.99
20	173.68	257.11	340.81	424.22	106	218.62	324.12	430.24	537.34
21	177.07	264.17	350.27	436.03	107	217.95	324.49	432.06	536.48
22	182.16	270.51	358.77	445.76	108	217.83	325.16	431.55	534.99
23	185.04	275.33	365.63	456.54	109	218.56	325.30	429.91	535.14
24	189.79	281.26	372.77	464.98	110	218.12	324.26	430.83	535.72
25	192.11	285.65	378.64	472.16	111	218.77	324.56	430.39	536.71
26	195.84	289.38	384.04	479.25	112	218.94	324.49	430.32	536.48
27	198.58	293.79	389.08	486.47	113	217.96	324.35	429.26	536.09
28	199.70	297.86	393.17	490.86	114	218.30	324.39	430.55	536.13
29	201.92	300.07	397.89	497.03	115	217.73	323.42	429.97	535.75
30	204.05	303.23	401.90	500.00	116	217.11	323.80	431.21	535.49
31	205.68	306.11	406.12	504.21	117	218.06	323.01	430.21	535.55
32	206.80	307.22	408.28	508.84	118	216.91	324.66	430.20	535.99
33	207.33	309.53	410.53	512.72	119	217.92	323.09	430.46	535.13
34	209.26	310.90	413.46	516.42	120	218.09	325.37	430.20	534.49
35	210.35	313.15	416.65	518.43	121	217.97	324.01	429.81	535.99
36	211.68	314.49	418.62	519.98	122	217.75	324.37	429.88	536.85
37	212.48	315.93	420.14	522.84	123	217.41	324.35	430.29	535.40
38	213.07	316.93	421.30	525.58	124	217.71	323.01	429.51	535.98
39	213.59	317.98	422.52	526.64	125	218.25	323.86	430.79	534.13
40	215.41	319.70	422.84	528.25	126	218.06	324.30	428.35	535.66
41	215.24	320.32	425.18	531.20	127	217.62	323.29	429.60	535.79
42	215.37	321.81	425.91	528.66	128	217.62	323.20	429.29	535.10
43	216.02	321.74	426.83	531.70	129	217.53	324.73	429.24	534.81
44	216.72	321.66	428.02	532.03	130	216.74	322.93	429.06	534.91
45	216.81	322.83	429.44	533.37	131	216.78	322.87	428.76	534.71
46	216.85	322.53	428.78	534.38	132	216.75	323.45	429.82	535.05
47	218.04	323.39	428.91	534.40	133	217.18	322.94	428.33	534.37
48	217.41	323.51	430.18	536.08	134	217.38	323.29	428.63	535.57
49	218.57	324.25	429.70	536.25	135	217.54	323.07	427.96	535.04
50	218.76	325.02	430.30	536.92	136	216.82	323.33	429.28	535.59
51	219.08	325.37	430.90	537.11	137	217.45	324.30	427.99	534.36
52	218.95	324.91	432.13	537.06	138	217.10	322.87	429.57	534.19
53	218.36	324.94	431.42	537.82	139	217.32	323.58	428.81	535.99
54	218.89	325.32	433.68	538.37	140	217.49	322.99	428.04	534.41
55	219.00	325.58	431.80	539.02	141	217.69	323.14	427.87	534.97
56	218.79	326.14	433.58	538.99	142	216.60	323.73	429.43	534.99
57	218.09	325.94	433.28	539.96	143	217.30	324.70	427.96	534.37
58	217.95	325.91	432.97	537.37	144	217.18	323.20	429.24	535.82
59	218.09	326.18	433.33	539.18	145	218.03	322.66	428.79	535.17
60	219.28	325.51	433.61	539.42	146	216.80	323.24	428.74	534.03
61	218.61	326.60	433.33	538.82	147	217.03	322.81	428.15	534.45
62	218.54	325.69	432.91	538.61	148	217.10	322.11	426.91	534.12
63	219.97	325.66	432.92	539.45	149	217.17	322.97	428.90	533.97
64	218.56	326.10	431.48	538.49	150	217.04	321.94	427.75	534.44
65	219.31	327.09	432.52	539.12	151	217.53	322.13	428.74	533.96
66	218.84	324.99	433.03	537.88	152	217.10	321.86	428.67	533.93
67	218.73	327.27	431.63	538.82	153	217.03	323.55	427.73	534.37
68	220.39	326.43	432.72	538.46	154	217.26	322.76	427.45	533.76
69	219.80	326.51	433.85	540.38	155	217.52	322.76	429.11	533.46
70	219.20	326.14	432.47	539.05	156	216.08	321.90	429.52	532.55
71	220.07	326.61	431.44	539.95	157	218.06	322.77	428.38	533.50
72	219.09	326.72	432.83	538.89	158	216.81	322.41	428.67	532.25
73	219.69	326.25	433.03	538.94	159	217.25	322.64	429.17	533.32
74	219.68	325.93	433.57	538.37	160	216.16	322.32	427.64	533.14
75	218.20	326.97	433.07	541.03	161	215.96	322.69	428.20	534.01
76	219.43	326.80	432.07	537.81	162	216.84	321.60	428.32	533.38
77	218.55	326.40	432.43	538.91	163	216.38	322.53	428.79	531.77
78	219.69	326.13	432.99	538.98	164	217.10	323.49	427.17	532.89
79	219.05	325.61	432.29	539.19	165	216.62	322.11	427.82	533.73
80	218.47	325.76	431.92	537.60	166	216.95	322.80	427.30	532.97
81	219.57	324.82	433.51	537.77	167	216.43	323.24	428.14	533.62
82	218.37	325.91	431.49	538.82	168	216.74	323.22	427.87	533.00
83	218.65	326.07	432.48	537.69	169	216.10	322.43	428.54	532.47
84	218.44	324.92	432.63	539.23	170	217.42	321.50	428.54	533.85
85	219.20	325.81	431.48	536.59	171	216.01	322.38	427.24	533.05
86	218.25	325.66	431.76	538.79	172	217.26	322.39	428.83	531.25
87	219.26	325.80	430.57	537.89	173	216.90	322.27	427.29	533.66
88	218.69	325.88	431.71	538.28	174	217.47	322.25	427.84	532.95
89	217.93	325.51	430.17	537.29	175	217.69	322.18	427.20	532.74
90	218.83	324.64	432.17	538.34	176	217.05	322.42	427.55	533.21
91	217.97	325.27	431.81	538.06	177	216.56	323.24	426.90	531.69

DFEWMA Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 100$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
178	216.50	321.66	427.96	532.71	264	214.92	320.38	425.07	530.31
179	216.98	322.58	427.70	532.82	265	215.84	320.16	424.66	530.07
180	216.62	323.77	428.85	532.81	266	216.41	320.49	424.89	529.31
181	216.16	322.96	427.96	532.83	267	215.06	320.40	425.68	529.68
182	216.66	321.05	427.35	530.98	268	215.06	320.35	425.17	530.71
183	216.59	322.95	428.23	532.03	269	215.70	321.64	426.33	529.27
184	215.84	322.46	427.48	533.02	270	215.25	319.75	425.58	531.22
185	216.49	322.26	428.04	533.52	271	215.33	321.06	425.98	529.53
186	215.94	322.24	428.21	531.51	272	215.72	321.56	425.61	531.23
187	215.92	322.47	428.20	531.02	273	215.26	321.51	426.00	529.75
188	216.61	322.44	427.40	531.97	274	214.79	320.27	425.13	530.03
189	216.74	321.94	427.54	531.91	275	215.31	320.25	425.34	529.59
190	216.69	321.79	427.96	532.62	276	215.22	321.32	426.05	529.55
191	216.54	323.09	426.30	530.86	277	215.47	320.13	425.72	529.41
192	216.29	321.48	428.73	531.46	278	214.87	321.61	425.08	529.02
193	215.98	322.28	426.91	535.10	279	215.22	320.66	424.00	529.44
194	215.31	322.05	426.36	531.68	280	215.00	320.18	425.06	529.90
195	217.15	322.05	426.68	531.46	281	214.66	320.24	424.02	529.79
196	215.81	321.34	426.92	532.84	282	215.66	320.81	425.79	529.47
197	216.04	320.70	425.87	532.86	283	216.34	319.60	424.60	529.85
198	216.08	323.01	426.78	532.09	284	215.76	320.80	425.77	528.74
199	216.32	321.58	426.79	532.84	285	215.02	320.38	426.07	528.54
200	215.30	321.91	427.35	532.03	286	215.53	320.27	424.72	528.02
201	215.55	321.95	427.29	531.86	287	215.49	319.92	424.65	528.45
202	216.00	321.43	427.67	530.94	288	215.01	319.69	423.34	530.49
203	216.60	321.10	427.25	531.36	289	216.38	320.21	425.22	530.67
204	216.41	320.61	426.11	531.15	290	215.24	320.53	424.12	528.61
205	215.89	321.95	426.53	530.92	291	215.58	319.79	424.75	529.79
206	215.74	321.10	426.32	532.21	292	215.55	320.16	424.10	529.33
207	216.28	321.92	425.84	532.76	293	215.13	320.42	425.35	529.55
208	216.13	322.86	426.15	531.62	294	215.86	321.05	426.26	528.49
209	215.98	322.40	427.12	531.73	295	215.32	320.46	425.23	529.16
210	216.22	321.71	426.08	531.61	296	216.46	320.23	426.62	529.86
211	215.56	322.63	426.22	531.60	297	215.22	320.47	425.33	529.23
212	216.54	321.59	426.51	532.12	298	215.30	320.80	424.90	529.27
213	215.25	321.16	426.26	531.50	299	215.79	320.22	423.79	530.97
214	216.65	320.89	428.13	530.86	300	215.10	319.83	424.37	529.81
215	216.50	322.08	426.74	531.95	301	216.06	319.96	425.45	529.94
216	216.10	321.62	426.15	530.45	302	214.68	319.54	424.18	529.10
217	215.81	321.69	428.43	531.78	303	215.94	320.69	425.81	528.87
218	215.91	320.13	426.31	531.55	304	214.79	320.97	424.71	527.84
219	216.12	321.12	425.51	531.48	305	215.34	319.96	425.70	530.18
220	216.21	321.47	425.04	532.19	306	216.08	320.49	425.10	529.93
221	216.48	321.13	426.84	530.83	307	215.84	318.74	423.87	529.72
222	216.15	321.75	427.17	531.20	308	215.13	321.30	424.49	528.77
223	216.02	321.60	427.12	532.36	309	215.10	320.70	424.40	528.83
224	215.22	321.45	426.63	530.89	310	215.00	319.90	423.42	529.50
225	216.14	320.27	426.22	531.67	311	215.47	321.24	425.09	529.02
226	216.18	321.76	425.60	529.84	312	214.35	320.19	424.56	527.68
227	216.55	322.28	424.34	531.89	313	214.98	319.38	425.67	528.52
228	216.58	320.81	427.62	531.84	314	215.60	319.61	424.11	529.07
229	215.40	320.81	425.84	529.96	315	215.21	319.63	424.20	528.69
230	215.92	321.05	425.66	531.18	316	214.48	319.72	426.31	528.86
231	216.61	320.50	425.79	532.05	317	215.74	320.60	424.89	528.59
232	215.72	321.32	426.53	532.43	318	215.63	320.70	424.42	529.31
233	216.22	320.86	426.54	530.77	319	215.47	319.86	425.11	529.92
234	216.03	321.51	426.80	529.65	320	215.04	320.50	424.58	528.66
235	216.07	320.57	425.45	530.17	321	214.60	320.10	424.64	528.55
236	215.14	321.25	426.60	529.78	322	214.08	319.87	425.29	529.33
237	216.46	319.83	424.66	531.40	323	215.06	321.65	424.38	527.95
238	215.99	321.22	425.54	531.19	324	214.47	319.20	425.81	529.12
239	215.90	320.00	425.37	530.75	325	215.22	320.57	424.24	528.74
240	215.48	320.02	425.72	531.37	326	215.43	320.04	424.96	528.30
241	216.04	320.87	425.99	531.21	327	214.94	320.01	425.25	528.58
242	215.42	320.98	425.84	531.79	328	215.82	320.03	424.83	528.72
243	215.14	321.23	426.84	530.09	329	215.47	320.35	424.26	529.41
244	214.38	320.11	426.20	529.60	330	215.28	319.88	424.67	529.49
245	216.17	320.28	425.35	531.11	331	214.86	318.89	424.71	529.29
246	215.36	321.29	425.45	530.10	332	215.48	319.45	424.93	528.38
247	215.48	321.01	425.83	529.75	333	214.79	319.21	424.67	527.43
248	215.51	321.08	425.86	531.12	334	216.27	320.07	424.67	529.18
249	216.57	321.50	425.23	529.06	335	215.21	320.30	424.46	527.63
250	216.23	321.64	426.10	530.25	336	215.28	319.79	423.95	528.95
251	216.70	321.61	426.10	530.73	337	215.01	319.78	425.43	526.89
252	216.08	320.97	425.14	529.68	338	215.29	319.35	424.00	529.31
253	215.96	321.09	425.98	529.88	339	215.97	318.81	424.32	529.12
254	216.24	320.70	425.41	530.88	340	215.34	319.91	425.32	529.10
255	215.64	320.17	425.27	530.54	341	215.16	319.79	425.05	529.07
256	216.48	320.10	426.29	529.17	342	215.50	319.87	424.37	527.89
257	215.56	321.36	424.74	532.07	343	213.83	319.14	424.89	527.06
258	215.72	321.10	424.93	530.87	344	214.24	319.89	423.69	528.61
259	215.62	321.00	425.58	530.23	345	214.83	320.50	424.29	529.08
260	215.37	320.03	424.25	529.88	346	214.81	319.27	424.48	528.35
261	215.49	320.67	425.08	528.69	347	215.35	319.55	423.79	528.27
262	215.37	321.48	425.99	531.06	348	214.95	320.01	424.44	527.82
263	215.58	321.01	425.37	529.08	349	214.97	319.53	423.88	527.57

DFEWMA Cut-off values $N_p(\mathbf{0},\mathbb{I})$, $\alpha = 0.01$, $m_0 = 100$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
350	214.38	319.72	424.58	527.92	426	215.26	319.20	424.17	527.09
351	214.68	318.91	424.28	528.16	427	215.36	319.03	423.34	527.93
352	215.71	320.01	424.82	528.31	428	214.30	318.97	423.18	526.64
353	214.63	318.85	425.17	528.89	429	214.03	319.27	424.86	526.36
354	215.54	319.90	423.98	528.19	430	213.71	318.66	424.76	527.11
355	215.44	319.09	424.15	528.47	431	214.75	319.06	424.12	526.73
356	214.46	319.16	424.21	527.95	432	213.86	318.67	423.88	528.27
357	214.26	319.66	423.46	528.05	433	214.48	319.48	422.95	527.21
358	214.46	319.95	425.72	528.14	434	215.06	320.21	423.04	526.77
359	215.12	319.96	425.73	527.39	435	214.98	318.68	423.68	527.89
360	215.00	320.61	424.16	528.06	436	214.44	318.50	423.25	527.09
361	214.58	319.34	425.28	529.05	437	214.72	318.74	422.96	526.58
362	214.92	318.57	424.48	527.53	438	214.40	318.61	423.11	527.44
363	215.00	319.78	423.91	528.54	439	214.11	318.91	423.48	528.17
364	214.77	318.32	423.81	528.47	440	214.54	319.51	423.12	526.46
365	214.00	318.65	423.89	528.70	441	213.99	319.81	423.99	528.07
366	215.64	319.47	424.64	527.05	442	213.95	319.01	421.69	526.83
367	215.03	319.19	425.19	528.14	443	214.83	319.25	424.16	526.46
368	215.42	319.56	423.81	529.85	444	213.84	319.41	423.42	526.97
369	214.88	318.80	424.56	528.17	445	215.53	318.81	422.23	527.26
370	214.81	319.42	422.88	530.19	446	214.43	320.29	423.65	525.61
371	214.09	319.12	423.16	527.67	447	214.44	318.76	424.18	526.63
372	215.38	319.07	424.02	528.18	448	213.40	319.30	423.83	527.40
373	214.57	318.93	423.92	528.22	449	215.37	319.26	422.75	528.92
374	214.08	319.67	424.20	527.74	450	214.74	320.15	423.33	526.34
375	215.00	320.13	424.21	527.31	451	214.76	319.27	423.97	526.51
376	213.70	319.79	424.67	527.01	452	214.38	319.15	422.56	527.05
377	214.51	319.45	423.18	528.99	453	214.97	319.03	423.18	526.26
378	214.54	320.00	423.11	528.35	454	214.28	318.98	422.78	527.64
379	214.06	319.17	423.31	528.46	455	214.33	319.97	422.76	526.30
380	214.61	321.02	424.38	527.55	456	213.97	319.24	422.97	525.83
381	215.07	319.92	423.42	527.00	457	214.64	319.34	424.11	527.27
382	214.44	320.47	423.83	527.53	458	215.22	319.62	422.98	526.57
383	214.76	319.20	424.21	527.81	459	214.17	318.22	422.28	528.53
384	214.78	318.77	423.97	527.56	460	214.92	318.44	422.75	527.85
385	214.05	319.14	423.63	527.98	461	214.23	318.21	423.46	527.29
386	214.57	319.50	423.23	527.85	462	214.08	318.87	422.49	527.33
387	215.27	319.51	423.36	527.58	463	214.65	318.31	422.85	525.79
388	214.19	320.11	423.24	527.53	464	214.97	318.46	422.90	527.09
389	214.94	318.42	423.51	526.33	465	213.74	319.11	422.82	526.72
390	215.05	320.13	424.31	527.51	466	214.93	319.33	422.88	525.94
391	215.39	319.18	423.84	528.55	467	214.92	319.31	423.02	526.72
392	214.65	319.31	423.36	527.97	468	213.34	318.20	424.91	526.42
393	213.97	319.49	423.66	528.14	469	214.50	318.38	423.68	527.60
394	215.04	318.99	422.26	528.51	470	213.02	319.02	422.61	527.35
395	214.46	318.98	423.29	527.59	471	214.18	318.53	422.81	526.84
396	214.94	319.05	423.62	528.18	472	214.16	319.56	422.76	527.70
397	215.00	317.68	423.34	527.21	473	213.95	319.10	423.58	526.92
398	214.32	319.27	423.81	529.52	474	213.51	319.39	422.37	525.58
399	214.71	319.17	423.72	527.08	475	214.90	319.55	421.99	527.42
400	214.18	319.37	423.56	527.65	476	214.16	319.29	422.57	526.54
401	215.75	318.84	424.34	527.17	477	214.65	319.03	423.10	527.03
402	214.83	319.20	423.00	527.96	478	214.37	318.37	421.95	526.74
403	214.50	319.21	424.14	528.80	479	214.97	319.33	423.36	526.96
404	214.22	318.70	423.51	528.63	480	213.95	318.76	423.74	526.77
405	214.46	318.95	425.45	527.81	481	213.64	318.80	422.82	526.43
406	214.77	318.33	425.05	526.26	482	214.19	318.58	422.49	527.44
407	215.33	319.41	422.46	527.59	483	214.18	317.81	421.76	526.69
408	215.30	319.76	422.88	527.39	484	214.57	318.85	423.42	526.87
409	214.58	319.19	424.08	527.71	485	213.84	318.99	422.39	526.62
410	214.13	318.62	423.36	527.15	486	214.14	317.33	423.41	526.79
411	214.54	319.29	424.29	527.86	487	214.29	317.79	423.52	527.76
412	214.58	319.36	423.57	526.49	488	214.19	318.30	423.11	525.71
413	214.46	319.16	424.20	528.08	489	214.58	318.40	422.33	527.04
414	214.35	319.26	422.91	527.74	490	214.31	318.97	421.99	525.64
415	214.98	318.94	423.77	528.60	491	214.90	319.72	422.72	527.93
416	214.44	319.30	423.32	528.27	492	214.27	319.91	422.79	525.64
417	215.00	319.10	424.17	526.82	493	214.94	316.78	421.72	526.84
418	214.54	319.39	424.31	527.19	494	213.71	317.82	423.74	526.95
419	215.09	319.86	423.87	527.97	495	215.43	318.59	424.32	525.99
420	214.52	319.36	422.88	526.32	496	214.10	319.62	422.63	526.23
421	214.24	319.29	424.39	527.13	497	213.84	318.47	422.78	526.46
422	214.67	318.82	423.62	526.57	498	214.49	319.66	423.33	526.93
423	214.70	319.29	423.20	528.46	499	215.37	317.69	422.75	527.73
424	215.08	319.14	422.92	527.06	500	214.40	318.95	422.34	526.07
425	215.18	319.35	422.42	526.31					

Table A.5: DFEWMA Cut-off values $\alpha = 0.05$, $m_0 = 100$, for $p = 2$, $X_1 \sim N(0,1)$, $X_2 \sim$ Uniform or t_3, for $p = 5$, $X_1, X_2, X_3 \sim N(0,1)$, $X_4, X_5 \sim$ both Uniform or t_3

	Uniform		t_3			Uniform		t_3	
n	$p = 2$	$p = 5$	$p = 2$	$p = 5$	n	$p = 2$	$p = 5$	$p = 2$	$p = 5$
6	49.91	122.26	50.11	122.97	54	216.41	534.58	215.97	535.16
7	61.73	150.94	61.44	152.08	55	215.63	533.46	216.12	535.41
8	73.12	179.38	73.24	180.86	56	216.82	534.23	216.38	536.70
9	84.28	207.22	84.36	208.10	57	216.25	534.88	216.02	535.76
10	95.35	234.06	95.09	235.50	58	216.3	534.82	216.47	536.32
11	105.31	258.95	105.12	260.64	59	216.46	534.07	215.97	535.34
12	115.11	282.05	114.74	284.07	60	216.9	534.99	216.83	536.19
13	124.12	305.08	123.91	307.04	61	216.21	534.67	216.68	536.00
14	131.96	325.55	132.31	327.94	62	216.71	535.26	215.93	536.56
15	140.09	345.65	140.14	346.83	63	216.21	535.11	216.35	536.49
16	147.14	362.94	147.18	364.18	64	216.48	535.00	216.50	536.42
17	153.61	378.87	153.70	380.66	65	216.8	535.30	216.88	535.80
18	159.67	394.2	159.52	395.74	66	217.03	535.46	216.23	536.44
19	165.13	408.34	165.23	409.24	67	216.45	534.57	216.79	536.44
20	170.28	419.72	170.39	421.67	68	216.62	534.72	216.38	536.62
21	175.08	432.46	175.13	433.24	69	216.54	535.77	216.45	535.85
22	178.93	441.85	178.98	443.43	70	216.85	534.36	215.97	536.61
23	183.03	451.39	182.80	453.09	71	216.19	535.06	216.01	536.68
24	186.46	460.34	186.92	462.59	72	215.99	535.50	216.25	535.45
25	189.45	468.3	189.46	469.59	73	216.44	535.56	216.53	535.48
26	192.00	474.13	192.60	476.52	74	216.29	534.19	216.90	535.72
27	194.76	481.22	194.58	482.71	75	216.62	534.43	216.25	535.13
28	197.59	486.88	197.29	488.68	76	216.41	534.53	215.97	535.60
29	199.02	491.38	199.01	493.49	77	216.16	534.97	215.84	535.34
30	200.94	496.83	200.98	499.27	78	216.39	534.05	216.45	536.42
31	202.86	499.87	202.49	502.72	79	216.52	533.99	216.03	536.69
32	204.78	504.83	204.11	506.46	80	215.69	534.46	216.06	535.82
33	205.44	508.04	205.35	510.09	81	216.14	533.19	215.96	535.29
34	207.06	511.14	206.52	513.11	82	216.28	533.50	216.16	536.07
35	208.81	513.71	208.01	515.70	83	216.56	534.18	216.03	535.20
36	208.88	515.64	208.83	518.30	84	215.5	534.39	216.12	535.58
37	210.42	518.45	209.87	520.10	85	215.77	532.93	216.40	534.75
38	210.91	520.4	210.99	521.85	86	216.24	533.81	215.54	534.90
39	211.03	521.8	211.11	524.81	87	216.05	533.37	216.37	535.22
40	211.98	523.89	211.53	524.92	88	215.74	533.65	215.77	534.79
41	212.05	524.91	213.00	526.90	89	215.53	533.25	215.47	534.98
42	213.22	526.78	212.77	528.52	90	215.61	533.51	215.97	535.13
43	213.51	527.88	213.16	529.83	91	216.09	533.3	215.49	535.94
44	214.32	528.22	213.66	529.45	92	215.65	533.05	216.20	534.26
45	214.51	529.48	213.96	530.79	93	215.5	533.18	215.73	533.78
46	214.67	530.1	214.10	532.04	94	216.09	533.44	215.26	534.15
47	214.7	531.2	214.68	533.84	95	215.9	532.61	215.94	534.80
48	215.27	531.59	214.76	533.38	96	215.59	532.79	215.15	533.89
49	214.91	531.61	215.66	533.61	97	215.97	531.9	215.28	533.95
50	215.37	532.66	215.17	534.12	98	215.26	532.65	216.29	534.32
51	215.58	533.71	215.32	533.84	99	215.81	533.38	216.00	534.38
52	216.36	532.44	215.96	534.84	100	216.05	532.15	215.87	533.69
53	215.89	533.68	216.22	534.90					

Table A.6: DFEWMA Cut-off values $\alpha = 0.01$, $m_0 = 100$, for $p = 2$, $X_1 \sim N(0, 1)$, $X_2 \sim$ Uniform or t_3, for $p = 5$, $X_1, X_2, X_3 \sim N(0, 1)$, $X_4, X_5 \sim$ both Uniform or t_3

	Uniform		t_3			Uniform		t_3	
n	$p = 2$	$p = 5$	$p = 2$	$p = 5$	n	$p = 2$	$p = 5$	$p = 2$	$p = 5$
6	51.24	124.1	51.06	124.89	89	218.43	537.42	218.51	537.86
7	62.95	153.38	62.75	153.65	90	218.58	537.15	219.14	539.76
8	74.57	182.17	74.59	182.81	91	218.21	538.9	218.07	539.79
9	86.02	210.15	85.75	210.67	92	219.36	538.56	218.97	539.48
10	97.03	237.64	96.84	238.84	93	218.34	537.24	218.19	539.32
11	107.02	262.66	106.91	264.89	94	219.16	538.49	217.77	538.61
12	116.74	284.35	116.64	287.31	95	218.48	536.56	219.11	540.35
13	125.9	308.03	125.69	310.26	96	219.2	537.7	217.81	539.84
14	134.57	328.7	134.27	332.25	97	218.92	536.64	218.58	540.16
15	142.16	348.34	141.81	350.6	98	217.48	536.8	218.62	539.54
16	149.28	366.88	149.2	367.95	99	218.71	538.02	219.4	539.68
17	155.94	383.74	155.67	384.32	100	219.73	535.62	219.25	539.43
18	162.06	399.05	161.62	399.04	101	218.57	536.01	217.68	539.08
19	167.55	412.13	168.24	413.91	102	219.05	538.01	218.69	538.1
20	172.65	423.25	172.55	426.18	103	218.38	536.18	217.89	537.84
21	177.89	436.51	177.24	438.45	104	218.45	535.88	217.92	537.85
22	181.71	445.45	182.23	447.14	105	218.31	536.76	218.17	538.05
23	185.21	454.93	184.56	457.12	106	217.31	537.01	218.26	536.5
24	188.91	463.63	189.38	467.48	107	217.97	536.09	217.58	537.72
25	192.04	472.18	191.62	474.57	108	218.96	535.98	217.62	536.08
26	194.96	478.98	195.64	481.45	109	217.56	536.07	218.04	535.91
27	197.3	486.11	197.16	486.34	110	218.2	535.79	217.54	536.42
28	199.78	491.17	199.83	494.36	111	217.66	535.61	218.13	536.37
29	201.46	495.15	201.81	500.69	112	218.74	536.56	217.54	537.73
30	203.44	501.56	203.51	503.85	113	218.56	536.45	218.84	538.15
31	205.5	503.96	205.72	507.35	114	217.91	534.8	217.61	537.09
32	206.74	508.49	207.39	511.85	115	217.84	537.82	218.07	535.69
33	208.35	513.35	207.9	515.42	116	218.5	535.13	217.67	535.66
34	209.63	515.16	208.63	517.81	117	217.69	534.95	217.6	536.09
35	211.02	518.81	210.44	519.52	118	218.13	537.03	217.4	536.25
36	211.46	520.21	210.96	522.84	119	217.86	537.29	217.98	537.33
37	213.32	522.33	212.23	523.52	120	217.98	535.88	218.05	535.79
38	213.46	523.9	213.52	527.49	121	217.62	535.48	217.89	535.72
39	214.32	525.72	213.94	529.78	122	217.36	535.63	217.02	536.49
40	214.79	529.15	213.81	530.64	123	218.19	534.67	216.79	536.56
41	215.45	528.7	215.98	530.96	124	217.68	535.13	217.23	536.49
42	216.26	530.59	215.64	532.81	125	217.49	535.44	217.54	535.76
43	216.51	531.82	215.76	533.72	126	217.39	536.08	217.45	535.26
44	217.65	532.68	216.41	534.91	127	217.14	534.95	216.99	535.22
45	216.67	533.77	216.53	535.75	128	216.99	534.87	217.49	535.89
46	217.51	534.72	217.26	536.31	129	218.61	533.99	217.07	535.68
47	217.84	535.82	217.48	537.37	130	217.01	535.08	217.29	536.51
48	218.03	535.97	216.8	539.26	131	216.98	535.54	216.97	535.74
49	217.92	536.17	218.38	537.77	132	217.78	534.38	217.37	535.6
50	217.5	536.42	217.99	537.75	133	216.69	534.24	217.19	535.31
51	218.87	537.79	218.33	538.48	134	217.77	535.55	217.77	535.75
52	219.55	536.53	218.57	540.59	135	217.55	533.93	216.9	533.56
53	218.08	537.83	218.65	539.5	136	217.52	534.4	216.97	534.87
54	218.78	538.99	218.19	539.85	137	218.09	535.01	217.58	535.94
55	217.94	538.1	218.52	539.82	138	216.66	533.62	217.53	535.55
56	218.96	538.13	218.9	540.83	139	217.07	534.81	216.82	535.91
57	220.1	538.6	219.12	540.44	140	217.83	533.7	217.46	535.09
58	219.19	539.02	219.09	541.31	141	217.27	534.06	216.87	534.75
59	218.9	538.03	219.82	540.61	142	216.91	535.52	218.34	535.96
60	219.18	539.53	219.54	541.3	143	217.17	533.19	218.5	534.38
61	218.82	539.74	219.32	540.68	144	217.59	534.34	218.03	534.31
62	219.33	539.56	218.89	542.26	145	217.19	532.01	217.01	534.93
63	218.77	539.17	219.23	542.59	146	217.68	534.42	216.9	534.59
64	219.19	540.57	219.54	540.74	147	216.64	534.27	217.02	533.91
65	219.61	539.17	219.63	540.04	148	216.81	532.86	217.08	534.7
66	219.49	539.38	218.77	541.31	149	217.22	533.16	217.36	533.48
67	219.59	538.59	219.35	540.72	150	216.87	534.11	217.44	535.44
68	219.73	539.2	219.18	541.67	151	217.43	532.96	215.98	534.75
69	219.29	540.6	218.76	540.66	152	217.9	532.48	217.02	534.19
70	219.38	540.17	219.03	540.95	153	217.12	533.09	217.45	535.76
71	218.74	539.15	218.23	541.59	154	216.66	533.57	217.41	533.6
72	218.56	539.7	219.17	540.08	155	215.68	534.25	216.37	535.27
73	218.66	540.02	219.46	539.21	156	217.76	535.52	216.51	532.86
74	219.65	538.91	220.27	539.92	157	216.91	532.67	217.64	533.28
75	218.86	538.12	219.46	541.83	158	217.22	534.2	217.25	535.01
76	219.64	539.88	218.19	539.78	159	216.3	533.08	216.47	533.12
77	218.7	540.12	218.34	539.99	160	216.81	533.15	216.62	533.84
78	219.6	538.11	218.47	541.13	161	218.15	533.15	217.11	533.77
79	218.65	538.16	217.91	543.21	162	216.76	532.13	216.72	533.6
80	218.09	538.5	219.65	540.27	163	216.48	533.77	216.54	532.88
81	218.95	537.49	219.39	541.22	164	216.46	533.24	216.43	532.77
82	219.8	537.26	218.67	541.13	165	216.52	532.87	217.24	533.08
83	219.18	537.83	218.92	540.01	166	216.18	531.95	217.97	533.3
84	218.24	538.54	218.73	539.76	167	216.61	532.78	216.84	532.7
85	218.59	539.33	219.45	539.76	168	216.61	532.94	216.22	532.84
86	219.05	538.84	218.73	540.19	169	216.17	533.27	217.06	533.9
87	219.3	538.97	219.52	540.35	170	217.43	533.85	216.62	533.68
88	218.67	537.88	218.69	539.45	171	216.46	533.69	216.94	533.31

DFEWMA Cut-off values $\alpha = 0.01$, $m_0 = 100$, for $p = 2$, $X_1 \sim N(0,1)$, $X_2 \sim$ Uniform or t_3, for $p = 5$, $X_1, X_2, X_3 \sim N(0,1)$, $X_4, X_5 \sim$ both Uniform or t_3

	Uniform		t_3			Uniform		t_3	
n	$p=2$	$p=5$	$p=2$	$p=5$	n	$p=2$	$p=5$	$p=2$	$p=5$
172	216.49	533.5	217.52	532.43	255	215.7	529.17	215.5	530.61
173	216.51	533.01	216.73	534.41	256	215.43	529.79	214.9	531.62
174	216.36	533.12	216.79	533.1	257	214.95	529.09	215.3	531.88
175	216.71	532.52	216.67	533.47	258	215.73	530.09	216.21	529.51
176	217.04	532.56	217.05	533.72	259	215.38	530.74	215.17	530.56
177	217.31	531.14	216.89	533.07	260	215.07	531.06	215.48	530.70
178	216.72	532.75	216.26	532.16	261	215.44	530.17	216.29	528.16
179	216.49	533.47	217.11	533.42	262	215.56	529.69	216.06	530.43
180	216.51	532.45	216.48	532.27	263	217.3	531.02	216.06	529.51
181	216.46	532.43	215.45	532.69	264	215.68	530.15	215.39	530.10
182	217.00	531.88	216.46	533.02	265	215.38	529.72	215.5	528.94
183	216.45	531.65	216.11	532.47	266	214.79	529.99	215.93	530.63
184	216.53	531.22	216.49	532.76	267	216.29	531.42	215.65	529.41
185	216.42	533.44	217.12	534.17	268	215.49	529.79	216.33	530.20
186	216.17	531.71	217.09	532.99	269	215.86	531.02	215.97	530.49
187	216.98	532.87	216.87	531.49	270	215.65	530.67	215.39	531.59
188	216.22	532.78	216.91	532.8	271	216.06	529.29	215.49	530.57
189	215.86	532.4	216.14	530.66	272	216.08	529.21	215.51	529.78
190	216.69	531.6	216.32	531.41	273	214.82	530.91	215.35	530.05
191	216.51	532.55	216.69	533.02	274	214.59	529.98	214.51	531.00
192	216.75	532.51	216.6	532.67	275	215.45	530.52	215.19	530.87
193	216.36	531.11	215.99	532.95	276	215.57	530.97	215.47	530.18
194	215.94	531.43	215.93	533.63	277	214.93	529.27	215.7	530.25
195	216.74	532.92	215.9	532.45	278	214.95	529.22	215.21	529.75
196	216.35	534.03	215.42	532.55	279	216.05	530.68	214.49	529.52
197	216.4	531.16	214.9	532.82	280	215.2	529.07	215.18	529.96
198	216.18	530.56	216.18	531.79	281	216.04	529.54	216.16	530
199	216.31	531.09	216.05	532.47	282	215.47	528.69	214.62	530.07
200	216.21	531.54	215.64	531.89	283	215.25	530.42	215.47	531.21
201	216.36	532.53	217.12	533.27	284	215.52	529.89	214.86	529.92
202	217.62	531.29	216.38	532.39	285	215.82	528.76	215.29	529
203	216.13	532.61	215.84	531.38	286	215.39	529.02	215.14	529.74
204	215.91	531.05	216.35	531.59	287	215.31	529.69	215.26	530.66
205	215.87	531.65	216.02	531.35	288	214.92	529.87	215.12	529.93
206	215.71	530.75	216.06	531.98	289	214.21	530.29	215.21	529.98
207	215.2	532.06	216.24	531.5	290	215.66	532.86	215.57	530.61
208	216.17	532.65	217.03	532.32	291	215.12	529.79	215.33	528.97
209	216.32	530.72	216.35	531.79	292	215.3	528.82	215.67	528.79
210	217.02	531.52	215.9	531.78	293	215.05	529.44	215.46	530.55
211	216.56	531.49	216.2	531.93	294	215.36	528.92	216.39	528.57
212	216.44	531.86	216.21	531.25	295	216.24	530.21	214.8	528.67
213	216.53	531.4	215.98	531.43	296	215.29	529.65	215.49	529.64
214	216.02	532.77	215.62	531.42	297	214.62	530.09	215.07	530.54
215	215.93	530.45	216.52	531.39	298	215.62	528.57	215.95	529.28
216	216.32	530.4	215.43	530.8	299	215.74	529.74	216.16	528.87
217	216.55	530.49	216.07	532.6	300	214.85	528.89	215.6	529.78
218	215.84	530.69	215.87	531.85	301	215.54	529.22	215.29	529.66
219	216.11	531.6	216.58	531.73	302	215.65	528.47	215.4	529.51
220	216.56	530.78	215.39	532.31	303	215.15	529.48	214.91	530.21
221	215.64	531.1	215.58	531.36	304	215.16	528.53	214.77	529.19
222	215.82	530.54	216.46	531.9	305	215.29	529.78	215.33	529.88
223	216.02	529.76	216.46	531.38	306	215.66	529.24	215.54	529.89
224	215.8	530.16	215.49	531.4	307	215.48	529.33	214.73	529.53
225	215.44	530.72	215.81	530.3	308	214.94	529.52	215.65	530.26
226	215.49	531.38	215.19	531.89	309	215.9	528.71	215.66	529.23
227	216.17	531.22	215.88	530.08	310	215.18	528.71	216.03	530.43
228	215.8	532.2	216.09	530.41	311	215.56	528.74	215.09	530.32
229	216.23	530.88	215.8	530.91	312	214.96	528.54	214.86	530.19
230	216.28	530.6	216.78	530.61	313	214.59	528.67	215.83	529.54
231	216.39	530.77	216.24	531.25	314	215.24	528.06	214.6	529.17
232	215.55	530.24	215.35	531.01	315	215.21	529.87	214.67	529.6
233	216.41	531.3	215.53	530.81	316	215.89	529.29	214.97	529.32
234	216.06	532.41	215.28	531.52	317	215.49	528.16	214.4	528.81
235	216.25	531.02	215.8	531.88	318	215.26	527.69	215.07	528.18
236	215.76	530.91	215.01	531.14	319	215.32	529.82	214.72	530.18
237	216.1	529.45	215.64	531.64	320	215.82	529.3	214.74	528.37
238	215.68	530.01	215.48	529.45	321	214.98	528.42	214.73	529.46
239	216.37	531	215.53	530.44	322	214.68	527.69	215.53	530.15
240	216.31	530.22	215.66	531.36	323	214.6	528.82	215.01	528.45
241	216.04	529.9	216.38	531.29	324	215.24	528.53	215.2	529.63
242	215.89	532.19	215.37	529.84	325	214.75	528.02	215.08	528.59
243	215.17	529.81	215.44	530.72	326	215.17	527.59	214.44	529.08
244	215.14	530.66	215.08	531.49	327	215.45	528.72	214.13	528.62
245	215.51	529.5	216.19	530.9	328	215.69	528.49	214.58	528.94
246	216.01	529.96	214.6	529.61	329	214.66	529.85	215.69	530.37
247	215.83	531.29	215.2	531.05	330	215.57	530.76	215.63	528.32
248	216.09	530.14	215.8	529.64	331	215.92	528.43	215.17	527.99
249	215.38	529.85	215.49	530.41	332	214.53	528.49	214.7	528.78
250	215.32	529.34	215.56	529.81	333	215.44	530.41	214.64	528.97
251	215.23	529.46	214.87	530.53	334	215.05	529.36	215.44	529.41
252	215.2	531.13	217.08	530.63	335	215.39	529.75	215.53	528.63
253	215.6	529.65	215.82	530.17	336	214.81	529.59	214.86	528.32
254	215.75	530.89	215.76	529.88	337	215.06	528.83	214.79	529.99

DFEWMA Cut-off values $\alpha = 0.01$, $m_0 = 100$, for $p = 2$, $X_1 \sim N(0,1)$, $X_2 \sim$ Uniform or t_3, for $p = 5$, $X_1, X_2, X_3 \sim N(0,1)$, $X_4, X_5 \sim$ both Uniform or t_3

	Uniform		t_3			Uniform		t_3	
n	$p = 2$	$p = 5$	$p = 2$	$p = 5$	n	$p = 2$	$p = 5$	$p = 2$	$p = 5$
338	215.2	528.06	215.28	529.86	420	214.42	528.11	214.34	527.82
339	215.27	528.81	215.81	528.64	421	215.03	526.99	214.93	530.08
340	215.34	529.23	213.98	528.55	422	214.83	526.87	214.33	526.28
341	215.57	528.78	214.89	529.17	423	215.18	527.8	214.18	530.85
342	215.51	527.01	215.86	528.32	424	214.64	526.68	214.33	528.07
343	215.08	528.17	214.13	528.6	425	214.89	527.06	215.08	526.42
344	215.13	528.37	214.67	529.59	426	215.18	528.28	214.09	526.76
345	215.32	528.81	214.05	529.29	427	214.4	527.62	213.93	525.69
346	214.15	527.73	214.58	528.83	428	214.36	526.66	214.41	526.73
347	214.34	527.9	214.34	529.69	429	213.98	528.39	214.81	527.47
348	214.9	528.41	215.36	529.35	430	214.7	528.26	214.75	527.43
349	214.64	528.52	214.25	528.88	431	214.49	527.31	214.87	526.69
350	214.99	528.31	214.54	528.13	432	213.63	527.48	214.54	527.04
351	214.87	528.73	216.02	528.81	433	214.84	527.57	214.77	526.59
352	214.93	528.22	214.36	527.37	434	214.67	527.05	214.67	528.16
353	215.77	528.11	215.23	528.06	435	214.86	528.72	214.8	528.05
354	214.75	528.16	215.62	528.63	436	213.72	527.54	214.67	528
355	214.63	529.29	214.05	528.51	437	215.09	527.12	214.28	528.91
356	214.69	527.17	214.46	528.72	438	213.53	528.38	213.93	527.7
357	215.41	529.36	215.58	529.23	439	213.87	526.43	214.13	526.98
358	214.59	528.09	214.17	528.76	440	214.08	525.93	213.93	526.32
359	214.65	529.14	215.2	528.97	441	214.84	527.36	214.45	526.63
360	214.91	529.25	214.77	528.88	442	213.74	527.87	214.15	526.86
361	215.01	528.22	214.96	528.51	443	214.94	525.54	214.88	527.42
362	214.9	528.42	214.74	527.43	444	214.94	526.62	214.77	526.66
363	214.97	528.34	215.09	528.46	445	214.95	527.54	214.97	527.13
364	214.58	528.62	214.6	528.53	446	213.61	526.91	214.45	527.27
365	214.34	529.13	214.22	527.16	447	214.21	526.92	213.78	527.45
366	214.53	527.87	214.65	527.3	448	214.05	526.39	214.32	526.55
367	215.23	527.98	215.02	527.64	449	214.97	527.47	214.34	526.61
368	215.4	527.29	214.95	527.15	450	214.54	527.06	214.95	526.35
369	215.53	527.6	213.76	527.72	451	214.45	527.39	214.48	527.47
370	214.06	528.23	215.09	528.55	452	214.05	527.42	214.85	526.98
371	214.65	529.57	214.53	528.61	453	214.52	527.04	214.31	529.10
372	214.61	528.24	215.87	528.79	454	214.7	528.38	214.78	527.34
373	214.43	528.22	214.25	528.66	455	213.77	527.53	214.18	525.71
374	214.46	528.37	215.09	527.85	456	214.69	526.9	213.51	526.52
375	214.74	527.91	215.3	527.26	457	214.63	527.03	214.56	526.76
376	214.74	526.39	214.65	528.29	458	215.15	527.35	214.44	527.09
377	214.77	528.22	214.2	528.35	459	215.16	528.3	214.47	526.93
378	214.26	527.73	214.43	527.91	460	213.54	526.9	214.02	527.02
379	214.73	528.32	214.69	528.56	461	214.38	525.61	214.62	525.89
380	214.48	528.28	214.04	528.01	462	213.92	527.76	214.41	526.72
381	214.79	528.39	214.7	528.48	463	214.12	527.88	214.9	527.09
382	214.74	528.53	214.75	528.68	464	214.75	528.19	215.09	527.06
383	214.41	528.92	215.33	528.31	465	213.63	527.64	214.1	527.00
384	214.49	527.69	214.75	527.54	466	214.14	526.00	214.49	528.9
385	214.32	527.94	214.94	527.05	467	213.93	526.3	213.52	528.46
386	215.07	527.8	215.6	526.72	468	214.77	525.53	214.75	527.71
387	215.04	527.25	214.85	529.21	469	214.86	526.07	213.95	527.16
388	214.5	527.3	214.89	527.5	470	214.41	525.73	214.09	528.74
389	214.9	527.74	214.04	527.39	471	214.47	527.67	214.3	528.05
390	215.32	527.5	214.74	528.26	472	213.36	527.22	214.21	527.7
391	214.57	528.93	214.66	527.39	473	214.05	527.37	215.03	527.82
392	215.92	528.26	214.93	529.94	474	214.19	525.74	214.99	527.1
393	215.19	527.92	214.74	528.35	475	213.53	526.45	214.29	527.62
394	214.92	528.28	213.94	529.93	476	214.09	526.02	214.76	527.16
395	214.66	528.12	214.37	527.73	477	214.05	526.17	214.97	526.34
396	214.19	526.86	214.73	528.22	478	214.41	526.38	214.11	526.88
397	214.6	528.23	213.67	527.63	479	214.61	526.74	213.85	526.83
398	214.67	526.17	215.25	527.59	480	213.31	527.57	213.89	527.24
399	214.62	529.56	215.14	527.49	481	214.3	526.29	214.09	526.97
400	214.86	527.19	214.40	526.56	482	214.73	526.43	213.57	526.28
401	214.57	527.03	214.89	527.9	483	214.61	526.13	214.15	527.35
402	214.91	528.76	214.22	529.44	484	215.09	527.19	214.23	526.98
403	214.28	527.12	215.20	527.26	485	215.2	528.68	214.84	526.34
404	214.78	526.41	213.92	528.07	486	214.3	526.99	213.35	526.12
405	214.36	527.5	214.62	528.79	487	213.62	526.92	213.6	527.1
406	214.69	527.89	214.00	528.28	488	213.9	526.95	214.72	527.51
407	214.01	527.93	215.38	527.93	489	214.26	526.84	214.41	527.71
408	213.9	527.24	213.97	526.88	490	215.24	527.21	213.15	526.45
409	214.55	527.57	214.83	526.65	491	213.45	526.27	214.67	527.42
410	214.86	527.79	214.99	528.59	492	214.65	528.64	213.49	526.51
411	214.87	528.35	214.44	527.14	493	213.91	527.42	215.03	526.58
412	215.44	527.24	214.51	528.62	494	214.43	527.01	213.74	526.54
413	215.15	527.21	214.06	528.36	495	214.90	525.58	213.83	527.18
414	215.43	527.08	214.59	528.19	496	214.49	526.66	214.09	528.17
415	214.14	527.16	214.84	527.58	497	215.64	527.21	214.69	527.26
416	214.61	527.19	214.54	527.02	498	214.80	527.45	213.85	527.03
417	214.92	527.68	213.5	527.87	499	214.00	527.19	214.65	528.13
418	214.03	527.11	214.58	528.24	500	213.63	527.28	214.6	526.08
419	214.22	527.03	215.53	528.4					

Table A.7: EWAEL Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.05$, $m_0 = 50$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
1	13.33	20.31	25.41	33.10	51	13.01	18.50	27.57	30.65
2	12.55	19.38	24.13	31.97	52	14.61	20.49	26.20	31.87
3	14.52	19.78	24.10	30.90	53	13.79	18.88	24.87	30.45
4	13.70	18.99	24.93	30.02	54	14.98	20.18	26.09	32.38
5	14.18	19.46	26.06	32.79	55	13.97	19.88	24.67	31.72
6	13.58	20.81	25.02	32.26	56	14.31	19.48	23.03	32.42
7	12.60	20.30	26.66	30.18	57	12.94	20.16	23.56	32.16
8	13.92	20.34	23.69	34.07	58	12.90	18.32	28.36	32.47
9	14.92	19.34	25.25	32.31	59	13.93	19.18	24.86	30.82
10	15.13	18.86	25.43	32.45	60	13.13	18.73	24.04	31.16
11	14.22	19.66	24.01	30.92	61	14.69	19.07	25.98	31.97
12	13.29	20.38	25.81	31.61	62	13.98	18.11	26.11	33.18
13	14.28	20.55	23.85	31.97	63	13.73	18.35	27.22	31.65
14	13.01	20.31	25.99	32.06	64	14.73	19.46	27.53	32.16
15	13.49	19.21	27.30	30.82	65	14.49	19.43	27.33	28.87
16	13.94	19.79	29.08	31.46	66	12.21	19.17	27.59	30.50
17	12.81	19.78	26.44	31.53	67	13.24	18.61	26.29	31.97
18	14.40	18.65	25.49	31.86	68	13.84	19.19	26.13	31.89
19	13.65	19.51	24.37	29.30	69	13.81	18.73	24.63	29.82
20	13.56	18.30	24.97	31.40	70	13.54	18.65	24.81	32.41
21	12.78	19.86	24.72	33.88	71	14.65	20.20	25.38	31.19
22	13.57	18.52	28.04	32.14	72	13.93	18.58	26.11	31.93
23	11.92	18.94	26.70	32.05	73	15.03	19.65	25.29	30.80
24	14.69	20.02	25.33	33.30	74	13.56	20.29	25.65	31.41
25	13.77	18.46	24.68	30.52	75	12.76	20.68	27.13	33.23
26	13.77	20.94	25.19	31.86	76	13.65	19.07	28.78	32.11
27	13.26	19.92	26.74	31.75	77	14.08	19.52	26.41	30.72
28	13.41	21.15	25.36	32.01	78	14.71	21.16	26.66	33.49
29	12.79	19.41	25.68	33.42	79	14.27	19.37	25.82	31.91
30	12.95	19.15	27.12	32.43	80	13.62	19.76	24.58	29.77
31	13.00	19.15	26.78	32.20	81	13.62	19.15	25.89	32.89
32	14.21	18.64	24.61	30.80	82	13.64	18.68	26.67	32.25
33	13.17	18.90	27.18	32.75	83	13.50	19.38	25.61	33.59
34	13.97	18.67	25.42	33.03	84	14.81	20.91	23.98	30.92
35	13.97	19.77	25.32	30.74	85	13.02	19.24	25.28	31.11
36	14.78	20.68	25.94	30.51	86	14.42	19.90	24.15	31.65
37	13.10	18.98	25.71	32.60	87	14.23	19.25	25.67	31.34
38	14.41	20.49	25.01	33.37	88	13.27	22.17	25.79	31.00
39	15.85	20.81	26.25	31.45	89	14.36	19.79	26.31	33.39
40	12.86	20.17	25.98	32.10	90	13.66	18.59	25.53	32.89
41	13.06	20.12	24.64	32.28	91	13.97	18.10	24.75	33.10
42	14.37	19.30	25.81	31.95	92	13.38	21.54	24.88	32.82
43	14.11	20.30	24.55	32.73	93	15.18	18.46	26.10	33.11
44	14.22	18.83	24.68	31.99	94	13.68	18.32	25.15	30.56
45	13.79	17.40	26.03	32.52	95	13.49	19.91	26.00	31.47
46	13.57	20.48	24.41	33.23	96	12.38	19.87	26.39	32.55
47	13.87	19.13	28.30	31.63	97	14.08	18.71	23.65	31.23
48	13.64	18.74	25.96	33.56	98	14.01	19.02	25.18	31.50
49	13.32	18.76	26.14	31.01	99	13.24	21.04	24.96	31.97
50	12.66	18.12	24.52	31.54	100	12.60	19.10	23.88	29.57

Table A.8: EWAEL Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 50$

	p variables					p variables					p variables			
n	2	3	4	5	n	2	3	4	5	n	2	3	4	5
1	19.42	30.71	36.98	41.43	85	20.98	28.98	37.82	44.45	169	20.30	28.55	34.81	47.24
2	21.62	29.89	36.51	47.03	86	23.87	27.48	38.12	44.06	170	20.91	31.87	36.45	42.78
3	22.99	31.29	35.92	47.93	87	22.95	32.30	37.39	40.98	171	20.89	29.93	37.90	45.13
4	23.18	26.87	36.47	48.41	88	21.80	31.95	37.42	43.39	172	20.66	34.22	41.38	45.12
5	21.28	26.68	33.61	44.98	89	22.53	29.31	38.27	43.38	173	22.76	28.66	38.55	43.86
6	21.22	29.57	33.93	42.66	90	21.81	31.07	35.01	45.21	174	20.28	32.12	35.81	46.09
7	19.97	30.61	38.56	39.77	91	23.82	29.12	37.74	49.51	175	20.25	30.34	36.62	39.16
8	19.30	30.78	33.08	44.33	92	23.17	34.21	35.77	44.44	176	20.23	33.71	38.19	42.00
9	22.71	29.90	32.94	48.09	93	22.82	26.43	37.42	44.63	177	22.94	29.74	37.02	47.60
10	22.09	27.66	35.48	41.41	94	22.53	28.73	33.96	43.39	178	22.52	31.64	37.47	45.60
11	22.68	29.73	33.28	40.78	95	22.08	29.07	38.81	43.52	179	22.73	28.07	35.89	47.17
12	21.11	29.52	34.28	43.72	96	18.50	25.13	38.60	44.86	180	21.66	29.18	36.96	46.02
13	24.84	32.09	36.18	40.59	97	21.90	31.86	31.86	40.28	181	24.74	26.91	38.31	45.08
14	18.77	33.34	34.83	45.57	98	21.51	31.87	34.16	39.93	182	20.16	27.05	31.98	42.66
15	23.37	30.40	37.11	43.30	99	19.60	29.90	35.92	47.03	183	21.98	30.30	34.61	46.63
16	21.01	27.78	40.41	44.63	100	21.28	28.40	39.39	44.41	184	21.69	28.46	38.43	49.13
17	21.53	29.21	40.42	46.90	101	23.30	27.18	34.13	44.47	185	22.92	30.45	34.52	42.77
18	24.92	26.69	36.59	42.45	102	23.67	28.46	38.41	40.76	186	21.86	31.65	37.02	44.32
19	19.81	31.75	33.26	40.52	103	21.51	28.99	33.90	43.02	187	21.34	29.08	34.20	42.25
20	21.84	28.44	35.22	40.26	104	24.62	29.63	38.59	45.94	188	25.92	30.68	33.94	42.89
21	22.13	30.37	39.24	47.24	105	20.01	27.43	38.14	43.11	189	21.73	28.04	36.15	44.74
22	22.86	29.37	41.27	43.69	106	20.63	34.33	35.56	45.92	190	21.62	28.66	35.46	48.62
23	22.59	30.59	39.03	42.74	107	17.91	25.13	36.94	41.92	191	21.73	33.66	37.81	46.93
24	23.75	28.19	31.34	44.19	108	22.00	26.87	31.43	44.73	192	19.86	28.24	35.97	41.52
25	21.45	26.41	36.94	43.17	109	20.81	29.06	40.90	44.36	193	22.87	26.24	38.37	48.41
26	22.91	29.27	37.17	42.94	110	21.24	31.92	33.17	43.00	194	25.90	31.93	35.46	45.48
27	25.16	29.21	36.93	44.75	111	19.04	29.65	35.34	44.40	195	19.97	25.64	35.40	48.22
28	22.68	31.14	34.10	45.61	112	21.38	27.98	36.76	42.32	196	21.04	26.47	38.27	45.23
29	20.78	29.80	39.99	48.49	113	23.88	27.18	35.51	44.95	197	22.13	29.46	37.15	43.06
30	19.41	30.36	37.59	44.69	114	25.54	30.81	34.48	46.59	198	23.23	29.51	37.09	47.08
31	20.16	25.65	39.33	45.52	115	21.75	28.11	34.19	41.50	199	19.37	29.43	40.24	43.39
32	24.47	31.70	36.33	43.49	116	20.33	27.76	40.94	46.76	200	21.19	31.53	34.40	43.73
33	19.60	27.57	41.65	48.08	117	20.43	29.56	36.20	47.00	201	21.40	31.06	38.54	46.19
34	22.16	27.79	38.14	46.42	118	22.91	28.85	39.27	40.86	202	21.82	29.98	35.55	41.73
35	21.95	28.97	36.63	46.30	119	22.37	30.94	33.97	48.46	203	22.02	29.33	36.96	41.96
36	23.97	29.15	36.26	48.06	120	21.11	30.68	36.50	48.91	204	20.30	32.21	35.97	44.74
37	20.95	27.87	35.04	42.02	121	20.03	29.48	36.19	47.78	205	20.62	27.33	34.68	42.14
38	23.74	27.27	36.20	43.02	122	19.10	26.22	36.37	41.89	206	22.54	27.38	36.37	43.66
39	22.29	30.77	35.40	43.98	123	21.41	28.34	37.08	44.19	207	20.15	29.74	37.67	40.74
40	22.57	27.70	33.58	41.08	124	21.43	29.20	38.99	45.08	208	24.33	27.82	37.60	44.86
41	20.83	29.90	38.52	47.44	125	21.29	34.25	34.00	45.15	209	19.29	30.93	36.12	45.88
42	25.71	30.06	35.51	43.49	126	22.53	30.87	35.97	46.48	210	20.39	25.88	35.52	47.02
43	22.15	30.00	35.33	46.41	127	24.00	29.48	34.75	45.36	211	20.82	31.40	36.61	45.66
44	22.09	28.30	35.33	44.26	128	19.26	29.28	36.64	51.80	212	20.59	30.08	37.79	46.98
45	21.79	28.67	36.22	43.70	129	21.46	24.82	38.07	45.46	213	19.33	28.58	39.30	42.50
46	19.97	30.04	32.96	45.74	130	19.39	25.62	36.74	43.69	214	20.83	25.40	35.51	42.95
47	24.57	28.83	36.62	42.04	131	21.28	29.04	39.96	43.23	215	25.50	26.84	37.97	44.92
48	21.72	27.98	33.65	42.48	132	19.86	30.04	39.56	47.50	216	23.01	32.38	38.45	46.63
49	21.36	29.01	37.40	44.00	133	25.37	31.40	40.01	41.97	217	21.76	29.04	34.60	45.68
50	20.56	27.95	37.37	45.11	134	24.26	26.96	34.57	45.08	218	20.40	27.57	38.93	43.39
51	22.46	25.98	40.29	41.81	135	20.42	29.37	35.45	43.64	219	22.55	28.10	37.62	44.52
52	23.75	32.30	33.65	41.55	136	20.45	29.82	33.47	47.65	220	23.18	26.95	34.93	43.22
53	22.98	28.80	36.73	40.75	137	19.79	30.25	37.53	42.76	221	21.54	28.39	36.93	47.62
54	21.76	30.61	38.61	41.66	138	21.05	31.11	35.71	48.05	222	22.85	28.94	35.76	45.30
55	23.48	29.74	37.26	43.68	139	19.82	29.77	36.09	46.88	223	22.12	31.05	36.38	41.99
56	24.70	28.06	34.59	44.82	140	20.85	27.43	37.64	44.24	224	19.56	27.92	35.09	43.57
57	20.82	31.12	36.02	40.64	141	19.58	29.09	40.40	44.38	225	19.61	26.29	39.40	46.93
58	21.05	29.84	38.92	44.38	142	25.88	30.51	38.61	43.54	226	20.11	30.13	36.37	45.12
59	23.14	31.63	36.96	42.23	143	20.08	31.05	33.99	41.66	227	19.97	30.57	36.99	44.40
60	20.44	28.20	36.04	42.66	144	21.35	28.43	35.48	43.57	228	20.58	30.04	35.16	44.24
61	25.82	29.47	39.61	47.77	145	20.42	30.43	38.78	42.67	229	19.92	30.59	37.74	48.02
62	21.20	25.09	38.18	44.91	146	23.10	28.72	35.99	45.58	230	21.75	27.77	35.92	46.65
63	23.13	28.44	37.24	44.21	147	23.17	28.17	40.33	45.13	231	20.68	28.86	40.63	41.86
64	22.08	27.14	36.18	44.66	148	22.09	31.52	37.03	43.70	232	21.45	28.74	38.29	42.80
65	24.44	30.15	39.96	40.81	149	21.71	30.12	38.44	40.27	233	21.56	27.15	40.53	45.66
66	21.53	32.31	37.03	40.75	150	19.29	28.57	35.80	48.76	234	21.97	28.82	36.33	39.51
67	20.58	28.79	39.26	44.19	151	21.41	30.27	35.22	41.95	235	24.93	29.69	39.60	44.76
68	21.05	28.31	38.81	44.22	152	21.31	28.92	35.24	44.39	236	23.47	28.71	39.59	48.20
69	20.89	27.48	36.75	39.88	153	23.35	26.26	37.87	42.63	237	20.46	25.89	35.81	44.19
70	23.79	32.42	35.42	43.48	154	26.63	27.20	35.76	43.46	238	23.28	27.19	35.21	44.86
71	24.08	28.81	36.28	42.21	155	20.87	30.27	34.16	44.04	239	22.20	32.14	37.97	42.09
72	22.58	30.83	33.86	45.19	156	23.66	30.13	39.97	44.28	240	20.87	26.71	32.45	43.45
73	26.35	30.30	40.92	39.22	157	21.12	29.20	35.49	43.15	241	22.78	32.29	37.79	43.61
74	20.18	30.40	39.77	41.20	158	21.47	27.38	37.50	42.43	242	20.37	31.29	36.47	41.68
75	22.39	29.26	40.40	44.29	159	20.66	28.17	32.39	44.78	243	21.91	29.67	31.78	47.69
76	23.04	31.16	38.61	44.64	160	22.19	28.52	34.18	43.41	244	21.11	27.63	38.05	41.59
77	22.27	28.62	36.32	45.93	161	21.64	31.33	39.50	45.88	245	24.80	30.68	37.62	46.73
78	20.63	32.00	39.19	43.73	162	22.72	26.27	35.70	47.90	246	21.73	27.98	39.15	45.67
79	22.77	32.45	33.96	44.32	163	21.52	26.80	35.23	46.98	247	23.43	30.20	38.09	38.52
80	22.43	28.61	34.66	39.90	164	23.67	29.34	38.27	43.00	248	23.85	27.65	36.40	41.59
81	22.56	28.69	35.34	45.89	165	21.54	26.32	31.65	40.73	249	23.21	27.32	38.22	45.93
82	20.30	30.91	35.12	41.07	166	20.05	28.53	37.79	48.95	250	22.25	31.36	33.41	44.03
83	23.63	30.29	35.68	52.87	167	22.85	29.84	37.30	45.61	251	21.34	30.11	40.44	43.68
84	26.99	28.29	34.00	42.02	168	22.73	28.58	39.00	44.15	252	23.52	33.02	37.55	48.00

EWAEL Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 50$

	p variables					p variables					p variables			
n	2	3	4	5	n	2	3	4	5	n	2	3	4	5
253	21.49	31.02	36.51	43.97	337	22.95	28.48	35.70	49.24	421	21.68	30.89	36.08	45.73
254	20.53	30.81	35.96	40.49	338	21.52	28.68	34.75	41.25	422	21.27	27.29	36.51	44.10
255	22.31	30.00	33.36	42.88	339	21.88	31.18	36.24	46.01	423	19.55	26.07	35.67	45.71
256	20.24	26.96	36.36	41.47	340	20.65	30.49	37.70	42.75	424	18.70	28.07	36.55	44.04
257	22.91	31.33	41.70	42.16	341	22.63	31.65	35.30	46.60	425	20.77	27.75	35.49	44.38
258	22.97	29.48	38.93	42.27	342	18.50	29.43	35.71	44.59	426	20.82	31.31	36.06	41.53
259	24.02	32.69	37.22	45.23	343	19.15	27.18	35.50	44.14	427	21.61	28.37	36.07	44.71
260	21.57	31.08	34.43	46.91	344	21.44	32.18	36.70	42.23	428	23.45	27.78	39.09	44.71
261	20.41	27.25	35.62	43.19	345	22.09	28.45	38.72	44.36	429	22.38	27.52	36.68	45.78
262	20.28	28.68	40.28	43.20	346	17.84	32.39	37.43	45.28	430	18.85	29.99	37.57	44.83
263	20.88	27.31	37.92	43.25	347	24.39	26.38	33.73	41.24	431	20.98	27.78	38.89	41.97
264	21.85	30.50	37.42	43.31	348	20.51	28.79	34.49	46.03	432	21.84	28.42	39.55	44.00
265	21.09	29.83	40.88	45.82	349	21.20	28.69	35.17	41.11	433	23.65	28.56	35.17	48.12
266	20.68	28.27	40.83	44.20	350	22.10	27.38	36.22	42.21	434	22.29	28.79	37.61	41.79
267	20.68	31.97	37.67	44.25	351	20.91	30.24	35.59	43.88	435	20.06	30.19	37.68	46.64
268	21.80	30.28	36.83	48.14	352	21.41	30.97	42.65	47.39	436	25.01	31.29	39.01	45.76
269	22.03	28.26	39.31	46.66	353	19.55	30.82	35.91	45.34	437	21.05	28.28	32.69	43.21
270	23.52	32.26	37.13	45.05	354	22.93	29.29	34.65	42.11	438	21.75	28.42	39.39	48.09
271	23.00	30.16	40.00	44.03	355	20.49	31.22	37.69	46.73	439	21.81	27.44	39.02	44.65
272	21.08	27.20	34.59	44.24	356	21.79	29.39	35.11	43.10	440	23.57	30.84	35.14	41.03
273	18.97	30.41	36.09	43.70	357	23.00	28.59	34.84	46.28	441	20.73	31.55	35.00	42.67
274	19.92	30.37	40.40	41.56	358	21.19	29.25	35.16	44.62	442	22.03	27.39	37.48	45.42
275	22.30	27.18	37.01	47.15	359	23.31	35.48	35.89	46.33	443	20.36	25.59	36.18	43.30
276	20.54	30.27	36.44	43.77	360	23.41	26.41	33.99	45.91	444	22.79	28.65	36.89	42.28
277	23.01	27.81	36.71	43.60	361	25.38	29.49	37.63	43.29	445	27.69	28.53	36.22	41.74
278	25.65	33.20	35.83	47.85	362	19.09	32.86	32.21	40.19	446	21.37	27.16	35.06	46.99
279	25.23	27.92	36.11	42.76	363	24.89	28.27	37.31	42.40	447	21.77	28.42	35.59	45.36
280	20.97	26.76	40.30	46.00	364	24.14	28.98	33.39	43.39	448	24.17	29.19	34.05	46.34
281	23.31	29.57	35.85	43.53	365	22.77	29.53	35.02	43.35	449	21.01	31.78	38.05	49.08
282	20.23	30.85	35.56	41.79	366	18.88	30.27	35.72	45.04	450	20.96	29.95	33.03	44.41
283	20.67	29.73	33.77	50.46	367	23.53	26.72	35.23	45.06	451	21.55	27.60	39.04	41.97
284	23.66	28.18	36.58	49.38	368	21.50	28.10	34.88	46.99	452	21.48	25.94	35.48	45.54
285	23.08	29.95	40.14	39.59	369	21.36	29.77	36.62	40.79	453	23.35	27.62	35.87	42.53
286	19.39	31.52	35.67	44.45	370	22.49	30.01	35.59	45.02	454	20.56	29.36	36.51	44.77
287	21.81	31.55	37.88	45.01	371	22.06	26.92	37.15	41.48	455	22.38	28.66	31.40	42.84
288	20.82	25.95	35.45	47.19	372	21.95	28.68	38.47	42.56	456	19.26	30.04	39.29	47.14
289	21.81	27.50	38.67	50.18	373	23.93	29.88	37.80	42.99	457	21.06	30.03	37.81	41.86
290	20.64	28.77	39.65	44.56	374	18.22	26.59	35.51	45.31	458	21.20	28.42	40.10	52.23
291	20.10	29.40	37.59	44.98	375	21.12	30.20	32.57	40.09	459	20.42	28.66	38.14	42.85
292	22.91	27.10	38.46	42.67	376	20.80	26.22	40.07	48.57	460	25.02	27.64	35.20	43.04
293	21.22	28.61	37.40	43.40	377	19.24	25.65	38.18	52.82	461	21.19	29.38	37.72	43.77
294	24.15	30.42	34.49	46.41	378	22.06	32.28	34.08	44.08	462	20.54	30.28	38.48	41.11
295	22.63	30.41	33.39	43.24	379	23.10	28.66	38.25	43.37	463	20.66	28.87	34.79	44.93
296	18.25	29.24	38.01	41.47	380	21.01	27.68	38.40	42.42	464	22.30	30.23	37.74	42.58
297	21.43	28.94	33.85	42.01	381	24.47	28.21	39.40	45.62	465	24.94	27.94	33.79	44.61
298	22.90	28.25	33.84	47.59	382	22.02	28.32	36.06	44.72	466	22.00	30.00	37.85	45.93
299	22.09	29.16	30.80	42.46	383	20.58	30.30	35.10	43.84	467	23.56	30.43	36.77	49.05
300	21.07	29.10	34.85	45.89	384	23.67	29.91	38.28	44.37	468	20.09	30.18	37.90	41.12
301	21.50	27.83	35.17	42.56	385	19.25	28.15	38.79	45.17	469	20.89	28.52	36.27	45.97
302	20.56	29.00	36.55	45.16	386	20.97	27.94	38.92	42.91	470	18.37	27.32	35.80	45.72
303	19.51	27.82	39.58	38.00	387	22.92	31.97	35.83	45.81	471	21.53	31.85	37.81	42.78
304	22.38	26.73	38.74	44.42	388	24.55	27.77	35.71	41.01	472	23.03	27.92	35.30	41.10
305	21.09	29.50	37.67	46.04	389	19.69	29.74	35.06	46.69	473	21.45	29.17	41.82	46.07
306	23.31	27.30	36.81	42.89	390	24.42	28.83	38.14	42.39	474	22.43	31.26	39.71	43.16
307	21.07	31.61	35.50	42.55	391	22.28	36.60	38.69	42.35	475	21.11	29.36	37.38	40.94
308	21.81	27.56	38.86	45.32	392	21.42	28.45	34.07	41.24	476	22.11	32.85	38.87	46.34
309	20.83	28.59	40.10	50.32	393	21.03	29.59	42.40	44.34	477	19.45	28.14	37.62	42.29
310	22.42	30.41	33.68	47.65	394	19.05	28.78	38.52	44.39	478	20.57	27.25	38.22	43.19
311	22.47	29.66	37.13	44.93	395	20.92	27.66	34.21	41.32	479	21.92	30.86	37.40	44.85
312	20.65	27.77	40.22	46.92	396	23.93	29.00	41.85	44.60	480	17.98	28.42	36.90	45.02
313	21.83	28.74	37.56	44.77	397	22.96	29.37	38.11	42.59	481	21.24	29.66	32.12	40.83
314	22.14	28.83	36.49	45.45	398	24.47	26.74	36.26	46.80	482	24.30	27.67	38.45	44.86
315	25.63	30.52	36.53	46.51	399	22.07	29.82	33.83	45.54	483	22.78	30.35	37.24	42.96
316	18.82	27.15	40.23	44.81	400	21.33	27.13	36.09	42.06	484	19.17	26.56	35.32	49.68
317	20.85	27.51	41.69	41.87	401	21.37	30.47	36.81	46.85	485	22.14	26.50	36.24	46.40
318	19.31	30.46	34.14	42.06	402	21.37	34.56	34.85	44.84	486	21.39	32.65	37.54	44.56
319	19.45	27.73	39.69	47.82	403	22.14	31.44	34.09	47.52	487	18.50	27.05	35.25	44.73
320	23.25	31.48	37.38	45.15	404	22.87	29.49	38.92	43.28	488	21.81	27.98	33.73	44.19
321	22.50	30.95	36.74	46.98	405	21.63	26.27	41.29	49.60	489	21.36	31.86	34.93	41.98
322	24.80	29.84	37.55	43.47	406	24.51	28.76	37.27	46.71	490	21.41	31.26	38.41	43.82
323	25.92	28.29	33.92	42.61	407	21.24	31.47	37.37	40.84	491	21.97	32.08	35.20	45.00
324	21.35	29.94	35.84	41.34	408	22.16	26.56	38.60	43.71	492	21.91	27.62	35.28	48.02
325	20.43	28.91	37.26	40.23	409	21.03	28.50	36.60	42.08	493	21.17	34.06	33.91	47.54
326	24.62	29.92	35.99	43.34	410	21.89	29.88	38.35	45.07	494	22.40	27.95	37.30	41.77
327	21.20	28.91	35.00	46.15	411	20.69	26.11	38.32	48.98	495	18.92	27.46	36.60	47.34
328	22.89	29.04	39.28	47.96	412	21.88	28.48	36.81	44.85	496	22.28	29.62	36.16	46.49
329	21.41	29.47	36.68	46.45	413	21.66	27.25	35.41	43.58	497	21.93	30.43	42.16	44.53
330	20.41	30.23	38.32	44.49	414	21.51	27.47	38.93	47.27	498	20.09	29.24	36.72	47.63
331	23.66	27.37	41.39	42.85	415	23.58	25.09	33.96	40.05	499	23.52	30.01	39.38	41.80
332	24.13	26.65	36.33	46.43	416	19.03	30.43	37.82	45.61	500	22.46	26.08	38.79	45.28
333	18.31	30.27	35.88	44.54	417	21.13	27.04	35.48	44.28					
334	22.00	27.13	38.59	47.71	418	22.36	27.51	36.59	43.45					
335	20.82	28.87	38.28	40.96	419	20.08	28.53	37.34	43.03					
336	20.41	31.56	34.97	39.09	420	22.32	29.40	34.70	44.25					

Table A.9: EWAEL Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.05$, $m_0 = 100$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
1	13.15	21.01	24.93	32.49	51	13.57	19.44	25.09	33.79
2	13.72	18.72	25.04	31.19	52	14.88	19.99	25.74	32.63
3	13.20	20.26	24.88	32.25	53	13.19	18.81	26.29	30.82
4	12.09	21.55	24.74	33.60	54	13.62	18.84	26.87	31.28
5	13.81	20.14	26.24	32.83	55	13.16	18.03	26.20	33.40
6	13.22	18.72	25.49	31.81	56	13.70	19.35	25.72	30.35
7	13.80	19.26	25.00	30.62	57	14.22	20.37	24.56	30.68
8	13.47	19.30	26.40	32.30	58	13.77	19.66	26.07	32.84
9	12.74	19.46	25.32	31.51	59	13.81	20.33	26.63	31.05
10	12.96	19.19	25.80	32.13	60	13.97	19.64	24.94	32.02
11	13.15	19.30	25.61	32.83	61	13.66	18.92	25.01	31.03
12	15.09	20.23	25.96	31.90	62	14.69	21.24	24.27	31.43
13	13.77	17.83	24.75	30.56	63	12.62	19.30	24.73	31.67
14	13.45	19.95	26.70	33.37	64	13.68	18.59	26.45	32.16
15	13.64	19.43	24.95	31.30	65	13.16	19.20	27.53	30.98
16	12.50	19.98	25.11	29.71	66	12.57	18.41	25.17	32.97
17	13.85	22.26	25.47	32.79	67	14.19	19.51	26.27	31.82
18	13.64	19.81	26.85	31.27	68	14.07	19.46	24.95	30.84
19	13.81	20.56	25.07	31.71	69	14.03	19.02	26.66	32.96
20	13.63	17.51	26.17	32.16	70	14.03	20.73	26.78	33.67
21	13.13	19.28	27.93	33.18	71	13.54	19.41	24.67	32.24
22	13.63	20.73	25.31	30.84	72	14.03	19.98	27.85	31.49
23	13.48	18.93	23.98	31.68	73	13.19	19.33	26.23	30.04
24	13.94	19.52	26.33	29.71	74	13.23	19.00	25.40	32.76
25	12.74	20.05	26.15	31.62	75	14.07	20.63	26.27	31.95
26	13.33	19.20	24.96	32.96	76	13.36	19.83	23.39	32.83
27	14.41	19.82	25.65	32.30	77	12.97	19.58	24.57	32.27
28	13.44	21.10	27.69	31.42	78	13.22	19.77	23.29	30.66
29	13.74	18.57	25.19	31.32	79	12.52	18.36	25.73	32.62
30	13.73	19.87	26.48	31.51	80	13.69	18.69	24.60	31.93
31	13.74	19.12	25.74	29.78	81	13.25	20.85	25.87	32.42
32	13.41	19.59	26.75	29.83	82	13.31	18.97	26.48	33.48
33	12.99	19.43	24.04	30.77	83	13.13	19.26	25.77	33.17
34	13.54	19.26	25.51	31.20	84	14.11	18.89	26.04	31.16
35	13.69	20.38	26.00	31.71	85	13.94	20.17	25.07	30.77
36	13.16	21.49	25.52	31.71	86	14.27	19.97	27.10	33.30
37	14.05	20.03	26.52	30.94	87	14.29	18.72	25.45	33.04
38	13.04	18.38	26.03	32.72	88	13.57	20.09	26.35	31.75
39	14.58	18.62	25.26	34.12	89	13.06	19.40	26.57	32.40
40	13.44	18.90	26.02	32.90	90	13.48	20.84	26.66	32.05
41	14.89	19.45	25.03	29.91	91	13.06	19.27	25.09	31.62
42	13.47	19.93	26.48	30.70	92	13.83	20.72	24.67	31.72
43	14.43	21.09	26.33	30.29	93	14.07	19.93	26.34	31.47
44	14.01	20.34	23.80	33.82	94	13.52	20.14	23.91	30.91
45	13.49	19.43	26.31	30.46	95	14.93	19.89	26.89	31.92
46	13.93	19.64	28.00	32.29	96	13.61	17.49	24.59	33.39
47	15.10	20.90	24.52	33.18	97	13.35	20.46	27.56	30.52
48	13.58	18.45	25.23	32.58	98	13.92	19.02	24.88	32.71
49	13.59	20.63	25.97	32.51	99	14.63	20.49	23.57	30.82
50	13.60	18.45	25.58	30.99	100	12.87	19.87	26.19	31.10

Table A.10: EWAEL Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 100$

	p variables					*p* variables					*p* variables			
n	2	3	4	5	n	2	3	4	5	n	2	3	4	5
1	21.33	28.70	34.89	41.35	85	22.65	29.18	35.58	42.93	169	23.92	27.91	36.38	44.22
2	20.73	26.65	34.91	41.85	86	22.28	29.14	39.17	43.46	170	18.87	28.04	34.86	48.02
3	22.03	26.98	38.70	45.60	87	20.59	28.83	34.34	47.16	171	21.00	28.48	36.28	42.99
4	21.38	25.65	34.59	44.76	88	24.06	27.81	42.57	45.60	172	21.38	30.09	36.50	44.54
5	20.57	32.88	38.92	42.72	89	21.00	28.07	33.98	46.54	173	17.70	29.81	33.17	41.80
6	22.59	27.34	36.43	40.91	90	23.31	28.35	40.21	43.02	174	22.49	28.62	34.78	42.88
7	20.58	27.52	37.31	45.83	91	19.75	28.67	34.33	44.04	175	21.64	29.83	36.41	43.51
8	18.73	24.95	37.28	45.06	92	19.00	27.78	33.49	43.47	176	19.67	27.65	38.05	43.14
9	22.06	27.95	37.81	44.40	93	20.44	26.31	39.03	43.15	177	19.85	30.91	38.36	42.80
10	25.12	28.06	34.48	43.98	94	21.41	26.22	36.21	47.08	178	20.22	29.41	38.44	42.60
11	22.56	29.22	33.44	46.34	95	22.41	27.12	35.52	39.72	179	19.48	27.81	34.71	44.44
12	24.74	33.64	37.72	42.75	96	21.49	29.20	41.52	48.36	180	19.77	32.24	36.46	40.40
13	21.05	28.87	33.39	46.33	97	20.20	30.59	34.98	42.80	181	25.61	26.93	39.13	44.84
14	24.33	27.06	35.92	43.96	98	21.93	32.73	36.99	42.61	182	19.73	28.55	41.77	45.94
15	20.45	33.43	38.43	42.56	99	21.08	28.77	39.39	45.31	183	20.36	30.00	36.42	45.29
16	19.91	24.91	41.92	44.00	100	21.62	33.77	38.38	46.78	184	24.07	28.89	34.13	44.80
17	19.40	33.88	42.47	41.85	101	23.45	27.57	37.78	48.13	185	21.85	29.40	38.91	43.46
18	24.26	30.83	38.38	46.48	102	20.99	30.42	35.61	48.81	186	20.12	28.33	37.29	43.62
19	21.08	30.12	35.29	45.24	103	20.65	28.29	34.84	44.15	187	24.26	27.59	34.18	42.73
20	20.65	29.06	38.50	45.59	104	23.60	32.07	38.36	42.83	188	23.75	29.13	39.47	42.10
21	21.53	27.55	36.08	43.76	105	18.86	29.93	38.11	45.47	189	20.56	32.08	33.66	43.86
22	20.39	27.90	37.91	43.32	106	21.52	28.25	38.26	44.82	190	18.25	30.38	36.37	43.09
23	21.54	28.96	36.72	41.87	107	21.08	27.23	36.66	45.82	191	22.29	28.90	36.51	42.15
24	21.71	28.79	34.18	44.23	108	21.42	34.27	38.54	42.80	192	21.52	28.19	42.03	42.40
25	22.37	27.31	34.98	42.28	109	21.65	28.35	38.98	46.75	193	23.20	31.65	42.13	43.44
26	20.34	28.43	36.30	45.50	110	21.41	28.80	35.26	43.94	194	22.76	29.31	36.87	45.52
27	17.68	30.07	35.42	41.73	111	23.55	27.35	36.78	43.15	195	21.29	28.61	36.45	40.78
28	24.19	29.20	37.05	46.06	112	22.32	29.18	38.69	42.80	196	24.10	30.93	33.82	44.63
29	20.98	29.11	36.12	43.92	113	22.77	27.59	35.84	38.93	197	19.77	26.74	38.75	50.60
30	21.10	29.43	33.73	43.74	114	21.56	30.38	34.18	44.74	198	20.38	28.70	35.36	43.38
31	20.59	26.81	34.73	43.58	115	26.68	27.20	39.94	45.39	199	22.33	26.86	37.81	44.29
32	22.38	27.02	37.76	39.31	116	21.26	26.72	34.73	43.10	200	23.16	31.58	35.33	42.44
33	23.43	28.91	35.38	43.12	117	21.68	31.35	38.06	46.12	201	21.90	30.91	38.55	46.00
34	22.36	28.16	38.81	40.58	118	21.92	27.76	37.10	43.18	202	20.75	28.59	36.45	42.75
35	19.60	26.85	37.53	42.62	119	19.58	29.95	35.31	49.47	203	21.37	29.80	36.04	44.83
36	20.93	28.43	40.54	44.29	120	21.55	32.03	35.96	45.27	204	18.92	30.13	37.20	45.57
37	24.60	30.86	32.80	43.83	121	22.97	27.39	35.04	46.52	205	20.87	32.27	37.99	44.98
38	24.00	29.08	34.80	44.06	122	19.98	27.61	37.58	48.52	206	21.29	29.10	39.09	42.44
39	20.71	28.65	40.92	42.44	123	22.19	29.73	34.76	42.94	207	23.51	29.37	37.77	45.81
40	20.11	26.73	40.04	43.41	124	23.73	28.45	37.88	45.38	208	22.42	29.55	38.27	47.92
41	20.77	28.83	37.15	46.08	125	19.62	29.46	35.09	43.11	209	22.95	31.50	35.49	45.30
42	19.17	28.93	37.32	42.21	126	18.31	28.68	35.56	41.35	210	19.41	30.21	36.75	47.75
43	24.64	28.42	35.63	45.33	127	21.93	27.48	36.88	42.65	211	22.41	31.37	36.94	47.71
44	21.61	30.28	35.16	45.45	128	21.99	29.97	39.30	46.94	212	20.32	29.69	36.43	42.97
45	22.09	27.36	37.84	45.15	129	21.69	29.92	37.39	42.06	213	19.27	26.71	32.60	41.87
46	22.29	32.23	35.28	39.17	130	19.74	26.06	35.17	40.88	214	23.30	30.09	38.30	45.72
47	23.42	26.20	38.09	43.00	131	19.17	29.33	38.08	45.60	215	21.54	32.86	37.52	43.62
48	21.94	30.54	35.82	40.48	132	20.13	32.63	37.73	41.55	216	24.51	30.34	32.95	41.55
49	22.53	28.51	35.48	43.52	133	21.22	30.83	35.90	43.28	217	22.02	26.23	34.39	42.79
50	24.24	30.96	37.60	46.91	134	21.37	31.93	37.43	43.51	218	21.30	28.94	34.33	40.88
51	18.17	28.53	33.09	44.22	135	22.97	26.09	32.15	43.64	219	23.93	29.24	36.58	50.64
52	23.02	28.95	37.39	42.17	136	21.95	30.06	38.31	47.02	220	20.47	30.40	34.73	44.58
53	24.32	30.97	36.59	42.56	137	22.22	28.57	35.72	49.28	221	18.14	28.40	38.77	48.13
54	21.39	30.19	35.73	46.23	138	25.19	32.95	37.95	45.95	222	22.14	31.82	36.10	44.58
55	19.78	26.64	38.35	46.36	139	24.14	29.52	36.34	46.37	223	23.93	28.86	37.83	43.74
56	26.21	30.29	39.11	43.74	140	20.70	30.34	37.05	46.08	224	22.30	26.24	35.67	42.25
57	21.10	30.31	35.08	42.50	141	24.41	26.63	35.37	42.24	225	22.89	26.75	35.80	44.25
58	22.43	34.47	36.60	45.74	142	19.81	29.03	40.40	40.41	226	20.77	29.61	37.89	43.97
59	21.02	30.86	39.61	50.01	143	20.79	28.77	37.27	42.79	227	23.50	32.70	34.58	44.02
60	20.18	29.22	38.62	44.92	144	24.19	28.58	41.63	42.51	228	19.07	27.79	32.43	46.64
61	20.12	32.40	37.12	48.23	145	20.37	26.52	34.00	42.81	229	21.74	32.36	38.39	43.57
62	23.84	28.82	36.32	41.43	146	23.28	29.43	37.96	40.60	230	20.53	26.82	36.57	44.06
63	19.14	32.10	38.15	41.32	147	21.74	28.33	38.80	47.25	231	21.60	30.09	35.28	48.34
64	19.26	31.83	35.23	47.17	148	22.32	30.72	37.26	43.54	232	21.13	30.78	37.00	44.30
65	22.62	30.27	34.00	43.88	149	22.75	26.39	39.84	42.33	233	20.12	28.39	35.79	42.25
66	22.68	28.69	35.74	43.59	150	20.67	27.84	34.45	46.21	234	22.46	28.26	37.36	40.28
67	23.36	28.59	37.53	43.22	151	24.45	28.69	33.33	44.41	235	23.58	29.49	35.69	43.54
68	24.85	27.44	38.05	44.79	152	20.62	27.05	34.27	50.02	236	23.61	32.10	34.68	48.16
69	24.92	28.01	37.27	44.78	153	20.20	28.08	37.78	45.40	237	20.55	29.45	36.77	39.97
70	23.92	33.77	33.15	41.01	154	20.79	28.91	37.36	40.16	238	22.60	26.61	36.54	44.16
71	20.26	30.60	36.31	43.71	155	20.56	28.32	37.33	43.17	239	18.83	27.95	31.37	44.60
72	22.35	28.45	38.27	40.70	156	21.71	26.13	37.28	43.39	240	27.87	29.17	37.02	45.25
73	24.30	28.04	38.31	44.01	157	25.00	28.08	36.03	42.10	241	21.44	31.27	36.10	46.31
74	21.71	27.85	35.46	41.44	158	24.73	29.31	38.87	47.62	242	22.49	30.15	37.99	46.39
75	20.38	31.16	36.43	42.39	159	19.76	26.46	38.06	41.45	243	23.29	28.02	37.59	41.63
76	21.78	28.48	33.70	43.78	160	21.61	28.05	36.30	44.17	244	18.62	30.78	34.12	43.27
77	21.26	25.80	33.97	39.99	161	20.50	30.11	38.44	45.32	245	20.70	27.01	38.51	50.43
78	25.66	30.01	34.23	45.30	162	25.71	30.22	36.85	42.76	246	20.75	29.97	40.75	44.35
79	21.79	27.21	39.80	45.89	163	21.33	32.36	40.29	43.20	247	20.88	27.49	38.87	43.62
80	22.48	27.92	38.05	44.28	164	21.56	26.19	35.40	42.84	248	21.86	30.33	36.51	46.38
81	20.48	30.46	35.85	43.83	165	23.05	28.61	38.96	44.11	249	20.74	28.49	34.09	44.91
82	20.40	27.26	35.14	41.09	166	19.41	28.99	37.18	42.71	250	19.42	26.63	36.04	40.82
83	22.72	26.37	36.72	47.57	167	22.14	25.10	38.05	43.87	251	22.71	29.72	34.14	45.99
84	19.92	28.57	37.25	44.56	168	24.83	31.82	36.63	50.55	252	20.62	27.90	33.89	43.55

EWAEL Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 100$

	p variables					p variables					p variables			
n	2	3	4	5	n	2	3	4	5	n	2	3	4	5
253	20.63	28.17	35.06	45.06	337	20.99	33.10	39.32	42.84	421	23.65	29.85	35.09	46.79
254	23.81	32.24	38.59	47.03	338	22.34	31.00	35.89	41.61	422	21.00	30.68	39.86	46.82
255	23.11	29.49	37.41	42.05	339	19.87	28.68	37.90	44.40	423	20.44	28.25	36.28	46.03
256	23.32	31.25	35.33	44.82	340	24.31	27.66	33.18	44.21	424	19.74	31.34	37.85	41.80
257	21.38	29.34	36.10	42.23	341	19.48	25.91	34.11	42.33	425	23.14	28.31	38.33	48.76
258	22.44	28.68	37.08	51.14	342	21.71	27.42	37.86	41.53	426	23.39	27.10	38.85	44.65
259	21.32	30.09	39.26	44.66	343	27.54	26.84	38.20	44.52	427	20.23	29.83	34.97	41.86
260	19.54	25.74	37.33	43.02	344	23.92	30.35	37.95	43.34	428	21.24	26.01	38.70	43.89
261	22.18	28.44	36.05	44.74	345	21.43	28.99	35.21	43.66	429	22.04	31.81	39.45	42.16
262	20.40	30.25	37.34	43.99	346	23.45	27.80	39.91	46.45	430	23.87	27.99	39.43	43.76
263	19.85	31.25	36.47	46.20	347	21.71	28.71	37.21	37.07	431	22.57	32.80	38.62	42.80
264	23.09	32.33	36.84	44.53	348	23.97	28.43	36.93	46.54	432	22.20	25.70	35.91	45.57
265	22.64	31.77	35.35	42.48	349	23.18	29.77	35.14	43.23	433	22.61	34.14	36.04	42.07
266	21.01	28.55	37.05	43.19	350	23.32	30.97	36.38	49.26	434	21.51	30.60	36.52	42.72
267	21.12	30.25	37.12	47.23	351	20.09	25.47	37.92	42.46	435	22.67	27.14	37.17	42.05
268	21.71	30.76	33.33	43.37	352	21.39	29.45	34.09	44.73	436	26.62	29.33	37.98	43.31
269	20.32	30.15	39.72	45.84	353	21.34	27.06	36.65	46.06	437	20.98	28.19	33.19	50.88
270	19.82	30.05	35.82	43.66	354	22.03	28.34	34.27	43.07	438	21.14	31.14	36.18	48.29
271	21.37	34.62	35.22	44.59	355	20.51	29.25	37.17	43.99	439	21.33	26.52	37.31	47.58
272	20.90	30.65	34.73	46.06	356	23.08	28.48	36.38	47.72	440	21.60	27.14	33.67	43.29
273	20.91	33.09	33.33	48.41	357	23.35	25.42	37.26	39.29	441	21.37	29.86	37.29	46.16
274	24.45	29.27	39.07	45.18	358	21.87	26.96	34.41	45.40	442	20.53	30.32	37.83	48.04
275	22.39	27.72	36.75	41.46	359	20.97	28.02	39.48	47.18	443	21.97	27.38	35.73	42.86
276	24.04	30.02	34.51	42.85	360	22.21	32.79	35.46	41.83	444	21.19	27.95	35.70	43.16
277	20.41	29.27	41.51	44.80	361	24.87	29.10	35.85	45.88	445	21.86	30.70	35.09	39.08
278	21.43	29.16	33.43	44.17	362	22.19	28.71	39.48	41.71	446	19.76	33.28	36.40	42.70
279	24.10	28.49	33.47	43.84	363	19.82	24.31	36.65	45.90	447	24.69	28.18	39.38	40.69
280	22.73	31.20	38.02	46.04	364	21.34	33.73	38.98	44.94	448	21.80	30.82	32.20	44.06
281	23.04	31.41	39.67	44.29	365	22.25	27.94	35.25	47.65	449	20.23	26.91	37.12	44.22
282	23.00	30.40	36.55	49.72	366	24.01	31.11	35.66	44.88	450	21.99	27.28	35.67	41.58
283	20.20	28.50	35.51	40.11	367	23.98	29.32	37.02	42.11	451	21.89	29.05	37.30	45.78
284	21.85	28.12	37.49	42.03	368	21.41	27.76	37.47	43.30	452	21.85	29.91	37.50	41.24
285	20.28	26.98	31.96	44.71	369	24.26	30.27	38.26	44.56	453	24.00	29.76	33.56	47.08
286	23.93	29.04	37.43	43.30	370	19.16	27.73	38.02	49.48	454	19.60	28.36	39.23	48.67
287	20.80	27.83	39.47	46.20	371	22.25	28.39	35.76	47.55	455	18.99	29.72	36.96	41.58
288	21.31	27.78	36.53	44.90	372	20.26	29.20	34.06	45.97	456	24.32	26.41	35.34	44.45
289	24.62	32.32	38.95	43.42	373	20.17	25.80	36.53	45.73	457	21.91	29.81	37.17	46.38
290	20.58	28.25	38.73	46.68	374	21.93	28.99	40.35	44.43	458	20.22	27.53	36.03	47.95
291	20.90	27.76	36.60	43.65	375	20.51	27.84	36.87	47.91	459	21.10	28.66	36.20	41.89
292	22.36	29.18	37.83	44.02	376	20.70	27.65	36.30	48.97	460	22.55	27.77	36.96	40.91
293	24.04	29.02	38.55	39.48	377	17.86	30.55	36.63	45.04	461	20.55	32.12	37.54	44.32
294	23.28	34.20	33.32	43.39	378	23.26	26.16	38.51	42.59	462	20.38	28.36	34.28	47.16
295	23.63	28.07	35.77	42.33	379	21.54	28.50	36.90	44.27	463	20.78	31.64	35.39	46.74
296	22.31	29.36	38.52	44.69	380	22.58	27.18	35.11	42.96	464	21.46	28.59	35.40	44.08
297	21.69	30.02	41.72	43.40	381	21.30	31.32	34.46	47.37	465	20.78	27.76	37.64	42.23
298	18.60	28.29	39.33	48.09	382	21.90	27.60	39.00	45.39	466	21.17	28.81	37.44	43.84
299	25.03	33.35	37.90	45.82	383	22.21	30.12	39.52	46.99	467	21.57	27.47	39.80	45.42
300	22.93	33.37	37.79	44.43	384	21.06	29.82	38.76	44.18	468	21.50	30.34	33.62	43.72
301	21.88	29.93	38.81	46.36	385	23.10	30.19	33.88	44.33	469	20.13	29.71	38.17	43.38
302	22.18	26.21	37.71	44.40	386	23.03	27.80	35.02	43.29	470	18.90	32.25	38.70	46.63
303	22.40	31.34	36.15	42.77	387	22.03	30.68	38.48	41.34	471	20.91	27.77	38.63	41.61
304	23.40	27.67	40.42	42.54	388	23.86	31.19	40.33	44.27	472	24.87	32.56	36.83	45.70
305	22.33	28.40	35.25	40.36	389	20.06	26.08	37.87	41.74	473	22.62	28.55	36.99	43.23
306	24.10	27.38	36.78	46.70	390	19.79	28.37	36.94	45.42	474	20.32	28.37	36.87	44.20
307	19.46	31.04	38.68	45.67	391	23.91	26.74	37.81	43.16	475	19.56	27.09	35.54	43.23
308	20.34	26.87	38.31	40.50	392	20.39	31.90	39.44	42.63	476	21.15	32.66	34.39	44.54
309	25.75	32.69	34.49	48.78	393	24.63	27.13	34.98	43.87	477	20.56	29.46	32.58	44.55
310	22.96	29.53	34.82	48.32	394	20.88	30.77	39.05	46.47	478	23.83	28.67	35.15	49.16
311	20.42	28.65	38.03	42.09	395	24.71	29.74	35.76	41.93	479	21.58	31.01	35.44	42.32
312	25.31	29.28	39.48	46.08	396	20.18	28.89	34.58	45.65	480	20.23	31.68	38.94	42.34
313	22.20	31.65	36.76	50.52	397	21.38	25.91	38.91	46.85	481	20.85	30.51	32.63	45.79
314	24.31	28.80	34.01	45.90	398	22.39	31.33	36.91	44.35	482	20.26	32.69	35.33	45.09
315	21.54	25.99	41.61	49.70	399	21.90	31.46	33.95	39.69	483	19.56	30.11	36.27	45.89
316	21.51	29.59	36.17	47.94	400	23.53	28.69	37.19	42.38	484	22.31	30.97	36.98	40.90
317	21.34	26.22	38.93	43.74	401	21.19	29.19	37.08	45.23	485	19.61	30.97	36.62	44.04
318	21.20	27.10	34.72	43.75	402	20.39	31.40	39.71	41.09	486	22.36	30.99	39.82	46.70
319	24.21	29.96	35.27	42.15	403	21.58	26.57	34.25	46.78	487	24.22	30.83	39.39	47.20
320	21.70	30.63	37.11	45.84	404	23.93	30.35	37.94	45.22	488	24.03	27.99	36.18	46.17
321	23.79	26.88	38.77	45.50	405	21.84	26.87	35.82	41.55	489	21.57	31.19	36.48	46.21
322	22.25	26.74	37.28	48.47	406	21.82	27.97	40.65	41.19	490	20.33	28.56	36.58	44.71
323	21.83	29.36	34.70	45.57	407	22.93	26.04	40.36	42.87	491	22.42	27.57	39.18	42.69
324	20.47	27.65	37.84	44.15	408	21.89	26.04	37.19	41.57	492	22.06	28.87	36.80	43.26
325	23.54	32.48	36.07	40.71	409	22.46	32.71	37.35	41.62	493	23.97	28.91	40.72	41.00
326	23.03	28.41	37.97	42.37	410	23.47	29.56	41.03	45.92	494	23.73	29.27	33.97	46.23
327	21.41	29.96	33.24	43.91	411	20.96	25.57	35.37	46.34	495	21.50	30.05	36.89	45.44
328	21.36	28.55	36.75	44.16	412	23.39	28.85	35.43	45.01	496	23.19	30.62	33.33	42.03
329	23.08	30.95	38.56	44.99	413	22.59	27.98	38.61	43.81	497	19.70	31.63	35.73	43.86
330	22.60	28.56	35.38	39.32	414	23.22	28.64	36.61	42.64	498	22.21	30.14	33.71	47.18
331	21.75	30.44	37.05	40.87	415	19.25	30.51	33.32	41.89	499	20.00	31.05	36.94	43.34
332	21.41	28.68	35.58	39.72	416	23.95	31.81	36.18	43.67	500	24.22	29.88	36.06	45.30
333	20.02	30.14	34.20	40.52	417	20.60	28.02	37.07	39.23					
334	22.36	30.85	37.09	44.26	418	20.64	29.65	38.63	43.83					
335	21.76	29.45	40.82	47.37	419	23.03	25.69	35.39	49.16					
336	22.01	29.74	36.36	48.16	420	22.06	29.09	33.74	46.46					

Table A.11: EWAEL Cut-off values $\alpha = 0.05$, $m_0 = 100$, for $p = 2$, $X_1 \sim N(0,1)$, $X_2 \sim$ Uniform or t_3, for $p = 5$, $X_1, X_2, X_3 \sim N(0,1)$, $X_4, X_5 \sim$ both Uniform or t_3

	Uniform		t_3			Uniform		t_3	
n	$p = 2$	$p = 5$	$p = 2$	$p = 5$	n	$p = 2$	$p = 5$	$p = 2$	$p = 5$
1	13.37	30.20	14.99	30.96	51	13.82	30.83	13.33	30.95
2	14.22	33.94	14.57	34.46	52	14.41	34.21	14.12	32.87
3	13.26	31.46	14.34	31.49	53	12.94	32.50	13.62	32.75
4	13.47	32.05	13.30	33.02	54	14.33	30.95	14.50	32.08
5	11.87	31.64	13.53	32.42	55	13.49	32.13	13.34	34.09
6	13.93	33.08	14.52	31.70	56	12.87	31.81	13.96	32.72
7	13.84	29.29	13.92	30.44	57	14.28	32.42	14.50	32.94
8	14.10	30.83	15.30	33.10	58	12.19	32.24	13.98	30.43
9	13.38	30.71	13.16	31.19	59	13.73	32.57	14.04	30.35
10	13.62	32.68	13.83	32.81	60	14.46	31.10	15.45	31.62
11	13.86	29.64	16.12	32.22	61	12.96	30.67	13.48	33.13
12	14.17	30.43	13.34	31.54	62	13.55	32.16	14.82	31.71
13	14.19	31.24	15.40	31.25	63	13.27	32.30	14.54	30.12
14	13.28	31.48	14.03	31.03	64	13.58	31.72	12.75	31.36
15	13.40	31.78	12.93	31.91	65	12.05	31.12	14.31	32.08
16	12.79	33.52	13.36	32.76	66	13.64	30.92	15.04	31.60
17	13.29	31.34	14.39	31.56	67	12.64	32.28	14.46	32.48
18	13.98	32.12	14.18	32.27	68	12.23	30.58	14.45	31.56
19	12.96	30.32	13.61	31.54	69	12.56	32.72	13.76	33.56
20	13.14	31.97	13.73	30.83	70	12.78	32.83	13.17	30.94
21	12.46	32.59	13.30	32.24	71	13.62	31.15	13.98	32.57
22	13.24	31.21	13.24	33.02	72	13.08	31.81	13.98	32.96
23	13.91	31.43	14.05	31.45	73	13.55	31.32	14.61	33.43
24	14.05	32.85	14.05	29.08	74	13.68	32.51	14.48	32.62
25	13.16	30.93	15.05	30.99	75	14.03	30.94	13.32	31.32
26	13.41	31.57	13.56	31.35	76	13.04	32.10	12.92	33.40
27	13.13	34.15	13.82	30.74	77	13.41	30.58	13.52	33.83
28	13.65	32.02	14.03	33.72	78	12.89	31.37	13.47	33.00
29	13.13	30.17	13.70	32.24	79	14.34	32.79	13.91	31.77
30	14.24	30.47	12.89	31.84	80	13.56	34.02	14.11	32.51
31	13.58	31.31	12.79	31.53	81	13.80	31.52	14.11	32.80
32	13.40	30.46	14.80	32.08	82	13.13	30.98	13.42	30.30
33	12.71	31.63	13.39	32.68	83	12.71	32.12	14.68	32.76
34	13.41	31.14	14.49	30.52	84	13.25	32.56	13.43	32.80
35	13.31	30.42	14.57	33.21	85	14.54	32.31	13.18	30.35
36	15.70	30.82	14.98	31.23	86	13.33	32.45	15.26	31.98
37	13.51	31.57	13.88	30.86	87	13.28	31.29	14.20	32.64
38	14.37	33.26	13.14	32.13	88	12.80	31.90	14.05	31.82
39	13.05	33.80	13.31	31.16	89	13.60	34.18	15.23	32.59
40	14.04	29.65	13.21	32.40	90	13.22	31.42	13.58	31.93
41	13.91	33.15	14.05	33.40	91	13.41	31.80	14.49	31.49
42	12.20	32.83	13.27	31.72	92	12.75	32.41	15.03	34.93
43	13.17	31.46	13.88	32.70	93	12.51	33.30	13.60	31.87
44	14.20	33.16	13.74	32.83	94	13.33	31.04	14.16	32.16
45	14.21	33.67	15.61	30.74	95	13.77	31.44	13.66	32.82
46	13.55	32.81	13.88	32.55	96	14.35	32.45	13.35	33.50
47	12.87	31.53	15.08	31.17	97	12.77	32.59	13.17	30.07
48	12.68	34.60	14.06	31.78	98	13.68	33.42	14.27	29.81
49	13.12	32.46	14.28	30.82	99	13.09	31.14	14.13	31.37
50	12.32	31.09	13.54	32.51	100	13.64	32.59	13.62	32.86

Table A.12: EWAEL Cut-off values $\alpha = 0.01$, $m_0 = 100$, for $p = 2$, $X_1 \sim N(0,1)$, $X_2 \sim$ Uniform or t_3, for $p = 5$, $X_1, X_2, X_3 \sim N(0,1)$, $X_4, X_5 \sim$ both Uniform or t_3

	Uniform		t_3			Uniform		t_3			Uniform		t_3	
n	$p = 2$	$p = 5$	$p = 2$	$p = 5$	n	$p = 2$	$p = 5$	$p = 2$	$p = 5$	n	$p = 2$	$p = 5$	$p = 2$	$p = 5$
1	22.51	48.64	22.51	43.05	85	24.76	48.77	24.76	42.01	169	20.33	48.15	20.33	42.60
2	22.35	44.84	22.35	48.89	86	22.50	45.01	22.50	44.60	170	23.15	40.94	23.15	44.27
3	21.77	41.66	21.77	43.16	87	21.80	40.83	21.80	43.59	171	21.08	45.83	21.08	46.66
4	21.98	44.20	21.98	47.13	88	22.02	43.27	22.02	41.59	172	23.18	46.06	23.18	45.73
5	20.66	42.50	20.66	43.86	89	22.25	45.19	22.25	42.24	173	22.05	42.39	22.05	44.47
6	20.11	43.16	20.11	42.35	90	20.53	44.52	20.53	44.23	174	19.09	41.75	19.09	39.85
7	20.85	45.56	20.85	41.15	91	21.58	44.45	21.58	43.23	175	21.21	43.17	21.21	46.64
8	22.24	43.27	22.24	50.43	92	20.13	43.93	20.13	46.52	176	19.77	47.12	19.77	42.85
9	20.82	40.80	20.82	42.79	93	22.05	46.93	22.05	44.04	177	21.19	47.53	21.19	46.35
10	20.97	44.19	20.97	45.03	94	22.67	40.49	22.67	43.76	178	21.10	43.84	21.10	46.26
11	21.82	38.43	21.82	42.88	95	20.24	47.47	20.24	45.32	179	22.57	42.62	22.57	44.03
12	21.85	42.51	21.85	47.47	96	22.74	47.05	22.74	45.79	180	20.39	43.08	20.39	43.29
13	20.64	44.54	20.64	41.17	97	19.45	45.84	19.45	46.96	181	22.69	45.61	22.69	42.25
14	21.59	44.58	21.59	42.50	98	24.73	47.75	24.73	40.93	182	18.61	45.01	18.61	43.84
15	18.82	45.77	18.82	45.48	99	21.82	44.66	21.82	44.25	183	23.67	42.27	23.67	43.54
16	22.97	44.41	22.97	48.87	100	22.19	44.34	22.19	45.22	184	21.93	44.32	21.93	46.07
17	22.05	43.01	22.05	45.65	101	21.90	46.35	21.90	45.22	185	21.93	45.60	21.93	47.08
18	23.51	41.28	23.51	40.74	102	17.75	44.19	17.75	41.26	186	19.12	46.55	19.12	42.15
19	20.55	40.19	20.55	45.73	103	21.14	44.47	21.14	45.70	187	21.75	48.40	21.75	43.25
20	26.25	44.93	26.25	44.67	104	19.77	42.43	19.77	42.51	188	23.65	43.35	23.65	44.28
21	19.56	42.54	19.56	44.00	105	19.14	47.25	19.14	44.31	189	20.07	48.80	20.07	48.76
22	19.51	46.19	19.51	44.95	106	23.03	41.23	23.03	43.31	190	23.59	48.65	23.59	46.36
23	20.45	43.02	20.45	42.64	107	22.52	43.84	22.52	42.18	191	22.03	45.32	22.03	47.11
24	20.45	46.54	20.45	38.89	108	20.53	45.52	20.53	45.12	192	22.34	41.83	22.34	45.84
25	19.83	43.71	19.83	44.87	109	20.67	42.15	20.67	46.61	193	22.98	43.92	22.98	46.52
26	21.06	43.98	21.06	50.22	110	23.60	45.47	23.60	43.43	194	20.63	47.17	20.63	42.77
27	21.06	48.64	21.06	42.72	111	21.34	45.28	21.34	42.81	195	24.07	46.80	24.07	44.60
28	21.59	43.92	21.59	48.00	112	20.99	46.59	20.99	41.02	196	24.95	44.75	24.95	40.05
29	21.42	41.42	21.42	45.62	113	21.72	47.13	21.72	40.14	197	22.45	43.31	22.45	40.18
30	24.09	41.96	24.09	46.58	114	23.36	41.54	23.36	43.74	198	20.90	44.06	20.90	42.00
31	19.25	40.37	19.25	43.97	115	20.56	45.65	20.56	42.60	199	21.96	50.71	21.96	46.55
32	21.08	44.35	21.08	45.17	116	22.48	44.67	22.48	43.57	200	20.85	44.83	20.85	44.23
33	20.65	45.35	20.65	42.43	117	22.28	46.62	22.28	48.19	201	20.72	47.63	20.72	40.32
34	20.95	44.39	20.95	40.08	118	22.68	42.54	22.68	47.79	202	22.26	44.05	22.26	46.86
35	19.68	45.39	19.68	45.26	119	20.28	47.00	20.28	47.70	203	19.70	44.56	19.70	46.67
36	22.94	41.73	22.94	47.23	120	23.40	41.29	23.40	44.30	204	20.31	43.75	20.31	47.37
37	22.65	42.49	22.65	45.16	121	21.02	45.17	21.02	46.87	205	21.77	42.36	21.77	46.21
38	21.34	43.25	21.34	43.52	122	20.92	45.52	20.92	44.52	206	19.93	43.71	19.93	44.81
39	21.08	46.24	21.08	42.01	123	21.70	47.47	21.70	41.88	207	19.61	40.20	19.61	41.77
40	22.80	45.68	22.80	44.55	124	21.66	43.12	21.66	40.25	208	21.55	42.55	21.55	43.07
41	23.12	46.25	23.12	49.20	125	19.25	46.36	19.25	43.85	209	21.74	44.57	21.74	48.49
42	18.73	44.74	18.73	43.92	126	18.04	48.92	18.04	48.18	210	22.43	42.50	22.43	44.73
43	20.25	43.10	20.25	44.97	127	21.78	40.34	21.78	48.91	211	22.74	42.00	22.74	45.06
44	21.88	46.80	21.88	43.82	128	19.81	47.43	19.81	41.62	212	18.41	47.38	18.41	45.17
45	20.62	49.75	20.62	42.61	129	22.63	45.24	22.63	41.12	213	23.33	45.63	23.33	41.39
46	21.10	44.42	21.10	47.70	130	22.93	44.67	22.93	48.14	214	19.95	44.67	19.95	44.00
47	22.08	48.94	22.08	43.49	131	21.31	46.20	21.31	46.00	215	22.21	42.78	22.21	46.16
48	23.88	47.16	23.88	50.29	132	20.06	47.06	20.06	40.93	216	21.57	46.09	21.57	48.56
49	19.48	43.49	19.48	44.19	133	23.86	41.45	23.86	44.81	217	26.13	45.56	26.13	46.52
50	21.88	43.19	21.88	41.49	134	20.08	43.95	20.08	42.02	218	23.05	41.43	23.05	43.57
51	22.30	43.12	22.30	44.37	135	21.90	42.55	21.90	46.23	219	18.35	41.61	18.35	41.60
52	20.51	46.98	20.51	43.49	136	19.78	43.46	19.78	42.07	220	21.09	49.25	21.09	42.43
53	17.62	43.06	17.62	42.92	137	18.72	43.95	18.72	46.17	221	18.55	43.61	18.55	44.37
54	23.01	38.78	23.01	44.83	138	23.89	43.94	23.89	49.90	222	21.02	47.14	21.02	45.26
55	20.49	45.33	20.49	46.18	139	19.26	41.37	19.26	46.62	223	20.38	42.52	20.38	41.92
56	21.73	42.08	21.73	48.93	140	22.41	41.69	22.41	48.22	224	21.62	43.13	21.62	46.81
57	21.62	49.27	21.62	43.07	141	21.72	44.32	21.72	43.40	225	20.33	47.68	20.33	44.34
58	17.70	45.57	17.70	41.46	142	19.96	48.49	19.96	46.49	226	20.76	44.87	20.76	46.27
59	20.67	44.25	20.67	41.96	143	22.15	47.75	22.15	42.36	227	21.11	44.75	21.11	41.32
60	23.07	40.52	23.07	41.84	144	21.59	43.26	21.59	42.63	228	21.34	47.23	21.34	43.45
61	20.94	45.83	20.94	43.03	145	22.46	43.00	22.46	41.38	229	19.00	49.40	19.00	46.14
62	21.52	45.19	21.52	46.11	146	20.20	40.79	20.20	45.67	230	17.81	41.70	17.81	42.10
63	22.67	42.60	22.67	43.68	147	21.21	47.81	21.21	43.88	231	21.96	45.60	21.96	40.90
64	23.37	38.87	23.37	45.23	148	21.53	43.71	21.53	48.19	232	22.96	47.29	22.96	43.79
65	21.85	43.70	21.85	46.37	149	20.84	42.39	20.84	44.56	233	21.94	43.76	21.94	44.14
66	23.10	46.19	23.10	44.51	150	19.27	45.06	19.27	42.14	234	20.74	41.13	20.74	45.97
67	24.77	42.65	24.77	45.43	151	20.22	46.38	20.22	49.01	235	24.03	47.55	24.03	48.71
68	19.60	42.92	19.60	41.95	152	20.09	41.49	20.09	44.90	236	23.31	43.49	23.31	44.47
69	19.87	42.26	19.87	45.74	153	21.22	46.77	21.22	38.50	237	23.86	39.61	23.86	48.02
70	18.57	44.95	18.57	41.67	154	20.54	46.76	20.54	44.47	238	22.17	44.15	22.17	49.66
71	24.02	41.03	24.02	43.78	155	18.74	43.93	18.74	47.06	239	23.02	46.08	23.02	43.98
72	22.86	46.85	22.86	47.18	156	21.24	43.16	21.24	46.32	240	22.88	45.51	22.88	47.08
73	19.52	40.54	19.52	45.87	157	20.88	45.38	20.88	46.98	241	21.36	45.59	21.36	43.43
74	20.53	43.31	20.53	45.16	158	22.95	47.09	22.95	42.46	242	22.88	43.25	22.88	45.92
75	24.31	42.19	24.31	42.05	159	24.16	47.40	24.16	42.22	243	21.07	43.24	21.07	45.84
76	21.78	44.11	21.78	43.55	160	22.69	47.10	22.69	42.85	244	21.29	43.32	21.29	46.14
77	18.90	40.04	18.90	47.37	161	20.11	43.11	20.11	46.75	245	23.07	42.87	23.07	43.38
78	20.15	46.86	20.15	46.46	162	21.23	47.09	21.23	43.14	246	20.63	42.61	20.63	41.31
79	23.03	50.07	23.03	44.55	163	22.96	42.63	22.96	42.83	247	20.62	46.87	20.62	45.25
80	21.01	43.97	21.01	39.96	164	20.22	49.31	20.22	45.67	248	21.68	44.43	21.68	41.91
81	20.30	41.30	20.30	44.30	165	19.22	47.93	19.22	51.43	249	19.36	46.94	19.36	44.17
82	21.61	39.95	21.61	39.69	166	20.15	45.09	20.15	44.45	250	20.78	46.67	20.78	47.94
83	19.32	45.98	19.32	41.76	167	20.55	43.16	20.55	41.06	251	21.12	44.12	21.12	46.90
84	22.24	43.19	22.24	45.46	168	19.55	42.35	19.55	47.78	252	24.29	46.04	24.29	48.54

EWAEL Cut-off values $\alpha = 0.01$, $m_0 = 100$, for $p = 2$, $X_1 \sim N(0,1)$, $X_2 \sim$ Uniform or t_3, for $p = 5$, $X_1, X_2, X_3 \sim N(0,1)$, $X_4, X_5 \sim$ both Uniform or t_3

	Uniform		t_3			Uniform		t_3			Uniform		t_3	
n	$p=2$	$p=5$	$p=2$	$p=5$	n	$p=2$	$p=5$	$p=2$	$p=5$	n	$p=2$	$p=5$	$p=2$	$p=5$
253	22.17	47.48	22.17	41.84	337	24.23	42.77	24.23	45.40	421	20.89	46.34	20.89	43.97
254	20.91	42.33	20.91	48.44	338	20.88	46.88	20.88	45.01	422	18.09	40.96	18.09	41.89
255	21.37	46.19	21.37	44.71	339	21.40	43.93	21.40	43.34	423	21.33	45.28	21.33	44.39
256	19.82	43.91	19.82	45.15	340	21.25	48.57	21.26	45.79	424	21.05	45.08	21.05	41.06
257	19.51	44.30	19.51	45.87	341	23.23	46.14	23.23	45.37	425	19.28	44.50	19.28	43.92
258	23.33	45.50	23.33	47.53	342	23.01	48.05	23.01	46.99	426	20.09	43.93	20.09	41.22
259	20.75	44.05	20.75	47.11	343	21.93	43.34	21.93	39.38	427	20.70	42.99	20.70	44.91
260	23.36	42.76	23.36	43.88	344	24.37	44.30	24.37	46.52	428	21.47	42.83	21.47	41.92
261	21.75	45.88	21.75	47.76	345	19.26	45.67	19.26	42.20	429	20.96	41.06	20.96	44.91
262	20.73	42.96	20.73	45.53	346	20.85	45.82	20.85	40.45	430	23.84	47.00	23.84	42.62
263	19.20	48.33	19.20	50.71	347	23.24	42.46	23.24	42.92	431	20.94	40.07	20.94	43.41
264	20.37	49.92	20.37	46.10	348	21.02	45.07	21.02	47.33	432	19.85	49.34	19.85	38.98
265	21.23	47.18	21.23	43.93	349	22.90	49.15	22.90	45.23	433	20.62	42.28	20.62	40.97
266	18.01	43.59	18.01	47.28	350	19.95	44.82	19.95	45.93	434	23.72	42.78	23.72	45.40
267	22.47	40.95	22.47	48.71	351	21.20	47.80	21.20	42.60	435	21.15	48.18	21.15	45.78
268	20.95	44.90	20.95	44.65	352	22.37	44.02	22.37	47.87	436	18.95	47.48	18.95	46.46
269	19.79	44.73	19.79	46.41	353	21.78	39.79	21.78	44.36	437	20.81	44.67	20.81	43.27
270	20.55	43.08	20.55	45.35	354	20.07	42.50	20.07	42.68	438	20.25	45.49	20.25	44.93
271	19.56	45.84	19.56	41.03	355	20.12	44.54	20.12	42.16	439	21.60	48.97	21.60	49.22
272	22.32	45.94	22.32	42.45	356	22.06	40.52	22.06	45.73	440	23.83	47.17	23.83	46.65
273	20.49	43.25	20.49	46.67	357	22.39	46.66	22.39	44.99	441	21.29	46.82	21.29	45.00
274	21.74	44.75	21.74	43.91	358	20.89	44.65	20.89	43.47	442	26.31	40.91	26.31	45.57
275	21.64	42.77	21.64	42.27	359	22.41	42.18	22.41	39.09	443	21.01	42.01	21.01	41.01
276	22.18	39.92	22.18	49.99	360	23.35	46.09	23.35	42.53	444	23.20	44.29	23.20	45.38
277	21.59	42.73	21.59	41.63	361	20.26	44.64	20.26	43.86	445	20.92	44.40	20.92	45.82
278	20.30	46.54	20.30	43.20	362	21.98	43.33	21.98	49.07	446	19.65	41.46	19.65	44.35
279	21.97	49.76	21.97	44.11	363	23.75	46.37	23.75	42.96	447	19.89	45.87	19.89	43.88
280	19.99	46.41	19.99	45.30	364	20.26	47.48	20.26	43.43	448	21.38	47.11	21.38	44.12
281	21.23	45.49	21.23	44.29	365	23.04	45.52	23.04	43.71	449	20.97	44.79	20.97	42.41
282	24.06	43.66	24.06	46.47	366	21.46	44.73	21.46	49.66	450	22.42	41.84	22.42	41.10
283	22.00	46.03	22.00	47.62	367	21.01	44.31	21.01	43.62	451	21.15	48.85	21.15	44.87
284	19.48	42.52	19.48	44.05	368	22.24	43.18	22.24	45.26	452	24.10	43.71	24.10	43.64
285	21.22	42.71	21.22	42.40	369	25.62	41.36	25.62	45.97	453	20.10	45.86	20.10	41.49
286	22.10	41.11	22.10	46.50	370	20.50	44.91	20.50	44.24	454	20.45	44.74	20.45	44.15
287	21.08	48.41	21.08	46.75	371	19.87	45.71	19.87	48.68	455	23.42	41.88	23.42	44.59
288	21.83	40.66	21.83	49.16	372	23.09	43.59	23.09	49.18	456	21.30	37.71	21.30	43.59
289	22.60	41.82	22.60	43.15	373	21.10	46.04	21.10	42.76	457	20.19	39.92	20.19	44.32
290	22.41	41.09	22.41	46.72	374	19.31	45.80	19.31	47.61	458	21.67	41.27	21.67	42.89
291	24.67	42.75	24.67	44.88	375	26.54	44.65	26.54	47.02	459	18.04	42.01	18.04	43.53
292	21.68	44.07	21.68	42.71	376	21.65	43.37	21.65	43.03	460	23.25	43.71	23.25	44.80
293	20.23	44.53	20.23	49.60	377	22.02	43.85	22.02	48.19	461	23.03	43.67	23.03	46.12
294	21.97	48.01	21.97	41.65	378	20.32	45.06	20.32	46.18	462	22.71	47.28	22.71	43.78
295	20.54	47.40	20.54	43.80	379	20.69	42.64	20.69	42.83	463	23.52	39.17	23.52	44.44
296	24.35	46.18	24.35	48.27	380	22.51	50.77	22.51	44.08	464	22.12	44.64	22.12	49.38
297	21.93	46.62	21.93	47.29	381	20.02	47.55	20.02	51.37	465	20.52	44.74	20.52	45.89
298	21.84	45.63	21.84	42.64	382	19.52	40.60	19.52	45.39	466	21.79	44.50	21.79	46.09
299	20.03	46.30	20.03	45.58	383	23.95	47.43	23.95	45.75	467	21.74	44.22	21.74	45.39
300	22.13	44.45	22.13	41.55	384	21.70	46.59	21.70	46.87	468	20.54	48.60	20.54	44.82
301	24.39	42.98	24.39	42.59	385	20.55	40.82	20.55	43.70	469	20.10	44.60	20.10	42.06
302	23.39	44.68	23.39	43.26	386	22.21	43.56	22.21	44.81	470	21.68	43.20	21.68	42.58
303	22.05	45.85	22.05	47.66	387	21.53	41.10	21.53	48.22	471	22.44	44.02	22.44	49.09
304	22.85	44.23	22.85	41.59	388	22.13	46.54	22.13	45.82	472	22.08	43.75	22.08	48.33
305	20.01	42.54	20.01	48.00	389	21.57	46.59	21.57	45.98	473	22.20	43.76	22.20	43.04
306	21.57	43.73	21.57	47.19	390	20.17	42.85	20.17	41.01	474	20.65	41.12	20.65	42.67
307	20.26	42.15	20.26	43.85	391	21.92	43.06	21.92	39.66	475	20.77	44.44	20.77	42.59
308	19.52	46.93	19.52	42.76	392	19.78	43.88	19.78	46.03	476	20.72	42.38	20.72	44.21
309	21.14	49.60	21.14	45.82	393	19.20	41.85	19.20	45.44	477	18.24	47.56	18.24	44.77
310	21.47	41.28	21.47	41.64	394	21.54	45.35	21.54	44.75	478	21.11	46.06	21.11	47.98
311	22.20	42.76	22.20	42.59	395	17.66	43.16	17.66	43.81	479	20.89	47.89	20.89	43.10
312	21.04	42.50	21.04	47.51	396	20.29	44.13	20.29	43.03	480	20.43	41.97	20.43	46.22
313	25.03	43.43	25.03	46.98	397	24.59	45.45	24.59	46.72	481	21.69	44.31	21.69	43.93
314	20.43	47.45	20.43	44.72	398	22.16	43.46	22.16	44.87	482	23.49	48.61	23.49	43.02
315	21.38	49.57	21.38	44.89	399	21.79	42.04	21.79	45.61	483	20.83	42.59	20.83	52.26
316	23.10	49.00	23.10	46.24	400	21.81	48.30	21.81	42.75	484	20.60	43.95	20.60	49.61
317	20.17	43.40	20.17	42.61	401	21.63	46.48	21.63	44.98	485	23.15	41.24	23.15	43.77
318	21.65	41.36	21.65	40.34	402	19.52	40.85	19.52	49.94	486	21.97	43.05	21.97	44.42
319	20.92	47.30	20.92	45.11	403	22.42	48.02	22.42	41.29	487	23.30	49.67	23.30	44.41
320	19.53	43.91	19.53	45.59	404	20.95	42.87	20.95	44.18	488	21.12	42.03	21.12	47.13
321	21.91	41.24	21.91	43.43	405	24.17	40.78	24.17	42.24	489	20.12	44.74	20.12	49.24
322	21.38	42.06	21.38	41.50	406	22.94	46.75	22.94	47.00	490	24.60	44.58	24.60	42.38
323	21.19	42.23	21.19	39.99	407	20.02	46.03	20.02	43.07	491	22.21	45.65	22.21	49.00
324	23.25	43.76	23.25	44.00	408	21.79	44.85	21.79	45.50	492	20.05	41.75	20.05	41.16
325	19.33	45.84	19.33	44.63	409	19.75	43.73	19.75	43.74	493	20.79	41.32	20.79	44.23
326	21.96	47.11	21.96	41.81	410	19.93	45.97	19.93	43.37	494	24.49	48.45	24.49	47.57
327	22.40	47.03	22.40	44.01	411	22.70	45.27	22.70	42.21	495	19.72	44.28	19.72	41.55
328	23.09	44.96	23.09	43.28	412	22.94	46.47	22.94	44.27	496	20.21	42.06	20.21	46.09
329	21.08	44.69	21.08	45.54	413	24.07	43.95	24.07	42.70	497	18.75	45.36	18.75	42.73
330	23.81	41.26	23.81	41.58	414	21.41	46.34	21.41	48.98	498	24.56	44.04	24.56	44.09
331	20.72	49.10	20.72	42.60	415	19.74	45.14	19.74	46.70	499	21.25	47.70	21.25	46.30
332	21.13	43.77	21.13	45.27	416	21.19	44.64	21.19	44.17	500	24.81	43.67	24.81	47.69
333	20.56	46.29	20.56	42.78	417	20.39	43.61	20.39	43.84					
334	22.48	44.59	22.48	42.70	418	20.57	43.60	20.57	46.03					
335	19.71	40.21	19.71	44.47	419	22.99	48.65	22.99	45.53					
336	23.85	43.57	23.85	42.49	420	20.55	41.31	20.55	41.33					

Table A.13: MST Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.05$, $m_0 = 50$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
1	0.938	1.070	1.229	1.332	55	0.325	0.382	0.427	0.459
2	0.820	0.956	1.079	1.178	56	0.329	0.376	0.425	0.467
3	0.756	0.899	0.980	1.077	57	0.326	0.386	0.419	0.463
4	0.707	0.811	0.911	1.003	58	0.328	0.374	0.429	0.464
5	0.660	0.774	0.859	0.944	59	0.320	0.379	0.423	0.457
6	0.619	0.718	0.829	0.897	60	0.326	0.374	0.414	0.463
7	0.575	0.691	0.771	0.855	61	0.325	0.373	0.421	0.461
8	0.572	0.655	0.748	0.811	62	0.325	0.374	0.412	0.451
9	0.564	0.647	0.728	0.775	63	0.318	0.370	0.411	0.448
10	0.533	0.622	0.692	0.765	64	0.315	0.371	0.417	0.450
11	0.509	0.594	0.678	0.736	65	0.308	0.367	0.403	0.452
12	0.501	0.581	0.655	0.711	66	0.314	0.360	0.408	0.447
13	0.486	0.562	0.636	0.695	67	0.310	0.365	0.419	0.443
14	0.477	0.552	0.621	0.682	68	0.315	0.365	0.400	0.446
15	0.467	0.541	0.605	0.670	69	0.312	0.357	0.418	0.447
16	0.466	0.540	0.599	0.660	70	0.309	0.364	0.405	0.448
17	0.446	0.529	0.587	0.640	71	0.303	0.372	0.412	0.438
18	0.434	0.505	0.580	0.632	72	0.309	0.359	0.403	0.442
19	0.435	0.512	0.567	0.612	73	0.304	0.350	0.398	0.438
20	0.433	0.498	0.551	0.607	74	0.308	0.357	0.403	0.435
21	0.419	0.491	0.551	0.597	75	0.313	0.355	0.403	0.431
22	0.403	0.481	0.540	0.589	76	0.308	0.352	0.402	0.440
23	0.405	0.484	0.537	0.579	77	0.307	0.355	0.399	0.432
24	0.395	0.475	0.520	0.583	78	0.305	0.359	0.403	0.432
25	0.400	0.473	0.516	0.564	79	0.303	0.351	0.395	0.434
26	0.399	0.461	0.517	0.567	80	0.305	0.357	0.404	0.435
27	0.384	0.449	0.512	0.555	81	0.303	0.357	0.389	0.431
28	0.392	0.451	0.507	0.551	82	0.301	0.355	0.397	0.428
29	0.377	0.445	0.500	0.563	83	0.306	0.357	0.388	0.433
30	0.387	0.451	0.496	0.538	84	0.305	0.358	0.388	0.437
31	0.380	0.434	0.493	0.533	85	0.300	0.347	0.395	0.428
32	0.381	0.434	0.496	0.526	86	0.299	0.347	0.389	0.423
33	0.369	0.430	0.488	0.526	87	0.303	0.351	0.387	0.432
34	0.358	0.434	0.473	0.524	88	0.299	0.349	0.388	0.426
35	0.360	0.420	0.472	0.516	89	0.304	0.347	0.386	0.426
36	0.361	0.420	0.469	0.519	90	0.302	0.350	0.388	0.420
37	0.361	0.418	0.462	0.508	91	0.300	0.345	0.392	0.423
38	0.358	0.413	0.460	0.513	92	0.301	0.343	0.384	0.424
39	0.350	0.415	0.464	0.508	93	0.295	0.345	0.387	0.422
40	0.354	0.414	0.462	0.494	94	0.290	0.342	0.385	0.418
41	0.346	0.415	0.447	0.492	95	0.300	0.343	0.384	0.422
42	0.338	0.404	0.455	0.507	96	0.298	0.339	0.387	0.413
43	0.348	0.401	0.449	0.484	97	0.292	0.344	0.389	0.423
44	0.342	0.406	0.449	0.496	98	0.292	0.337	0.381	0.422
45	0.349	0.388	0.441	0.486	99	0.295	0.343	0.389	0.418
46	0.338	0.389	0.443	0.488	100	0.293	0.341	0.380	0.418
47	0.341	0.396	0.433	0.481	101	0.291	0.336	0.384	0.409
48	0.342	0.391	0.437	0.481	102	0.292	0.340	0.382	0.413
49	0.330	0.385	0.436	0.475	103	0.288	0.345	0.381	0.419
50	0.326	0.386	0.433	0.473	104	0.296	0.342	0.377	0.419
51	0.328	0.386	0.429	0.471	105	0.288	0.343	0.375	0.419
52	0.334	0.385	0.431	0.478	106	0.286	0.337	0.377	0.415
53	0.330	0.385	0.426	0.466	107	0.285	0.337	0.376	0.415
54	0.327	0.386	0.432	0.470	108	0.293	0.337	0.378	0.414

Table A.14: MST Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 50$

	p variables					p variables					p variables			
n	2	3	4	5	n	2	3	4	5	n	2	3	4	5
1	1.012	1.188	1.327	1.433	85	0.344	0.388	0.439	0.475	169	0.314	0.358	0.390	0.433
2	0.944	1.043	1.196	1.282	86	0.344	0.383	0.434	0.461	170	0.317	0.363	0.391	0.429
3	0.850	0.974	1.068	1.163	87	0.349	0.399	0.416	0.462	171	0.320	0.359	0.388	0.436
4	0.831	0.888	0.968	1.082	88	0.333	0.392	0.432	0.465	172	0.315	0.363	0.402	0.425
5	0.767	0.865	0.939	1.012	89	0.342	0.394	0.432	0.465	173	0.320	0.362	0.399	0.438
6	0.682	0.807	0.908	0.959	90	0.358	0.390	0.428	0.465	174	0.319	0.355	0.390	0.423
7	0.623	0.777	0.851	0.927	91	0.339	0.397	0.433	0.456	175	0.319	0.354	0.389	0.425
8	0.644	0.728	0.825	0.896	92	0.340	0.381	0.425	0.462	176	0.311	0.355	0.396	0.426
9	0.652	0.712	0.811	0.849	93	0.322	0.388	0.429	0.459	177	0.316	0.355	0.393	0.416
10	0.610	0.688	0.768	0.844	94	0.350	0.383	0.414	0.466	178	0.310	0.357	0.393	0.422
11	0.573	0.667	0.744	0.816	95	0.343	0.375	0.436	0.475	179	0.304	0.367	0.391	0.423
12	0.574	0.650	0.702	0.766	96	0.334	0.400	0.419	0.450	180	0.311	0.357	0.383	0.421
13	0.540	0.617	0.723	0.771	97	0.340	0.384	0.423	0.467	181	0.300	0.357	0.391	0.437
14	0.536	0.610	0.680	0.748	98	0.344	0.381	0.426	0.456	182	0.311	0.349	0.404	0.418
15	0.522	0.606	0.664	0.730	99	0.336	0.388	0.424	0.450	183	0.303	0.345	0.390	0.413
16	0.539	0.614	0.666	0.718	100	0.346	0.386	0.421	0.454	184	0.309	0.361	0.391	0.427
17	0.516	0.585	0.651	0.696	101	0.330	0.367	0.426	0.461	185	0.306	0.361	0.393	0.425
18	0.486	0.562	0.626	0.699	102	0.343	0.383	0.427	0.447	186	0.317	0.353	0.380	0.423
19	0.503	0.569	0.626	0.668	103	0.334	0.385	0.421	0.472	187	0.314	0.351	0.406	0.418
20	0.483	0.539	0.603	0.667	104	0.332	0.382	0.412	0.458	188	0.313	0.349	0.392	0.428
21	0.488	0.554	0.609	0.647	105	0.330	0.391	0.415	0.461	189	0.307	0.368	0.384	0.420
22	0.465	0.559	0.590	0.661	106	0.328	0.376	0.426	0.456	190	0.302	0.357	0.400	0.421
23	0.457	0.551	0.583	0.631	107	0.333	0.377	0.411	0.451	191	0.304	0.348	0.382	0.408
24	0.455	0.542	0.578	0.628	108	0.328	0.378	0.418	0.445	192	0.304	0.347	0.398	0.415
25	0.465	0.530	0.567	0.612	109	0.342	0.380	0.420	0.450	193	0.302	0.356	0.387	0.414
26	0.486	0.516	0.576	0.624	110	0.330	0.372	0.422	0.452	194	0.298	0.357	0.378	0.427
27	0.452	0.502	0.567	0.594	111	0.325	0.374	0.408	0.446	195	0.314	0.353	0.391	0.430
28	0.439	0.503	0.541	0.622	112	0.329	0.367	0.413	0.451	196	0.304	0.349	0.386	0.414
29	0.428	0.508	0.554	0.602	113	0.325	0.375	0.412	0.454	197	0.301	0.350	0.390	0.419
30	0.428	0.504	0.556	0.595	114	0.317	0.365	0.413	0.456	198	0.302	0.364	0.388	0.424
31	0.437	0.491	0.547	0.591	115	0.333	0.365	0.421	0.437	199	0.313	0.348	0.401	0.422
32	0.443	0.501	0.538	0.579	116	0.327	0.374	0.404	0.448	200	0.314	0.357	0.390	0.421
33	0.413	0.490	0.531	0.596	117	0.325	0.391	0.414	0.439	201	0.298	0.353	0.381	0.419
34	0.415	0.481	0.525	0.575	118	0.326	0.375	0.410	0.449	202	0.306	0.350	0.403	0.410
35	0.413	0.460	0.519	0.559	119	0.335	0.369	0.419	0.439	203	0.299	0.349	0.386	0.429
36	0.420	0.475	0.533	0.570	120	0.321	0.374	0.402	0.452	204	0.299	0.342	0.402	0.413
37	0.406	0.489	0.516	0.568	121	0.326	0.368	0.406	0.451	205	0.303	0.350	0.386	0.429
38	0.419	0.473	0.492	0.560	122	0.325	0.391	0.412	0.444	206	0.313	0.343	0.378	0.428
39	0.398	0.471	0.511	0.561	123	0.335	0.373	0.405	0.446	207	0.302	0.352	0.392	0.422
40	0.400	0.466	0.496	0.536	124	0.337	0.383	0.407	0.442	208	0.299	0.344	0.385	0.407
41	0.418	0.472	0.493	0.547	125	0.324	0.364	0.404	0.451	209	0.303	0.335	0.383	0.418
42	0.401	0.461	0.497	0.552	126	0.337	0.364	0.411	0.439	210	0.307	0.352	0.376	0.417
43	0.402	0.448	0.492	0.534	127	0.325	0.372	0.418	0.457	211	0.301	0.349	0.375	0.416
44	0.393	0.454	0.493	0.547	128	0.328	0.382	0.401	0.452	212	0.306	0.353	0.381	0.416
45	0.401	0.439	0.493	0.518	129	0.317	0.371	0.415	0.441	213	0.295	0.346	0.390	0.411
46	0.395	0.432	0.493	0.527	130	0.315	0.372	0.419	0.456	214	0.304	0.339	0.386	0.416
47	0.390	0.432	0.485	0.514	131	0.327	0.368	0.403	0.442	215	0.299	0.361	0.377	0.425
48	0.384	0.434	0.478	0.535	132	0.310	0.373	0.402	0.439	216	0.318	0.354	0.379	0.417
49	0.390	0.423	0.485	0.523	133	0.319	0.358	0.403	0.443	217	0.295	0.358	0.388	0.415
50	0.372	0.427	0.469	0.513	134	0.319	0.368	0.398	0.448	218	0.305	0.341	0.384	0.413
51	0.375	0.428	0.482	0.527	135	0.314	0.365	0.402	0.432	219	0.315	0.346	0.383	0.413
52	0.369	0.424	0.478	0.528	136	0.320	0.359	0.403	0.441	220	0.303	0.349	0.391	0.411
53	0.373	0.432	0.476	0.515	137	0.319	0.362	0.396	0.434	221	0.299	0.351	0.393	0.409
54	0.365	0.422	0.473	0.512	138	0.327	0.374	0.399	0.445	222	0.310	0.351	0.389	0.418
55	0.370	0.429	0.474	0.518	139	0.311	0.354	0.396	0.437	223	0.306	0.339	0.377	0.413
56	0.393	0.424	0.465	0.518	140	0.327	0.361	0.411	0.435	224	0.305	0.345	0.387	0.417
57	0.369	0.423	0.459	0.512	141	0.316	0.365	0.395	0.417	225	0.314	0.346	0.378	0.413
58	0.367	0.424	0.474	0.511	142	0.321	0.369	0.402	0.443	226	0.306	0.356	0.383	0.431
59	0.366	0.415	0.470	0.504	143	0.326	0.379	0.404	0.435	227	0.293	0.338	0.376	0.414
60	0.367	0.415	0.446	0.501	144	0.314	0.365	0.398	0.446	228	0.311	0.348	0.380	0.416
61	0.372	0.429	0.464	0.503	145	0.316	0.381	0.402	0.430	229	0.302	0.339	0.394	0.414
62	0.358	0.416	0.454	0.495	146	0.315	0.362	0.397	0.440	230	0.311	0.339	0.387	0.416
63	0.364	0.419	0.470	0.491	147	0.322	0.369	0.393	0.429	231	0.304	0.350	0.371	0.400
64	0.347	0.416	0.463	0.487	148	0.312	0.358	0.389	0.430	232	0.312	0.351	0.392	0.418
65	0.346	0.408	0.438	0.484	149	0.308	0.355	0.414	0.440	233	0.315	0.350	0.382	0.415
66	0.365	0.410	0.446	0.494	150	0.321	0.361	0.395	0.438	234	0.307	0.346	0.386	0.416
67	0.362	0.421	0.457	0.506	151	0.308	0.374	0.400	0.431	235	0.298	0.349	0.392	0.411
68	0.358	0.402	0.450	0.493	152	0.323	0.362	0.405	0.430	236	0.301	0.340	0.378	0.403
69	0.355	0.393	0.465	0.483	153	0.321	0.345	0.387	0.440	237	0.305	0.350	0.391	0.418
70	0.347	0.402	0.452	0.482	154	0.318	0.348	0.400	0.442	238	0.302	0.341	0.382	0.414
71	0.350	0.404	0.445	0.476	155	0.312	0.350	0.401	0.421	239	0.303	0.352	0.377	0.415
72	0.365	0.392	0.446	0.483	156	0.313	0.359	0.399	0.446	240	0.301	0.343	0.403	0.412
73	0.346	0.390	0.445	0.487	157	0.303	0.363	0.392	0.429	241	0.301	0.334	0.377	0.419
74	0.350	0.415	0.444	0.468	158	0.324	0.349	0.407	0.428	242	0.305	0.355	0.389	0.417
75	0.360	0.395	0.455	0.485	159	0.316	0.370	0.393	0.435	243	0.293	0.349	0.377	0.409
76	0.367	0.399	0.439	0.489	160	0.310	0.358	0.395	0.432	244	0.308	0.350	0.384	0.412
77	0.345	0.390	0.442	0.468	161	0.322	0.343	0.403	0.427	245	0.306	0.342	0.379	0.407
78	0.358	0.408	0.446	0.460	162	0.313	0.372	0.393	0.431	246	0.299	0.356	0.375	0.418
79	0.340	0.394	0.430	0.468	163	0.314	0.352	0.393	0.422	247	0.301	0.346	0.375	0.413
80	0.347	0.394	0.445	0.465	164	0.312	0.362	0.382	0.419	248	0.303	0.354	0.382	0.411
81	0.352	0.402	0.427	0.474	165	0.298	0.355	0.394	0.427	249	0.310	0.336	0.391	0.402
82	0.334	0.393	0.443	0.471	166	0.312	0.360	0.392	0.425	250	0.306	0.341	0.393	0.415
83	0.348	0.392	0.424	0.468	167	0.309	0.358	0.390	0.420	251	0.303	0.342	0.374	0.418
84	0.350	0.408	0.436	0.483	168	0.316	0.363	0.390	0.432	252	0.309	0.329	0.375	0.427

MST Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 50$

	p variables					p variables					p variables			
n	2	3	4	5	n	2	3	4	5	n	2	3	4	5
253	0.310	0.344	0.386	0.419	337	0.291	0.340	0.372	0.415	421	0.285	0.333	0.368	0.333
254	0.303	0.333	0.372	0.403	338	0.295	0.341	0.365	0.404	422	0.295	0.330	0.371	0.330
255	0.296	0.330	0.381	0.416	339	0.294	0.346	0.366	0.407	423	0.286	0.348	0.370	0.348
256	0.296	0.347	0.378	0.411	340	0.303	0.336	0.367	0.419	424	0.304	0.328	0.358	0.328
257	0.292	0.348	0.377	0.415	341	0.299	0.348	0.380	0.405	425	0.296	0.338	0.372	0.338
258	0.295	0.334	0.383	0.407	342	0.286	0.356	0.376	0.403	426	0.292	0.335	0.377	0.335
259	0.290	0.340	0.377	0.402	343	0.301	0.327	0.373	0.410	427	0.316	0.325	0.365	0.325
260	0.303	0.345	0.389	0.414	344	0.303	0.335	0.378	0.406	428	0.283	0.339	0.376	0.339
261	0.298	0.356	0.375	0.398	345	0.298	0.345	0.365	0.403	429	0.275	0.325	0.369	0.325
262	0.309	0.341	0.379	0.401	346	0.307	0.327	0.364	0.393	430	0.291	0.334	0.370	0.334
263	0.297	0.341	0.368	0.405	347	0.297	0.329	0.372	0.404	431	0.283	0.336	0.379	0.336
264	0.298	0.350	0.385	0.410	348	0.301	0.337	0.375	0.391	432	0.293	0.329	0.360	0.329
265	0.296	0.342	0.386	0.405	349	0.288	0.341	0.367	0.402	433	0.286	0.334	0.362	0.334
266	0.306	0.350	0.387	0.411	350	0.293	0.335	0.371	0.403	434	0.293	0.331	0.363	0.331
267	0.303	0.358	0.375	0.413	351	0.298	0.329	0.363	0.407	435	0.293	0.337	0.377	0.337
268	0.287	0.342	0.376	0.405	352	0.290	0.335	0.370	0.401	436	0.285	0.325	0.361	0.325
269	0.298	0.340	0.374	0.421	353	0.303	0.332	0.367	0.397	437	0.292	0.321	0.378	0.321
270	0.301	0.330	0.372	0.404	354	0.292	0.346	0.375	0.402	438	0.289	0.338	0.370	0.338
271	0.297	0.337	0.366	0.401	355	0.296	0.335	0.373	0.392	439	0.291	0.339	0.365	0.339
272	0.296	0.348	0.376	0.412	356	0.304	0.349	0.364	0.406	440	0.307	0.327	0.361	0.327
273	0.309	0.336	0.369	0.401	357	0.282	0.335	0.376	0.403	441	0.286	0.348	0.368	0.348
274	0.301	0.344	0.383	0.415	358	0.295	0.345	0.373	0.408	442	0.293	0.322	0.364	0.322
275	0.303	0.346	0.389	0.396	359	0.295	0.332	0.368	0.412	443	0.302	0.331	0.365	0.331
276	0.308	0.336	0.373	0.420	360	0.283	0.338	0.375	0.403	444	0.300	0.326	0.358	0.326
277	0.303	0.341	0.372	0.404	361	0.289	0.336	0.382	0.414	445	0.292	0.326	0.370	0.326
278	0.302	0.356	0.370	0.411	362	0.301	0.327	0.363	0.398	446	0.297	0.321	0.369	0.321
279	0.300	0.344	0.381	0.404	363	0.295	0.332	0.373	0.397	447	0.282	0.329	0.365	0.329
280	0.299	0.349	0.376	0.418	364	0.288	0.319	0.379	0.394	448	0.290	0.335	0.354	0.335
281	0.301	0.338	0.372	0.406	365	0.315	0.338	0.372	0.408	449	0.295	0.336	0.364	0.336
282	0.294	0.337	0.371	0.401	366	0.290	0.340	0.362	0.401	450	0.291	0.332	0.358	0.332
283	0.304	0.340	0.363	0.408	367	0.302	0.344	0.374	0.404	451	0.303	0.331	0.365	0.331
284	0.301	0.333	0.374	0.412	368	0.303	0.336	0.377	0.399	452	0.288	0.328	0.364	0.328
285	0.301	0.338	0.383	0.408	369	0.314	0.340	0.387	0.397	453	0.298	0.330	0.374	0.330
286	0.304	0.321	0.389	0.405	370	0.291	0.345	0.362	0.400	454	0.293	0.326	0.375	0.326
287	0.302	0.339	0.385	0.405	371	0.302	0.334	0.380	0.401	455	0.298	0.333	0.366	0.333
288	0.300	0.346	0.374	0.409	372	0.294	0.339	0.376	0.412	456	0.288	0.337	0.365	0.337
289	0.296	0.344	0.369	0.400	373	0.292	0.334	0.379	0.401	457	0.291	0.327	0.364	0.327
290	0.288	0.337	0.373	0.419	374	0.290	0.339	0.366	0.394	458	0.281	0.336	0.366	0.336
291	0.287	0.341	0.367	0.402	375	0.285	0.334	0.369	0.403	459	0.292	0.337	0.371	0.337
292	0.294	0.349	0.373	0.408	376	0.287	0.339	0.364	0.403	460	0.279	0.344	0.364	0.344
293	0.294	0.332	0.373	0.410	377	0.288	0.333	0.374	0.396	461	0.283	0.327	0.365	0.327
294	0.296	0.349	0.378	0.407	378	0.296	0.337	0.378	0.396	462	0.293	0.344	0.361	0.344
295	0.292	0.332	0.376	0.398	379	0.303	0.341	0.366	0.384	463	0.283	0.329	0.366	0.329
296	0.278	0.338	0.380	0.407	380	0.286	0.329	0.365	0.406	464	0.293	0.332	0.360	0.332
297	0.295	0.339	0.371	0.407	381	0.296	0.323	0.365	0.389	465	0.282	0.327	0.363	0.327
298	0.292	0.347	0.374	0.400	382	0.290	0.343	0.371	0.401	466	0.280	0.340	0.368	0.340
299	0.284	0.344	0.375	0.403	383	0.298	0.339	0.367	0.401	467	0.289	0.322	0.369	0.322
300	0.293	0.335	0.375	0.408	384	0.288	0.341	0.369	0.407	468	0.297	0.333	0.368	0.333
301	0.296	0.348	0.372	0.393	385	0.295	0.320	0.366	0.394	469	0.300	0.326	0.377	0.326
302	0.304	0.329	0.372	0.407	386	0.295	0.343	0.369	0.411	470	0.287	0.328	0.366	0.328
303	0.294	0.336	0.367	0.400	387	0.292	0.340	0.366	0.394	471	0.287	0.328	0.363	0.328
304	0.312	0.340	0.378	0.406	388	0.295	0.333	0.376	0.399	472	0.292	0.329	0.357	0.329
305	0.293	0.342	0.376	0.411	389	0.307	0.332	0.375	0.400	473	0.291	0.331	0.364	0.331
306	0.297	0.340	0.378	0.407	390	0.295	0.331	0.359	0.401	474	0.283	0.328	0.372	0.328
307	0.302	0.330	0.369	0.412	391	0.296	0.331	0.382	0.398	475	0.295	0.329	0.366	0.329
308	0.290	0.342	0.383	0.398	392	0.292	0.325	0.379	0.402	476	0.291	0.334	0.369	0.334
309	0.282	0.326	0.372	0.413	393	0.285	0.327	0.375	0.399	477	0.297	0.331	0.361	0.331
310	0.291	0.345	0.370	0.403	394	0.294	0.340	0.379	0.402	478	0.287	0.334	0.365	0.334
311	0.302	0.340	0.364	0.405	395	0.288	0.326	0.373	0.408	479	0.301	0.332	0.370	0.332
312	0.309	0.343	0.372	0.402	396	0.295	0.331	0.370	0.399	480	0.300	0.323	0.371	0.323
313	0.293	0.331	0.372	0.410	397	0.291	0.334	0.365	0.400	481	0.287	0.338	0.366	0.338
314	0.295	0.335	0.373	0.399	398	0.295	0.334	0.376	0.394	482	0.297	0.323	0.382	0.323
315	0.290	0.353	0.375	0.398	399	0.288	0.331	0.369	0.394	483	0.291	0.329	0.372	0.329
316	0.293	0.343	0.374	0.399	400	0.299	0.335	0.368	0.402	484	0.302	0.340	0.370	0.340
317	0.289	0.343	0.368	0.403	401	0.284	0.329	0.368	0.392	485	0.295	0.323	0.367	0.323
318	0.297	0.337	0.379	0.407	402	0.297	0.332	0.361	0.397	486	0.289	0.335	0.372	0.335
319	0.288	0.342	0.378	0.400	403	0.292	0.336	0.362	0.397	487	0.291	0.324	0.362	0.324
320	0.287	0.338	0.381	0.407	404	0.297	0.330	0.372	0.394	488	0.303	0.326	0.375	0.326
321	0.305	0.341	0.369	0.409	405	0.300	0.324	0.363	0.404	489	0.291	0.328	0.368	0.328
322	0.292	0.336	0.375	0.408	406	0.289	0.326	0.366	0.398	490	0.293	0.330	0.362	0.330
323	0.291	0.346	0.366	0.399	407	0.288	0.335	0.362	0.397	491	0.288	0.323	0.373	0.323
324	0.293	0.341	0.368	0.407	408	0.304	0.333	0.380	0.406	492	0.301	0.334	0.363	0.334
325	0.291	0.333	0.372	0.403	409	0.284	0.337	0.376	0.404	493	0.286	0.336	0.367	0.336
326	0.293	0.342	0.379	0.411	410	0.295	0.332	0.371	0.393	494	0.296	0.327	0.357	0.327
327	0.289	0.341	0.378	0.395	411	0.289	0.331	0.377	0.411	495	0.285	0.322	0.364	0.322
328	0.291	0.336	0.364	0.409	412	0.295	0.337	0.364	0.397	496	0.292	0.346	0.360	0.346
329	0.297	0.351	0.367	0.401	413	0.294	0.334	0.367	0.400	497	0.307	0.325	0.359	0.325
330	0.283	0.335	0.368	0.411	414	0.280	0.338	0.370	0.405	498	0.300	0.326	0.361	0.326
331	0.287	0.338	0.367	0.403	415	0.305	0.337	0.376	0.410	499	0.292	0.337	0.371	0.337
332	0.292	0.338	0.373	0.402	416	0.297	0.333	0.374	0.399	500	0.296	0.319	0.360	0.319
333	0.307	0.343	0.370	0.403	417	0.281	0.338	0.371	0.392					
334	0.291	0.338	0.373	0.406	418	0.289	0.329	0.355	0.395					
335	0.307	0.334	0.383	0.400	419	0.288	0.343	0.364	0.393					
336	0.299	0.329	0.365	0.405	420	0.288	0.363	0.384	0.340					

Table A.15: MST Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.05$, $m_0 = 100$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
1	0.941	1.069	1.186	1.318	55	0.276	0.328	0.365	0.398
2	0.820	0.965	1.057	1.169	56	0.274	0.328	0.370	0.392
3	0.744	0.854	0.969	1.070	57	0.277	0.327	0.362	0.397
4	0.678	0.797	0.891	0.967	58	0.282	0.333	0.362	0.393
5	0.650	0.735	0.819	0.913	59	0.271	0.322	0.358	0.399
6	0.592	0.719	0.803	0.849	60	0.270	0.317	0.357	0.393
7	0.576	0.672	0.749	0.825	61	0.273	0.317	0.353	0.391
8	0.544	0.638	0.706	0.786	62	0.277	0.319	0.351	0.383
9	0.525	0.608	0.685	0.759	63	0.273	0.318	0.351	0.386
10	0.515	0.592	0.656	0.723	64	0.269	0.320	0.351	0.384
11	0.491	0.564	0.632	0.699	65	0.274	0.316	0.347	0.382
12	0.470	0.549	0.613	0.670	66	0.264	0.317	0.344	0.387
13	0.466	0.529	0.604	0.657	67	0.264	0.315	0.346	0.381
14	0.444	0.525	0.592	0.641	68	0.263	0.312	0.340	0.378
15	0.437	0.514	0.572	0.624	69	0.267	0.304	0.342	0.377
16	0.429	0.504	0.563	0.616	70	0.265	0.309	0.348	0.373
17	0.413	0.480	0.555	0.596	71	0.263	0.313	0.339	0.371
18	0.412	0.476	0.539	0.590	72	0.256	0.303	0.337	0.372
19	0.403	0.465	0.532	0.586	73	0.259	0.299	0.338	0.372
20	0.400	0.468	0.525	0.565	74	0.266	0.305	0.338	0.372
21	0.389	0.464	0.521	0.553	75	0.261	0.303	0.328	0.369
22	0.383	0.444	0.505	0.539	76	0.256	0.299	0.336	0.372
23	0.385	0.438	0.482	0.533	77	0.254	0.294	0.336	0.361
24	0.367	0.430	0.484	0.526	78	0.246	0.297	0.331	0.364
25	0.362	0.419	0.477	0.523	79	0.254	0.300	0.323	0.360
26	0.358	0.418	0.478	0.511	80	0.256	0.289	0.326	0.362
27	0.357	0.413	0.464	0.514	81	0.256	0.300	0.331	0.363
28	0.358	0.403	0.467	0.505	82	0.253	0.294	0.332	0.357
29	0.344	0.409	0.454	0.496	83	0.250	0.291	0.329	0.358
30	0.340	0.397	0.449	0.487	84	0.245	0.292	0.329	0.355
31	0.345	0.394	0.446	0.484	85	0.246	0.292	0.324	0.359
32	0.336	0.391	0.442	0.475	86	0.247	0.289	0.323	0.354
33	0.330	0.397	0.437	0.471	87	0.247	0.284	0.321	0.358
34	0.326	0.374	0.434	0.465	88	0.246	0.287	0.315	0.348
35	0.322	0.374	0.424	0.466	89	0.246	0.294	0.312	0.350
36	0.323	0.379	0.431	0.460	90	0.242	0.284	0.324	0.349
37	0.320	0.381	0.416	0.453	91	0.245	0.286	0.317	0.352
38	0.309	0.366	0.412	0.451	92	0.248	0.289	0.320	0.350
39	0.315	0.363	0.413	0.451	93	0.239	0.283	0.321	0.348
40	0.309	0.358	0.407	0.443	94	0.246	0.283	0.323	0.347
41	0.315	0.360	0.405	0.443	95	0.245	0.277	0.317	0.348
42	0.302	0.359	0.396	0.434	96	0.240	0.275	0.316	0.340
43	0.299	0.350	0.395	0.434	97	0.241	0.283	0.312	0.344
44	0.302	0.345	0.392	0.431	98	0.239	0.273	0.315	0.347
45	0.304	0.351	0.393	0.426	99	0.238	0.276	0.318	0.349
46	0.301	0.348	0.387	0.423	100	0.241	0.282	0.310	0.344
47	0.297	0.342	0.383	0.427	101	0.236	0.276	0.308	0.338
48	0.297	0.335	0.380	0.420	102	0.239	0.278	0.313	0.339
49	0.292	0.343	0.383	0.427	103	0.239	0.279	0.312	0.341
50	0.293	0.339	0.379	0.411	104	0.241	0.280	0.310	0.337
51	0.291	0.337	0.374	0.417	105	0.236	0.275	0.302	0.336
52	0.290	0.341	0.379	0.413	106	0.237	0.277	0.311	0.338
53	0.285	0.334	0.369	0.403	107	0.234	0.272	0.306	0.339
54	0.287	0.331	0.367	0.400					

Table A.16: MST Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 100$

	p variables					p variables					p variables			
n	2	3	4	5	n	2	3	4	5	n	2	3	4	5
1	1.021	1.195	1.310	1.423	57	0.324	0.356	0.389	0.443	113	0.270	0.300	0.344	0.359
2	0.916	1.054	1.166	1.244	58	0.325	0.348	0.394	0.444	114	0.273	0.308	0.328	0.363
3	0.799	0.928	1.093	1.133	59	0.319	0.359	0.407	0.433	115	0.261	0.302	0.346	0.357
4	0.761	0.864	0.966	1.028	60	0.316	0.350	0.389	0.431	116	0.264	0.301	0.331	0.367
5	0.757	0.828	0.924	0.993	61	0.307	0.372	0.404	0.426	117	0.259	0.300	0.341	0.350
6	0.691	0.783	0.873	0.936	62	0.309	0.353	0.386	0.425	118	0.273	0.307	0.327	0.360
7	0.675	0.743	0.829	0.876	63	0.309	0.359	0.390	0.420	119	0.261	0.304	0.334	0.354
8	0.616	0.730	0.795	0.846	64	0.301	0.338	0.390	0.435	120	0.251	0.294	0.331	0.354
9	0.621	0.678	0.759	0.842	65	0.297	0.353	0.383	0.427	121	0.269	0.299	0.342	0.361
10	0.559	0.640	0.724	0.781	66	0.311	0.341	0.388	0.414	122	0.263	0.300	0.328	0.351
11	0.581	0.634	0.703	0.746	67	0.305	0.355	0.380	0.420	123	0.258	0.307	0.335	0.355
12	0.553	0.631	0.672	0.726	68	0.301	0.337	0.373	0.412	124	0.283	0.301	0.319	0.349
13	0.548	0.608	0.668	0.713	69	0.304	0.339	0.387	0.425	125	0.272	0.307	0.325	0.365
14	0.511	0.593	0.659	0.691	70	0.298	0.343	0.381	0.410	126	0.263	0.301	0.335	0.355
15	0.493	0.574	0.633	0.684	71	0.300	0.344	0.383	0.399	127	0.262	0.302	0.331	0.356
16	0.485	0.539	0.631	0.676	72	0.291	0.339	0.376	0.401	128	0.263	0.285	0.335	0.355
17	0.473	0.560	0.607	0.653	73	0.300	0.352	0.378	0.407	129	0.258	0.297	0.322	0.360
18	0.457	0.545	0.592	0.653	74	0.289	0.332	0.369	0.406	130	0.264	0.294	0.328	0.353
19	0.461	0.536	0.587	0.632	75	0.293	0.335	0.367	0.391	131	0.262	0.289	0.319	0.351
20	0.462	0.506	0.575	0.633	76	0.292	0.335	0.362	0.394	132	0.248	0.301	0.323	0.362
21	0.425	0.491	0.568	0.600	77	0.303	0.343	0.374	0.400	133	0.262	0.293	0.335	0.349
22	0.440	0.498	0.543	0.582	78	0.279	0.337	0.369	0.391	134	0.263	0.294	0.326	0.352
23	0.431	0.484	0.536	0.594	79	0.299	0.330	0.386	0.399	135	0.259	0.294	0.328	0.348
24	0.422	0.500	0.531	0.577	80	0.282	0.321	0.365	0.381	136	0.254	0.294	0.318	0.345
25	0.417	0.477	0.528	0.563	81	0.285	0.340	0.378	0.405	137	0.257	0.289	0.319	0.338
26	0.402	0.467	0.508	0.547	82	0.294	0.339	0.364	0.398	138	0.254	0.295	0.324	0.341
27	0.397	0.460	0.510	0.560	83	0.283	0.325	0.357	0.392	139	0.255	0.283	0.317	0.346
28	0.406	0.462	0.523	0.549	84	0.281	0.328	0.364	0.389	140	0.241	0.292	0.319	0.348
29	0.392	0.460	0.495	0.538	85	0.287	0.325	0.359	0.404	141	0.251	0.295	0.316	0.354
30	0.385	0.448	0.502	0.527	86	0.288	0.319	0.357	0.397	142	0.253	0.293	0.315	0.346
31	0.391	0.445	0.479	0.530	87	0.277	0.325	0.359	0.383	143	0.260	0.296	0.319	0.340
32	0.382	0.440	0.488	0.523	88	0.278	0.324	0.348	0.391	144	0.249	0.291	0.318	0.343
33	0.397	0.440	0.491	0.498	89	0.275	0.328	0.358	0.387	145	0.253	0.304	0.316	0.339
34	0.381	0.440	0.480	0.519	90	0.272	0.330	0.347	0.383	146	0.243	0.280	0.315	0.354
35	0.361	0.430	0.477	0.515	91	0.282	0.317	0.353	0.383	147	0.251	0.286	0.316	0.352
36	0.361	0.439	0.460	0.519	92	0.278	0.315	0.347	0.378	148	0.251	0.288	0.317	0.336
37	0.367	0.418	0.461	0.488	93	0.274	0.314	0.350	0.374	149	0.260	0.283	0.315	0.342
38	0.373	0.416	0.445	0.510	94	0.275	0.317	0.355	0.383	150	0.258	0.287	0.314	0.339
39	0.357	0.405	0.459	0.496	95	0.270	0.323	0.344	0.382	151	0.246	0.289	0.315	0.335
40	0.352	0.416	0.447	0.479	96	0.276	0.315	0.341	0.379	152	0.248	0.283	0.311	0.337
41	0.351	0.397	0.441	0.491	97	0.273	0.315	0.343	0.379	153	0.255	0.293	0.316	0.336
42	0.356	0.389	0.444	0.491	98	0.269	0.317	0.355	0.374	154	0.245	0.284	0.320	0.339
43	0.336	0.384	0.436	0.463	99	0.281	0.305	0.336	0.374	155	0.259	0.286	0.318	0.341
44	0.353	0.388	0.429	0.474	100	0.277	0.314	0.338	0.377	156	0.246	0.291	0.318	0.340
45	0.329	0.400	0.439	0.460	101	0.274	0.310	0.340	0.369	157	0.252	0.282	0.305	0.341
46	0.336	0.389	0.431	0.462	102	0.277	0.312	0.349	0.368	158	0.247	0.286	0.315	0.344
47	0.336	0.390	0.417	0.464	103	0.276	0.310	0.345	0.371	159	0.248	0.277	0.317	0.347
48	0.331	0.381	0.414	0.461	104	0.271	0.300	0.345	0.374	160	0.248	0.276	0.312	0.341
49	0.341	0.363	0.429	0.447	105	0.277	0.314	0.342	0.364	161	0.248	0.282	0.306	0.336
50	0.330	0.382	0.423	0.450	106	0.276	0.310	0.349	0.373	162	0.249	0.279	0.304	0.338
51	0.333	0.376	0.412	0.449	107	0.284	0.311	0.336	0.371	163	0.251	0.274	0.309	0.344
52	0.336	0.359	0.413	0.441	108	0.260	0.315	0.336	0.361	164	0.242	0.278	0.302	0.336
53	0.313	0.372	0.402	0.444	109	0.268	0.309	0.350	0.371	165	0.251	0.277	0.306	0.335
54	0.322	0.370	0.407	0.440	110	0.264	0.304	0.342	0.367	166	0.242	0.284	0.313	0.337
55	0.318	0.362	0.393	0.445	111	0.265	0.310	0.336	0.365	167	0.244	0.278	0.306	0.339
56	0.319	0.371	0.399	0.439	112	0.270	0.305	0.340	0.363	168	0.247	0.287	0.317	0.334

MST Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 100$

	p variables					p variables					p variables			
n	2	3	4	5	n	2	3	4	5	n	2	3	4	5
169	0.246	0.279	0.310	0.339	226	0.231	0.261	0.296	0.312	283	0.231	0.262	0.292	0.305
170	0.246	0.273	0.318	0.342	227	0.235	0.266	0.294	0.316	284	0.229	0.259	0.281	0.310
171	0.237	0.290	0.307	0.339	228	0.233	0.273	0.309	0.317	285	0.231	0.260	0.292	0.309
172	0.258	0.280	0.327	0.336	229	0.228	0.260	0.302	0.320	286	0.231	0.264	0.282	0.321
173	0.258	0.292	0.306	0.334	230	0.234	0.270	0.296	0.326	287	0.225	0.260	0.282	0.305
174	0.249	0.282	0.313	0.330	231	0.227	0.269	0.292	0.329	288	0.227	0.269	0.294	0.310
175	0.245	0.281	0.312	0.335	232	0.231	0.267	0.312	0.320	289	0.240	0.257	0.283	0.316
176	0.241	0.287	0.312	0.334	233	0.232	0.276	0.298	0.320	290	0.232	0.260	0.286	0.318
177	0.243	0.283	0.307	0.338	234	0.232	0.262	0.308	0.321	291	0.234	0.259	0.287	0.312
178	0.251	0.280	0.310	0.333	235	0.218	0.265	0.298	0.314	292	0.225	0.257	0.281	0.309
179	0.235	0.283	0.310	0.331	236	0.232	0.263	0.293	0.317	293	0.229	0.252	0.284	0.313
180	0.236	0.285	0.304	0.329	237	0.235	0.274	0.297	0.335	294	0.226	0.265	0.286	0.298
181	0.246	0.274	0.302	0.329	238	0.231	0.268	0.291	0.320	295	0.230	0.256	0.292	0.321
182	0.244	0.284	0.305	0.326	239	0.227	0.269	0.290	0.317	296	0.223	0.251	0.287	0.322
183	0.249	0.276	0.323	0.324	240	0.231	0.263	0.293	0.319	297	0.227	0.256	0.278	0.321
184	0.237	0.282	0.304	0.335	241	0.228	0.275	0.293	0.314	298	0.231	0.263	0.293	0.304
185	0.242	0.282	0.302	0.334	242	0.239	0.263	0.293	0.318	299	0.221	0.258	0.293	0.314
186	0.243	0.279	0.317	0.330	243	0.229	0.262	0.292	0.314	300	0.231	0.255	0.285	0.307
187	0.247	0.275	0.311	0.326	244	0.229	0.265	0.290	0.309	301	0.222	0.258	0.285	0.306
188	0.248	0.283	0.305	0.328	245	0.236	0.265	0.290	0.324	302	0.224	0.257	0.291	0.303
189	0.243	0.275	0.307	0.328	246	0.249	0.255	0.297	0.323	303	0.234	0.249	0.286	0.303
190	0.243	0.269	0.307	0.329	247	0.245	0.266	0.293	0.321	304	0.227	0.254	0.280	0.305
191	0.254	0.279	0.304	0.339	248	0.227	0.265	0.294	0.316	305	0.220	0.253	0.287	0.316
192	0.246	0.274	0.302	0.328	249	0.223	0.263	0.300	0.310	306	0.223	0.257	0.281	0.305
193	0.247	0.278	0.305	0.328	250	0.234	0.272	0.292	0.318	307	0.231	0.262	0.280	0.298
194	0.243	0.281	0.301	0.332	251	0.231	0.267	0.291	0.313	308	0.233	0.264	0.279	0.307
195	0.246	0.273	0.302	0.326	252	0.225	0.264	0.290	0.320	309	0.228	0.265	0.277	0.304
196	0.248	0.270	0.299	0.324	253	0.237	0.279	0.293	0.313	310	0.232	0.253	0.288	0.304
197	0.243	0.272	0.304	0.328	254	0.229	0.261	0.291	0.315	311	0.221	0.259	0.285	0.307
198	0.236	0.283	0.301	0.322	255	0.230	0.266	0.289	0.314	312	0.232	0.264	0.281	0.305
199	0.240	0.273	0.305	0.327	256	0.237	0.264	0.288	0.302	313	0.233	0.253	0.274	0.316
200	0.233	0.277	0.302	0.321	257	0.228	0.261	0.291	0.304	314	0.224	0.254	0.288	0.306
201	0.250	0.277	0.300	0.329	258	0.226	0.260	0.291	0.306	315	0.227	0.262	0.285	0.310
202	0.252	0.271	0.304	0.324	259	0.230	0.261	0.289	0.322	316	0.227	0.259	0.288	0.303
203	0.235	0.271	0.292	0.326	260	0.235	0.259	0.293	0.321	317	0.221	0.260	0.281	0.309
204	0.236	0.273	0.303	0.326	261	0.227	0.273	0.287	0.315	318	0.221	0.252	0.279	0.307
205	0.238	0.279	0.304	0.322	262	0.234	0.269	0.290	0.314	319	0.227	0.259	0.275	0.300
206	0.232	0.273	0.303	0.333	263	0.232	0.264	0.286	0.306	320	0.224	0.252	0.284	0.319
207	0.232	0.271	0.307	0.326	264	0.226	0.251	0.291	0.311	321	0.224	0.259	0.284	0.307
208	0.231	0.270	0.298	0.329	265	0.232	0.258	0.294	0.311	322	0.224	0.254	0.281	0.307
209	0.239	0.268	0.304	0.325	266	0.235	0.272	0.289	0.319	323	0.224	0.262	0.275	0.305
210	0.234	0.266	0.307	0.320	267	0.228	0.258	0.290	0.309	324	0.217	0.255	0.286	0.305
211	0.243	0.282	0.292	0.330	268	0.222	0.262	0.293	0.315	325	0.227	0.249	0.284	0.305
212	0.238	0.268	0.301	0.319	269	0.225	0.261	0.292	0.309	326	0.217	0.257	0.279	0.305
213	0.233	0.260	0.309	0.323	270	0.227	0.258	0.285	0.317	327	0.226	0.253	0.279	0.316
214	0.225	0.270	0.309	0.323	271	0.236	0.260	0.289	0.319	328	0.223	0.253	0.278	0.304
215	0.231	0.286	0.294	0.324	272	0.230	0.269	0.292	0.313	329	0.215	0.258	0.283	0.304
216	0.235	0.267	0.298	0.319	273	0.221	0.256	0.286	0.313	330	0.225	0.260	0.280	0.313
217	0.232	0.281	0.304	0.332	274	0.227	0.264	0.295	0.320	331	0.227	0.265	0.278	0.294
218	0.233	0.268	0.306	0.317	275	0.228	0.256	0.279	0.309	332	0.229	0.261	0.284	0.304
219	0.229	0.267	0.294	0.315	276	0.231	0.261	0.288	0.308	333	0.238	0.254	0.279	0.313
220	0.235	0.270	0.297	0.320	277	0.228	0.257	0.289	0.307	334	0.214	0.249	0.273	0.304
221	0.235	0.269	0.302	0.321	278	0.228	0.260	0.288	0.319	335	0.221	0.251	0.274	0.307
222	0.241	0.270	0.289	0.327	279	0.225	0.266	0.283	0.314	336	0.217	0.252	0.286	0.307
223	0.233	0.279	0.293	0.318	280	0.232	0.266	0.280	0.313	337	0.215	0.257	0.281	0.298
224	0.235	0.267	0.291	0.334	281	0.232	0.263	0.285	0.307	338	0.224	0.251	0.277	0.302
225	0.232	0.270	0.297	0.324	282	0.225	0.258	0.295	0.316	339	0.223	0.254	0.280	0.307

MST Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 100$

	p variables					*p* variables					*p* variables			
n	2	3	4	5	n	2	3	4	5	n	2	3	4	5
340	0.220	0.262	0.274	0.302	396	0.225	0.263	0.275	0.315	452	0.213	0.251	0.275	0.290
341	0.216	0.251	0.295	0.302	397	0.218	0.247	0.281	0.298	453	0.225	0.243	0.277	0.303
342	0.224	0.256	0.288	0.320	398	0.217	0.259	0.275	0.302	454	0.216	0.255	0.271	0.297
343	0.218	0.251	0.280	0.302	399	0.213	0.253	0.280	0.303	455	0.218	0.246	0.279	0.293
344	0.222	0.252	0.276	0.309	400	0.226	0.245	0.272	0.300	456	0.211	0.242	0.272	0.297
345	0.227	0.252	0.286	0.303	401	0.223	0.253	0.272	0.304	457	0.207	0.243	0.273	0.295
346	0.216	0.253	0.290	0.312	402	0.217	0.256	0.274	0.294	458	0.212	0.246	0.270	0.292
347	0.221	0.252	0.280	0.306	403	0.218	0.249	0.277	0.308	459	0.214	0.248	0.277	0.293
348	0.234	0.254	0.280	0.296	404	0.217	0.246	0.273	0.295	460	0.211	0.253	0.272	0.298
349	0.222	0.246	0.268	0.303	405	0.215	0.258	0.275	0.298	461	0.218	0.242	0.271	0.299
350	0.226	0.259	0.282	0.304	406	0.224	0.253	0.275	0.301	462	0.225	0.242	0.272	0.297
351	0.228	0.254	0.276	0.300	407	0.214	0.244	0.278	0.296	463	0.221	0.240	0.278	0.299
352	0.222	0.257	0.274	0.307	408	0.219	0.246	0.281	0.295	464	0.219	0.249	0.267	0.300
353	0.223	0.256	0.274	0.313	409	0.221	0.253	0.277	0.307	465	0.215	0.251	0.264	0.293
354	0.222	0.260	0.276	0.301	410	0.222	0.259	0.278	0.303	466	0.221	0.243	0.277	0.298
355	0.214	0.254	0.276	0.304	411	0.216	0.248	0.285	0.302	467	0.214	0.243	0.265	0.290
356	0.215	0.253	0.280	0.307	412	0.214	0.244	0.276	0.297	468	0.218	0.245	0.271	0.297
357	0.215	0.250	0.282	0.309	413	0.216	0.242	0.283	0.299	469	0.217	0.254	0.269	0.292
358	0.230	0.265	0.273	0.308	414	0.218	0.249	0.279	0.297	470	0.213	0.245	0.270	0.290
359	0.214	0.254	0.290	0.304	415	0.225	0.246	0.274	0.301	471	0.213	0.245	0.283	0.293
360	0.219	0.251	0.273	0.305	416	0.208	0.248	0.276	0.303	472	0.221	0.254	0.265	0.294
361	0.216	0.253	0.288	0.302	417	0.222	0.253	0.281	0.310	473	0.215	0.248	0.270	0.293
362	0.222	0.251	0.278	0.305	418	0.216	0.252	0.273	0.293	474	0.217	0.251	0.272	0.300
363	0.216	0.247	0.276	0.297	419	0.217	0.253	0.280	0.301	475	0.223	0.248	0.271	0.296
364	0.221	0.254	0.279	0.304	420	0.215	0.254	0.284	0.296	476	0.223	0.240	0.278	0.291
365	0.219	0.255	0.282	0.310	421	0.220	0.240	0.273	0.290	477	0.219	0.249	0.272	0.294
366	0.223	0.254	0.276	0.298	422	0.219	0.242	0.271	0.298	478	0.212	0.246	0.276	0.296
367	0.216	0.246	0.286	0.304	423	0.226	0.258	0.265	0.288	479	0.206	0.247	0.281	0.288
368	0.215	0.248	0.287	0.310	424	0.217	0.244	0.268	0.295	480	0.213	0.247	0.266	0.296
369	0.228	0.253	0.284	0.300	425	0.216	0.250	0.270	0.295	481	0.211	0.240	0.265	0.296
370	0.226	0.252	0.279	0.303	426	0.221	0.246	0.274	0.297	482	0.215	0.246	0.270	0.293
371	0.224	0.259	0.278	0.305	427	0.218	0.245	0.276	0.306	483	0.223	0.246	0.268	0.296
372	0.228	0.259	0.276	0.300	428	0.210	0.248	0.272	0.295	484	0.213	0.248	0.272	0.297
373	0.221	0.250	0.276	0.292	429	0.222	0.257	0.273	0.295	485	0.211	0.245	0.272	0.289
374	0.220	0.246	0.276	0.296	430	0.217	0.246	0.280	0.305	486	0.218	0.243	0.269	0.289
375	0.223	0.249	0.279	0.303	431	0.212	0.250	0.270	0.300	487	0.217	0.242	0.263	0.298
376	0.231	0.261	0.274	0.304	432	0.215	0.247	0.283	0.298	488	0.219	0.244	0.271	0.294
377	0.221	0.250	0.282	0.305	433	0.219	0.243	0.270	0.289	489	0.214	0.247	0.271	0.287
378	0.233	0.250	0.279	0.299	434	0.219	0.242	0.275	0.291	490	0.216	0.247	0.264	0.297
379	0.221	0.252	0.275	0.309	435	0.222	0.240	0.268	0.298	491	0.212	0.244	0.272	0.296
380	0.221	0.255	0.273	0.304	436	0.219	0.251	0.279	0.298	492	0.223	0.238	0.268	0.307
381	0.224	0.255	0.291	0.301	437	0.216	0.257	0.269	0.294	493	0.217	0.248	0.266	0.296
382	0.219	0.249	0.274	0.293	438	0.209	0.245	0.270	0.291	494	0.212	0.252	0.264	0.294
383	0.229	0.257	0.283	0.304	439	0.214	0.252	0.273	0.294	495	0.217	0.242	0.272	0.293
384	0.221	0.248	0.275	0.300	440	0.224	0.244	0.278	0.301	496	0.225	0.248	0.267	0.299
385	0.223	0.245	0.278	0.299	441	0.225	0.251	0.270	0.300	497	0.209	0.246	0.266	0.290
386	0.222	0.253	0.277	0.304	442	0.222	0.252	0.282	0.299	498	0.214	0.242	0.276	0.297
387	0.218	0.261	0.286	0.294	443	0.220	0.250	0.278	0.293	499	0.211	0.245	0.263	0.291
388	0.220	0.247	0.269	0.305	444	0.214	0.255	0.273	0.297	500	0.213	0.253	0.267	0.291
389	0.220	0.245	0.275	0.299	445	0.215	0.251	0.275	0.295	501	0.218	0.251	0.269	0.292
390	0.218	0.260	0.275	0.297	446	0.219	0.244	0.277	0.296	502	0.216	0.243	0.275	0.296
391	0.220	0.251	0.277	0.312	447	0.220	0.244	0.277	0.293	503	0.216	0.239	0.267	0.296
392	0.227	0.252	0.279	0.303	448	0.212	0.245	0.267	0.299	504	0.207	0.235	0.274	0.290
393	0.221	0.244	0.283	0.305	449	0.226	0.244	0.271	0.291	505	0.220	0.249	0.277	0.292
394	0.224	0.256	0.271	0.299	450	0.220	0.245	0.278	0.297	506	0.212	0.239	0.273	0.287
395	0.216	0.249	0.286	0.295	451	0.214	0.246	0.281	0.296	507	0.206	0.239	0.265	0.285

Table A.17: MST Cut-off values $\alpha = 0.05$, $m_0 = 100$, for $p = 2$, $X_1 \sim N(0,1)$, $X_2 \sim$ Uniform or t_3, for $p = 5$, $X_1, X_2, X_3 \sim N(0,1)$, $X_4, X_5 \sim$ both Uniform or t_3

	Uniform		t_3			Uniform		t_3	
n	$p = 2$	$p = 5$	$p = 2$	$p = 5$	n	$p = 2$	$p = 5$	$p = 2$	$p = 5$
1	0.901	1.302	0.930	1.317	51	0.280	0.409	0.293	0.418
2	0.823	1.165	0.785	1.156	52	0.287	0.407	0.284	0.407
3	0.723	1.044	0.722	1.047	53	0.285	0.397	0.287	0.405
4	0.695	0.978	0.685	0.973	54	0.284	0.403	0.287	0.413
5	0.642	0.922	0.641	0.921	55	0.282	0.403	0.281	0.402
6	0.601	0.869	0.590	0.867	56	0.280	0.401	0.275	0.402
7	0.585	0.826	0.575	0.823	57	0.283	0.394	0.283	0.401
8	0.536	0.781	0.552	0.778	58	0.272	0.396	0.283	0.396
9	0.535	0.743	0.532	0.747	59	0.274	0.388	0.272	0.391
10	0.498	0.719	0.512	0.727	60	0.272	0.393	0.273	0.398
11	0.487	0.704	0.493	0.695	61	0.274	0.396	0.269	0.390
12	0.481	0.676	0.458	0.682	62	0.273	0.386	0.277	0.385
13	0.458	0.654	0.465	0.662	63	0.272	0.383	0.268	0.387
14	0.453	0.635	0.442	0.653	64	0.268	0.385	0.266	0.390
15	0.445	0.631	0.441	0.624	65	0.272	0.383	0.264	0.384
16	0.441	0.615	0.428	0.615	66	0.261	0.382	0.267	0.384
17	0.409	0.598	0.415	0.600	67	0.264	0.383	0.271	0.377
18	0.412	0.594	0.416	0.584	68	0.264	0.375	0.261	0.377
19	0.402	0.577	0.403	0.571	69	0.258	0.377	0.269	0.377
20	0.392	0.564	0.392	0.559	70	0.264	0.378	0.261	0.381
21	0.382	0.552	0.392	0.558	71	0.262	0.369	0.261	0.370
22	0.385	0.549	0.380	0.544	72	0.264	0.377	0.256	0.371
23	0.374	0.534	0.381	0.532	73	0.259	0.373	0.260	0.375
24	0.368	0.528	0.367	0.529	74	0.255	0.376	0.255	0.369
25	0.360	0.522	0.365	0.520	75	0.257	0.370	0.261	0.365
26	0.358	0.515	0.356	0.518	76	0.257	0.368	0.249	0.367
27	0.351	0.509	0.352	0.498	77	0.260	0.366	0.255	0.360
28	0.350	0.499	0.353	0.503	78	0.253	0.362	0.256	0.364
29	0.343	0.490	0.354	0.491	79	0.251	0.363	0.253	0.368
30	0.340	0.488	0.343	0.495	80	0.248	0.365	0.250	0.364
31	0.337	0.486	0.345	0.481	81	0.250	0.360	0.250	0.359
32	0.332	0.479	0.335	0.477	82	0.255	0.358	0.259	0.356
33	0.335	0.471	0.331	0.480	83	0.252	0.357	0.247	0.360
34	0.334	0.472	0.318	0.468	84	0.253	0.359	0.247	0.362
35	0.324	0.468	0.325	0.468	85	0.244	0.358	0.247	0.357
36	0.316	0.460	0.322	0.462	86	0.249	0.360	0.248	0.356
37	0.315	0.460	0.323	0.458	87	0.255	0.351	0.247	0.353
38	0.308	0.447	0.320	0.454	88	0.244	0.355	0.245	0.352
39	0.313	0.443	0.311	0.448	89	0.250	0.355	0.248	0.352
40	0.310	0.446	0.307	0.449	90	0.246	0.345	0.248	0.353
41	0.313	0.438	0.301	0.445	91	0.244	0.350	0.251	0.350
42	0.305	0.437	0.307	0.442	92	0.242	0.348	0.243	0.349
43	0.308	0.435	0.310	0.429	93	0.246	0.346	0.241	0.351
44	0.302	0.430	0.307	0.436	94	0.242	0.344	0.246	0.346
45	0.300	0.419	0.301	0.427	95	0.245	0.352	0.241	0.345
46	0.297	0.425	0.294	0.420	96	0.241	0.350	0.241	0.339
47	0.296	0.420	0.294	0.422	97	0.240	0.343	0.239	0.340
48	0.292	0.426	0.292	0.420	98	0.234	0.344	0.242	0.335
49	0.293	0.417	0.295	0.415	99	0.238	0.341	0.243	0.348
50	0.293	0.411	0.285	0.411	100	0.235	0.340	0.241	0.335

Table A.18: MST Cut-off values $\alpha = 0.01$, $m_0 = 100$, for $p = 2$, $X_1 \sim N(0,1)$, $X_2 \sim$ Uniform or t_3, for $p = 5$, $X_1, X_2, X_3 \sim N(0,1)$, $X_4, X_5 \sim$ both Uniform or t_3

	Uniform		t_3			Uniform		t_3			Uniform		t_3	
n	$p = 2$	$p = 5$	$p = 2$	$p = 5$	n	$p = 2$	$p = 5$	$p = 2$	$p = 5$	n	$p = 2$	$p = 5$	$p = 2$	$p = 5$
1	1.034	1.430	1.013	1.407	85	0.297	0.397	0.293	0.391	169	0.245	0.332	0.251	0.340
2	0.899	1.258	0.913	1.248	86	0.277	0.398	0.279	0.384	170	0.250	0.335	0.241	0.337
3	0.838	1.165	0.851	1.136	87	0.275	0.377	0.288	0.381	171	0.254	0.339	0.244	0.339
4	0.782	1.095	0.763	1.078	88	0.288	0.385	0.271	0.380	172	0.243	0.326	0.246	0.341
5	0.690	0.989	0.707	1.006	89	0.272	0.388	0.276	0.389	173	0.241	0.331	0.243	0.329
6	0.664	0.941	0.667	0.967	90	0.280	0.374	0.276	0.387	174	0.243	0.339	0.246	0.333
7	0.663	0.898	0.649	0.909	91	0.286	0.385	0.276	0.386	175	0.238	0.341	0.229	0.333
8	0.644	0.859	0.637	0.861	92	0.280	0.381	0.280	0.380	176	0.240	0.335	0.239	0.330
9	0.611	0.824	0.588	0.822	93	0.281	0.382	0.286	0.386	177	0.249	0.332	0.248	0.337
10	0.569	0.769	0.599	0.782	94	0.277	0.377	0.274	0.379	178	0.240	0.334	0.241	0.336
11	0.560	0.768	0.581	0.760	95	0.274	0.381	0.263	0.378	179	0.239	0.335	0.257	0.334
12	0.539	0.740	0.544	0.742	96	0.291	0.384	0.278	0.370	180	0.228	0.330	0.251	0.326
13	0.521	0.716	0.536	0.733	97	0.269	0.383	0.273	0.370	181	0.251	0.342	0.253	0.327
14	0.504	0.695	0.533	0.706	98	0.273	0.379	0.272	0.365	182	0.249	0.329	0.244	0.331
15	0.500	0.704	0.489	0.670	99	0.281	0.371	0.272	0.388	183	0.251	0.329	0.247	0.339
16	0.476	0.671	0.476	0.655	100	0.284	0.378	0.271	0.362	184	0.234	0.328	0.245	0.332
17	0.474	0.654	0.475	0.652	101	0.268	0.364	0.270	0.359	185	0.251	0.319	0.239	0.331
18	0.474	0.645	0.476	0.640	102	0.266	0.372	0.264	0.365	186	0.240	0.334	0.249	0.330
19	0.457	0.618	0.463	0.613	103	0.274	0.372	0.282	0.370	187	0.252	0.327	0.240	0.328
20	0.432	0.616	0.472	0.603	104	0.264	0.372	0.271	0.392	188	0.235	0.329	0.250	0.334
21	0.458	0.621	0.451	0.604	105	0.264	0.373	0.269	0.372	189	0.240	0.339	0.244	0.327
22	0.441	0.605	0.431	0.602	106	0.269	0.368	0.256	0.363	190	0.245	0.330	0.235	0.329
23	0.438	0.593	0.442	0.582	107	0.275	0.360	0.269	0.370	191	0.253	0.328	0.240	0.334
24	0.422	0.573	0.440	0.586	108	0.273	0.355	0.272	0.367	192	0.249	0.326	0.231	0.324
25	0.407	0.578	0.416	0.555	109	0.277	0.365	0.263	0.370	193	0.236	0.326	0.235	0.337
26	0.419	0.560	0.400	0.575	110	0.262	0.365	0.268	0.365	194	0.238	0.323	0.240	0.326
27	0.415	0.561	0.402	0.555	111	0.263	0.373	0.269	0.363	195	0.242	0.329	0.246	0.327
28	0.402	0.553	0.396	0.545	112	0.263	0.368	0.260	0.357	196	0.249	0.323	0.243	0.327
29	0.401	0.552	0.390	0.558	113	0.263	0.354	0.264	0.369	197	0.234	0.328	0.235	0.331
30	0.389	0.538	0.399	0.544	114	0.271	0.374	0.267	0.367	198	0.240	0.333	0.234	0.316
31	0.382	0.533	0.407	0.546	115	0.267	0.353	0.255	0.366	199	0.247	0.328	0.239	0.328
32	0.389	0.529	0.384	0.524	116	0.262	0.364	0.262	0.361	200	0.239	0.328	0.246	0.328
33	0.377	0.519	0.376	0.516	117	0.273	0.367	0.260	0.359	201	0.239	0.326	0.247	0.326
34	0.370	0.519	0.365	0.508	118	0.273	0.366	0.270	0.357	202	0.242	0.320	0.241	0.329
35	0.377	0.513	0.371	0.510	119	0.264	0.360	0.262	0.369	203	0.244	0.316	0.237	0.331
36	0.377	0.503	0.377	0.503	120	0.255	0.358	0.257	0.351	204	0.234	0.318	0.238	0.327
37	0.372	0.490	0.363	0.498	121	0.259	0.360	0.259	0.368	205	0.244	0.331	0.236	0.319
38	0.365	0.498	0.368	0.499	122	0.262	0.354	0.259	0.354	206	0.245	0.333	0.241	0.340
39	0.354	0.505	0.359	0.496	123	0.257	0.356	0.270	0.361	207	0.244	0.319	0.243	0.333
40	0.364	0.502	0.347	0.491	124	0.268	0.355	0.266	0.348	208	0.236	0.323	0.254	0.318
41	0.350	0.487	0.351	0.488	125	0.267	0.358	0.269	0.357	209	0.227	0.326	0.236	0.324
42	0.348	0.476	0.346	0.485	126	0.260	0.367	0.260	0.353	210	0.237	0.321	0.235	0.322
43	0.343	0.478	0.347	0.478	127	0.256	0.348	0.260	0.358	211	0.246	0.331	0.235	0.326
44	0.355	0.480	0.339	0.478	128	0.264	0.355	0.261	0.351	212	0.232	0.324	0.230	0.323
45	0.342	0.460	0.346	0.463	129	0.261	0.353	0.262	0.355	213	0.238	0.319	0.249	0.324
46	0.335	0.468	0.348	0.457	130	0.252	0.355	0.254	0.353	214	0.232	0.326	0.242	0.335
47	0.340	0.479	0.335	0.458	131	0.259	0.349	0.257	0.350	215	0.236	0.323	0.234	0.329
48	0.340	0.459	0.333	0.463	132	0.256	0.356	0.255	0.349	216	0.236	0.321	0.229	0.324
49	0.333	0.457	0.342	0.453	133	0.248	0.352	0.260	0.351	217	0.233	0.317	0.226	0.328
50	0.334	0.455	0.332	0.447	134	0.257	0.343	0.246	0.348	218	0.242	0.321	0.233	0.327
51	0.334	0.449	0.319	0.462	135	0.266	0.355	0.257	0.364	219	0.228	0.317	0.244	0.323
52	0.331	0.449	0.330	0.445	136	0.255	0.351	0.252	0.356	220	0.239	0.322	0.240	0.322
53	0.328	0.428	0.323	0.437	137	0.250	0.344	0.259	0.348	221	0.232	0.319	0.237	0.324
54	0.315	0.437	0.318	0.459	138	0.268	0.343	0.253	0.355	222	0.234	0.330	0.236	0.328
55	0.326	0.431	0.329	0.436	139	0.246	0.342	0.256	0.357	223	0.236	0.315	0.229	0.321
56	0.312	0.440	0.327	0.437	140	0.255	0.353	0.251	0.335	224	0.236	0.316	0.229	0.308
57	0.330	0.435	0.320	0.437	141	0.257	0.348	0.247	0.357	225	0.232	0.319	0.232	0.325
58	0.321	0.434	0.318	0.436	142	0.248	0.351	0.257	0.348	226	0.231	0.323	0.234	0.312
59	0.314	0.416	0.305	0.426	143	0.254	0.346	0.253	0.342	227	0.227	0.318	0.234	0.318
60	0.325	0.436	0.318	0.437	144	0.252	0.346	0.259	0.342	228	0.230	0.320	0.228	0.326
61	0.313	0.432	0.318	0.418	145	0.254	0.343	0.255	0.342	229	0.240	0.325	0.232	0.317
62	0.309	0.416	0.318	0.434	146	0.251	0.337	0.250	0.354	230	0.234	0.317	0.238	0.318
63	0.313	0.419	0.317	0.422	147	0.254	0.351	0.249	0.348	231	0.232	0.326	0.237	0.316
64	0.323	0.435	0.318	0.433	148	0.255	0.343	0.250	0.339	232	0.224	0.315	0.241	0.330
65	0.308	0.421	0.304	0.423	149	0.256	0.351	0.251	0.340	233	0.232	0.322	0.233	0.310
66	0.310	0.422	0.312	0.427	150	0.246	0.343	0.249	0.349	234	0.233	0.320	0.230	0.319
67	0.308	0.427	0.309	0.419	151	0.252	0.343	0.258	0.338	235	0.239	0.321	0.234	0.324
68	0.293	0.412	0.304	0.404	152	0.248	0.349	0.255	0.336	236	0.231	0.317	0.233	0.313
69	0.307	0.414	0.289	0.404	153	0.250	0.340	0.256	0.346	237	0.232	0.326	0.232	0.313
70	0.317	0.411	0.306	0.413	154	0.257	0.338	0.257	0.348	238	0.234	0.324	0.241	0.315
71	0.302	0.403	0.297	0.405	155	0.247	0.344	0.249	0.339	239	0.239	0.331	0.235	0.323
72	0.296	0.409	0.303	0.411	156	0.257	0.342	0.240	0.335	240	0.234	0.316	0.231	0.313
73	0.303	0.413	0.303	0.411	157	0.254	0.342	0.236	0.334	241	0.233	0.313	0.236	0.318
74	0.286	0.411	0.302	0.393	158	0.246	0.336	0.245	0.341	242	0.233	0.321	0.236	0.314
75	0.288	0.397	0.298	0.397	159	0.264	0.347	0.254	0.334	243	0.235	0.324	0.234	0.312
76	0.284	0.393	0.285	0.403	160	0.243	0.329	0.247	0.336	244	0.235	0.325	0.227	0.318
77	0.304	0.401	0.295	0.389	161	0.253	0.337	0.260	0.342	245	0.226	0.317	0.226	0.319
78	0.291	0.404	0.293	0.408	162	0.250	0.336	0.249	0.339	246	0.233	0.311	0.238	0.320
79	0.295	0.389	0.291	0.413	163	0.248	0.337	0.245	0.335	247	0.240	0.314	0.236	0.322
80	0.290	0.402	0.307	0.394	164	0.245	0.337	0.248	0.349	248	0.231	0.323	0.226	0.319
81	0.292	0.397	0.296	0.392	165	0.244	0.337	0.258	0.335	249	0.230	0.319	0.228	0.321
82	0.286	0.387	0.284	0.396	166	0.253	0.341	0.247	0.333	250	0.224	0.314	0.235	0.316
83	0.280	0.387	0.295	0.392	167	0.240	0.338	0.255	0.338	251	0.236	0.310	0.220	0.317
84	0.296	0.388	0.281	0.402	168	0.246	0.334	0.245	0.332	252	0.227	0.319	0.227	0.317

MST Cut-off values $\alpha = 0.01$, $m_0 = 100$, for $p = 2$, $X_1 \sim N(0,1)$, $X_2 \sim$ Uniform or t_3, for $p = 5$, $X_1, X_2, X_3 \sim N(0,1)$, $X_4, X_5 \sim$ both Uniform or t_3

	Uniform		t_3			Uniform		t_3			Uniform		t_3	
n	$p=2$	$p=5$	$p=2$	$p=5$	n	$p=2$	$p=5$	$p=2$	$p=5$	n	$p=2$	$p=5$	$p=2$	$p=5$
253	0.229	0.314	0.233	0.317	337	0.215	0.310	0.212	0.303	421	0.223	0.302	0.216	0.292
254	0.228	0.317	0.224	0.308	338	0.222	0.304	0.222	0.310	422	0.214	0.300	0.215	0.303
255	0.231	0.320	0.233	0.309	339	0.216	0.304	0.227	0.309	423	0.206	0.295	0.217	0.295
256	0.228	0.315	0.235	0.317	340	0.228	0.304	0.220	0.305	424	0.222	0.296	0.224	0.295
257	0.228	0.319	0.227	0.313	341	0.225	0.301	0.221	0.297	425	0.216	0.296	0.210	0.291
258	0.223	0.320	0.223	0.315	342	0.226	0.306	0.233	0.305	426	0.223	0.294	0.215	0.300
259	0.234	0.313	0.231	0.310	343	0.221	0.303	0.224	0.306	427	0.221	0.301	0.216	0.295
260	0.236	0.319	0.231	0.319	344	0.218	0.305	0.222	0.303	428	0.218	0.292	0.222	0.299
261	0.234	0.307	0.234	0.310	345	0.225	0.303	0.221	0.302	429	0.220	0.301	0.217	0.296
262	0.225	0.322	0.232	0.313	346	0.213	0.311	0.223	0.298	430	0.217	0.295	0.224	0.299
263	0.233	0.308	0.231	0.315	347	0.222	0.312	0.222	0.305	431	0.211	0.292	0.217	0.292
264	0.222	0.325	0.240	0.309	348	0.223	0.310	0.223	0.299	432	0.227	0.294	0.221	0.296
265	0.228	0.307	0.225	0.316	349	0.216	0.303	0.224	0.303	433	0.212	0.297	0.214	0.300
266	0.221	0.312	0.228	0.320	350	0.223	0.299	0.219	0.307	434	0.214	0.291	0.214	0.301
267	0.228	0.319	0.232	0.305	351	0.223	0.303	0.214	0.307	435	0.220	0.300	0.216	0.295
268	0.227	0.317	0.236	0.316	352	0.210	0.304	0.224	0.301	436	0.212	0.297	0.221	0.293
269	0.233	0.313	0.229	0.309	353	0.220	0.299	0.229	0.295	437	0.221	0.297	0.208	0.296
270	0.225	0.304	0.232	0.311	354	0.216	0.305	0.223	0.302	438	0.215	0.292	0.208	0.306
271	0.232	0.307	0.229	0.313	355	0.215	0.298	0.221	0.308	439	0.219	0.294	0.214	0.300
272	0.234	0.315	0.233	0.313	356	0.221	0.306	0.224	0.303	440	0.217	0.299	0.212	0.296
273	0.235	0.305	0.229	0.313	357	0.221	0.304	0.215	0.311	441	0.221	0.300	0.229	0.298
274	0.239	0.314	0.237	0.316	358	0.223	0.311	0.220	0.308	442	0.215	0.297	0.218	0.291
275	0.218	0.318	0.233	0.315	359	0.222	0.303	0.223	0.305	443	0.219	0.303	0.217	0.293
276	0.234	0.318	0.229	0.316	360	0.227	0.310	0.222	0.300	444	0.216	0.300	0.227	0.309
277	0.222	0.305	0.226	0.314	361	0.219	0.303	0.222	0.308	445	0.216	0.297	0.216	0.302
278	0.222	0.311	0.227	0.313	362	0.231	0.307	0.219	0.306	446	0.220	0.299	0.215	0.306
279	0.235	0.310	0.236	0.316	363	0.223	0.303	0.219	0.296	447	0.215	0.295	0.213	0.290
280	0.230	0.321	0.225	0.317	364	0.229	0.300	0.218	0.299	448	0.221	0.294	0.217	0.302
281	0.224	0.309	0.235	0.312	365	0.219	0.317	0.219	0.304	449	0.206	0.295	0.219	0.294
282	0.229	0.315	0.229	0.317	366	0.229	0.303	0.231	0.303	450	0.221	0.303	0.217	0.292
283	0.225	0.313	0.228	0.315	367	0.223	0.297	0.222	0.306	451	0.215	0.297	0.225	0.291
284	0.226	0.314	0.225	0.312	368	0.222	0.305	0.213	0.300	452	0.218	0.296	0.220	0.288
285	0.218	0.315	0.241	0.311	369	0.218	0.303	0.219	0.312	453	0.218	0.304	0.221	0.289
286	0.239	0.308	0.229	0.307	370	0.221	0.306	0.219	0.303	454	0.217	0.290	0.208	0.298
287	0.237	0.311	0.219	0.313	371	0.213	0.309	0.220	0.310	455	0.214	0.288	0.210	0.301
288	0.229	0.310	0.220	0.309	372	0.217	0.301	0.220	0.300	456	0.207	0.298	0.220	0.301
289	0.236	0.313	0.230	0.319	373	0.229	0.302	0.220	0.299	457	0.218	0.298	0.213	0.297
290	0.232	0.310	0.226	0.315	374	0.233	0.299	0.226	0.302	458	0.220	0.293	0.225	0.298
291	0.220	0.306	0.219	0.311	375	0.216	0.294	0.218	0.301	459	0.217	0.294	0.215	0.299
292	0.236	0.310	0.227	0.309	376	0.219	0.300	0.215	0.304	460	0.216	0.291	0.214	0.303
293	0.223	0.315	0.231	0.309	377	0.220	0.297	0.225	0.307	461	0.222	0.295	0.204	0.286
294	0.230	0.310	0.229	0.308	378	0.219	0.301	0.218	0.301	462	0.214	0.288	0.223	0.296
295	0.219	0.310	0.220	0.311	379	0.225	0.304	0.224	0.301	463	0.210	0.293	0.213	0.297
296	0.228	0.305	0.221	0.307	380	0.225	0.295	0.229	0.292	464	0.221	0.289	0.210	0.291
297	0.225	0.303	0.223	0.304	381	0.222	0.301	0.221	0.293	465	0.221	0.288	0.222	0.298
298	0.227	0.311	0.224	0.314	382	0.223	0.292	0.214	0.298	466	0.214	0.295	0.219	0.291
299	0.224	0.306	0.226	0.310	383	0.226	0.303	0.222	0.294	467	0.212	0.296	0.210	0.295
300	0.226	0.312	0.226	0.313	384	0.217	0.302	0.217	0.314	468	0.218	0.292	0.221	0.296
301	0.226	0.320	0.232	0.309	385	0.214	0.297	0.225	0.302	469	0.218	0.291	0.221	0.299
302	0.221	0.305	0.233	0.310	386	0.228	0.298	0.223	0.294	470	0.211	0.301	0.215	0.292
303	0.221	0.313	0.225	0.308	387	0.227	0.302	0.210	0.298	471	0.218	0.291	0.216	0.293
304	0.228	0.310	0.222	0.306	388	0.222	0.305	0.220	0.297	472	0.215	0.288	0.221	0.290
305	0.224	0.304	0.211	0.309	389	0.225	0.295	0.215	0.297	473	0.220	0.297	0.220	0.293
306	0.230	0.312	0.222	0.310	390	0.220	0.298	0.217	0.294	474	0.218	0.291	0.222	0.294
307	0.222	0.310	0.228	0.306	391	0.218	0.297	0.228	0.297	475	0.210	0.297	0.211	0.291
308	0.222	0.303	0.227	0.311	392	0.223	0.304	0.222	0.295	476	0.209	0.290	0.205	0.293
309	0.217	0.306	0.219	0.304	393	0.230	0.301	0.221	0.291	477	0.224	0.293	0.211	0.296
310	0.227	0.310	0.225	0.313	394	0.223	0.296	0.223	0.293	478	0.210	0.301	0.213	0.293
311	0.222	0.296	0.229	0.308	395	0.218	0.300	0.229	0.309	479	0.215	0.297	0.220	0.291
312	0.225	0.312	0.218	0.309	396	0.214	0.303	0.221	0.296	480	0.215	0.303	0.218	0.304
313	0.225	0.302	0.227	0.307	397	0.216	0.298	0.227	0.298	481	0.219	0.291	0.216	0.290
314	0.220	0.305	0.228	0.308	398	0.218	0.290	0.224	0.306	482	0.212	0.301	0.215	0.295
315	0.219	0.301	0.224	0.309	399	0.213	0.296	0.220	0.298	483	0.212	0.293	0.211	0.298
316	0.222	0.310	0.220	0.309	400	0.227	0.299	0.221	0.295	484	0.212	0.292	0.219	0.303
317	0.225	0.304	0.220	0.310	401	0.213	0.305	0.212	0.297	485	0.210	0.293	0.208	0.295
318	0.219	0.301	0.227	0.306	402	0.218	0.302	0.216	0.290	486	0.215	0.297	0.213	0.293
319	0.221	0.305	0.225	0.302	403	0.211	0.309	0.216	0.303	487	0.215	0.297	0.219	0.291
320	0.224	0.306	0.225	0.303	404	0.216	0.302	0.214	0.300	488	0.220	0.291	0.225	0.298
321	0.218	0.306	0.219	0.307	405	0.223	0.297	0.208	0.301	489	0.214	0.295	0.208	0.298
322	0.224	0.313	0.220	0.301	406	0.210	0.306	0.227	0.305	490	0.220	0.294	0.220	0.296
323	0.226	0.307	0.217	0.308	407	0.230	0.304	0.205	0.304	491	0.223	0.297	0.210	0.290
324	0.225	0.317	0.234	0.302	408	0.220	0.309	0.220	0.301	492	0.214	0.294	0.218	0.297
325	0.224	0.315	0.216	0.305	409	0.216	0.306	0.225	0.298	493	0.210	0.291	0.218	0.287
326	0.218	0.306	0.216	0.309	410	0.222	0.296	0.219	0.295	494	0.209	0.290	0.218	0.289
327	0.226	0.302	0.221	0.301	411	0.209	0.299	0.230	0.292	495	0.217	0.294	0.218	0.294
328	0.227	0.294	0.218	0.305	412	0.218	0.297	0.223	0.300	496	0.212	0.296	0.219	0.291
329	0.220	0.299	0.228	0.307	413	0.214	0.295	0.222	0.299	497	0.212	0.300	0.217	0.293
330	0.219	0.310	0.226	0.307	414	0.210	0.298	0.219	0.291	498	0.217	0.290	0.222	0.302
331	0.227	0.311	0.217	0.304	415	0.216	0.302	0.217	0.298	499	0.215	0.289	0.211	0.308
332	0.220	0.310	0.224	0.307	416	0.217	0.304	0.212	0.301	500	0.211	0.307	0.212	0.288
333	0.226	0.305	0.217	0.308	417	0.225	0.300	0.217	0.300					
334	0.228	0.311	0.222	0.305	418	0.213	0.297	0.224	0.296					
335	0.222	0.306	0.226	0.318	419	0.224	0.294	0.212	0.293					
336	0.222	0.310	0.220	0.302	420	0.217	0.294	0.219	0.301					

Table A.19: SREWMA Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.05$, $m_0 = 50$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
1	0.46	1.10	2.00	3.18	51	43.76	103.95	189.35	299.70
2	1.76	4.19	7.64	12.12	52	43.41	102.86	187.48	296.87
3	4.08	9.71	17.71	28.09	53	42.99	102.03	185.69	294.05
4	6.98	16.62	30.31	48.03	54	42.61	101.12	183.91	291.49
5	10.24	24.38	44.45	70.43	55	42.27	100.19	182.29	288.70
n=6	13.67	32.57	59.40	94.07	56	41.84	99.27	180.49	285.89
7	17.17	40.86	74.52	118.09	57	41.45	98.38	178.84	283.29
8	20.63	49.09	89.51	141.79	58	41.09	97.43	177.21	280.47
9	23.98	57.00	103.93	164.78	59	40.70	96.53	175.57	277.94
10	27.15	64.56	117.74	186.54	60	40.31	95.64	173.84	275.24
11	30.12	71.62	130.52	206.85	61	39.94	94.71	172.27	272.79
12	32.87	78.18	142.52	225.70	62	39.54	93.81	170.69	270.11
13	35.38	84.17	153.48	243.08	63	39.20	92.94	169.03	267.60
14	37.70	89.60	163.29	258.79	64	38.83	92.08	167.47	265.16
15	39.75	94.48	172.28	272.96	65	38.47	91.15	165.96	262.65
16	41.58	98.86	180.23	285.54	66	38.14	90.33	164.44	260.26
17	43.23	102.76	187.34	296.77	67	37.78	89.43	163.02	257.73
18	44.66	106.18	193.43	306.35	68	37.39	88.61	161.32	255.26
19	45.86	109.04	198.82	314.95	69	37.12	87.77	159.77	252.86
20	46.91	111.59	203.37	322.26	70	36.68	86.98	158.22	250.66
21	47.80	113.70	207.32	328.18	71	36.35	86.13	156.64	248.08
22	48.57	115.46	210.44	333.34	72	36.00	85.31	155.26	245.81
23	49.14	116.91	213.15	337.56	73	35.68	84.60	153.81	243.48
24	49.67	118.09	215.45	340.97	74	35.31	83.77	152.44	241.18
25	50.04	118.96	216.80	343.48	75	35.01	82.95	150.96	238.96
26	50.38	119.60	218.08	345.27	76	34.66	82.16	149.51	236.70
27	50.49	120.14	218.75	346.57	77	34.34	81.39	148.21	234.37
28	50.64	120.32	219.24	347.33	78	34.01	80.59	146.74	232.11
29	50.61	120.36	219.49	347.52	79	33.69	79.84	145.35	229.98
30	50.63	120.30	219.31	347.24	80	33.36	79.08	144.04	227.76
31	50.47	120.14	218.90	346.58	81	33.05	78.31	142.71	225.70
32	50.38	119.71	218.34	345.70	82	32.75	77.55	141.40	223.65
33	50.14	119.33	217.46	344.40	83	32.43	76.86	140.04	221.58
34	49.97	118.83	216.43	342.92	84	32.11	76.13	138.69	219.48
35	49.72	118.20	215.37	341.24	85	31.79	75.41	137.49	217.39
36	49.45	117.51	214.00	339.30	86	31.51	74.77	136.12	215.38
37	49.12	116.81	212.79	337.12	87	31.20	74.01	134.84	213.38
38	48.79	116.12	211.30	334.91	88	30.92	73.32	133.56	211.38
39	48.42	115.27	209.85	332.33	89	30.63	72.62	132.31	209.40
40	48.07	114.39	208.17	329.90	90	30.33	71.93	130.99	207.45
41	47.78	113.48	206.63	327.26	91	30.02	71.33	129.86	205.57
42	47.34	112.60	205.04	324.69	92	29.75	70.61	128.64	203.62
43	47.00	111.70	203.27	322.06	93	29.47	69.98	127.43	201.76
44	46.61	110.80	201.57	319.17	94	29.20	69.36	126.25	199.95
45	46.19	109.78	199.83	316.57	95	28.93	68.74	125.08	198.08
46	45.82	108.81	198.14	313.69	96	28.66	68.10	123.96	196.23
47	45.42	107.82	196.35	310.92	97	28.40	67.49	122.78	194.40
48	44.96	106.81	194.45	308.15	98	28.12	66.84	121.69	192.68
49	44.56	105.86	192.73	305.25	99	27.86	66.28	120.54	190.93
50	44.15	104.92	190.97	302.49	100	27.60	65.64	119.50	189.17

Table A.20: SREWMA Cut-off values $N_p(\mathbf{0},\mathbb{I})$, $\alpha = 0.01$, $m_0 = 50$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
1	0.46	1.10	2.01	3.19	85	43.11	102.36	186.46	295.29
2	1.88	4.47	8.16	12.93	86	42.84	101.75	185.03	293.33
3	4.64	11.04	20.13	31.91	87	42.5	100.93	183.84	291.2
4	8.15	19.41	35.4	56.11	88	42.21	100.23	182.6	289.09
5	12.13	28.91	52.72	83.54	89	41.93	99.65	181.36	287.22
6	16.36	38.96	71.1	112.73	90	41.64	98.82	180.18	285.21
7	20.67	49.28	89.86	142.59	91	41.37	98.16	178.97	283.44
8	24.98	59.39	108.48	171.88	92	41.1	97.52	177.74	281.61
9	29.09	69.27	126.42	200.33	93	40.79	96.86	176.63	279.7
10	33.03	78.66	143.54	227.58	94	40.52	96.17	175.4	277.81
11	36.77	87.47	159.71	252.94	95	40.22	95.54	174.34	275.8
12	40.13	95.75	174.58	276.77	96	39.95	94.9	173.05	274.11
13	43.32	103.15	188.22	298.42	97	39.7	94.25	171.9	272.24
14	46.21	109.98	200.62	318.08	98	39.43	93.62	170.84	270.45
15	48.73	116.12	212.05	335.76	99	39.18	93	169.53	268.7
16	51.02	121.61	222.11	351.91	100	38.91	92.42	168.56	266.91
17	53.15	126.44	230.88	365.85	101	38.69	91.84	167.35	265.12
18	54.93	130.68	238.5	378.16	102	38.39	91.26	166.5	263.27
19	56.48	134.35	245.34	388.96	103	38.16	90.65	165.33	261.61
20	57.80	137.56	251.28	397.87	104	37.91	90.07	164.27	259.92
21	58.94	140.33	256.3	405.87	105	37.66	89.53	163.28	258.31
22	59.89	142.56	260.21	412.43	106	37.42	88.91	162.25	256.7
23	60.61	144.4	263.53	417.82	107	37.2	88.32	161.13	255.05
24	61.31	145.94	266.44	422.37	108	36.96	87.82	160.15	253.47
25	61.87	147.18	268.66	425.67	109	36.73	87.3	159.11	251.75
26	62.21	148.03	270.27	428.28	110	36.52	86.79	158.2	250.16
27	62.48	148.55	271.35	429.88	111	36.27	86.15	157.24	248.68
28	62.65	148.93	272.01	431.27	112	36.05	85.58	156.2	247.2
29	62.75	149.12	272.59	431.55	113	35.82	85.04	155.19	245.6
30	62.77	149.33	272.27	431.64	114	35.6	84.52	154.26	244.19
31	62.66	149.03	271.9	431.14	115	35.4	84.1	153.26	242.65
32	62.51	148.66	271.55	430.14	116	35.19	83.56	152.42	241.2
33	62.35	148.29	270.41	428.9	117	34.96	83.09	151.51	239.65
34	62.04	147.65	269.5	427.32	118	34.72	82.58	150.63	238.21
35	61.8	146.98	268.24	425.19	119	34.51	82.07	149.72	236.88
36	61.49	146.21	266.92	423.16	120	34.34	81.56	148.88	235.37
37	61.13	145.48	265.54	420.92	121	34.12	81.06	147.94	234.02
38	60.86	144.58	263.99	418.17	122	33.92	80.6	147.06	232.59
39	60.49	143.64	262.21	415.59	123	33.71	80.08	146.21	231.23
40	60.08	142.75	260.55	412.87	124	33.51	79.66	145.23	229.88
41	59.67	141.57	258.72	409.9	125	33.31	79.12	144.39	228.46
42	59.22	140.7	257.1	406.94	126	33.13	78.7	143.51	227.07
43	58.8	139.76	255.01	404.08	127	32.91	78.27	142.76	225.8
44	58.4	138.61	253.28	401.14	128	32.74	77.76	141.87	224.47
45	58.02	137.62	251.21	397.88	129	32.52	77.34	141.05	223.11
46	57.52	136.54	249.3	395.14	130	32.36	76.9	140.29	221.82
47	57	135.52	247.38	391.67	131	32.19	76.44	139.49	220.57
48	56.57	134.4	245.61	388.69	132	31.97	76.02	138.67	219.32
49	56.15	133.36	243.58	385.58	133	31.81	75.6	137.86	218.08
50	55.74	132.22	241.62	382.63	134	31.6	75.13	137.09	216.89
51	55.39	131.29	239.75	379.54	135	31.44	74.7	136.32	215.71
52	54.9	130.26	237.84	376.69	136	31.26	74.3	135.51	214.4
53	54.48	129.24	236.11	373.63	137	31.1	73.86	134.71	213.25
54	54.09	128.28	234.16	370.62	138	30.89	73.46	133.95	212.09
55	53.82	127.29	232.57	367.9	139	30.74	73.01	133.26	210.92
56	53.36	126.4	230.75	364.88	140	30.58	72.59	132.5	209.74
57	52.93	125.41	228.93	362.34	141	30.4	72.18	131.74	208.57
58	52.59	124.51	227.21	359.42	142	30.21	71.77	131.03	207.36
59	52.16	123.64	225.56	356.8	143	30.06	71.37	130.35	206.29
60	51.8	122.67	223.85	354.09	144	29.88	70.98	129.6	205.18
61	51.41	121.82	222.16	351.43	145	29.73	70.62	128.93	204.06
62	50.97	120.8	220.49	348.98	146	29.56	70.25	128.22	202.98
63	50.62	119.97	218.68	346.21	147	29.41	69.86	127.52	201.9
64	50.25	119.17	217.21	343.56	148	29.27	69.48	126.84	200.88
65	49.88	118.2	215.5	340.92	149	29.12	69.15	126.19	199.85
66	49.50	117.43	214.05	338.47	150	28.96	68.74	125.49	198.68
67	49.15	116.54	212.66	335.95	151	28.82	68.38	124.84	197.61
68	48.79	115.61	210.9	333.37	152	28.65	68.04	124.17	196.58
69	48.37	114.81	209.32	330.88	153	28.49	67.67	123.5	195.56
70	48.00	113.87	207.77	328.66	154	28.33	67.35	122.92	194.51
71	47.66	113.06	206.11	326.31	155	28.19	66.97	122.25	193.47
72	47.33	112.17	204.61	323.98	156	28.06	66.62	121.62	192.5
73	46.98	111.44	203.27	321.49	157	27.91	66.26	120.98	191.51
74	46.70	110.65	201.73	319.07	158	27.77	65.93	120.36	190.63
75	46.38	109.85	200.31	316.94	159	27.63	65.58	119.72	189.63
76	45.97	109.07	198.78	314.46	160	27.49	65.25	119.11	188.57
77	45.63	108.25	197.43	312.42	161	27.34	64.92	118.5	187.61
78	45.28	107.46	196.01	310.03	162	27.21	64.59	117.9	186.63
79	44.99	106.76	194.59	307.78	163	27.06	64.25	117.22	185.7
80	44.64	106.02	193.31	305.84	164	26.92	63.93	116.67	184.73
81	44.37	105.24	191.78	303.73	165	26.78	63.61	116.06	183.82
82	44.05	104.47	190.45	301.78	166	26.66	63.30	115.47	182.86
83	43.73	103.8	189.09	299.55	167	26.52	62.97	114.88	181.97
84	43.47	103.04	187.71	297.28	168	26.39	62.66	114.3	181.05

SREWMA Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 50$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
169	26.24	62.36	113.7	180.13	253	18.09	43.08	78.58	124.49
170	26.11	62.04	113.13	179.29	254	18.02	42.9	78.27	124.03
171	25.98	61.74	112.57	178.38	255	17.96	42.73	77.94	123.5
172	25.86	61.4	112.01	177.49	256	17.88	42.55	77.64	123.04
173	25.71	61.15	111.48	176.67	257	17.81	42.4	77.34	122.55
174	25.59	60.83	110.96	175.77	258	17.74	42.23	77.05	122.1
175	25.49	60.53	110.32	174.92	259	17.66	42.07	76.73	121.6
176	25.36	60.22	109.85	174.04	260	17.61	41.89	76.47	121.14
177	25.24	59.9	109.3	173.2	261	17.53	41.75	76.15	120.69
178	25.12	59.62	108.77	172.31	262	17.46	41.61	75.88	120.19
179	24.97	59.33	108.26	171.47	263	17.4	41.44	75.56	119.8
180	24.86	59.06	107.76	170.69	264	17.32	41.26	75.28	119.34
181	24.75	58.78	107.25	169.86	265	17.26	41.08	75	118.9
182	24.64	58.51	106.71	169.06	266	17.2	40.94	74.73	118.42
183	24.51	58.23	106.25	168.25	267	17.13	40.77	74.47	117.96
184	24.4	57.95	105.7	167.45	268	17.08	40.62	74.17	117.55
185	24.28	57.68	105.24	166.65	269	17.01	40.46	73.91	117.09
186	24.16	57.41	104.74	165.92	270	16.95	40.32	73.62	116.61
187	24.04	57.14	104.24	165.11	271	16.89	40.15	73.33	116.17
188	23.95	56.88	103.71	164.33	272	16.81	40.01	73.07	115.73
189	23.83	56.61	103.26	163.54	273	16.76	39.86	72.77	115.31
190	23.72	56.33	102.75	162.74	274	16.68	39.71	72.52	114.83
191	23.6	56.06	102.31	162.03	275	16.63	39.58	72.24	114.43
192	23.49	55.82	101.81	161.25	276	16.56	39.42	71.99	113.97
193	23.39	55.57	101.37	160.52	277	16.5	39.29	71.71	113.56
194	23.29	55.34	100.91	159.73	278	16.43	39.12	71.44	113.12
195	23.19	55.08	100.44	158.97	279	16.37	38.98	71.16	112.7
196	23.06	54.84	99.98	158.25	280	16.31	38.82	70.87	112.31
197	22.98	54.6	99.49	157.55	281	16.25	38.68	70.6	111.87
198	22.86	54.37	99.04	156.88	282	16.19	38.56	70.36	111.45
199	22.75	54.12	98.62	156.12	283	16.13	38.41	70.06	111.07
200	22.64	53.85	98.17	155.43	284	16.07	38.28	69.79	110.69
201	22.55	53.62	97.74	154.73	285	16.01	38.14	69.56	110.3
202	22.44	53.38	97.3	154	286	15.95	37.99	69.3	109.88
203	22.33	53.11	96.86	153.35	287	15.89	37.86	69.05	109.49
204	22.24	52.9	96.43	152.61	288	15.85	37.73	68.82	109.11
205	22.15	52.65	96.04	151.98	289	15.79	37.59	68.56	108.71
206	22.05	52.41	95.58	151.3	290	15.72	37.45	68.32	108.33
207	21.95	52.18	95.15	150.67	291	15.67	37.31	68.07	107.9
208	21.83	51.97	94.74	149.99	292	15.62	37.18	67.83	107.53
209	21.73	51.72	94.29	149.33	293	15.56	37.04	67.58	107.14
210	21.64	51.51	93.88	148.67	294	15.5	36.91	67.35	106.75
211	21.53	51.26	93.48	148.01	295	15.45	36.77	67.11	106.39
212	21.44	51.05	93.08	147.44	296	15.39	36.64	66.86	106
213	21.34	50.84	92.68	146.73	297	15.34	36.5	66.65	105.63
214	21.26	50.58	92.27	146.12	298	15.29	36.37	66.4	105.23
215	21.16	50.36	91.88	145.5	299	15.24	36.24	66.15	104.88
216	21.07	50.16	91.47	144.84	300	15.18	36.12	65.92	104.48
217	20.99	49.93	91.08	144.22	301	15.12	36	65.71	104.12
218	20.89	49.72	90.7	143.64	302	15.08	35.87	65.46	103.76
219	20.81	49.51	90.3	143.01	303	15.03	35.76	65.24	103.4
220	20.72	49.3	89.91	142.38	304	14.98	35.63	65	103.03
221	20.63	49.07	89.54	141.76	305	14.92	35.5	64.76	102.68
222	20.54	48.89	89.14	141.19	306	14.87	35.37	64.54	102.29
223	20.46	48.67	88.78	140.58	307	14.82	35.25	64.33	101.93
224	20.39	48.48	88.38	139.95	308	14.76	35.13	64.11	101.6
225	20.29	48.25	88.02	139.38	309	14.71	35	63.88	101.25
226	20.22	48.04	87.64	138.78	310	14.66	34.88	63.69	100.88
227	20.12	47.84	87.27	138.23	311	14.6	34.76	63.43	100.53
228	20.05	47.65	86.89	137.69	312	14.56	34.65	63.22	100.22
229	19.96	47.42	86.53	137.1	313	14.5	34.54	62.99	99.86
230	19.87	47.23	86.19	136.55	314	14.45	34.42	62.78	99.53
231	19.79	47.05	85.84	135.99	315	14.4	34.3	62.57	99.19
232	19.71	46.86	85.5	135.42	316	14.35	34.18	62.34	98.85
233	19.64	46.67	85.13	134.82	317	14.3	34.07	62.15	98.51
234	19.55	46.45	84.79	134.3	318	14.25	33.94	61.94	98.16
235	19.47	46.27	84.42	133.78	319	14.19	33.82	61.71	97.84
236	19.39	46.08	84.07	133.2	320	14.15	33.71	61.51	97.48
237	19.33	45.9	83.74	132.68	321	14.1	33.6	61.28	97.19
238	19.23	45.71	83.4	132.12	322	14.06	33.48	61.07	96.82
239	19.16	45.55	83.1	131.6	323	14	33.37	60.85	96.49
240	19.07	45.35	82.71	131.07	324	13.96	33.26	60.64	96.15
241	19	45.17	82.4	130.55	325	13.92	33.16	60.45	95.84
242	18.92	44.99	82.06	130.01	326	13.87	33.03	60.24	95.49
243	18.84	44.79	81.73	129.5	327	13.83	32.94	60.04	95.19
244	18.77	44.61	81.41	128.99	328	13.78	32.82	59.84	94.92
245	18.68	44.43	81.07	128.48	329	13.73	32.71	59.63	94.57
246	18.61	44.25	80.76	127.94	330	13.68	32.6	59.43	94.25
247	18.54	44.07	80.43	127.43	331	13.64	32.48	59.23	93.92
248	18.46	43.9	80.12	126.94	332	13.6	32.38	59.05	93.6
249	18.4	43.73	79.8	126.43	333	13.55	32.26	58.85	93.28
250	18.32	43.56	79.48	125.92	334	13.5	32.17	58.67	92.96
251	18.25	43.4	79.2	125.46	335	13.46	32.05	58.47	92.64
252	18.18	43.24	78.88	124.97	336	13.41	31.95	58.26	92.33

SREWMA Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 50$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
337	13.37	31.84	58.09	92.03	419	10.35	24.65	45	71.3
338	13.32	31.73	57.88	91.73	420	10.32	24.6	44.87	71.1
339	13.28	31.62	57.68	91.43	421	10.29	24.51	44.72	70.88
340	13.23	31.52	57.48	91.12	422	10.26	24.44	44.6	70.68
341	13.19	31.42	57.31	90.83	423	10.23	24.37	44.47	70.46
342	13.15	31.32	57.12	90.52	424	10.2	24.3	44.34	70.27
343	13.11	31.2	56.93	90.21	425	10.17	24.22	44.21	70.04
344	13.06	31.1	56.74	89.94	426	10.14	24.15	44.07	69.87
345	13.02	31.01	56.56	89.64	427	10.12	24.08	43.94	69.66
346	12.97	30.89	56.37	89.35	428	10.09	24.01	43.81	69.46
347	12.93	30.8	56.2	89.09	429	10.06	23.94	43.68	69.26
348	12.89	30.7	56.01	88.77	430	10.02	23.86	43.55	69.05
349	12.85	30.6	55.82	88.49	431	9.99	23.8	43.42	68.86
350	12.81	30.5	55.63	88.2	432	9.97	23.73	43.3	68.68
351	12.77	30.39	55.47	87.92	433	9.94	23.66	43.19	68.47
352	12.73	30.29	55.28	87.63	434	9.91	23.59	43.08	68.26
353	12.68	30.19	55.1	87.35	435	9.89	23.53	42.94	68.06
354	12.64	30.1	54.92	87.07	436	9.86	23.46	42.81	67.85
355	12.6	30.01	54.75	86.8	437	9.83	23.4	42.69	67.66
356	12.56	29.9	54.58	86.53	438	9.8	23.33	42.56	67.46
357	12.52	29.81	54.4	86.25	439	9.77	23.26	42.47	67.26
358	12.48	29.71	54.21	85.97	440	9.74	23.2	42.34	67.09
359	12.43	29.62	54.05	85.69	441	9.71	23.13	42.23	66.89
360	12.39	29.51	53.87	85.42	442	9.69	23.07	42.11	66.69
361	12.36	29.43	53.71	85.17	443	9.66	23.01	41.99	66.5
362	12.32	29.33	53.55	84.88	444	9.63	22.94	41.86	66.31
363	12.29	29.24	53.38	84.6	445	9.6	22.87	41.74	66.12
364	12.25	29.15	53.21	84.35	446	9.57	22.81	41.64	65.94
365	12.21	29.06	53.06	84.07	447	9.55	22.75	41.53	65.76
366	12.17	28.96	52.88	83.8	448	9.52	22.68	41.4	65.56
367	12.13	28.88	52.74	83.54	449	9.5	22.61	41.29	65.39
368	12.09	28.78	52.57	83.29	450	9.47	22.55	41.15	65.21
369	12.05	28.68	52.41	83.01	451	9.44	22.48	41.04	65.03
370	12.01	28.6	52.26	82.76	452	9.42	22.42	40.91	64.84
371	11.98	28.51	52.08	82.49	453	9.39	22.36	40.81	64.68
372	11.94	28.41	51.92	82.23	454	9.36	22.29	40.69	64.49
373	11.91	28.33	51.74	81.98	455	9.34	22.23	40.58	64.32
374	11.86	28.24	51.58	81.74	456	9.31	22.17	40.47	64.12
375	11.83	28.16	51.43	81.49	457	9.28	22.11	40.35	63.94
376	11.79	28.07	51.27	81.23	458	9.25	22.05	40.24	63.77
377	11.76	27.98	51.11	80.99	459	9.23	21.98	40.13	63.59
378	11.72	27.89	50.94	80.72	460	9.2	21.92	40.02	63.41
379	11.69	27.81	50.79	80.48	461	9.18	21.86	39.9	63.22
380	11.65	27.72	50.63	80.23	462	9.15	21.79	39.79	63.05
381	11.62	27.63	50.48	79.98	463	9.13	21.73	39.69	62.88
382	11.58	27.54	50.31	79.72	464	9.1	21.67	39.58	62.71
383	11.54	27.47	50.14	79.49	465	9.07	21.61	39.46	62.54
384	11.51	27.38	50	79.24	466	9.05	21.55	39.34	62.37
385	11.47	27.3	49.83	79.01	467	9.03	21.49	39.23	62.2
386	11.43	27.22	49.69	78.77	468	9	21.42	39.13	62.02
387	11.4	27.13	49.54	78.54	469	8.97	21.36	39.02	61.83
388	11.36	27.06	49.4	78.29	470	8.95	21.31	38.9	61.67
389	11.33	26.97	49.25	78.04	471	8.92	21.25	38.8	61.49
390	11.29	26.89	49.09	77.82	472	8.9	21.19	38.68	61.33
391	11.26	26.8	48.94	77.57	473	8.87	21.13	38.58	61.13
392	11.22	26.72	48.78	77.32	474	8.84	21.07	38.46	60.97
393	11.19	26.64	48.63	77.1	475	8.83	21.01	38.36	60.8
394	11.15	26.56	48.47	76.87	476	8.8	20.95	38.26	60.63
395	11.12	26.48	48.35	76.65	477	8.77	20.89	38.15	60.46
396	11.08	26.4	48.2	76.42	478	8.75	20.84	38.05	60.3
397	11.05	26.31	48.04	76.18	479	8.72	20.77	37.94	60.13
398	11.02	26.24	47.89	75.95	480	8.7	20.72	37.84	59.99
399	10.99	26.17	47.77	75.7	481	8.68	20.66	37.72	59.79
400	10.95	26.08	47.61	75.49	482	8.65	20.61	37.62	59.64
401	10.92	26	47.46	75.26	483	8.63	20.55	37.52	59.48
402	10.89	25.92	47.31	75	484	8.6	20.49	37.42	59.3
403	10.85	25.84	47.18	74.78	485	8.58	20.43	37.31	59.14
404	10.82	25.77	47.05	74.55	486	8.56	20.37	37.21	58.97
405	10.79	25.69	46.91	74.36	487	8.54	20.32	37.11	58.8
406	10.75	25.61	46.77	74.14	488	8.52	20.26	37.01	58.66
407	10.72	25.55	46.63	73.91	489	8.49	20.21	36.92	58.49
408	10.69	25.47	46.5	73.69	490	8.47	20.15	36.8	58.35
409	10.66	25.4	46.34	73.46	491	8.44	20.1	36.71	58.16
410	10.63	25.33	46.22	73.25	492	8.42	20.04	36.61	58.01
411	10.59	25.25	46.08	73.03	493	8.4	19.99	36.51	57.87
412	10.57	25.18	45.94	72.82	494	8.37	19.93	36.41	57.69
413	10.53	25.1	45.81	72.61	495	8.35	19.88	36.32	57.53
414	10.5	25.02	45.68	72.37	496	8.33	19.83	36.22	57.38
415	10.47	24.94	45.54	72.13	497	8.31	19.77	36.11	57.22
416	10.44	24.88	45.4	71.95	498	8.28	19.72	36.03	57.06
417	10.41	24.8	45.27	71.74	499	8.26	19.66	35.92	56.92
418	10.38	24.73	45.14	71.51	500	8.23	19.61	35.83	56.75

Table A.21: SREWMA Cut-off values $N_p(\mathbf{0},\mathbb{I})$, $\alpha = 0.05$, $m_0 = 100$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
1	0.46	1.10	2.00	3.18	51	43.91	104.40	190.25	301.81
2	1.69	4.05	7.38	11.73	52	43.66	103.79	189.17	300.09
3	3.73	8.88	16.19	25.72	53	43.40	103.17	188.02	298.25
4	6.22	14.82	27.04	42.94	54	43.16	102.57	187.00	296.50
5	9.01	21.45	39.14	62.13	55	42.89	101.92	185.82	294.74
6	11.96	28.44	51.87	82.36	56	42.58	101.31	184.65	292.95
7	14.95	35.56	64.85	102.88	57	42.34	100.71	183.47	291.12
8	17.90	42.57	77.63	123.26	58	42.06	100.10	182.44	289.48
9	20.76	49.41	90.12	142.91	59	41.83	99.44	181.15	287.51
10	23.54	55.90	102.00	161.83	60	41.57	98.85	180.15	285.69
11	26.13	62.09	113.23	179.69	61	41.33	98.20	179.02	283.99
12	28.56	67.93	123.75	196.37	62	41.04	97.63	177.95	282.21
13	30.80	73.23	133.48	211.69	63	40.78	97.00	176.75	280.44
14	32.84	78.13	142.42	225.98	64	40.55	96.40	175.60	278.72
15	34.74	82.63	150.63	238.92	65	40.30	95.74	174.49	276.97
16	36.49	86.68	158.01	250.73	66	40.06	95.16	173.42	275.09
17	38.02	90.32	164.72	261.34	67	39.78	94.56	172.24	273.47
18	39.39	93.65	170.71	270.86	68	39.51	93.95	171.16	271.72
19	40.63	96.59	176.05	279.26	69	39.26	93.38	170.09	270.05
20	41.69	99.15	180.80	286.92	70	39.02	92.83	169.04	268.36
21	42.69	101.46	184.98	293.41	71	38.80	92.24	167.94	266.53
22	43.52	103.43	188.57	299.10	72	38.52	91.66	166.92	264.80
23	44.22	105.13	191.60	304.04	73	38.30	91.11	165.88	263.13
24	44.85	106.59	194.43	308.44	74	38.06	90.54	164.82	261.53
25	45.34	107.87	196.62	312.03	75	37.81	89.99	163.84	259.94
26	45.86	108.87	198.55	314.89	76	37.58	89.41	162.74	258.45
27	46.16	109.76	200.21	317.43	77	37.36	88.83	161.71	256.75
28	46.45	110.47	201.41	319.46	78	37.11	88.27	160.84	255.16
29	46.68	110.96	202.45	321.05	79	36.90	87.75	159.79	253.71
30	46.86	111.32	203.13	322.03	80	36.65	87.24	158.82	252.14
31	46.96	111.64	203.57	322.88	81	36.43	86.64	157.83	250.48
32	47.03	111.77	203.89	323.43	82	36.18	86.13	156.85	249.10
33	47.07	111.89	203.98	323.60	83	35.98	85.61	155.90	247.45
34	47.05	111.89	203.99	323.50	84	35.75	85.09	154.89	245.95
35	47.02	111.77	203.78	323.17	85	35.56	84.53	153.90	244.43
36	46.98	111.55	203.38	322.61	86	35.34	84.03	152.92	242.99
37	46.82	111.39	202.96	322.09	87	35.11	83.50	152.02	241.53
38	46.69	111.06	202.42	321.17	88	34.90	82.98	151.12	240.05
39	46.57	110.72	201.84	320.05	89	34.67	82.45	150.23	238.52
40	46.43	110.29	201.08	319.12	90	34.46	81.97	149.33	237.07
41	46.23	109.92	200.44	317.88	91	34.32	81.45	148.36	235.70
42	46.04	109.45	199.50	316.58	92	34.07	80.96	147.50	234.20
43	45.87	108.97	198.64	315.09	93	33.87	80.48	146.59	232.73
44	45.57	108.41	197.69	313.65	94	33.69	80.04	145.75	231.46
45	45.42	107.89	196.67	312.02	95	33.49	79.53	144.84	229.97
46	45.18	107.38	195.70	310.53	96	33.28	79.05	144.04	228.59
47	44.91	106.83	194.70	308.90	97	33.06	78.55	143.19	227.25
48	44.68	106.24	193.55	307.18	98	32.87	78.06	142.28	225.89
49	44.42	105.60	192.46	305.39	99	32.67	77.60	141.45	224.49
50	44.15	104.99	191.32	303.71	100	32.49	77.12	140.57	223.28

Table A.22: SREWMA Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 100$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
1	0.46	1.1	2.01	3.19	85	51.48	122.33	223.65	354.46
2	1.84	4.39	8.02	12.72	86	51.2	121.69	222.52	352.82
3	4.47	10.66	19.47	30.88	87	50.94	121.19	221.44	351.15
4	7.81	18.63	34.03	53.95	88	50.74	120.57	220.48	349.37
5	11.6	27.67	50.54	80.17	89	50.5	120.11	219.4	347.82
6	15.63	37.3	68.12	108.06	90	50.27	119.53	218.45	346
7	19.76	47.18	86.12	136.65	91	50.05	118.92	217.32	344.58
8	23.85	56.98	104.04	164.99	92	49.82	118.41	216.34	343.03
9	27.88	66.5	121.49	192.68	93	49.62	117.9	215.28	341.31
10	31.74	75.67	138.29	219.33	94	49.42	117.3	214.32	339.73
11	35.36	84.39	154.15	244.39	95	49.18	116.83	213.36	338.21
12	38.8	92.49	168.94	267.94	96	48.99	116.3	212.36	336.72
13	41.95	100	182.75	289.95	97	48.75	115.78	211.62	335.2
14	44.87	107.03	195.59	309.93	98	48.53	115.26	210.69	333.84
15	47.53	113.4	207.05	328.49	99	48.31	114.76	209.64	332.26
16	49.93	119.11	217.55	345.14	100	48.11	114.28	208.73	330.87
17	52.16	124.34	227.1	360.49	101	47.87	113.77	207.81	329.5
18	54.13	129.06	235.78	373.91	102	47.69	113.32	206.74	328.12
19	55.89	133.31	243.39	386.01	103	47.48	112.84	206.06	326.72
20	57.43	136.96	250.17	396.92	104	47.31	112.38	205.36	325.48
21	58.83	140.17	255.98	406.21	105	47.13	111.95	204.31	323.96
22	60	143.16	261.27	414.64	106	46.89	111.47	203.44	322.63
23	61.06	145.6	265.82	421.71	107	46.72	110.98	202.77	321.31
24	61.93	147.7	269.66	427.86	108	46.56	110.55	201.82	319.94
25	62.7	149.6	272.95	433.11	109	46.37	110.04	200.95	318.64
26	63.32	151.12	275.89	437.64	110	46.2	109.62	200.27	317.46
27	63.89	152.32	278.2	441.24	111	45.97	109.13	199.47	316.11
28	64.28	153.42	280.13	444.42	112	45.84	108.74	198.65	314.86
29	64.6	154.15	281.47	446.57	113	45.65	108.33	197.83	313.64
30	64.93	154.84	282.74	448.49	114	45.43	107.84	197.09	312.3
31	65.09	155.23	283.54	449.76	115	45.28	107.45	196.33	311.1
32	65.19	155.5	284.11	450.56	116	45.08	107.04	195.53	309.81
33	65.26	155.57	284.38	451.09	117	44.86	106.61	194.72	308.45
34	65.3	155.67	284.55	450.96	118	44.71	106.2	193.96	307.34
35	65.25	155.59	284.26	450.75	119	44.54	105.77	193.14	306.12
36	65.17	155.41	284.06	450.25	120	44.35	105.37	192.46	304.86
37	65.07	155.17	283.41	449.39	121	44.15	104.88	191.57	303.69
38	64.94	154.84	282.93	448.37	122	43.98	104.5	190.86	302.55
39	64.69	154.42	281.94	447.28	123	43.78	104.04	190.04	301.33
40	64.6	154.01	280.96	445.87	124	43.62	103.62	189.26	300.18
41	64.4	153.46	280.15	444.12	125	43.47	103.23	188.6	298.92
42	64.06	152.83	279.15	442.53	126	43.28	102.82	187.8	297.74
43	63.84	152.18	278.09	440.89	127	43.11	102.42	186.98	296.66
44	63.56	151.6	276.88	438.91	128	42.95	101.98	186.25	295.48
45	63.31	150.93	275.56	436.91	129	42.78	101.58	185.48	294.34
46	63.02	150.25	274.32	435.03	130	42.64	101.19	184.81	293.1
47	62.77	149.58	272.99	433.08	131	42.48	100.86	184.12	292
48	62.41	148.78	271.63	430.83	132	42.31	100.46	183.3	290.88
49	62.15	148.02	270.37	428.74	133	42.13	100.05	182.56	289.73
50	61.79	147.29	268.91	426.37	134	41.91	99.67	181.95	288.66
51	61.51	146.44	267.5	424.09	135	41.79	99.28	181.13	287.46
52	61.12	145.65	266.12	421.93	136	41.61	98.91	180.51	286.3
53	60.81	144.93	264.71	419.62	137	41.44	98.6	179.75	285.28
54	60.51	144.18	263.27	417.22	138	41.28	98.23	179.03	284.28
55	60.18	143.31	262.03	415.14	139	41.13	97.85	178.37	283.16
56	59.81	142.56	260.65	412.87	140	40.95	97.45	177.65	281.96
57	59.49	141.72	258.99	410.57	141	40.8	97.06	177.14	280.84
58	59.14	140.95	257.66	408.23	142	40.63	96.68	176.35	279.81
59	58.78	140.28	256.23	406.25	143	40.48	96.36	175.69	278.68
60	58.5	139.49	254.72	403.76	144	40.33	95.98	175	277.65
61	58.17	138.68	253.39	401.66	145	40.18	95.62	174.36	276.53
62	57.82	137.93	252.02	399.38	146	40.03	95.27	173.74	275.53
63	57.51	137.16	250.58	397.34	147	39.86	94.9	173.11	274.48
64	57.2	136.44	249.13	395.1	148	39.72	94.55	172.42	273.4
65	56.92	135.65	247.92	393.05	149	39.58	94.17	171.78	272.41
66	56.63	134.92	246.59	390.85	150	39.43	93.85	171.14	271.34
67	56.32	134.12	245.18	388.65	151	39.28	93.44	170.43	270.44
68	56.02	133.45	243.88	386.56	152	39.15	93.15	169.82	269.37
69	55.76	132.79	242.6	384.64	153	39.04	92.78	169.16	268.34
70	55.46	132.1	241.28	382.68	154	38.87	92.43	168.57	267.24
71	55.18	131.36	240.07	380.53	155	38.72	92.04	167.94	266.21
72	54.89	130.69	238.63	378.35	156	38.57	91.70	167.31	265.28
73	54.60	129.95	237.5	376.57	157	38.43	91.38	166.72	264.29
74	54.31	129.29	236.17	374.44	158	38.30	91.03	166.07	263.22
75	54.07	128.63	235.08	372.54	159	38.17	90.69	165.58	262.32
76	53.76	128.05	233.88	370.71	160	38.01	90.38	164.81	261.34
77	53.47	127.31	232.74	368.83	161	37.88	89.98	164.26	260.35
78	53.24	126.67	231.46	366.83	162	37.74	89.65	163.59	259.33
79	52.96	125.98	230.32	365.07	163	37.61	89.32	162.92	258.43
80	52.71	125.41	229.15	363.17	164	37.45	89.00	162.39	257.47
81	52.46	124.72	228.03	361.42	165	37.29	88.67	161.88	256.49
82	52.20	124.14	226.83	359.63	166	37.15	88.36	161.28	255.56
83	51.93	123.55	225.96	357.86	167	37.01	88.03	160.63	254.64
84	51.70	122.9	224.82	356.24	168	36.87	87.72	160.12	253.66
85	51.48	122.33	223.65	354.46	169	36.73	87.36	159.57	252.76

SREWMA Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 100$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
170	36.61	87.09	159.01	251.82	254	27.51	65.46	119.5	189.42
171	36.48	86.77	158.32	250.93	255	27.43	65.26	119.13	188.85
172	36.36	86.42	157.76	249.98	256	27.33	65.07	118.72	188.3
173	36.21	86.14	157.2	249.06	257	27.25	64.85	118.39	187.74
174	36.09	85.79	156.58	248.17	258	27.17	64.7	118.03	187.19
175	35.95	85.49	156.05	247.31	259	27.08	64.46	117.63	186.57
176	35.82	85.2	155.56	246.33	260	26.99	64.26	117.26	185.94
177	35.71	84.88	154.93	245.46	261	26.91	64.05	116.89	185.35
178	35.57	84.57	154.41	244.62	262	26.82	63.85	116.54	184.81
179	35.43	84.29	153.8	243.69	263	26.74	63.64	116.11	184.22
180	35.32	83.98	153.28	242.84	264	26.67	63.45	115.76	183.68
181	35.18	83.66	152.67	242.03	265	26.58	63.25	115.43	183.14
182	35.07	83.34	152.16	241.12	266	26.49	63.08	115.05	182.57
183	34.94	83.05	151.59	240.31	267	26.42	62.86	114.69	181.98
184	34.82	82.74	151.12	239.46	268	26.35	62.64	114.37	181.41
185	34.69	82.48	150.63	238.57	269	26.25	62.44	114.03	180.87
186	34.58	82.13	150.1	237.74	270	26.16	62.28	113.69	180.25
187	34.45	81.83	149.62	236.86	271	26.09	62.08	113.36	179.73
188	34.34	81.56	149.02	236.14	272	26.01	61.89	112.95	179.2
189	34.2	81.22	148.46	235.24	273	25.93	61.7	112.64	178.6
190	34.07	80.98	147.96	234.5	274	25.85	61.51	112.25	178.11
191	33.95	80.68	147.43	233.65	275	25.77	61.34	111.91	177.47
192	33.83	80.4	146.9	232.86	276	25.68	61.15	111.56	176.99
193	33.7	80.15	146.42	232.06	277	25.6	60.95	111.26	176.42
194	33.59	79.85	145.9	231.3	278	25.53	60.74	110.91	175.92
195	33.47	79.56	145.42	230.53	279	25.45	60.57	110.55	175.38
196	33.36	79.32	144.93	229.68	280	25.37	60.39	110.21	174.8
197	33.24	79.03	144.32	228.89	281	25.28	60.19	109.87	174.27
198	33.12	78.79	143.85	228.08	282	25.21	60.02	109.56	173.8
199	33.02	78.52	143.35	227.3	283	25.14	59.85	109.22	173.3
200	32.9	78.25	142.88	226.45	284	25.07	59.68	108.91	172.78
201	32.79	77.97	142.39	225.7	285	24.98	59.52	108.58	172.33
202	32.71	77.7	141.9	224.77	286	24.92	59.3	108.27	171.8
203	32.57	77.47	141.31	224.11	287	24.84	59.15	107.94	171.28
204	32.48	77.17	140.85	223.35	288	24.77	58.96	107.61	170.73
205	32.35	76.96	140.37	222.57	289	24.7	58.79	107.3	170.17
206	32.24	76.64	139.87	221.87	290	24.62	58.61	106.97	169.66
207	32.12	76.39	139.4	221.13	291	24.55	58.45	106.66	169.11
208	32.00	76.13	138.97	220.42	292	24.47	58.25	106.31	168.62
209	31.89	75.85	138.51	219.56	293	24.38	58.13	106.02	168.16
210	31.80	75.61	138.01	218.93	294	24.32	57.95	105.7	167.65
211	31.70	75.32	137.61	218.2	295	24.23	57.79	105.36	167.18
212	31.58	75.1	137.11	217.38	296	24.17	57.6	105.04	166.67
213	31.48	74.82	136.62	216.67	297	24.1	57.38	104.76	166.25
214	31.36	74.59	136.1	215.89	298	24.04	57.25	104.43	165.68
215	31.29	74.32	135.7	215.16	299	23.96	57.05	104.12	165.21
216	31.18	74.09	135.21	214.45	300	23.9	56.9	103.84	164.73
217	31.06	73.86	134.77	213.67	301	23.82	56.72	103.53	164.22
218	30.98	73.59	134.36	212.97	302	23.77	56.54	103.22	163.79
219	30.86	73.34	133.9	212.26	303	23.7	56.36	102.92	163.29
220	30.75	73.14	133.45	211.5	304	23.61	56.19	102.63	162.84
221	30.62	72.88	132.99	210.81	305	23.55	56.04	102.35	162.3
222	30.54	72.65	132.54	210.09	306	23.48	55.85	102.03	161.81
223	30.44	72.41	132.13	209.42	307	23.42	55.69	101.7	161.34
224	30.34	72.15	131.68	208.69	308	23.35	55.53	101.43	160.87
225	30.25	71.93	131.24	208.09	309	23.28	55.37	101.12	160.42
226	30.16	71.65	130.82	207.43	310	23.2	55.2	100.81	159.96
227	30.05	71.41	130.4	206.68	311	23.15	55.06	100.52	159.45
228	29.96	71.20	129.96	206	312	23.08	54.87	100.24	158.99
229	29.86	70.97	129.53	205.23	313	23	54.72	99.92	158.58
230	29.77	70.72	129.1	204.58	314	22.94	54.57	99.63	158.05
231	29.66	70.49	128.7	203.91	315	22.86	54.43	99.37	157.64
232	29.55	70.26	128.29	203.25	316	22.79	54.26	99.09	157.14
233	29.45	70.05	127.86	202.58	317	22.73	54.09	98.77	156.71
234	29.37	69.81	127.42	201.93	318	22.66	53.94	98.48	156.23
235	29.26	69.59	127.01	201.23	319	22.6	53.79	98.19	155.85
236	29.16	69.35	126.6	200.59	320	22.53	53.63	97.93	155.37
237	29.09	69.13	126.17	200	321	22.48	53.48	97.67	154.92
238	28.99	68.91	125.81	199.28	322	22.41	53.32	97.35	154.52
239	28.90	68.7	125.4	198.66	323	22.34	53.19	97.1	154.08
240	28.79	68.45	124.96	198.05	324	22.28	53.02	96.82	153.58
241	28.68	68.28	124.55	197.39	325	22.22	52.86	96.54	153.17
242	28.6	68	124.16	196.79	326	22.15	52.72	96.28	152.72
243	28.49	67.78	123.77	196.16	327	22.09	52.57	96.02	152.27
244	28.40	67.57	123.39	195.56	328	22.03	52.41	95.73	151.89
245	28.32	67.35	122.91	194.89	329	21.98	52.28	95.45	151.48
246	28.22	67.13	122.54	194.26	330	21.9	52.12	95.19	151.05
247	28.13	66.93	122.18	193.66	331	21.85	51.98	94.91	150.62
248	28.05	66.71	121.83	193.02	332	21.78	51.84	94.62	150.19
249	27.96	66.48	121.43	192.46	333	21.73	51.68	94.37	149.79
250	27.88	66.31	120.98	191.86	334	21.66	51.54	94.08	149.35
251	27.77	66.09	120.64	191.3	335	21.61	51.39	93.84	148.9
252	27.69	65.86	120.24	190.71	336	21.53	51.24	93.55	148.51
253	27.60	65.66	119.89	190.03	337	21.48	51.10	93.28	148.09

SREWMA Cut-off values $N_p(\mathbf{0},\mathbb{I})$, $\alpha = 0.01$, $m_0 = 100$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
338	21.42	50.95	93.05	147.68	420	17.17	40.91	74.68	118.5
339	21.36	50.81	92.8	147.21	421	17.13	40.8	74.49	118.19
340	21.29	50.69	92.54	146.84	422	17.08	40.7	74.31	117.92
341	21.24	50.51	92.26	146.45	423	17.04	40.6	74.11	117.57
342	21.18	50.39	92	145.99	424	17	40.49	73.93	117.29
343	21.12	50.25	91.74	145.6	425	16.96	40.39	73.75	117
344	21.04	50.12	91.5	145.2	426	16.91	40.27	73.56	116.67
345	20.99	49.98	91.23	144.78	427	16.87	40.19	73.35	116.37
346	20.93	49.84	90.99	144.34	428	16.83	40.07	73.18	116.1
347	20.85	49.7	90.74	143.98	429	16.78	39.96	72.98	115.78
348	20.81	49.57	90.5	143.54	430	16.74	39.87	72.81	115.46
349	20.75	49.42	90.24	143.14	431	16.7	39.78	72.64	115.19
350	20.68	49.29	90	142.79	432	16.65	39.67	72.45	114.91
351	20.63	49.17	89.75	142.37	433	16.61	39.57	72.24	114.63
352	20.58	49.03	89.53	141.99	434	16.56	39.47	72.05	114.36
353	20.52	48.88	89.26	141.58	435	16.52	39.37	71.88	114.05
354	20.46	48.74	89.01	141.21	436	16.47	39.28	71.69	113.77
355	20.4	48.62	88.78	140.8	437	16.43	39.18	71.51	113.47
356	20.34	48.49	88.53	140.45	438	16.38	39.1	71.31	113.19
357	20.29	48.35	88.29	140.04	439	16.35	38.99	71.16	112.91
358	20.24	48.2	88.03	139.65	440	16.31	38.9	70.98	112.65
359	20.18	48.06	87.8	139.29	441	16.27	38.79	70.8	112.35
360	20.12	47.94	87.58	138.89	442	16.22	38.7	70.62	112.05
361	20.07	47.79	87.32	138.51	443	16.19	38.61	70.46	111.77
362	20.02	47.68	87.09	138.14	444	16.15	38.5	70.28	111.49
363	19.97	47.55	86.86	137.75	445	16.1	38.39	70.1	111.2
364	19.91	47.42	86.62	137.4	446	16.06	38.3	69.9	110.92
365	19.85	47.28	86.35	137.1	447	16.02	38.2	69.72	110.65
366	19.8	47.17	86.14	136.68	448	15.99	38.11	69.55	110.36
367	19.74	47.04	85.92	136.33	449	15.95	38.01	69.37	110.08
368	19.69	46.94	85.69	135.95	450	15.9	37.92	69.2	109.8
369	19.64	46.8	85.45	135.62	451	15.86	37.82	69.04	109.55
370	19.59	46.69	85.21	135.32	452	15.82	37.72	68.85	109.28
371	19.54	46.55	84.96	134.87	453	15.78	37.63	68.7	109.01
372	19.48	46.42	84.77	134.52	454	15.74	37.53	68.51	108.76
373	19.43	46.3	84.54	134.16	455	15.71	37.44	68.32	108.45
374	19.37	46.17	84.31	133.82	456	15.66	37.33	68.15	108.19
375	19.32	46.07	84.08	133.43	457	15.62	37.24	67.99	107.95
376	19.26	45.95	83.86	133.07	458	15.58	37.15	67.81	107.67
377	19.22	45.82	83.63	132.72	459	15.54	37.06	67.64	107.39
378	19.16	45.7	83.42	132.37	460	15.5	36.95	67.46	107.13
379	19.11	45.57	83.19	132.01	461	15.47	36.85	67.29	106.86
380	19.07	45.44	82.98	131.64	462	15.42	36.78	67.13	106.58
381	19.02	45.3	82.75	131.28	463	15.39	36.68	66.96	106.34
382	18.97	45.17	82.53	130.93	464	15.35	36.59	66.81	106.07
383	18.91	45.06	82.28	130.58	465	15.32	36.5	66.64	105.84
384	18.86	44.95	82.09	130.25	466	15.28	36.39	66.46	105.54
385	18.81	44.83	81.87	129.85	467	15.24	36.31	66.31	105.27
386	18.77	44.7	81.64	129.56	468	15.21	36.22	66.15	105.01
387	18.73	44.58	81.44	129.2	469	15.17	36.14	65.98	104.77
388	18.67	44.47	81.23	128.89	470	15.13	36.05	65.81	104.49
389	18.62	44.36	81.02	128.54	471	15.09	35.96	65.66	104.21
390	18.57	44.23	80.79	128.19	472	15.06	35.86	65.5	103.97
391	18.52	44.14	80.59	127.85	473	15.02	35.78	65.34	103.72
392	18.47	44.04	80.36	127.5	474	14.99	35.68	65.17	103.46
393	18.42	43.9	80.13	127.16	475	14.95	35.6	65.02	103.19
394	18.37	43.8	79.93	126.84	476	14.91	35.52	64.86	102.95
395	18.32	43.66	79.73	126.47	477	14.88	35.43	64.7	102.68
396	18.27	43.56	79.5	126.14	478	14.84	35.34	64.53	102.41
397	18.23	43.45	79.29	125.82	479	14.81	35.28	64.37	102.17
398	18.19	43.33	79.08	125.49	480	14.77	35.18	64.23	101.92
399	18.14	43.22	78.87	125.18	481	14.73	35.09	64.06	101.66
400	18.09	43.11	78.66	124.83	482	14.7	35.01	63.92	101.42
401	18.05	42.99	78.46	124.49	483	14.66	34.92	63.76	101.18
402	18.00	42.86	78.24	124.2	484	14.62	34.85	63.6	100.93
403	17.96	42.76	78.05	123.84	485	14.59	34.76	63.49	100.69
404	17.91	42.66	77.85	123.53	486	14.55	34.67	63.31	100.42
405	17.86	42.55	77.65	123.17	487	14.52	34.59	63.16	100.2
406	17.81	42.44	77.43	122.87	488	14.48	34.51	62.99	99.94
407	17.76	42.31	77.25	122.58	489	14.44	34.42	62.85	99.68
408	17.72	42.21	77.04	122.23	490	14.41	34.33	62.7	99.44
409	17.68	42.10	76.84	121.93	491	14.37	34.25	62.54	99.2
410	17.62	41.98	76.66	121.61	492	14.34	34.17	62.4	98.99
411	17.58	41.87	76.47	121.29	493	14.30	34.09	62.25	98.73
412	17.54	41.76	76.26	120.97	494	14.26	34.02	62.09	98.51
413	17.48	41.65	76.07	120.64	495	14.23	33.93	61.93	98.26
414	17.43	41.55	75.85	120.34	496	14.20	33.85	61.77	98.03
415	17.38	41.44	75.65	120.04	497	14.16	33.76	61.62	97.79
416	17.34	41.33	75.44	119.74	498	14.12	33.68	61.5	97.55
417	17.31	41.22	75.27	119.43	499	14.09	33.6	61.34	97.3
418	17.26	41.11	75.04	119.12	500	14.06	33.53	61.19	97.07
419	17.21	41.02	74.85	118.84					

Table A.23: SREWMA Cut-off values $\alpha = 0.05$, $m_0 = 100$, for $p = 2$, $X_1 \sim N(0,1)$, $X_2 \sim$ Uniform or t_3, for $p = 5$, $X_1, X_2, X_3 \sim N(0,1)$, $X_4, X_5 \sim$ both Uniform or t_3

	Uniform		t_3			Uniform		t_3	
n	$p = 2$	$p = 5$	$p = 2$	$p = 5$	n	$p = 2$	$p = 5$	$p = 2$	$p = 5$
1	0.46	3.19	0.46	3.16	51	61.69	425.53	60.78	420.38
2	1.85	12.75	1.82	12.61	52	61.36	423.34	60.44	418.10
3	4.48	30.98	4.42	30.61	53	61.04	420.99	60.15	415.90
4	7.84	54.14	7.73	53.49	54	60.73	418.67	59.80	413.65
5	11.65	80.42	11.47	79.45	55	60.37	416.29	59.50	411.39
n=6	15.70	108.41	15.47	107.13	56	60.06	414.09	59.17	409.08
7	19.85	137.04	19.56	135.40	57	59.72	411.83	58.85	406.83
8	23.99	165.52	23.63	163.59	58	59.42	409.54	58.51	404.61
9	28.02	193.32	27.58	191.00	59	59.09	407.31	58.20	402.34
10	31.87	219.94	31.40	217.34	60	58.74	404.99	57.86	400.23
11	35.53	245.21	34.99	242.28	61	58.45	402.91	57.57	397.88
12	38.94	268.85	38.36	265.60	62	58.12	400.63	57.24	395.77
13	42.13	290.71	41.49	287.28	63	57.79	398.47	56.92	393.56
14	45.07	311.05	44.37	307.34	64	57.48	396.22	56.63	391.43
15	47.73	329.56	47.01	325.59	65	57.18	394.07	56.31	389.17
16	50.18	346.34	49.41	342.15	66	56.88	391.89	56.00	387.09
17	52.39	361.48	51.58	357.20	67	56.58	389.80	55.69	385.00
18	54.36	375.14	53.52	370.62	68	56.28	387.72	55.39	382.98
19	56.11	387.22	55.26	382.68	69	55.97	385.57	55.09	380.85
20	57.67	397.94	56.80	393.28	70	55.68	383.52	54.81	378.81
21	59.05	407.49	58.17	402.55	71	55.38	381.41	54.51	376.82
22	60.23	415.83	59.33	410.87	72	55.08	379.36	54.25	374.80
23	61.30	423.11	60.37	417.94	73	54.79	377.35	53.97	372.91
24	62.23	429.22	61.24	424.15	74	54.50	375.39	53.68	370.93
25	62.96	434.57	62.04	429.38	75	54.23	373.49	53.40	368.86
26	63.61	438.95	62.65	433.60	76	53.95	371.54	53.12	367.08
27	64.14	442.61	63.18	437.26	77	53.71	369.62	52.86	365.13
28	64.59	445.64	63.59	440.25	78	53.42	367.77	52.58	363.31
29	64.91	448.01	63.95	442.60	79	53.14	365.91	52.32	361.49
30	65.18	449.79	64.22	444.42	80	52.86	364.06	52.06	359.60
31	65.36	451.08	64.40	445.63	81	52.63	362.31	51.81	357.89
32	65.50	451.91	64.51	446.45	82	52.35	360.46	51.56	356.09
33	65.56	452.40	64.57	446.85	83	52.10	358.76	51.30	354.32
34	65.59	452.41	64.61	447.05	84	51.84	356.94	51.06	352.65
35	65.53	452.23	64.56	446.77	85	51.59	355.27	50.81	350.90
36	65.44	451.72	64.48	446.25	86	51.35	353.47	50.58	349.16
37	65.36	450.85	64.38	445.39	87	51.10	351.86	50.32	347.60
38	65.21	449.86	64.23	444.43	88	50.84	350.19	50.09	345.90
39	65.03	448.70	64.04	443.25	89	50.64	348.45	49.85	344.23
40	64.82	447.32	63.85	441.87	90	50.38	346.84	49.63	342.64
41	64.61	445.80	63.61	440.40	91	50.16	345.28	49.39	340.94
42	64.40	444.13	63.41	438.81	92	49.93	343.60	49.18	339.43
43	64.09	442.28	63.14	437.02	93	49.69	342.04	48.92	337.86
44	63.84	440.38	62.90	435.15	94	49.48	340.41	48.72	336.31
45	63.56	438.48	62.60	433.23	95	49.27	338.95	48.49	334.66
46	63.29	436.45	62.34	431.08	96	49.05	337.43	48.28	333.27
47	62.96	434.35	62.04	429.06	97	48.82	336.03	48.08	331.78
48	62.66	432.21	61.73	426.90	98	48.61	334.41	47.87	330.23
49	62.35	430.02	61.41	424.79	99	48.38	332.94	47.63	328.87
50	62.03	427.82	61.11	422.58	100	48.21	331.46	47.45	327.35

Table A.24: SREWMA Cut-off values $\alpha = 0.01$, $m_0 = 100$, for $p = 2$, $X_1 \sim N(0,1)$, $X_2 \sim$ Uniform or t_3, for $p = 5$, $X_1, X_2, X_3 \sim N(0,1)$, $X_4, X_5 \sim$ both Uniform or t_3

	Uniform		t_3			Uniform		t_3			Uniform		t_3	
n	$p=2$	$p=5$	$p=2$	$p=5$	n	$p=2$	$p=5$	$p=2$	$p=5$	n	$p=2$	$p=5$	$p=2$	$p=5$
1	0.46	3.20	0.46	3.17	85	51.84	356.18	51.04	351.76	169	37.01	253.99	36.46	250.93
2	1.85	12.77	1.83	12.64	86	51.55	354.33	50.80	350.12	170	36.89	253.09	36.30	249.95
3	4.50	31.02	4.43	30.67	87	51.31	352.59	50.55	348.30	171	36.74	252.14	36.19	249.03
4	7.87	54.22	7.75	53.60	88	51.04	350.89	50.30	346.76	172	36.58	251.25	36.08	248.09
5	11.68	80.54	11.51	79.63	89	50.81	349.30	50.12	345.13	173	36.47	250.38	35.93	247.18
n=6	15.75	108.59	15.52	107.32	90	50.55	347.68	49.85	343.58	174	36.31	249.39	35.80	246.31
7	19.94	137.22	19.64	135.74	91	50.34	346.08	49.63	341.93	175	36.17	248.59	35.67	245.41
8	24.06	165.85	23.70	163.92	92	50.13	344.49	49.40	340.41	176	36.02	247.65	35.53	244.56
9	28.10	193.64	27.71	191.51	93	49.87	342.77	49.19	338.88	177	35.93	246.79	35.40	243.62
10	31.96	220.31	31.51	217.72	94	49.68	341.21	48.92	337.28	178	35.78	245.86	35.27	242.75
11	35.63	245.58	35.12	242.73	95	49.46	339.84	48.71	335.82	179	35.65	244.95	35.13	241.89
12	39.06	269.24	38.52	266.24	96	49.23	338.31	48.55	334.47	180	35.53	244.11	35.04	241.10
13	42.26	291.30	41.67	287.75	97	49.04	336.69	48.31	333.00	181	35.38	243.15	34.89	240.30
14	45.23	311.53	44.54	307.88	98	48.81	335.10	48.11	331.55	182	35.26	242.30	34.76	239.45
15	47.90	330.07	47.20	326.17	99	48.62	333.70	47.92	329.90	183	35.14	241.51	34.63	238.61
16	50.31	346.80	49.60	342.82	100	48.43	332.39	47.75	328.44	184	35.00	240.63	34.53	237.75
17	52.59	361.96	51.76	357.95	101	48.21	330.90	47.56	327.00	185	34.87	239.75	34.39	236.89
18	54.54	375.75	53.74	371.19	102	48.02	329.35	47.34	325.63	186	34.76	238.93	34.28	236.12
19	56.30	388.04	55.49	383.43	103	47.81	328.05	47.15	324.32	187	34.66	238.04	34.16	235.34
20	57.85	398.81	56.99	393.93	104	47.66	326.74	46.94	322.91	188	34.53	237.24	34.06	234.38
21	59.27	408.06	58.36	403.24	105	47.49	325.46	46.79	321.39	189	34.40	236.36	33.91	233.62
22	60.47	416.42	59.55	411.59	106	47.24	324.16	46.59	320.25	190	34.29	235.57	33.80	232.78
23	61.49	423.75	60.59	418.75	107	47.07	322.80	46.39	318.96	191	34.19	234.72	33.67	232.01
24	62.38	429.89	61.52	425.03	108	46.92	321.38	46.22	317.71	192	34.08	233.88	33.56	231.19
25	63.13	435.13	62.23	430.06	109	46.66	320.27	46.06	316.55	193	33.95	233.09	33.44	230.39
26	63.83	439.74	62.86	434.48	110	46.53	318.99	45.82	315.22	194	33.84	232.29	33.37	229.49
27	64.37	443.37	63.43	438.30	111	46.32	317.61	45.62	313.93	195	33.73	231.53	33.20	228.73
28	64.79	446.30	63.83	440.96	112	46.12	316.37	45.50	312.46	196	33.61	230.71	33.10	227.99
29	65.13	448.67	64.15	443.37	113	45.92	315.03	45.30	311.37	197	33.47	229.87	32.97	227.12
30	65.42	450.58	64.49	445.42	114	45.74	313.75	45.11	310.15	198	33.35	229.14	32.86	226.33
31	65.66	451.71	64.65	446.63	115	45.58	312.56	44.93	308.86	199	33.26	228.34	32.74	225.55
32	65.71	452.66	64.75	447.34	116	45.40	311.28	44.73	307.53	200	33.13	227.59	32.62	224.77
33	65.76	453.01	64.80	447.79	117	45.18	310.11	44.57	306.29	201	33.04	226.84	32.50	224.08
34	65.81	453.20	64.82	447.88	118	45.02	308.82	44.41	305.23	202	32.90	225.98	32.40	223.29
35	65.76	452.94	64.76	447.55	119	44.85	307.47	44.24	303.89	203	32.79	225.26	32.28	222.47
36	65.72	452.44	64.66	447.12	120	44.70	306.40	44.05	302.81	204	32.69	224.49	32.18	221.76
37	65.59	451.46	64.57	446.35	121	44.51	305.15	43.84	301.69	205	32.59	223.77	32.06	220.99
38	65.43	450.52	64.42	445.32	122	44.29	303.89	43.72	300.42	206	32.47	222.95	31.96	220.31
39	65.23	449.24	64.31	444.27	123	44.10	302.68	43.51	299.01	207	32.40	222.19	31.85	219.50
40	65.05	447.91	64.07	442.75	124	43.95	301.42	43.33	298.02	208	32.26	221.38	31.72	218.79
41	64.86	446.47	63.85	441.20	125	43.73	300.28	43.16	296.75	209	32.15	220.73	31.62	218.13
42	64.59	444.68	63.65	439.65	126	43.55	298.99	42.98	295.74	210	32.04	219.98	31.52	217.28
43	64.32	442.95	63.37	437.93	127	43.37	297.85	42.82	294.71	211	31.93	219.23	31.40	216.54
44	64.08	441.06	63.11	436.15	128	43.18	296.83	42.70	293.47	212	31.81	218.42	31.30	215.87
45	63.77	439.04	62.84	433.97	129	43.06	295.54	42.50	292.27	213	31.69	217.73	31.20	215.06
46	63.45	437.16	62.57	431.96	130	42.91	294.42	42.30	291.11	214	31.57	217.00	31.09	214.40
47	63.17	434.98	62.25	429.88	131	42.72	293.28	42.14	290.03	215	31.48	216.22	30.98	213.68
48	62.84	432.77	61.96	427.91	132	42.55	292.18	41.97	288.91	216	31.37	215.43	30.87	212.90
49	62.54	430.65	61.63	425.62	133	42.38	291.00	41.78	287.76	217	31.28	214.76	30.76	212.16
50	62.24	428.58	61.34	423.42	134	42.23	289.88	41.62	286.73	218	31.18	214.03	30.67	211.54
51	61.88	426.40	61.02	421.31	135	42.04	288.82	41.50	285.51	219	31.07	213.29	30.56	210.83
52	61.60	424.03	60.78	419.16	136	41.90	287.78	41.31	284.29	220	30.96	212.60	30.46	210.13
53	61.23	421.68	60.43	416.73	137	41.73	286.49	41.16	283.41	221	30.86	211.94	30.35	209.37
54	60.97	419.57	60.07	414.43	138	41.60	285.47	41.01	282.19	222	30.76	211.25	30.26	208.79
55	60.58	417.33	59.78	412.31	139	41.39	284.24	40.87	281.11	223	30.63	210.51	30.14	208.09
56	60.30	414.85	59.42	410.00	140	41.25	283.16	40.69	280.09	224	30.54	209.81	30.05	207.32
57	59.91	412.58	59.15	407.86	141	41.09	281.97	40.52	278.96	225	30.43	209.11	29.95	206.70
58	59.58	410.18	58.75	405.67	142	40.93	280.96	40.39	277.77	226	30.34	208.41	29.85	205.92
59	59.28	407.96	58.46	403.24	143	40.77	279.93	40.19	276.82	227	30.23	207.65	29.75	205.29
60	58.99	405.74	58.14	401.14	144	40.63	278.88	40.07	275.59	228	30.14	207.07	29.65	204.65
61	58.67	403.55	57.81	398.75	145	40.46	277.80	39.89	274.61	229	30.04	206.32	29.57	203.95
62	58.30	401.37	57.49	396.83	146	40.29	276.77	39.73	273.60	230	29.92	205.66	29.46	203.30
63	57.99	399.14	57.23	394.50	147	40.17	275.75	39.58	272.42	231	29.83	204.98	29.36	202.61
64	57.72	396.82	56.87	392.35	148	39.99	274.70	39.44	271.30	232	29.75	204.30	29.26	201.89
65	57.39	394.78	56.58	390.30	149	39.83	273.51	39.26	270.40	233	29.64	203.71	29.18	201.23
66	57.09	392.62	56.24	388.13	150	39.67	272.66	39.14	269.32	234	29.56	203.00	29.06	200.57
67	56.78	390.29	55.97	386.02	151	39.54	271.50	38.99	268.30	235	29.45	202.42	28.98	199.89
68	56.49	388.38	55.65	383.93	152	39.39	270.41	38.88	267.36	236	29.35	201.74	28.87	199.29
69	56.22	386.21	55.40	381.79	153	39.22	269.50	38.76	266.29	237	29.26	201.09	28.77	198.61
70	55.91	384.27	55.08	379.74	154	39.10	268.47	38.59	265.28	238	29.16	200.37	28.70	197.96
71	55.60	382.22	54.75	377.76	155	38.96	267.48	38.44	264.24	239	29.05	199.78	28.60	197.38
72	55.29	380.23	54.45	375.84	156	38.78	266.54	38.28	263.29	240	28.96	199.17	28.49	196.69
73	54.99	378.23	54.23	373.75	157	38.67	265.58	38.13	262.23	241	28.88	198.51	28.42	196.06
74	54.74	376.27	53.92	371.85	158	38.53	264.48	37.99	261.36	242	28.78	197.86	28.32	195.42
75	54.48	374.33	53.69	370.02	159	38.36	263.47	37.85	260.29	243	28.69	197.22	28.25	194.81
76	54.19	372.55	53.40	368.04	160	38.23	262.53	37.73	259.40	244	28.61	196.57	28.15	194.14
77	53.92	370.56	53.13	366.15	161	38.09	261.57	37.59	258.29	245	28.50	196.01	28.06	193.63
78	53.66	368.55	52.85	364.24	162	37.94	260.66	37.43	257.41	246	28.39	195.40	27.95	192.94
79	53.33	366.77	52.61	362.54	163	37.82	259.70	37.26	256.49	247	28.31	194.78	27.88	192.34
80	53.10	364.89	52.32	360.74	164	37.68	258.73	37.14	255.59	248	28.21	194.19	27.78	191.73
81	52.84	363.01	52.05	358.95	165	37.54	257.77	36.97	254.55	249	28.12	193.64	27.69	191.10
82	52.56	361.34	51.76	357.07	166	37.40	256.82	36.85	253.70	250	28.04	193.02	27.61	190.50
83	52.32	359.65	51.54	355.09	167	37.30	255.86	36.71	252.75	251	27.95	192.36	27.52	189.97
84	52.08	357.79	51.29	353.43	168	37.14	254.98	36.59	251.74	252	27.88	191.74	27.44	189.34

SREWMA Cut-off values $\alpha = 0.01$, $m_0 = 100$, for $p = 2$, $X_1 \sim N(0,1)$, $X_2 \sim$ Uniform or t_3, for $p = 5$, $X_1, X_2, X_3 \sim N(0,1)$, $X_4, X_5 \sim$ both Uniform or t_3

	Uniform		t_3			Uniform		t_3			Uniform		t_3	
n	$p=2$	$p=5$	$p=2$	$p=5$	n	$p=2$	$p=5$	$p=2$	$p=5$	n	$p=2$	$p=5$	$p=2$	$p=5$
253	27.79	191.16	27.35	188.75	337	21.62	148.77	21.31	146.90	421	17.23	118.75	17.01	117.28
254	27.70	190.56	27.27	188.16	338	21.56	148.33	21.25	146.49	422	17.20	118.48	16.97	117.00
255	27.61	190.01	27.18	187.58	339	21.49	147.91	21.17	146.12	423	17.14	118.17	16.92	116.67
256	27.53	189.40	27.08	186.91	340	21.44	147.48	21.12	145.67	424	17.10	117.86	16.87	116.40
257	27.45	188.83	26.99	186.34	341	21.36	147.07	21.07	145.25	425	17.06	117.54	16.84	116.10
258	27.36	188.17	26.91	185.74	342	21.32	146.67	20.99	144.86	426	17.02	117.22	16.79	115.76
259	27.25	187.61	26.83	185.23	343	21.24	146.31	20.94	144.44	427	16.98	116.94	16.75	115.49
260	27.17	186.96	26.74	184.65	344	21.18	145.86	20.89	144.04	428	16.93	116.64	16.72	115.20
261	27.08	186.42	26.66	184.05	345	21.12	145.42	20.82	143.64	429	16.89	116.36	16.66	114.91
262	27.01	185.80	26.58	183.41	346	21.07	145.05	20.77	143.27	430	16.84	116.06	16.61	114.64
263	26.93	185.21	26.48	182.86	347	21.01	144.62	20.71	142.87	431	16.80	115.74	16.57	114.37
264	26.83	184.64	26.39	182.28	348	20.95	144.24	20.64	142.50	432	16.76	115.49	16.53	114.07
265	26.75	184.07	26.31	181.72	349	20.89	143.82	20.60	142.13	433	16.72	115.19	16.49	113.75
266	26.67	183.51	26.24	181.12	350	20.84	143.43	20.53	141.77	434	16.68	114.92	16.45	113.45
267	26.59	182.91	26.16	180.52	351	20.78	143.03	20.48	141.36	435	16.63	114.63	16.41	113.17
268	26.51	182.36	26.08	180.02	352	20.73	142.67	20.43	140.94	436	16.60	114.33	16.37	112.89
269	26.43	181.79	25.98	179.41	353	20.68	142.27	20.37	140.53	437	16.55	114.02	16.33	112.61
270	26.35	181.26	25.91	178.86	354	20.62	141.87	20.32	140.11	438	16.51	113.73	16.28	112.32
271	26.27	180.70	25.85	178.36	355	20.56	141.50	20.27	139.74	439	16.47	113.44	16.25	112.05
272	26.19	180.12	25.78	177.82	356	20.50	141.10	20.21	139.37	440	16.42	113.16	16.20	111.75
273	26.11	179.56	25.71	177.22	357	20.46	140.69	20.16	138.99	441	16.38	112.89	16.16	111.49
274	26.02	179.00	25.61	176.73	358	20.39	140.32	20.09	138.65	442	16.34	112.61	16.12	111.19
275	25.96	178.40	25.54	176.17	359	20.34	139.93	20.03	138.26	443	16.29	112.32	16.08	110.89
276	25.88	177.87	25.45	175.65	360	20.29	139.55	19.99	137.86	444	16.25	112.02	16.03	110.60
277	25.81	177.37	25.38	175.17	361	20.22	139.15	19.93	137.49	445	16.22	111.74	16.00	110.36
278	25.74	176.83	25.32	174.62	362	20.16	138.78	19.87	137.11	446	16.18	111.48	15.95	110.05
279	25.65	176.27	25.24	174.05	363	20.11	138.40	19.82	136.75	447	16.14	111.18	15.91	109.80
280	25.57	175.76	25.16	173.55	364	20.05	138.05	19.77	136.37	448	16.10	110.91	15.88	109.53
281	25.49	175.24	25.08	173.01	365	20.00	137.66	19.71	135.99	449	16.05	110.63	15.84	109.26
282	25.42	174.74	25.00	172.46	366	19.95	137.32	19.66	135.64	450	16.01	110.35	15.79	108.97
283	25.35	174.24	24.92	171.97	367	19.88	136.93	19.60	135.26	451	15.98	110.09	15.75	108.70
284	25.28	173.70	24.86	171.46	368	19.83	136.56	19.55	134.87	452	15.94	109.82	15.70	108.42
285	25.18	173.18	24.77	170.96	369	19.78	136.20	19.50	134.51	453	15.89	109.55	15.66	108.16
286	25.11	172.68	24.71	170.44	370	19.73	135.83	19.45	134.15	454	15.86	109.28	15.62	107.87
287	25.02	172.11	24.62	169.91	371	19.68	135.46	19.39	133.77	455	15.82	109.02	15.58	107.61
288	24.96	171.58	24.56	169.39	372	19.62	135.09	19.34	133.40	456	15.78	108.74	15.55	107.34
289	24.88	171.11	24.49	168.86	373	19.57	134.73	19.29	133.10	457	15.74	108.46	15.50	107.07
290	24.80	170.61	24.45	168.39	374	19.52	134.34	19.23	132.71	458	15.70	108.22	15.47	106.80
291	24.74	170.10	24.37	167.91	375	19.46	134.02	19.19	132.40	459	15.65	107.96	15.42	106.53
292	24.65	169.60	24.29	167.37	376	19.42	133.67	19.14	132.02	460	15.62	107.68	15.39	106.28
293	24.58	169.10	24.21	166.88	377	19.36	133.30	19.08	131.66	461	15.58	107.37	15.35	106.03
294	24.51	168.60	24.12	166.42	378	19.31	132.96	19.03	131.33	462	15.54	107.11	15.30	105.74
295	24.44	168.09	24.05	165.90	379	19.25	132.57	18.98	131.01	463	15.50	106.83	15.27	105.47
296	24.36	167.61	23.98	165.45	380	19.20	132.24	18.93	130.65	464	15.46	106.60	15.23	105.24
297	24.29	167.09	23.92	164.94	381	19.15	131.88	18.89	130.32	465	15.42	106.32	15.20	104.98
298	24.22	166.60	23.83	164.45	382	19.10	131.52	18.84	129.94	466	15.39	106.05	15.16	104.73
299	24.16	166.08	23.76	164.04	383	19.04	131.17	18.79	129.66	467	15.35	105.79	15.12	104.45
300	24.08	165.61	23.69	163.51	384	19.00	130.86	18.74	129.25	468	15.31	105.53	15.08	104.19
301	24.00	165.10	23.62	163.02	385	18.95	130.51	18.70	128.91	469	15.27	105.27	15.05	103.92
302	23.93	164.61	23.57	162.53	386	18.91	130.15	18.64	128.57	470	15.23	105.00	15.01	103.68
303	23.87	164.12	23.49	162.00	387	18.86	129.81	18.59	128.21	471	15.19	104.74	14.97	103.43
304	23.79	163.58	23.41	161.58	388	18.81	129.46	18.55	127.87	472	15.17	104.50	14.94	103.17
305	23.72	163.12	23.34	161.07	389	18.77	129.12	18.50	127.53	473	15.13	104.20	14.90	102.94
306	23.64	162.64	23.28	160.60	390	18.72	128.78	18.44	127.18	474	15.09	103.96	14.86	102.67
307	23.57	162.15	23.22	160.13	391	18.67	128.46	18.40	126.90	475	15.06	103.68	14.82	102.41
308	23.51	161.73	23.15	159.67	392	18.62	128.11	18.35	126.52	476	15.02	103.43	14.79	102.17
309	23.45	161.25	23.07	159.19	393	18.57	127.79	18.30	126.20	477	14.98	103.20	14.75	101.91
310	23.38	160.82	23.01	158.72	394	18.53	127.42	18.26	125.88	478	14.95	102.95	14.72	101.66
311	23.30	160.32	22.95	158.29	395	18.48	127.10	18.21	125.54	479	14.91	102.69	14.68	101.43
312	23.24	159.84	22.87	157.80	396	18.42	126.75	18.16	125.20	480	14.87	102.44	14.64	101.19
313	23.16	159.44	22.81	157.32	397	18.37	126.45	18.11	124.86	481	14.83	102.22	14.60	100.92
314	23.10	158.94	22.74	156.87	398	18.32	126.09	18.07	124.55	482	14.80	101.94	14.56	100.67
315	23.04	158.52	22.70	156.46	399	18.27	125.75	18.02	124.24	483	14.76	101.67	14.53	100.44
316	22.97	158.06	22.62	156.01	400	18.22	125.43	17.98	123.93	484	14.72	101.41	14.49	100.17
317	22.91	157.56	22.56	155.54	401	18.17	125.09	17.92	123.60	485	14.69	101.19	14.46	99.93
318	22.83	157.11	22.48	155.09	402	18.13	124.82	17.88	123.27	486	14.65	100.94	14.42	99.70
319	22.77	156.66	22.42	154.64	403	18.08	124.47	17.83	122.91	487	14.61	100.69	14.39	99.43
320	22.72	156.22	22.34	154.21	404	18.03	124.15	17.79	122.64	488	14.57	100.44	14.35	99.21
321	22.66	155.74	22.29	153.77	405	17.99	123.82	17.74	122.32	489	14.54	100.18	14.32	98.97
322	22.61	155.29	22.22	153.29	406	17.94	123.49	17.69	121.96	490	14.50	99.95	14.28	98.73
323	22.54	154.84	22.16	152.86	407	17.89	123.15	17.64	121.65	491	14.47	99.70	14.24	98.50
324	22.47	154.41	22.10	152.42	408	17.84	122.82	17.60	121.33	492	14.43	99.47	14.22	98.27
325	22.39	153.94	22.04	151.98	409	17.79	122.49	17.56	121.01	493	14.39	99.26	14.18	98.06
326	22.33	153.47	21.98	151.58	410	17.75	122.19	17.50	120.70	494	14.36	98.99	14.14	97.82
327	22.25	153.06	21.91	151.13	411	17.70	121.87	17.46	120.36	495	14.33	98.75	14.11	97.57
328	22.19	152.62	21.85	150.71	412	17.64	121.52	17.41	120.05	496	14.29	98.51	14.07	97.32
329	22.13	152.21	21.79	150.30	413	17.60	121.22	17.38	119.75	497	14.26	98.27	14.04	97.10
330	22.05	151.79	21.73	149.87	414	17.56	120.93	17.33	119.44	498	14.22	98.03	14.00	96.86
331	22.00	151.33	21.68	149.40	415	17.52	120.64	17.28	119.12	499	14.19	97.79	13.97	96.63
332	21.92	150.89	21.61	149.02	416	17.47	120.32	17.24	118.82	500	14.16	97.54	13.93	96.38
333	21.87	150.47	21.55	148.59	417	17.42	120.02	17.20	118.51					
334	21.80	150.04	21.48	148.16	418	17.37	119.68	17.14	118.21					
335	21.74	149.65	21.43	147.79	419	17.33	119.36	17.11	117.88					
336	21.68	149.21	21.36	147.34	420	17.28	119.07	17.07	117.58					

Table A.25: MSS Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.05$, $m_0 = 50$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
n=1	1.97	3.70	5.13	5.57	51	88.93	184.52	209.85	280.53
2	4.16	6.97	9.46	11.22	52	97.37	157.93	246.74	280.06
3	5.35	10.90	13.53	17.25	53	100.67	177.74	230.04	297.69
4	8.87	16.18	18.44	22.69	54	112.81	168.58	216.33	292.08
5	11.63	15.63	22.90	28.00	55	111.27	196.03	237.73	296.39
6	13.28	19.44	27.21	35.63	56	111.56	194.16	255.89	298.32
7	13.10	24.33	30.76	40.31	57	127.08	193.49	233.10	307.23
8	17.27	24.72	34.68	49.51	58	122.07	207.22	254.35	291.37
9	18.50	29.96	40.21	47.82	59	141.21	175.99	236.66	317.26
10	22.25	34.88	44.02	57.88	60	133.80	204.45	261.01	309.80
11	19.87	34.57	47.27	68.49	61	135.45	180.21	291.48	331.65
12	23.48	43.73	55.74	67.13	62	129.15	219.24	263.26	322.37
13	26.31	46.12	57.45	70.82	63	121.40	214.44	274.11	325.23
14	30.78	48.48	67.38	79.60	64	124.21	208.33	285.20	343.33
15	30.40	54.94	66.71	82.60	65	138.82	211.34	285.04	333.70
16	30.33	56.30	78.17	90.05	66	127.65	206.33	293.87	359.45
17	36.75	55.42	70.60	98.62	67	121.67	241.27	283.14	365.02
18	35.51	61.83	82.03	101.85	68	165.59	233.00	310.87	374.74
19	40.59	61.46	90.02	107.13	69	144.33	224.40	311.61	349.05
20	41.62	66.15	86.40	101.69	70	149.23	227.17	314.73	397.61
21	43.35	64.46	92.58	115.34	71	130.18	224.18	300.20	375.20
22	43.76	73.70	98.05	129.64	72	129.74	252.80	292.56	377.10
23	41.15	74.01	105.96	122.05	73	129.57	250.31	301.22	370.83
24	48.01	83.36	113.05	132.32	74	168.49	242.74	349.69	394.64
25	57.34	83.70	115.18	142.19	75	164.75	227.81	338.32	383.69
26	56.62	97.67	114.29	144.81	76	154.56	247.75	338.50	418.27
27	57.13	93.45	118.34	148.63	77	159.92	237.62	356.26	424.46
28	58.36	92.71	114.81	152.87	78	156.36	247.98	344.29	402.51
29	59.34	90.80	121.70	153.06	79	173.49	278.40	336.03	418.04
30	51.86	108.07	141.48	167.11	80	149.79	255.75	347.09	410.31
31	68.84	100.43	129.90	164.89	81	164.60	273.04	350.14	422.10
32	65.76	107.56	152.99	174.26	82	160.29	268.24	372.62	407.82
33	56.40	117.54	144.03	171.14	83	189.87	263.65	335.79	423.25
34	66.52	105.62	152.15	190.22	84	169.93	284.16	359.98	430.42
35	69.15	119.35	166.03	195.61	85	171.52	319.31	344.62	410.93
36	71.79	114.02	165.09	200.52	86	144.75	303.02	390.39	441.15
37	79.27	126.35	172.28	213.23	87	172.81	278.07	376.78	461.76
38	89.08	134.84	177.12	191.09	88	217.85	274.13	398.66	443.01
39	79.90	132.57	176.15	204.01	89	194.72	294.15	371.42	474.14
40	97.98	116.59	183.69	211.72	90	163.27	290.27	350.59	468.10
41	86.44	150.49	192.44	224.23	91	181.17	309.29	380.41	465.15
42	84.38	143.92	186.07	221.61	92	208.27	315.56	396.86	485.56
43	88.71	141.43	197.20	253.49	93	195.07	331.14	408.47	506.62
44	88.21	165.47	191.00	232.39	94	233.95	302.69	424.32	500.87
45	103.18	144.56	191.09	241.86	95	180.91	334.71	387.43	485.39
46	84.32	158.79	193.78	253.76	96	193.45	309.48	379.53	497.43
47	90.17	153.98	198.41	250.57	97	191.60	333.32	436.70	518.86
48	96.07	158.12	208.46	240.94	98	164.88	334.95	411.60	535.38
49	105.93	175.94	216.96	251.10	99	198.83	305.64	434.91	521.93
50	100.29	146.25	229.92	277.61	100	185.73	334.26	419.31	498.53

Table A.26: MSS Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 50$

	p variables					*p* variables			
n	2	3	4	5	n	2	3	4	5
n=1	4.04	5.27	7.02	9.28	85	359.04	452.70	607.71	714.8
2	9.50	11.95	14.56	17.47	86	375.18	551.30	606.08	677.3
3	11.35	19.40	20.29	25.90	87	294.99	518.60	700.97	674.7
4	14.93	22.85	27.79	31.34	88	406.09	611.20	596.99	676.8
5	17.85	27.39	34.82	42.17	89	374.81	579.94	601.04	664.5
6	22.45	37.33	42.59	54.69	90	382.10	509.64	682.73	715.1
7	31.27	43.60	55.04	61.07	91	417.08	505.32	606.28	685.0
8	41.97	45.50	69.64	70.12	92	471.84	531.32	688.27	663.7
9	36.23	44.35	61.80	73.36	93	410.67	675.43	659.80	683.6
10	44.45	55.55	68.12	86.01	94	432.60	540.31	674.55	712.1
11	45.92	69.21	79.54	88.80	95	472.48	547.49	620.85	954.1
12	60.04	75.07	82.53	95.39	96	367.97	593.15	554.05	677.2
13	46.04	79.48	94.53	121.88	97	382.42	487.71	656.93	743.6
14	59.84	82.28	103.08	112.43	98	422.51	567.13	671.04	754.3
15	72.33	92.95	115.83	121.08	99	376.37	509.70	681.86	838.8
16	78.84	77.12	120.79	127.26	100	368.54	554.63	674.42	695.1
17	77.69	98.49	121.53	155.93	101	456.36	547.26	674.34	826.2
18	61.53	96.97	122.01	154.07	102	442.34	582.56	722.18	754.6
19	79.65	113.01	131.03	153.80	103	349.39	556.44	780.80	783.4
20	72.46	128.29	146.64	163.14	104	430.00	647.05	694.34	825.2
21	86.08	116.70	154.74	167.15	105	521.54	544.13	757.59	765.5
22	86.55	126.41	154.59	169.46	106	384.56	566.50	691.00	795.1
23	116.49	160.76	154.12	201.18	107	420.40	571.95	786.10	914.0
24	114.29	129.84	185.11	213.85	108	531.21	571.42	648.40	927.0
25	107.78	180.19	190.52	232.23	109	444.59	744.86	728.30	893.6
26	124.30	122.18	227.87	216.26	110	517.03	640.11	699.85	903.8
27	111.54	172.91	166.80	222.90	111	569.00	656.50	743.59	906.2
28	108.43	169.73	186.50	222.72	112	550.49	586.86	817.95	936.4
29	112.30	163.24	189.10	207.24	113	530.26	680.54	799.28	948.2
30	139.07	186.04	221.64	259.97	114	535.47	775.67	707.14	819.1
31	134.45	198.93	220.94	264.52	115	555.60	608.54	774.40	908.3
32	141.79	165.97	252.96	264.67	116	395.20	664.66	805.14	852.8
33	118.44	189.02	238.62	268.26	117	555.95	541.66	854.61	821.1
34	139.61	186.06	229.23	279.99	118	575.69	704.53	785.60	916.1
35	139.54	234.71	242.06	303.80	119	508.81	655.86	790.35	980.3
36	197.50	215.66	269.68	274.95	120	532.06	686.91	732.38	917.4
37	221.46	195.82	286.25	320.61	121	591.25	614.31	740.12	911.5
38	172.98	207.92	280.10	309.20	122	517.69	663.02	818.87	898.9
39	139.78	218.50	292.41	301.66	123	522.97	858.61	783.15	885.0
40	238.70	250.61	309.09	322.25	124	515.68	600.47	865.35	981.7
41	189.23	216.88	287.58	346.68	125	562.61	735.56	892.03	986.5
42	197.52	232.56	292.72	352.73	126	584.96	682.84	864.67	962.7
43	162.76	248.72	295.99	373.89	127	523.96	664.48	965.73	887.5
44	188.53	270.76	309.41	354.34	128	403.69	794.82	837.96	1007.3
45	177.03	266.84	332.85	354.38	129	587.80	679.79	1008.57	1053.0
46	182.20	265.04	312.78	365.76	130	515.41	668.66	839.90	1054.2
47	163.13	288.84	314.46	389.55	131	613.50	705.84	906.25	940.5
48	264.86	278.82	334.47	387.83	132	523.22	803.73	873.77	1133.6
49	177.94	304.41	370.89	390.13	133	527.71	814.58	827.78	987.5
50	203.10	289.54	347.75	411.81	134	648.98	697.99	913.47	980.4
51	198.30	296.04	374.47	386.49	135	610.18	821.43	840.97	1009.3
52	205.22	319.68	460.45	381.43	136	530.65	855.57	1091.21	1038.5
53	250.38	325.17	338.42	418.71	137	617.09	789.01	936.84	1117.5
54	219.63	309.73	329.26	465.41	138	557.33	1008.16	923.44	1071.8
55	249.53	337.50	381.17	415.14	139	566.33	831.80	939.93	1177.7
56	242.81	378.12	418.97	420.85	140	582.47	830.48	906.34	1050.3
57	263.23	344.56	429.25	464.17	141	601.30	760.48	974.87	1118.6
58	201.43	308.02	429.07	480.90	142	536.87	951.64	950.94	1156.1
59	269.81	361.02	409.90	516.00	143	591.92	825.04	964.80	1103.4
60	188.68	276.10	438.25	504.12	144	685.72	862.21	1000.43	1155.0
61	273.81	338.87	437.63	473.55	145	499.27	887.93	902.45	944.3
62	218.14	378.86	401.73	501.82	146	536.90	840.59	952.36	1281.8
63	255.48	410.99	389.32	556.19	147	512.77	699.71	903.91	1084.1
64	270.86	379.69	448.63	604.20	148	603.50	952.22	1175.95	1062.2
65	294.87	382.73	493.44	533.02	149	565.61	759.65	1079.54	1154.7
66	307.75	389.36	474.80	581.11	150	538.08	936.18	1074.68	1241.1
67	257.75	432.39	448.81	551.02	151	595.39	926.06	1033.24	1226.5
68	257.21	370.51	469.02	518.40	152	562.75	891.31	1023.29	1153.1
69	277.36	385.50	510.38	551.30	153	650.73	968.94	1087.88	1083.7
70	289.73	344.92	495.04	627.70	154	696.59	801.32	998.66	1139.0
71	291.97	351.64	415.51	543.58	155	693.92	872.31	996.61	1275.2
72	274.67	442.87	472.61	587.98	156	746.92	803.70	946.82	1205.2
73	292.23	417.27	490.69	561.10	157	590.93	920.17	1014.48	1257.5
74	305.51	448.93	617.82	655.92	158	685.63	920.87	1096.20	1414.8
75	338.23	404.13	506.50	540.67	159	666.29	889.40	1012.29	1113.3
76	376.74	448.13	588.64	633.50	160	816.77	867.52	1065.70	1185.0
77	375.05	415.51	563.04	655.92	161	613.69	870.30	1069.07	1159.7
78	314.96	489.67	515.55	612.49	162	709.20	842.17	1007.76	1404.8
79	357.32	534.00	573.38	648.09	163	668.40	768.94	1224.41	1268.8
80	328.38	445.79	619.56	758.91	164	707.57	888.22	1056.87	1236.1
81	362.64	443.61	525.03	650.11	165	750.66	964.21	1328.20	1269.6
82	345.34	450.71	521.71	679.54	166	624.20	941.09	1040.97	1319.5
83	367.84	432.52	622.01	653.83	167	664.52	1003.42	1195.16	1242.7
84	376.74	451.46	647.05	691.35	168	808.16	954.85	1147.39	1245.0

MSS Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 50$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
169	709.70	883.63	1220.0	1319.5	253	1154.6	1424.5	1685.6	1830.4
170	612.18	866.38	1152.6	1304.1	254	1020.4	1434.3	1878.6	2179.2
171	991.68	965.71	1171.6	1403.3	255	1012.5	1275.5	1709.5	2051.2
172	797.24	973.99	1093.0	1368.8	256	1056.9	1627.9	1633.2	2035.1
173	778.24	834.37	1268.8	1305.4	257	972.1	1568.9	1698.7	2064.6
174	770.56	1081.94	1249.4	1376.2	258	1032.4	1218.7	1565.6	1822.9
175	602.34	905.96	1192.0	1319.1	259	1183.1	1635.0	1815.2	2054.7
176	618.28	907.16	1062.2	1311.5	260	1110.0	1822.7	1920.8	1837.7
177	744.68	920.32	1100.1	1363.8	261	977.7	1263.5	1718.9	2155.9
178	693.97	1076.83	1166.2	1530.3	262	1154.5	1578.2	2088.3	1782.5
179	854.29	984.51	1200.5	1271.6	263	1242.9	1353.5	1773.4	2100.1
180	823.09	1082.08	1165.5	1388.8	264	1112.2	1495.2	1815.7	1992.7
181	907.95	1067.08	1287.4	1481.2	265	1353.5	1531.6	1495.2	2060.0
182	734.14	1089.98	1223.4	1417.2	266	1217.6	1431.5	1587.3	1912.3
183	876.00	1016.55	1106.9	1334.7	267	1359.4	1777.0	1736.3	2184.4
184	774.75	1131.92	1221.5	1300.4	268	1238.3	1695.7	1724.1	1910.5
185	921.72	1147.67	1279.5	1458.6	269	1266.6	1221.7	1802.8	2093.6
186	697.86	1052.51	1174.5	1408.5	270	1223.6	1345.7	1809.6	2172.4
187	872.14	1082.63	1279.5	1306.4	271	1182.0	1753.0	1845.7	1935.9
188	1072.70	1199.97	1281.2	1521.1	272	1450.1	1578.4	2021.4	1860.6
189	852.76	1149.51	1292.7	1324.9	273	1492.8	1551.6	1740.2	1843.6
190	848.31	1041.82	1338.2	1601.6	274	1229.4	1409.3	1968.3	2349.0
191	792.00	1254.95	1285.9	1304.1	275	1189.4	1426.7	1924.5	2266.3
192	775.94	969.94	1206.1	1381.0	276	1222.0	1666.8	2077.4	1969.3
193	741.07	1197.30	1397.4	1622.9	277	1467.4	1344.1	1671.8	2216.5
194	824.90	1146.17	1467.8	1583.5	278	1059.8	1404.1	1907.4	1971.1
195	904.51	1212.88	1305.6	1662.2	279	1258.8	1571.2	1948.7	2170.3
196	809.91	1169.82	1325.4	1488.8	280	1043.8	1736.8	1736.1	2070.7
197	1041.53	972.39	1301.5	1672.5	281	1077.8	1550.8	1741.3	2244.7
198	759.92	1157.09	1390.4	1748.9	282	1144.9	1406.3	1682.2	2141.7
199	748.57	1061.87	1387.5	1567.2	283	1143.1	1452.5	1757.2	2235.5
200	896.76	927.47	1382.6	1489.3	284	1281.6	1807.3	1894.2	2536.0
201	967.24	1070.08	1354.2	1556.9	285	1265.0	1457.3	1952.8	2258.3
202	962.67	1149.76	1441.6	1519.2	286	1025.1	1515.5	1858.4	2181.4
203	834.16	1150.73	1407.2	1599.6	287	1508.5	1645.3	1773.4	2202.5
204	881.61	1109.71	1150.9	1393.0	288	1238.7	1579.1	2060.2	2055.5
205	811.12	1024.64	1420.3	1579.8	289	1281.3	2007.7	2005.2	2026.5
206	773.05	925.12	1277.9	1572.0	290	1140.7	1440.0	1932.6	2305.5
207	855.55	1275.89	1495.6	1512.4	291	1460.5	1454.1	2218.4	2072.5
208	832.30	1022.81	1439.7	1555.8	292	1096.0	1714.0	1941.5	2449.1
209	903.43	1428.37	1437.8	1717.0	293	1266.6	1531.7	1899.0	2093.0
210	850.14	1247.33	1565.0	1558.3	294	890.7	1951.3	2045.6	2213.7
211	842.82	1024.00	1481.1	1505.0	295	1386.0	1444.4	1856.7	2075.9
212	936.56	1236.88	1256.9	1599.5	296	1260.0	1513.6	1891.3	2086.1
213	838.60	1294.65	1500.4	1707.6	297	1371.3	1570.3	2020.0	2628.0
214	884.71	1449.33	1346.8	1473.8	298	1316.6	1762.9	2027.2	2443.2
215	936.77	1392.59	1347.4	1688.1	299	1470.6	1607.8	2280.5	2111.2
216	873.51	1344.25	1486.9	1672.9	300	1265.1	1687.8	2532.0	2290.1
217	943.59	1163.84	1762.3	1676.7	301	1208.9	1591.1	2294.6	2367.3
218	1053.70	1260.38	1523.0	1674.2	302	1249.5	1493.2	2068.4	2281.0
219	757.37	1343.44	1460.0	1742.1	303	1433.3	1710.4	2113.2	2474.8
220	880.07	1197.82	1488.4	1781.1	304	1154.1	1390.0	1859.6	2590.4
221	930.81	1266.87	1550.4	1813.6	305	1209.0	1673.4	2056.8	2139.6
222	918.41	1229.43	1398.3	1762.3	306	1202.4	1637.9	2102.3	2195.5
223	923.96	1427.50	1610.8	1796.0	307	1484.4	1748.8	2277.5	2262.7
224	972.96	1344.78	1713.6	1714.7	308	1168.1	1583.9	2035.1	2533.6
225	857.43	1231.47	1637.1	1833.8	309	1433.0	1717.3	1811.8	2418.9
226	913.41	1336.10	1517.7	1798.3	310	1352.8	1747.7	2176.8	2198.0
227	1111.15	1417.90	1529.3	1740.4	311	1376.5	1872.8	2320.1	2259.0
228	921.94	1196.95	1508.9	1599.3	312	1304.1	1739.3	2044.9	2303.5
229	1074.19	1234.25	1784.1	1716.4	313	1491.0	1550.4	2150.9	2366.2
230	884.68	1301.32	1662.7	1844.9	314	1457.7	1892.4	1988.1	2375.5
231	1024.33	1196.97	1511.4	1888.5	315	1718.2	1818.0	2070.9	2410.9
232	878.60	1449.99	1556.0	1649.4	316	1487.7	1989.2	2469.5	2317.7
233	1098.49	1373.07	1832.3	1634.9	317	1413.8	1885.8	2205.2	2468.1
234	1110.66	1586.36	1607.9	1722.7	318	1234.5	1667.1	1981.3	2203.5
235	1106.31	1350.01	1695.0	1694.8	319	1258.1	1567.1	2081.2	2419.4
236	1040.74	1404.96	1388.2	1745.8	320	1202.1	1808.2	2021.5	2493.9
237	959.32	1109.31	1652.1	1763.4	321	1615.1	1680.2	1867.0	2234.9
238	1046.15	1388.10	1644.0	1884.0	322	1272.8	1909.8	1930.4	2312.8
239	1202.08	1523.08	1490.9	2062.6	323	1344.1	1773.3	1975.6	2320.1
240	1071.05	1190.08	1891.0	1665.4	324	1480.0	2054.0	2262.6	2555.0
241	983.57	1302.21	1744.8	1828.1	325	1474.7	1690.4	2311.7	2567.0
242	951.83	1562.20	1853.4	1690.0	326	1307.2	1716.5	2435.6	2332.1
243	1164.49	1526.79	1662.2	1985.2	327	1366.1	1602.6	2024.0	2309.4
244	868.01	1468.37	1532.8	1970.0	328	1229.7	1746.8	2473.8	2500.5
245	988.94	1418.27	1930.0	1784.9	329	1607.5	1972.4	2471.0	2469.1
246	973.19	1353.54	1805.5	1953.8	330	1544.6	1973.3	2354.0	2459.8
247	1126.27	1315.22	1615.1	1914.0	331	1486.0	1596.0	2023.3	2805.8
248	977.88	1326.28	1653.2	1974.7	332	1410.5	1755.7	2038.8	2359.7
249	1101.05	1289.75	1951.7	1753.7	333	1553.6	2094.6	2318.4	2812.6
250	1169.31	1473.05	1444.5	1919.1	334	1476.5	1854.3	2557.5	2302.2
251	997.83	1467.21	1640.3	2108.7	335	1654.8	2199.1	2177.6	2510.1
252	937.81	1227.90	1733.0	2042.1	336	1707.7	1671.4	2197.9	2808.6

MSS Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 50$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
337	1688.3	2261.4	2198.4	2308.4	419	1627.2	2166.5	2805.2	3182.4
338	1662.5	1638.3	2417.0	2811.4	420	1909.7	2576.3	3071.0	3360.8
339	1769.7	1912.8	2270.6	2621.5	421	1936.6	2275.9	3016.6	3382.3
340	1777.2	1975.5	2104.3	2660.9	422	1844.7	2355.4	2943.3	3129.4
341	1463.1	1996.9	2373.6	2661.0	423	2151.8	2613.3	2915.0	3209.3
342	1733.8	2187.9	2301.2	2457.6	424	1907.7	2124.0	2794.9	3338.2
343	1498.2	2001.9	2315.9	2903.6	425	1996.2	2577.0	2677.5	3701.4
344	1417.8	1846.9	2024.3	2427.5	426	1684.9	2495.3	2711.1	3051.9
345	1429.1	1878.5	2241.4	2622.4	427	2040.3	2057.6	2811.3	3644.6
346	1685.8	1916.9	2177.5	2696.7	428	2275.3	2871.2	2820.2	3383.2
347	1944.2	2185.9	2338.2	2449.3	429	2066.6	2407.7	2747.0	2805.7
348	1493.9	1777.4	2100.9	2625.6	430	1757.2	2523.4	2765.3	3479.8
349	1420.8	2001.8	2299.7	2776.1	431	1905.4	2341.4	3319.8	3176.6
350	1412.5	2114.5	2410.9	2604.9	432	1488.6	2402.4	2878.7	3435.3
351	1412.8	2543.2	2280.7	2816.9	433	2060.3	2901.7	3218.7	3298.9
352	1437.4	2239.8	2228.5	2705.1	434	2097.2	2344.8	2936.6	3270.3
353	1559.6	1929.0	2448.8	2454.3	435	1985.0	2293.2	3198.7	3300.9
354	1339.1	1960.9	2572.1	2876.2	436	1877.3	2428.6	2755.5	3511.0
355	1438.9	2203.0	2073.9	2463.4	437	2048.2	2394.6	3229.0	3308.2
356	1431.6	1879.8	2478.0	2775.5	438	2123.8	2782.7	2949.4	3677.4
357	1608.5	1734.1	2533.1	2770.4	439	1753.9	2855.5	2965.6	3227.6
358	1794.2	1661.8	2381.0	2731.9	440	2379.7	2757.7	3012.8	3260.5
359	1576.2	1983.8	2111.9	2764.1	441	2843.1	3101.7	2943.2	3370.2
360	1693.2	2107.8	2379.8	2740.8	442	1769.1	2318.4	3392.4	3168.2
361	1531.2	1902.0	2247.6	2796.1	443	2078.5	2226.9	2800.7	3473.8
362	1651.7	1731.6	2421.8	2709.7	444	1687.5	2232.9	2934.1	3648.4
363	1526.9	2293.2	2322.9	3033.4	445	2334.6	2486.5	2894.3	3534.2
364	1435.0	1906.2	2589.2	2827.7	446	2420.6	2164.1	2748.0	3416.7
365	1403.3	2078.3	2436.1	2970.3	447	1738.5	2437.5	3156.4	3228.3
366	1503.3	2211.9	2497.9	2829.4	448	2150.7	2759.4	2988.9	3459.3
367	1385.1	2258.5	2193.5	2859.9	449	1786.3	2432.8	3239.9	3408.6
368	1553.7	2110.7	2833.6	2897.1	450	2468.6	2816.6	2944.8	3441.0
369	1658.0	2404.0	2370.7	2951.8	451	2053.5	1931.8	2816.1	3120.0
370	1928.6	2491.8	2886.3	2585.8	452	1768.8	2658.1	2900.4	3411.3
371	1565.9	2195.8	3006.3	2756.6	453	2157.6	2528.0	2550.9	3059.5
372	1420.5	2340.5	2714.0	2879.1	454	2950.1	2363.5	2866.4	3188.4
373	1922.2	2230.0	2806.4	2485.2	455	2340.2	3004.9	2713.9	3818.2
374	1801.4	2263.5	2424.6	3278.1	456	2239.2	2647.8	3153.5	3272.6
375	1498.3	2482.4	2269.5	3126.5	457	2337.0	2669.0	3082.2	3843.6
376	1471.7	1831.8	2367.7	3278.7	458	1851.6	3341.9	3038.9	3334.4
377	1835.7	2368.1	2497.7	2744.3	459	1839.8	2753.4	3127.3	3482.9
378	1599.2	1732.2	2583.2	2759.3	460	2341.4	2601.9	2866.7	3265.0
379	1601.4	2001.7	2731.6	2664.2	461	1988.0	2441.1	3090.3	3710.2
380	2003.5	1974.1	2604.9	3037.1	462	2304.3	2847.1	2836.6	3598.5
381	1735.7	2250.2	2573.7	2779.2	463	2672.2	2513.1	2908.3	3786.1
382	1916.8	2462.8	2455.4	3076.4	464	2554.8	2556.6	3159.8	3637.0
383	1590.0	2476.3	2318.6	2717.1	465	2017.6	2700.7	2691.1	3852.3
384	1395.6	2070.1	2595.6	2800.5	466	2073.6	2438.7	3255.5	3812.6
385	1517.6	2197.7	2677.5	2805.2	467	2041.8	2548.6	3183.3	3198.0
386	1813.8	2205.9	2830.1	3110.2	468	2027.7	3115.3	3314.8	3643.5
387	1825.1	1877.8	2617.7	3116.3	469	2317.9	2798.2	3150.7	3477.7
388	1679.1	2260.8	2428.3	2824.9	470	2049.2	3001.3	3155.6	3487.7
389	1803.7	2067.6	2543.5	2717.2	471	2398.0	2704.9	3167.3	3619.4
390	1770.0	2411.0	2642.4	2700.4	472	2454.5	2240.9	3012.1	3735.0
391	1878.4	2061.1	2482.8	3344.5	473	2343.2	2369.5	3097.4	3715.8
392	1857.4	2411.3	2703.9	2829.2	474	1873.2	3062.9	3443.5	3759.1
393	1868.6	2371.2	2689.9	3256.4	475	2529.6	2646.7	3030.6	3702.3
394	1639.5	2204.3	2563.7	2920.9	476	2248.3	2606.2	3528.5	3647.3
395	1637.4	2099.9	2498.4	2911.3	477	2616.6	2593.0	3349.1	3426.4
396	1709.5	2166.8	3080.6	3454.2	478	2050.9	3160.0	2772.0	3725.3
397	2074.4	2074.7	2535.7	3409.5	479	2885.2	2369.0	3171.3	3716.6
398	1515.8	2255.1	2528.4	2723.8	480	2584.3	2703.1	2936.5	4114.1
399	1681.0	2078.8	3034.6	3127.6	481	2555.2	2869.9	3625.4	3423.3
400	2038.0	2251.2	2661.7	3144.9	482	2701.9	2632.3	3060.7	3778.9
401	1463.2	2718.0	2596.8	3222.0	483	2592.4	2360.7	3738.2	4123.8
402	1490.2	2362.6	2476.4	3273.9	484	2108.8	2374.7	3616.6	3572.7
403	2361.7	2215.8	2527.1	3413.0	485	2239.4	2440.4	3195.8	3527.4
404	1753.3	2336.9	2288.2	3552.3	486	2409.4	2642.4	3046.4	3405.2
405	2068.7	2284.7	2666.1	3017.3	487	2554.5	2464.9	3425.5	3486.1
406	1678.9	2094.4	3197.4	2846.4	488	2054.4	2652.6	3119.4	4399.6
407	1753.5	2205.4	2753.1	2822.1	489	2663.0	3077.4	3305.0	3636.7
408	2037.5	2525.0	2788.6	3071.7	490	2246.4	2946.8	3038.8	3465.6
409	1520.4	2303.8	2573.7	3068.9	491	2194.4	3092.9	3522.4	3505.1
410	1752.0	2443.9	2849.5	3422.9	492	1775.3	2728.7	3813.4	3687.5
411	2446.7	2431.4	2857.8	3427.2	493	2108.5	2892.2	3358.0	3695.1
412	1558.6	2453.6	2745.2	3366.5	494	2071.2	3022.2	3382.1	3900.6
413	2310.4	2017.5	2585.8	2992.2	495	1933.5	2570.1	3580.0	3821.4
414	1756.0	2281.6	2887.6	2789.1	496	2287.1	2806.7	3773.0	3785.4
415	2652.9	2535.8	2840.8	3505.2	497	2440.1	3069.0	3305.7	3819.8
416	1674.6	2075.0	3323.9	3001.9	498	2201.0	2765.2	3395.7	3572.6
417	2381.6	2069.3	2619.7	3361.4	499	2329.5	2967.0	3059.3	3448.5
418	1822.3	2476.4	2904.9	3044.6	500	2491.9	3548.9	3007.5	3521.5

Table A.27: MSS Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.05$, $m_0 = 100$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
n=1	2.30	3.16	4.13	5.55	51	115.30	166.79	217.77	260.31
2	4.32	6.74	9.02	10.72	52	112.04	168.22	215.69	277.12
3	6.87	10.02	13.34	16.95	53	105.02	166.69	234.04	260.92
4	8.20	14.77	17.89	22.54	54	109.74	187.17	227.85	275.29
5	10.68	16.82	22.10	26.84	55	105.43	185.95	243.12	283.46
6	12.43	22.33	26.64	30.91	56	116.56	202.76	253.54	289.93
7	14.61	23.25	29.16	37.51	57	118.75	194.32	251.33	291.64
8	12.24	25.53	33.82	45.48	58	127.34	191.37	251.63	280.64
9	19.02	30.20	40.26	47.47	59	117.83	225.82	251.88	320.48
10	18.84	32.95	40.92	54.23	60	127.86	183.49	234.38	300.12
11	23.34	38.68	49.25	58.79	61	134.79	201.10	251.37	304.90
12	26.80	38.80	55.07	62.81	62	120.78	208.91	283.06	328.36
13	25.81	43.21	57.62	65.95	63	130.52	224.81	287.15	319.07
14	26.81	42.42	59.78	70.78	64	125.46	208.65	286.27	325.26
15	32.88	48.38	68.81	79.39	65	125.97	198.38	296.05	338.39
16	32.11	47.69	71.09	80.00	66	125.24	223.76	272.04	328.67
17	37.01	57.10	73.09	90.79	67	139.46	221.85	289.37	352.11
18	37.21	52.79	83.43	98.64	68	138.16	230.14	301.87	370.10
19	41.58	66.95	78.20	106.76	69	146.27	237.74	313.68	354.92
20	52.21	70.05	90.41	103.92	70	133.83	230.77	298.19	364.64
21	51.11	66.52	88.45	97.99	71	148.46	210.05	337.92	364.92
22	46.21	76.80	97.72	122.06	72	161.64	223.41	361.18	370.49
23	45.77	77.06	109.23	121.01	73	174.08	265.34	324.92	406.19
24	52.74	78.49	101.20	130.21	74	144.80	231.12	299.18	354.04
25	51.23	84.35	114.02	140.21	75	149.82	234.65	344.60	390.07
26	52.61	85.40	115.86	150.86	76	158.64	256.45	330.67	397.32
27	56.67	86.72	114.26	143.16	77	154.88	250.40	328.35	435.16
28	63.05	92.86	131.40	155.84	78	189.52	247.56	303.83	414.33
29	57.76	100.28	120.95	157.83	79	157.59	248.50	354.98	444.01
30	65.70	95.84	151.59	161.32	80	154.21	266.38	360.27	414.30
31	71.39	102.76	151.30	161.23	81	148.13	258.05	375.87	414.25
32	74.41	111.26	148.16	167.14	82	184.50	296.25	350.59	410.03
33	71.59	111.01	143.12	181.59	83	151.21	278.36	331.56	428.29
34	72.34	111.63	146.51	179.32	84	173.46	260.97	364.21	432.95
35	66.26	102.01	142.45	200.21	85	174.54	274.17	362.51	442.72
36	68.81	118.12	141.55	181.31	86	186.09	297.54	379.60	461.87
37	77.54	120.36	160.45	194.94	87	195.26	292.37	353.29	441.56
38	75.70	121.86	153.96	200.17	88	190.57	296.92	387.27	409.92
39	86.42	117.79	180.94	201.35	89	178.43	322.65	390.37	452.42
40	73.11	136.46	177.55	204.34	90	174.11	282.30	409.06	486.40
41	80.22	134.41	163.47	208.31	91	199.53	309.07	411.55	472.19
42	83.22	130.20	186.08	223.71	92	177.14	325.45	415.11	487.88
43	89.31	133.47	188.42	234.35	93	191.90	288.33	384.68	474.81
44	89.08	141.73	195.21	237.78	94	190.58	343.26	406.13	503.40
45	102.38	149.50	191.22	226.37	95	191.63	323.36	392.68	433.23
46	95.25	140.15	191.98	237.13	96	203.08	282.93	381.14	478.20
47	103.87	150.99	206.45	234.17	97	216.84	323.17	427.63	521.30
48	93.91	157.77	213.85	258.16	98	209.89	341.03	445.97	506.95
49	82.32	163.82	215.51	258.44	99	205.61	297.62	416.36	534.25
50	102.37	151.95	211.05	267.96	100	188.08	333.31	424.39	513.64

Table A.28: MSS Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 100$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
1	4.29	6.02	5.90	8.29	85	334.73	510.2	582.9	608.0
2	9.08	10.34	15.55	14.27	86	375.78	487.1	548.3	823.0
3	13.10	15.72	19.01	24.48	87	440.12	541.2	607.6	653.7
4	14.99	22.89	27.56	30.92	88	342.32	449.7	586.1	692.3
5	19.68	32.44	37.10	36.23	89	349.86	532.7	645.6	703.5
6	22.39	38.02	41.08	43.09	90	325.32	543.5	653.2	689.3
7	28.41	44.46	45.11	56.58	91	360.61	507.3	702.4	665.6
8	31.75	47.46	55.63	58.80	92	448.21	559.0	654.7	759.8
9	41.21	50.05	60.96	77.71	93	339.12	515.9	654.8	795.1
10	33.80	51.31	71.99	86.91	94	450.91	563.2	693.4	701.5
11	49.96	66.52	67.85	90.90	95	433.79	585.5	637.5	691.1
12	53.67	77.63	83.50	88.47	96	496.68	543.9	781.1	856.8
13	56.12	66.52	94.35	115.05	97	405.34	522.6	608.5	784.7
14	58.63	80.11	102.82	112.36	98	444.74	671.4	680.9	773.9
15	64.22	99.99	119.08	112.75	99	467.59	643.2	679.7	744.9
16	68.89	101.63	106.19	131.03	100	391.93	540.9	640.4	761.0
17	67.53	88.05	127.90	139.71	101	438.41	504.8	657.0	835.6
18	76.74	105.13	109.47	152.40	102	421.12	534.2	636.3	717.1
19	87.50	103.38	135.59	156.96	103	480.94	598.1	661.5	802.8
20	93.42	116.50	132.50	131.59	104	465.70	532.9	659.6	737.7
21	106.62	102.85	158.85	168.63	105	440.23	546.3	840.1	776.1
22	84.57	138.00	159.53	197.09	106	500.76	591.9	670.3	813.7
23	110.04	130.04	150.67	186.93	107	461.08	515.2	673.0	808.0
24	109.39	122.21	153.49	212.51	108	423.09	741.3	699.8	926.2
25	101.78	157.50	159.84	196.40	109	407.28	515.9	727.7	776.1
26	105.78	145.54	179.13	196.19	110	469.38	577.9	718.6	855.4
27	124.85	162.21	172.47	219.43	111	433.74	656.3	773.2	815.4
28	123.04	156.33	188.68	226.12	112	613.19	597.4	825.7	858.6
29	113.76	206.97	210.49	243.38	113	534.12	634.8	784.4	890.7
30	151.18	161.85	204.58	249.63	114	467.15	694.0	751.9	743.8
31	152.93	180.20	237.27	250.24	115	460.35	636.8	727.0	913.4
32	151.44	171.00	204.51	250.55	116	503.19	751.1	729.4	960.3
33	142.22	174.67	248.72	278.85	117	532.22	630.7	752.0	843.0
34	155.78	181.75	206.58	281.88	118	562.75	757.7	855.4	975.4
35	186.89	201.38	284.34	283.99	119	374.95	765.7	921.4	966.6
36	133.76	202.57	255.06	298.86	120	587.00	662.0	846.1	850.2
37	143.09	226.89	218.26	296.67	121	541.71	630.1	873.7	1068.6
38	148.45	217.64	297.54	335.36	122	400.36	621.3	824.4	881.7
39	155.68	236.85	291.36	299.20	123	544.99	651.6	923.5	963.7
40	157.14	207.73	284.00	359.73	124	474.03	737.9	829.5	904.8
41	140.40	257.52	246.43	309.00	125	479.41	771.0	968.6	933.2
42	185.14	294.28	285.93	320.53	126	531.82	642.5	869.9	965.0
43	195.70	246.76	298.17	336.54	127	548.89	796.0	763.7	948.0
44	170.70	258.06	290.07	369.86	128	563.54	637.5	893.5	939.2
45	201.71	273.44	318.89	379.83	129	528.71	662.4	757.4	1044.9
46	254.13	280.60	321.33	329.11	130	492.28	699.3	846.3	913.7
47	268.54	239.72	363.17	354.25	131	548.30	642.3	898.9	987.8
48	179.22	272.05	346.40	390.82	132	554.27	685.7	1063.8	937.5
49	233.71	286.45	347.69	389.08	133	561.59	841.3	943.2	984.9
50	210.58	262.74	324.09	395.10	134	583.22	703.4	858.1	972.5
51	194.99	324.58	347.60	403.32	135	683.07	860.4	881.5	1068.7
52	201.46	346.46	371.17	434.29	136	623.63	702.2	861.6	1123.7
53	249.64	324.10	354.43	415.55	137	594.07	727.0	921.8	1043.9
54	247.31	267.23	346.61	402.68	138	546.09	795.9	1066.5	1117.6
55	200.55	277.63	353.10	449.14	139	642.73	884.4	924.0	982.8
56	217.20	329.27	386.86	402.06	140	580.78	744.7	932.0	1133.8
57	250.60	326.82	325.22	425.76	141	497.65	798.0	916.0	1013.1
58	253.90	303.80	394.44	465.55	142	637.52	778.2	863.6	999.9
59	230.41	406.92	422.05	472.74	143	745.10	812.5	920.1	1146.2
60	290.97	363.85	387.22	497.41	144	720.14	853.3	892.0	1126.7
61	271.63	358.78	412.30	503.53	145	672.14	773.2	862.6	1160.3
62	279.75	342.28	419.88	517.86	146	690.47	705.7	1029.0	1026.8
63	266.41	349.40	398.63	457.97	147	550.23	810.5	1013.7	1201.1
64	235.85	337.14	377.78	477.61	148	661.61	762.7	1010.2	1178.5
65	283.33	356.10	423.57	542.69	149	544.26	863.5	948.4	1068.2
66	300.54	356.83	426.75	528.83	150	793.52	1029.7	1024.4	1077.4
67	269.22	364.99	519.08	508.22	151	537.75	831.9	1036.5	1112.4
68	286.15	335.47	535.70	510.66	152	653.21	836.3	1076.6	1088.6
69	279.10	344.25	444.26	502.18	153	628.42	811.1	907.3	1185.4
70	276.78	382.22	477.02	589.82	154	645.96	802.8	973.6	1152.7
71	272.71	434.68	508.16	506.14	155	747.89	1029.6	985.3	1137.2
72	312.81	408.16	455.40	654.58	156	832.09	844.8	1041.9	1161.5
73	358.21	374.88	514.08	609.12	157	714.40	860.5	1050.8	1253.6
74	308.67	457.77	582.94	607.94	158	663.80	800.1	1047.1	1204.4
75	366.36	416.59	496.40	577.03	159	695.91	1034.8	984.9	1140.8
76	379.29	414.84	505.31	603.51	160	806.42	895.7	1217.8	1314.7
77	330.47	458.11	546.05	589.41	161	772.63	945.9	1220.2	1213.9
78	322.79	469.12	569.18	675.18	162	684.13	954.6	1114.9	1217.3
79	299.54	485.38	472.03	561.94	163	613.23	1010.4	1119.1	1521.4
80	389.15	478.08	615.94	717.23	164	691.60	817.2	1103.3	1243.2
81	381.45	468.56	598.35	632.77	165	576.12	1153.0	1071.6	1173.0
82	325.60	445.96	501.86	629.17	166	660.00	924.4	1040.9	1458.5
83	365.14	505.81	603.12	726.87	167	765.91	814.9	1224.5	1228.3
84	389.28	506.70	648.12	650.37	168	769.85	980.2	1221.3	1274.5

MSS Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 100$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
169	703.8	842.2	1070.0	1437.8	253	894.1	1228.6	1642.6	1854.2
170	799.5	874.3	1135.2	1321.5	254	1074.6	1427.1	1575.2	2099.8
171	547.7	919.2	1205.3	1285.1	255	1394.9	1375.2	1515.8	2097.9
172	681.5	1062.6	1131.7	1289.3	256	1373.8	1429.8	1688.2	1805.4
173	707.1	916.7	1149.5	1348.5	257	1087.4	1468.4	1711.2	1764.2
174	778.7	897.9	1134.4	1585.4	258	1011.9	1289.1	1740.7	1735.5
175	680.4	1019.2	1165.4	1615.2	259	1216.7	1574.9	1775.6	1869.1
176	661.5	885.3	1152.0	1465.3	260	1421.4	1975.4	1574.7	1831.2
177	819.9	888.8	1232.2	1350.4	261	1067.2	1617.8	2205.2	2200.6
178	769.0	1033.7	1154.0	1413.5	262	1198.8	1706.0	1919.1	2189.8
179	785.2	994.9	1371.5	1296.9	263	900.0	1430.7	1866.7	2089.2
180	732.2	925.0	1253.7	1542.2	264	1441.2	1800.6	1861.5	1874.5
181	762.5	989.4	1088.0	1405.2	265	1287.3	1702.1	1825.6	2137.6
182	797.1	1131.2	1165.2	1342.6	266	1150.4	1291.6	1732.1	2017.1
183	828.5	922.0	1315.0	1276.3	267	1261.6	1582.8	1719.6	1935.3
184	872.7	831.5	1527.3	1340.5	268	1297.5	1336.6	2144.7	1820.2
185	735.5	1074.1	1500.2	1381.8	269	1047.7	1522.7	1830.5	2450.1
186	780.9	1050.7	1165.8	1536.5	270	1059.0	1613.3	1841.6	1934.0
187	784.6	1128.5	1249.2	1363.2	271	1267.4	1596.1	1833.7	2048.7
188	721.0	964.4	1238.8	1403.8	272	1164.9	1699.1	2012.1	2129.7
189	775.5	963.8	1357.7	1390.3	273	1402.3	1320.6	1717.2	1993.3
190	913.4	1017.9	1170.4	1463.2	274	1256.8	1655.5	1984.7	1941.0
191	835.2	1046.5	1230.5	1485.7	275	1069.1	1598.1	2187.5	2216.8
192	831.1	1284.1	1267.8	1476.4	276	1049.2	1759.9	1666.1	2284.8
193	966.6	1261.4	1320.6	1344.3	277	1314.7	1559.1	1702.3	2095.7
194	858.4	997.9	1316.8	1573.9	278	1288.2	1520.0	1762.1	2139.5
195	865.7	1041.3	1321.6	1461.3	279	1641.3	1645.7	2120.6	2219.9
196	907.5	1075.0	1516.7	1549.2	280	1202.2	1464.3	1994.8	1968.0
197	953.4	1035.1	1229.4	1357.8	281	1130.1	1786.7	1959.0	2544.2
198	807.3	995.1	1483.7	1560.0	282	1341.3	1815.4	1854.4	2226.6
199	1050.0	1093.4	1459.3	1642.4	283	1293.5	1417.6	2004.3	2413.8
200	633.9	1142.3	1343.6	1576.2	284	1240.9	1558.3	1820.3	2331.9
201	887.7	1067.5	1505.0	1504.8	285	1536.1	1505.4	1922.5	2111.5
202	830.9	1071.0	1342.8	1577.3	286	1361.1	1665.9	1720.2	2470.5
203	1006.8	1144.3	1308.3	1469.2	287	1195.2	1520.7	2028.4	2084.7
204	842.5	1241.5	1265.4	1467.9	288	1083.3	1933.1	2304.0	2453.1
205	793.0	1200.8	1400.8	1637.7	289	1122.9	1748.4	2002.4	2025.7
206	785.7	1144.5	1409.3	1867.8	290	1147.5	1710.6	1977.6	2296.0
207	945.0	1277.9	1534.5	1405.7	291	1200.1	1545.9	2248.4	1972.6
208	862.1	1181.0	1444.5	1568.4	292	1223.4	1529.9	1944.7	2259.8
209	873.7	1310.5	1363.7	1549.0	293	1421.6	1585.3	2062.6	2335.9
210	753.7	1141.1	1649.7	1509.0	294	1591.4	1412.9	2305.5	2190.6
211	787.0	1081.8	1469.5	1402.2	295	1148.8	1907.0	2098.7	1965.2
212	886.8	1273.3	1322.5	1572.0	296	1158.3	1864.2	1961.5	2250.6
213	898.8	1039.6	1419.0	1773.9	297	1542.2	1497.2	1985.7	2254.6
214	898.8	1226.0	1424.9	1829.1	298	1301.0	1924.4	1895.2	2196.4
215	932.5	1014.1	1456.2	1574.3	299	1216.8	2094.8	2012.9	2336.5
216	1027.8	1138.4	1471.2	1785.2	300	1106.5	1771.4	1994.5	2094.7
217	1020.3	1236.1	1379.6	1549.6	301	1342.2	1477.7	2034.9	2036.4
218	864.8	1243.7	1643.2	1644.4	302	1255.1	1750.9	1762.7	2091.9
219	1086.0	1128.1	1488.9	1741.0	303	1434.5	1686.7	2102.3	2351.8
220	1018.9	1195.8	1387.2	1597.4	304	1318.3	1612.5	2181.8	2003.6
221	1067.3	1242.6	1358.5	1610.8	305	1248.7	1766.5	2020.5	2699.6
222	1061.4	1020.2	1548.8	1777.1	306	1644.2	1565.5	1797.5	2483.9
223	1054.1	1228.3	1454.4	1704.7	307	1628.4	1969.4	1801.3	2138.4
224	890.4	1296.5	1444.3	1674.0	308	1797.2	1773.5	1875.9	2162.0
225	992.1	1346.1	1343.6	1823.6	309	1396.1	1555.5	2097.9	2484.6
226	994.9	1436.8	1775.5	2019.4	310	1498.5	1775.8	1918.9	2439.4
227	1037.0	1310.3	1302.4	1853.5	311	1819.9	1770.5	2157.6	2480.6
228	891.3	1346.3	1521.9	1741.4	312	1399.6	1960.5	2129.5	2443.0
229	1075.5	1320.9	1313.8	1699.4	313	1421.5	1711.3	1875.5	2620.0
230	1010.5	1157.5	1552.7	1717.9	314	1476.0	1627.8	2104.5	2306.8
231	999.0	1227.9	1439.7	1650.9	315	1326.4	1730.8	1928.6	2412.0
232	762.7	1327.3	1608.6	1794.8	316	1267.6	1710.3	2217.2	2816.6
233	932.4	1438.3	1712.9	1621.0	317	1344.6	1811.8	2055.2	2249.5
234	1077.5	1319.5	1595.6	1913.6	318	1414.0	1844.4	2196.6	2471.7
235	987.8	1279.5	1506.4	1817.2	319	1606.5	1727.7	1929.2	2513.6
236	910.6	1306.8	1497.0	1821.5	320	1370.4	1850.0	2386.1	2597.7
237	1105.6	1518.1	1544.5	1756.6	321	1425.0	1715.3	2225.4	2561.5
238	1022.0	1449.7	1534.3	1804.6	322	1621.9	2001.8	2522.6	2383.3
239	1110.6	1293.6	1503.2	2004.2	323	1555.5	2023.8	1961.9	2704.2
240	983.0	1586.3	1858.0	1725.4	324	1540.1	2049.3	2287.5	2348.9
241	1109.7	1324.9	1622.5	1743.8	325	1431.4	2187.9	2470.9	2588.4
242	941.9	1152.4	1640.3	1753.7	326	1427.5	1758.0	2187.9	2629.7
243	1052.1	1232.4	1504.9	1799.3	327	1577.1	1758.4	2156.8	2351.3
244	1138.8	1203.2	1679.9	2071.3	328	1509.9	1529.1	2467.4	2488.2
245	1033.3	1389.5	1656.4	1979.9	329	1474.9	1832.3	2447.1	2735.1
246	1227.1	1359.8	1712.2	1677.7	330	1571.9	1744.5	2461.2	2605.4
247	1183.0	1365.3	1804.7	1803.0	331	1411.5	1963.5	2194.6	2540.2
248	917.4	1460.1	1460.6	1843.1	332	1187.8	1666.5	2110.6	2441.3
249	1184.9	1355.7	1665.8	2009.3	333	1814.1	2179.6	2003.2	2398.3
250	1060.7	1507.8	1643.1	1931.5	334	1396.9	2046.1	2311.9	2602.9
251	1407.1	1456.9	1631.0	1738.3	335	1277.3	1806.5	2273.3	2527.7
252	1141.2	1262.9	1893.1	1808.9	336	1434.8	2227.8	2453.5	2682.8

MSS Cut-off values $N_p(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 100$

	p variables					p variables			
n	2	3	4	5	n	2	3	4	5
337	1748.9	1907.5	2429.5	2295.6	421	1584.0	2268.2	3138.9	2942.0
338	1283.0	1894.4	2169.1	2421.7	422	1879.1	2690.2	2502.8	3155.1
339	1815.4	1942.3	2369.0	2557.8	423	2123.3	2056.4	3516.2	3326.4
340	1157.5	2299.1	2423.4	2951.4	424	1449.9	2580.2	2776.4	3160.2
341	1411.6	1906.5	2290.6	2489.1	425	2005.3	2323.3	2827.9	3052.8
342	1355.1	2124.3	2354.4	2717.4	426	2001.7	2006.0	2651.0	3065.6
343	1851.1	2251.7	2512.3	2609.2	427	2170.0	2397.1	2805.0	3039.3
344	1587.3	1866.7	2242.8	2712.5	428	2027.9	2836.4	2934.5	3061.0
345	1871.6	1762.7	2401.8	2633.6	429	2109.9	2350.7	2669.9	3059.8
346	1571.5	2043.1	2597.5	2746.0	430	2281.2	2505.8	2707.2	3437.7
347	1611.3	1951.5	2369.1	3119.3	431	2017.8	2953.2	2756.8	3560.2
348	1649.2	2071.5	2440.6	2528.9	432	2475.4	2705.8	2920.8	3273.0
349	1542.9	2461.7	2369.8	2512.2	433	2234.5	2328.2	3696.3	3279.8
350	1877.3	1963.3	2405.1	2689.9	434	1933.0	2328.5	2878.5	3668.7
351	1771.7	1915.5	2437.2	2773.8	435	1690.3	2293.4	2885.7	3597.3
352	1569.9	1951.0	2246.0	2550.4	436	1666.6	2514.1	2914.7	3909.9
353	1692.1	2173.0	2310.0	2666.5	437	2040.7	2357.3	2741.3	3618.1
354	1942.1	2373.8	2397.3	2546.6	438	2182.1	2430.6	2602.1	3493.0
355	1411.0	1838.1	2331.2	2623.3	439	2031.8	2768.5	2746.3	3350.6
356	1588.1	2121.1	2472.2	2532.8	440	2312.3	2326.7	3270.7	3263.3
357	1483.2	2220.3	2357.3	2802.3	441	2148.7	2375.8	2736.0	3208.9
358	1517.6	2152.0	2421.2	2527.5	442	1947.3	2602.8	3055.7	3232.3
359	1832.3	2018.8	2158.8	2549.7	443	1969.1	2203.1	2666.2	3408.6
360	1388.8	2080.2	2389.8	2865.1	444	2127.9	2269.4	2793.9	3397.4
361	2266.8	2040.0	2669.6	2459.5	445	2304.8	2626.5	2768.9	3737.9
362	1682.2	2339.7	2506.2	2794.6	446	2759.6	2572.3	3079.3	3508.3
363	1716.0	1930.0	2620.7	2725.0	447	2161.4	2517.6	3064.4	3096.3
364	1548.6	1875.2	2551.6	2617.7	448	2039.6	2503.5	3465.4	3267.4
365	1602.6	2106.3	2281.6	3097.3	449	2054.1	2697.5	3026.9	3504.0
366	2360.6	1908.3	2395.2	2829.7	450	2567.7	2575.5	2715.2	3310.5
367	1857.4	2096.7	2563.5	3009.4	451	2046.5	2745.4	2802.2	3629.2
368	1576.4	1924.0	2537.4	2586.2	452	2137.8	2468.1	3099.9	3337.5
369	1769.4	2153.3	2692.9	2729.9	453	1853.0	2474.1	3115.9	3744.9
370	1721.6	2151.5	2464.4	2779.2	454	1682.2	2287.8	3172.1	3291.7
371	1657.1	1872.6	2456.4	2860.8	455	2426.0	2712.3	3233.8	3620.7
372	1888.4	2563.2	2668.8	2760.4	456	2053.8	2545.1	3064.8	3415.6
373	1765.8	2240.8	2477.5	2984.5	457	1969.4	2361.7	2898.9	3416.8
374	1715.8	1945.0	2829.2	2893.8	458	1954.6	2258.5	2914.6	3567.8
375	1669.0	2031.9	2324.3	2827.0	459	2393.4	2307.2	2863.2	3393.1
376	1706.3	2622.4	2369.3	2766.6	460	2159.6	2990.4	3326.3	3395.7
377	1506.6	2270.0	2628.2	2796.0	461	2355.2	2276.2	2768.4	3896.8
378	1604.8	2308.3	2694.7	2938.5	462	2060.3	2388.1	3157.8	3429.1
379	1859.6	1970.0	2466.0	2785.2	463	2093.8	3021.6	3229.5	3571.1
380	1682.9	2167.1	2583.0	2971.1	464	2002.4	2707.2	3046.9	3999.0
381	1762.5	2127.7	2909.0	2928.2	465	2411.5	2707.8	3356.5	3310.7
382	1292.6	2084.5	2479.4	2669.4	466	2687.4	2616.4	3079.8	3190.9
383	1678.8	2404.5	2734.3	3002.9	467	2507.6	2474.8	3331.4	3834.4
384	1509.3	2299.6	2872.2	3293.0	468	2564.5	2855.3	3225.9	3589.7
385	2096.3	2205.0	2940.9	2983.3	469	2117.8	2413.7	3393.5	3885.4
386	2197.6	2051.2	2177.4	2979.4	470	2252.9	2712.9	3020.7	3746.1
387	1556.3	2402.7	2807.5	2965.6	471	2366.8	2375.9	2777.2	3664.9
388	1936.8	2007.5	3005.6	3017.0	472	2171.3	2214.2	3738.3	3343.9
389	1947.4	2470.8	2469.1	2667.8	473	2252.7	3247.0	3177.0	3727.0
390	1804.9	2325.8	2735.7	3033.9	474	1890.2	2867.9	3519.6	3321.1
391	1780.1	2067.8	2741.0	2906.9	475	2236.4	2516.7	3518.1	4020.2
392	1701.2	2685.5	2181.7	2790.8	476	2184.4	2634.7	3686.6	3581.0
393	1991.3	2269.2	2344.1	3315.7	477	2465.1	2876.8	3231.9	3573.2
394	2062.8	2238.9	2679.8	2983.3	478	2117.5	2258.5	3211.7	3347.2
395	2055.8	2115.3	2446.7	2906.6	479	2357.6	2607.3	2978.9	3768.8
396	1594.1	2026.7	2721.7	3128.2	480	2592.4	2688.6	2724.8	3784.5
397	1791.7	2270.3	3042.8	2864.8	481	2064.3	2997.2	3221.3	3913.5
398	1657.8	2219.0	2719.3	3190.1	482	2521.9	2331.6	3332.6	3261.7
399	1839.9	2131.7	2437.4	2895.2	483	2478.9	2740.6	3251.5	3765.0
400	1629.7	1962.1	2369.2	2895.5	484	1919.3	2714.6	3590.3	3912.4
401	1687.7	2320.8	2511.3	3035.4	485	2149.0	2838.1	3097.1	4003.2
402	1668.1	2365.1	2426.2	3187.6	486	2074.0	2946.1	3561.9	3735.8
403	2079.2	2586.4	3066.0	3022.6	487	2227.8	2849.4	3305.1	3969.0
404	1898.0	2138.7	2708.8	3015.5	488	2247.9	3138.0	3201.0	3251.5
405	1860.5	2261.4	2647.0	3207.4	489	2524.8	2756.0	3099.4	3470.4
406	2054.1	2271.2	2611.1	3064.7	490	2863.1	2453.9	2887.5	3621.2
407	1884.2	2492.7	3013.6	3206.2	491	2420.3	2781.2	3073.8	3607.2
408	1809.3	2495.7	2765.1	2751.9	492	2024.5	2751.2	3305.9	3890.0
409	1791.8	2658.5	2836.2	3288.6	493	2154.0	3329.9	3563.0	3600.1
410	1581.4	2227.5	2504.6	3011.2	494	2257.9	3012.8	3650.9	3520.7
411	2217.0	2355.9	2808.6	3241.6	495	2117.4	3004.7	3676.1	3808.6
412	2472.0	2301.6	2718.8	3301.1	496	2252.2	2744.1	3052.7	3395.2
413	1850.2	2821.5	2624.3	3092.8	497	2389.9	3394.3	3532.6	3748.2
414	1760.2	2579.7	2626.5	2875.5	498	2427.4	2662.5	2907.5	3767.0
415	2286.6	2426.2	2792.3	3125.4	499	1950.5	2747.3	3470.9	3809.5
416	1671.8	2304.0	2681.5	3244.4	500	2187.1	2648.8	3056.6	3880.2
417	2079.4	2622.5	2588.8	2858.5					
418	2179.8	2553.5	2637.2	3504.5					
419	1899.3	2295.7	2705.6	3315.3					
420	1823.3	2410.9	2652.6	3039.3					

Table A.29: MSS Cut-off values $\alpha = 0.05$, $m_0 = 100$, for $p = 2$, $X_1 \sim N(0, 1)$, $X_2 \sim$ Uniform or t_3, for $p = 5$, $X_1, X_2, X_3 \sim N(0, 1)$, $X_4, X_5 \sim$ both Uniform or t_3

	Uniform		t_3			Uniform		t_3	
n	$p = 2$	$p = 5$	$p = 2$	$p = 5$	n	$p = 2$	$p = 5$	$p = 2$	$p = 5$
2	6.58	12.4	15.89	24.4	52	240.18	335.3	615.17	772.8
3	10.57	18.2	25.90	33.7	53	222.28	337.4	568.48	768.1
4	13.66	23.9	32.58	46.1	54	246.45	341.3	727.85	805.7
5	19.15	31.0	40.82	58.7	55	218.86	324.2	616.08	836.2
6	21.55	36.2	49.24	75.6	56	245.00	380.5	588.30	857.6
7	25.98	40.1	66.10	81.1	57	217.97	378.2	577.24	852.7
8	30.84	49.6	69.99	99.8	58	269.59	371.4	647.10	837.4
9	32.16	51.7	68.85	119.9	59	250.77	372.3	670.36	844.0
10	40.04	63.6	82.18	127.7	60	247.68	375.9	626.09	909.7
11	40.50	69.1	87.39	134.7	61	306.60	379.8	737.29	949.5
12	44.83	71.0	113.36	148.0	62	260.75	403.6	797.32	931.4
13	45.86	77.2	107.19	164.6	63	302.59	414.8	874.28	965.4
14	57.17	86.0	110.77	172.3	64	272.58	390.1	744.48	922.3
15	54.94	94.3	138.74	170.5	65	280.13	386.6	795.97	1037.3
16	58.54	93.7	149.91	210.1	66	270.21	385.4	738.98	1014.3
17	60.44	104.7	156.90	211.9	67	297.35	408.6	826.46	961.3
18	67.04	113.1	179.58	230.5	68	330.85	422.4	753.39	1025.3
19	78.10	114.8	186.32	242.3	69	327.54	447.0	786.47	945.8
20	81.61	129.6	192.74	252.0	70	315.26	445.1	863.49	1068.0
21	90.52	122.8	191.33	268.6	71	319.29	431.9	842.96	1099.9
22	84.77	133.7	210.16	284.4	72	352.14	449.3	900.51	1094.6
23	96.39	148.4	222.73	292.0	73	298.81	463.8	818.89	1131.9
24	90.95	135.9	213.49	308.2	74	347.56	457.4	877.34	1129.4
25	97.32	156.9	218.58	334.9	75	304.15	445.3	977.81	1225.9
26	95.13	156.5	245.48	315.2	76	375.67	462.9	902.82	1304.8
27	101.23	156.7	279.18	353.8	77	351.86	481.6	983.85	1158.1
28	107.96	172.3	242.45	369.7	78	336.31	462.0	912.17	1305.7
29	124.58	185.1	252.67	358.6	79	366.14	513.1	925.68	1222.8
30	114.01	175.7	288.53	370.9	80	348.04	492.2	987.18	1286.5
31	115.93	210.9	327.56	433.0	81	356.11	503.4	1108.13	1301.2
32	127.27	196.3	272.36	450.1	82	378.41	517.6	990.52	1349.0
33	125.49	212.0	301.09	448.2	83	424.67	550.4	1013.95	1319.4
34	141.44	223.1	374.30	476.1	84	413.36	552.9	1078.04	1375.0
35	140.60	209.5	350.04	474.8	85	428.79	538.7	1105.36	1467.1
36	150.20	222.1	342.37	496.3	86	410.68	571.7	1112.50	1407.4
37	166.55	220.6	369.71	488.5	87	412.32	585.9	922.48	1369.7
38	137.80	234.2	419.83	516.7	88	474.43	587.8	1178.15	1433.1
39	159.99	240.5	409.90	552.9	89	451.88	569.3	1196.75	1497.5
40	161.56	230.5	431.72	542.3	90	430.50	580.8	1195.63	1551.6
41	154.99	259.5	397.10	561.0	91	459.65	589.6	1209.58	1519.3
42	175.26	263.9	421.95	592.4	92	443.55	617.9	1184.71	1597.5
43	190.89	274.7	507.76	606.6	93	487.38	633.2	1251.12	1562.3
44	166.08	267.0	467.91	608.0	94	401.39	617.8	1249.54	1527.4
45	182.76	268.7	436.35	642.9	95	466.80	599.0	1266.87	1581.3
46	198.58	284.3	460.97	625.7	96	438.03	584.6	1320.40	1539.1
47	220.42	307.4	549.65	620.1	97	432.47	596.2	1248.14	1566.2
48	233.93	292.4	504.73	671.3	98	523.32	641.2	1305.46	1737.0
49	214.56	290.6	573.50	677.9	99	458.78	622.4	1298.48	1659.9
50	205.54	322.8	593.64	759.8	100	515.20	627.0	1425.40	1817.5
51	220.72	305.4	570.01	679.1					

Table A.30: MSS Cut-off values $\alpha = 0.01$, $m_0 = 100$, for $p = 2$, $X_1 \sim N(0,1)$, $X_2 \sim$ Uniform or t_3, for $p = 5$, $X_1, X_2, X_3 \sim N(0,1)$, $X_4, X_5 \sim$ both Uniform or t_3

	Uniform		t_3			Uniform		t_3			Uniform		t_3	
n	$p = 2$	$p = 5$	$p = 2$	$p = 5$	n	$p = 2$	$p = 5$	$p = 2$	$p = 5$	n	$p = 2$	$p = 5$	$p = 2$	$p = 5$
2	13.491	18.928	30.307	40.287	86	829.89	906.99	1848.6	2039.0	170	1439.5	1672.0	4976.2	5633.0
3	20.419	27.714	44.160	50.268	87	688.49	879.77	1717.0	2032.3	171	1952.0	1489.1	4770.	5287.7
4	27.11	32.877	54.035	67.599	88	880.80	888.42	2097.8	2389.7	172	1601.6	1634.5	4924.8	5374.0
5	34.410	47.130	66.994	84.737	89	874.93	738.24	2262.8	2132.0	173	1660.2	1787.4	5534.7	5661.8
6	46.204	54.987	99.992	113.37	90	729.81	818.09	2006.1	2315.9	174	1673.4	1873.6	5179.6	5481.0
7	57.417	61.908	129.67	121.8	91	821.45	891.05	2260.0	2485.8	175	1603.6	1715.0	5475.6	6033.3
8	53.754	78.083	111.99	136.16	92	913.94	964.57	2056.8	2313.6	176	1994.4	1764.	4553.5	4813.4
9	58.51	76.452	128.70	183.73	93	883.20	957.64	2209.5	2718.8	177	2403.9	1579.9	4770.3	5205.4
10	78.420	84.176	151.20	201.84	94	738.62	822.13	2364.4	2192.3	178	1595.8	1739.3	6289.9	5725.2
11	81.017	103.55	181.94	211.31	95	877.16	948.57	2234.1	2255.7	179	1896.9	1686.4	5183.2	5682.8
12	82.161	113.63	173.29	206.66	96	850.36	819.77	2405	2278.9	180	2138.3	1981.3	5136.6	5239.3
13	98.624	111.91	216.32	229.14	97	937.30	883.7	2025.0	2319.7	181	1995.0	1747.0	5500.4	4860.2
14	108.79	133.48	210.6	271.	98	928.83	912.56	1975.7	2366.2	182	2169.4	1598.6	7112.8	5946.7
15	124.18	148.56	281.43	284.67	99	758.12	951.68	2016.7	2163.7	183	2091.3	1877.4	6040.9	6135.3
16	133.46	145.34	257.77	307.46	100	959.10	935.3	2592.0	2778.3	184	1728.7	1787.2	5635.6	6075.1
17	141.85	157.13	288.83	289.93	101	842.64	1062.5	2342.0	2902.6	185	2149.8	1856.0	5543.4	6965.6
18	145.18	171.56	323.22	350.64	102	1014.1	944.62	2198.3	2724.3	186	1821.9	1872.6	5625.1	5685.1
19	127.26	176.89	340.60	361.63	103	854.83	948.78	2572.4	2297.3	187	2083.9	1957.8	5660.8	604
20	156.89	206.19	318.87	378.39	104	1035.2	998.76	2560.6	2532.5	188	1979.6	1844.9	5243.3	5523.0
21	154.20	179.69	378.0	365.68	105	976.96	1114.5	2631.3	2998.8	189	2032.0	1900.5	7144.9	6297.
22	170.82	224.12	405.49	398.03	106	1141.0	983.10	2431.0	2813.2	190	2307.0	1846.7	5943.2	5998.9
23	154.04	244	390.35	398.74	107	973.63	994.18	2435.5	2754.8	191	2314.3	2009.7	6872.0	5574.2
24	169.60	218.72	429.55	511.76	108	1071.7	1160.6	2392.0	2686.7	192	1942.0	2006.6	6612.5	6392.1
25	177.65	207.47	413.31	443.78	109	1032.8	965.83	2659.6	2935.9	193	2031.6	2035.1	5525.1	7157.3
26	181.93	239.11	393.59	525.66	110	1059.3	1083.7	2206.0	2938.9	194	2429.0	1901.6	6442.	6053.
27	171.87	242.	489.13	533.62	111	1067.4	1205.0	2537.0	2762.0	195	2033	2156.	5711.5	6487.9
28	200.38	246.94	503.68	557.30	112	1232.6	1054.8	2575.2	2810.2	196	2180.9	2027.0	6480.1	6472.0
29	199.94	283.43	544.43	534.28	113	1084.0	1076.9	2695.7	3389.2	197	2415.8	2130.1	6114.1	6698.2
30	220.75	260.62	511.23	557.36	114	1121.5	1084.5	2807.8	3033.8	198	2076.7	1860.7	6147.5	6842.5
31	260.09	313.30	565.69	716.20	115	956.88	1180.4	2753.0	3149.2	199	2543.3	2044.6	6526.5	7062.4
32	232.34	303.93	535.01	597.69	116	924.36	1191.9	2961.1	3388.9	200	2516.1	2198.0	5798.0	5889.7
33	250.5	301.58	527.14	755.	117	1044.0	1144.6	2608.9	3243.2	201	2115.5	2096.2	6406.6	6584.
34	240.60	316.3	653.88	654.03	118	1076.4	1022.2	3255.5	3519.3	202	2719.0	2222.8	5812.6	6519.0
35	234.13	323.	600.85	676.80	119	1137.3	1101.4	3225.5	3116.0	203	2160.5	2323.7	5946.4	6903.9
36	350.27	313.7	571.50	800.2	120	1128.8	1026.7	2696.2	3062.8	204	2583.8	2293.4	6388.1	7026.0
37	288.79	358.89	602.51	728.48	121	1224.8	1221.7	3006.5	3183.9	205	2453.6	1787.4	6764.2	6248.7
38	282.96	349.13	810.73	808.01	122	1134.2	1434.1	3376.9	3242.3	206	2221.6	2016.0	7441.9	7407.
39	272.24	375.91	716.39	838.29	123	1182.3	1196.5	2963.5	3586.6	207	2388.2	2172.0	6918.3	6863.0
40	330.47	354.83	656.04	790.94	124	1237.6	1109.1	3068.0	3491.1	208	2622.3	2016.8	6779.6	6684.8
41	250.	385.30	830.22	829.16	125	1238.4	1197.4	3410.9	3204.9	209	2420.3	1974.2	5749.9	7116.5
42	289.66	419.02	794.69	919.60	126	1225.4	1180.3	3061.0	3404.0	210	2439.3	2203.2	6671.6	7424.0
43	369.70	386.47	835.19	946.59	127	1221.1	1173.3	3660.4	3509.	211	2563.8	2095.5	7063.	6916.2
44	311.52	377.94	807.59	945.70	128	1172.5	1335.2	3734.6	3578.2	212	2156.6	2107.6	8208.	7179.8
45	354.92	428.01	738.82	1060.2	129	1146.5	1218.4	3506.8	3365.1	213	2579.5	2090.3	7499.	8010.2
46	374.48	422.84	775.5	946.18	130	1131.7	1078.9	3528.4	3238.7	214	2566.8	2217	7980.9	7307.6
47	388.4	460.45	833.10	941.65	131	1498.	1306.8	3101.0	3812.2	215	3178.7	2551.1	7522.3	7636.2
48	428.26	504.55	939.47	1080.0	132	1455.1	1273.1	3138.2	3625.8	216	2521.1	2350.4	7154.8	8248.3
49	424.28	435.04	1017.2	1066.2	133	1431.8	1211.0	3277.4	3962.1	217	2327.3	2120.5	6786.2	8158.7
50	415.28	516.02	1109.1	1099.2	134	1377.7	1379.1	3292.5	4010.1	218	2899.8	2351.0	6922.3	7577.2
51	408.3	464.64	1000.4	961.11	135	1565.2	1264.2	4735.0	3800	219	2463.5	2018.8	7072.9	7412.9
52	446.02	504.48	1040.8	1172.3	136	1380.9	1283.9	3519.8	3756.3	220	2587.2	2099.1	7525.	7691.6
53	444.4	504.75	1069.5	1218.1	137	1226.	1249.5	2990.0	4005.5	221	2596.3	1929.5	7451.3	7406.4
54	384.85	516.47	1240.	1138.0	138	1304.2	1433.0	3630.2	3803.	222	2996.9	2155.1	7566.0	8550.4
55	431.96	545.22	1205.4	1251.9	139	1500.3	1324.5	3515.8	3977.6	223	2199.2	2577.7	7753.6	8062.3
56	518.21	542.66	1141.4	1316.1	140	1758.0	1353.2	3636.0	3906.2	224	2532.0	2367.9	8526.4	8148.9
57	438.93	597.14	948.28	1389.3	141	1544.1	1459.4	3816.7	3868.6	225	2681.	2507.8	7589.2	7777.0
58	477.64	545.70	1072.5	1173.0	142	1452.2	1372.6	4393.6	4286.0	226	2586.4	2487.8	9071.9	8300.6
59	485.91	547.43	1176.0	1264.4	143	1349.0	1560.1	3723.0	4263.0	227	2297.6	2341.5	8063.6	7149.3
60	451.36	528.64	1084.4	1397.0	144	1468.7	1394.5	4419.3	4005.8	228	2426.7	2174.1	6990.3	7904.1
61	613.14	521.02	1313.8	1380.5	145	1518	1413.8	4164.2	4076.	229	3193.5	2326.	8390.9	8177.4
62	517.25	580.81	1427.1	1497.7	146	1477.8	1620.8	3667.0	4055.7	230	2309.8	2545.5	7421.0	7781.0
63	546.8	605.38	1430.3	1360.1	147	1422.	1494.5	4340.6	4540.7	231	2770.6	2434.	7530.3	8637.4
64	500.00	567.48	1419.7	1395.5	148	1495.7	1513.4	5125.2	4063.9	232	2620.2	2160.3	8833.3	8599.1
65	465.15	559.60	1218.4	1412.3	149	1513.8	1554.	4628.	4175.3	233	2447.2	2561.1	7757.4	8054.4
66	516.51	645.61	1202.3	1595.7	150	1842.0	1514.4	3928.6	4577.4	234	2783.7	2688.9	9574.6	7817.1
67	601.77	592.35	1463.5	1462.2	151	1841.0	1443.2	4074.6	4387.	235	2557.0	2433.3	8301.2	8867.6
68	661.22	643.76	1429.0	1456	152	1487.7	1444.1	3770.6	4695.3	236	2982.7	2761.7	9429.9	8248
69	620.15	700.8	1306.8	1420.9	153	1483.9	1650.3	4912.2	4652.4	237	3188.0	2550.3	9087.8	8903.1
70	578.67	677.89	1506.6	1562.0	154	1659.6	1505.8	4016	4658.2	238	3710.4	2615.1	9115.8	8419.4
71	618.16	672.34	1590.9	1589.2	155	1514.8	1658.0	4571.1	4303.6	239	2685.0	2424.4	8761.4	8583.9
72	674.05	648.41	1557.8	1632.5	156	1573.9	1525	4539.8	4535.3	240	3376.0	2419.7	8221.7	8675.5
73	625.79	798.76	1542.5	1817.4	157	1766.3	1447.5	4693.3	4609.6	241	3451.3	2562.4	8640.1	8865.0
74	677.25	685.77	1666.6	1596.6	158	1388.	1474.0	5522.1	4631.6	242	2987.5	2587.3	8533.6	8977.1
75	610.34	619.62	1808.6	1819.7	159	2118.7	1516.0	5420.3	4827.3	243	3067.1	2446.1	8653.8	9297.2
76	640.00	675.92	1543.6	1777.0	160	1679.0	1498.0	4997.5	4715.8	244	3165.3	2619.0	8687.3	9195.3
77	577.57	738.98	1543.0	1554.0	161	1809.6	1613.5	4942.4	4797.	245	3137.9	2635.4	9037.5	8798.9
78	639.60	743.74	1545.8	2065.2	162	1732.5	1543.0	3983.7	5121.9	246	3275.1	2454.0	9153.3	9829.8
79	605.18	758.82	1753.3	1782.2	163	1734.9	1510.0	4464.9	5066.8	247	3238.1	2097.1	8873.0	9946.4
80	637.77	736.62	1841.6	1833.5	164	1791.7	1482.7	5468	4711.7	248	3043.1	2432.9	8395.4	9725.2
81	679.04	759.05	1796.0	1804.9	165	1855.8	1666.8	4312.6	5091.5	249	2859.3	2615.8	9640.9	9231.4
82	653.76	884.61	1636.1	1887.	166	1458.1	1655.3	5016.	4786.0	250	2697.6	2521.1	7933.6	9437.7
83	768.44	783.18	1957.9	1938.5	167	1958.7	1871.3	4655.9	5121.2	251	2820.7	2590.7	8968.8	8846.8
84	754.76	754.9	2282.4	2039.2	168	1828.7	1665.7	5414.0	5578.1	252	3525.0	2990.4	10288	9031.0
85	780.72	909.10	1800.4	2173.1	169	1996.8	2026.6	5228.3	5315.6	253	3137.1	2781.8	8612.7	9435.3

MSS Cut-off values $\alpha = 0.01$, $m_0 = 100$, for $p = 2$, $X_1 \sim N(0,1)$, $X_2 \sim$ Uniform or t_3, for $p = 5$, $X_1, X_2, X_3 \sim N(0,1)$, $X_4, X_5 \sim$ both Uniform or t_3

	Uniform		t_3			Uniform		t_3			Uniform		t_3	
n	$p = 2$	$p = 5$	$p = 2$	$p = 5$	n	$p = 2$	$p = 5$	$p = 2$	$p = 5$	n	$p = 2$	$p = 5$	$p = 2$	$p = 5$
254	2921.4	2860.9	8875.4	9800.8	338	5011.1	3825.4	17553	15811	422	6851.8	5219.0	24628	21912
255	3874.1	2839.	9094.1	9618.2	339	4611.8	3785.0	13975	15737	423	6453.3	4839.5	20871	21727
256	3438.6	2787.3	9006.6	9448.3	340	4010.9	3731.6	15459	16063	424	7120.5	5543.4	22270	20028
257	3321.7	2496.7	10186	9061.0	341	5400.1	3992.6	14714	14878	425	6781.4	5455.9	21411	21681
258	3378.5	2929.9	9940.4	9609.4	342	5221.3	3854.8	14232	15858	426	6269.6	5818.6	19179	21868
259	3272.5	2694.7	9473	10594.	343	5371	4344.7	14467	16304	427	7567.0	5248.5	19591	21112
260	3097.8	2405.5	10578	9289.0	344	4875.7	3941.5	16396	14721	428	6704.3	4702.3	26091	22219
261	4004.0	2816.5	9123.7	10563.	345	5330.8	4307.2	14596	15603	429	7282.9	5188.1	23178	22289
262	4125.7	3218.3	8462.0	10629.	346	4410.8	4358	17667	15400	430	8487.2	5591.2	22689	22057
263	3228.0	2678.1	8996.2	10203	347	5563.3	3861.1	15840	14022	431	6637.0	5373.2	25131	23196
264	2951.0	2677.3	10913	9954.9	348	5916.0	4292.0	15169	16878	432	7680.1	5439.6	20555	22544
265	3206.0	2964.2	9308.9	10533.	349	4776.	3935.1	17221	17343	433	6645.8	4924.5	22474	25755
266	3319.6	2580.4	8918	9802.8	350	5215.7	3813.8	16441	16112	434	8069.6	4680.5	21906	22821
267	3002.8	2951.0	10601	10522.	351	5192.7	4348.1	15926	15111	435	6448	5613.0	23739	22456
268	3291.2	2986.9	9066.7	9480.6	352	4768.	4355.8	18119	15726	436	6625.9	5344.4	21106	23065
269	3334.8	3223.	10487	9830	353	4359.5	4542.3	15130	16221	437	6349.2	4818.4	23283	22661
270	3914.0	3028.7	9595.5	11600.	354	6143	4116.7	18488	15501	438	6795	5084.4	22500	22633
271	3287.8	2970.6	8760.0	11333	355	5286.7	3717.0	15929	16498	439	7266.1	5263.6	24206	20752
272	3619.2	2992.7	10274	9738.9	356	5312.7	3617	16229	15376	440	7284.6	5929.2	23258	22440
273	3749.1	2880.0	11455	10287	357	6072.9	3821.4	16513	16904	441	7675.7	4561.7	23579	21652
274	3580.2	2733.3	10064	9666.4	358	5643.2	3945.9	18856	16858	442	7690.8	5581.1	20349	24577
275	3294.1	2951.6	9140.6	10854	359	5881.2	4624.7	15531	17034	443	7334.4	5238.5	24812	22972
276	3952.5	2873.9	11883	10038	360	5875.4	3608	16278	17894	444	7874.2	5338.8	22499	23084
277	3552.0	3064.8	10969	10462	361	5476.6	4872.6	18542	16667	445	8310.7	5192.1	24309	24988
278	3720.0	3524.7	11777	10078	362	5369.8	4428	17683	17600	446	7385.1	5514	26340	23062
279	3757.2	2969.	11429	10852	363	6326.3	4045	15056	18179	447	7563.4	5560	24843	22810
280	3981.4	3291.0	10117	11202	364	5626.2	4269.7	14837	16263	448	8433.4	5037.2	25218	24795
281	3924.1	3087.4	10878	10808	365	6426.1	4253.8	15071	14488	449	7997.1	5529.2	22215	22739
282	3470.1	2905.0	1099	10791	366	5770.0	3977.6	16928	18126	450	7249.1	5344.1	23505	22195
283	4305.3	2918.6	11250	11129	367	5366.8	4076.0	19345	17829	451	7405.7	5533.0	25261	24546
284	4070.2	2780.4	11206	12275	368	4898.8	4863.5	16739	16492	452	6831.8	5296.2	29308	24134
285	3624	3355.2	10703	11542	369	6682.3	4731.9	17904	15067	453	7394	5464.3	25914	23804
286	3572.6	3707.1	9978	12568	370	6151.6	3814.1	16908	18026	454	7767.7	5542	22202	24381
287	3774	3427.0	11095	11380	371	6586.4	4249.8	18659	17403	455	7179.1	5744.9	23906	25356
288	4200.4	2746.	9976	12513	372	5431.2	4031.6	18133	15300	456	9819.3	5365.9	27118	22363
289	3770.4	3357.	10941	11887	373	6031.2	3761.5	16260	18312	457	8393.5	6152.1	23232	26030
290	4076.1	2659.1	11487	12950	374	5163.6	4555.6	20362	17176	458	6724.7	5448.9	25041	23696
291	3924.0	3133.6	9915	10788	375	5708.7	4123.7	16803	15781	459	8363.1	5878.0	25149	23490
292	3781.2	3233.3	11965	12472	376	6135.0	4298.8	17908	17734	460	7734.6	6145.3	22909	25821
293	4873.9	3100.1	13114	11580	377	5884.1	4368.9	20891	17799	461	8543.6	5396.0	2500	24781
294	4048.0	3233.3	13341	12700	378	5547.5	4171.2	16354	20112	462	8931.9	5173	21930	24710
295	4213.8	3081.3	12418	12304	379	5246.6	4808.7	15706	18138	463	9483.8	6068.1	25624	24993
296	4154.2	3328.6	12421	11646	380	5944.0	4439.3	18812	17973	464	7383	5802.8	27204	25823
297	3908.3	3448.5	11140	11653	381	6759.6	3869.8	21805	19516	465	8053.1	5597.2	26539	23732
298	4002.3	3185.0	11380	12276	382	5262.0	4556.4	18271	18881	466	6872.9	5988.5	23097	26459
299	4762	3196.3	12217	12095	383	6094.6	4101.1	17547	20481	467	7428	5941.8	23931	23667
300	4208.3	2916.8	12104	11692	384	6053.8	4902	19245	19339	468	7083.2	6686.5	28042	26185
301	4235.9	3606.6	12313	12750	385	5995.4	4311	17232	19124	469	8565.7	5621.1	22746	27798
302	4089	3359.4	12774	12658	386	6010	4242.6	21190	18738	470	9220.8	5618.0	26651	23437
303	4729.8	2974.6	11251	12357	387	5081.5	4137.7	19981	18310	471	8266.7	5330.9	28854	25630
304	4692.8	3161.8	12531	13443	388	5255.1	4530.3	20203	17826	472	9747	5604.2	22738	25921
305	4704.6	3393.6	11794	12105	389	5292.3	4513.0	18098	21585	473	8009.2	5890.2	28921	28829
306	3948.5	3515.5	12305	14080	390	5453.1	4688.9	22966	17016	474	8470.2	6259.7	29085	25765
307	5299.6	3192.6	11548	13540	391	6567	4752	18605	19117	475	8565.5	5799.8	24349	24340
308	3905.1	3682.2	12998	12000	392	5448.9	4277.9	18115	19113	476	8302.5	6042.5	28930	27325
309	4588.1	3532.3	12320	12463	393	6141.1	4309.3	21438	20156	477	7756.6	5363.3	26509	25319
310	4305.3	3326.4	14398	14453	394	6094.5	4679.1	20302	17819	478	8378.2	5610.9	27183	27299
311	4073.4	3422.0	13019	13624	395	6975.2	4657.2	18692	18094	479	7796.8	5694.1	30794	28863
312	4297.4	3192.8	12859	13367	396	5942.2	4185.1	20525	20560	480	9038.4	5822.1	27033	23773
313	4274.9	3695.9	12286	15391	397	5873.5	4987.6	19379	20319	481	8853.8	5022.5	22281	28119
314	4528.0	3230.7	12315	12804	398	6307.5	4622.8	19001	21481	482	8551.5	5549.0	28419	23648
315	4482.2	3466.7	12924	11370	399	7376.5	4898	21281	19268	483	7522.5	6031.0	25465	25705
316	4343.4	3185.2	12055	12049	400	6427.1	4901.4	19793	20432	484	9519.7	5941.9	30670	27182
317	4412.8	3846.7	13042	13276	401	6130	4799.4	20065	19499	485	8321.9	6639.8	26156	25147
318	4549.7	3335.	14966	15069	402	6763.5	4215	20354	19666	486	8888.9	6835.8	30346	27179
319	5120.3	3126.9	13535	13394	403	5878.1	4778.9	19670	18696	487	9574.6	5990.3	31497	29283
320	4535.1	3849.9	12950	13069	404	7210.7	4672.0	20349	19994	488	8192	5891.6	29974	29724
321	4013.2	3648.5	12924	14296	405	5095.4	4803.1	19885	19317	489	8681.5	6078.9	25418	29737
322	3966.4	3480.6	15481	14157	406	6125.7	4587.6	19192	20149	490	8148.0	6296.1	31677	27583
323	3885.5	3819.4	14125	15632	407	7397.6	4908.0	21031	19878	491	8957.3	6981.7	28276	29787
324	5154.2	3363.8	13989	14553	408	6277.6	4378.5	19145	20231	492	9204	6319.4	2803	2811
325	4990.1	3506.2	14127	13752	409	5907.1	4804.9	21988	22291	493	8149.7	6429.4	32621	29158
326	5243.4	3723	14198	14271	410	6679.0	4938.3	17282	18720	494	8705	5657.2	25120	28736
327	4224.9	3234.7	15353	14775	411	6578.2	4977.3	18581	20711	495	9739.1	6472.1	28878	29399
328	4097.4	4138.6	14461	14222	412	6831.7	5199	21821	19919	496	8370.0	5953.9	28025	33028
329	4134.9	3736.5	16115	13532	413	7497	5075.3	20314	22558	497	9776.8	6039.7	31204	27367
330	4530.5	3787.4	13855	14337	414	6175.2	4896.2	20562	21059	498	8660.4	6243.	30404	31362
331	4733.7	4052.1	14768	17132	415	7269	5492.7	21838	19863	499	9493.7	5867.0	28459	30951
332	4927.9	3341.5	15712	13144	416	6861.3	4862.1	18715	19671	500	9174.5	6182.8	26242	32195
333	4644.1	3934	15142	14935	417	7429.0	5154.2	24409	19618					
334	3652.1	3712.9	16042	17805	418	7656.1	4694.6	21885	22265					
335	4698.4	3834.2	14916	14524	419	6396.4	4800.3	24766	21440					
336	4433.8	3502.1	16681	14918	420	6628	5336.9	22093	20458					
337	4929.1	3582.2	15035	14186	421	5998.9	4599.1	23194	21933					

Appendix B

ARL tables

Table B.1: ARL and sd(RL) values for $N_2(\mathbf{0}, \mathbb{I})$, $\alpha = 0.05$, $m_0 = 50$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ			ARL(sdRL)		
0	19.085(21.922)	18.950(23.607)	22.765 (15.791)	19.466(19.987)	18.695(25.111)
0.5	8.628(8.057)	6.594(2.697)	17.797 (11.512)	4.736(2.664)	17.791(14.104)
1	7.400(3.551)	5.387(2.291)	15.165 (8.676)	3.053(1.927)	15.073(13.464)
1.5	6.632(2.859)	4.695(1.928)	12.944(7.514)	2.343(1.253)	14.355(11.570)
2	6.289(4.232)	4.197(2.319)	11.561 (6.177)	2.039(0.801)	13.267(8.306)
3	5.788(3.356)	3.685(1.939)	9.532(4.651)	1.579(0.717)	11.816(6.106)
4	5.293(2.430)	3.351(1.133)	8.229(3.839)	1.353(0.479)	10.571(5.687)
5	4.970(3.062)	3.097(1.700)	7.272(2.817)	1.220(0.554)	9.685(5.948)

Table B.2: ARL and sd(RL) values for $N_2(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 50$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ			ARL(sdRL)		
0	88.607(76.785)	93.420(63.711)	97.292(151.535)	92.148(89.812)	96.132(62.810)
0.5	17.660(13.683)	12.774(7.497)	74.426(112.722)	7.989(3.622)	75.064(38.473)
1	13.453(6.397)	9.424(2.971)	58.359(82.040)	4.988(3.415)	55.981(56.600)
1.5	11.741(4.936)	7.853(2.834)	49.862(52.636)	3.717(2.014)	40.270(33.655)
2	11.040(5.951)	6.957(1.921)	43.732(57.364)	2.865(2.441)	33.028(19.445)
3	9.431(6.409)	5.926(2.814)	32.437(26.442)	2.066(1.074)	23.443(12.156)
4	8.946(4.462)	5.259(1.545)	28.196(20.753)	1.624(1.262)	19.392(10.882)
5	8.554(4.755)	4.932(1.742)	24.454(20.690)	1.403(0.457)	17.085(10.990)

Table B.3: ARL and sd(RL) values for $N_2(\mathbf{0}, \mathbb{I})$, $\alpha = 0.05$, $m_0 = 100$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ			ARL(sdRL)		
0	19.386(19.286)	19.245(17.961)	19.036(16.080)	20.264(13.319)	19.753(16.426)
0.5	10.658(10.125)	6.877(2.556)	15.669(26.632)	4.663(2.578)	18.459(22.095)
1	9.602(6.957)	5.406(2.136)	13.477(9.438)	3.184(2.871)	17.623(16.298)
1.5	8.941(7.222)	4.628(1.118)	11.800(6.109)	2.415(1.144)	15.989(9.085)
2	8.594(5.377)	4.392(1.563)	10.699(11.782)	2.027(0.976)	14.435(12.020)
3	7.645(3.593)	3.886(1.339)	9.067(3.445)	1.580(0.492)	12.447(8.892)
4	7.653(4.006)	3.605(1.289)	8.157(3.259)	1.324(0.486)	12.001(7.869)
5	7.116(2.053)	3.335(1.409)	7.343(2.561)	1.212(0.312)	11.174(7.452)

Table B.4: ARL and sd(RL) values for $N_2(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 100$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ			ARL(sdRL)		
0	93.970(64.181)	85.965(84.104)	99.758(114.800)	90.729(72.871)	99.523(88.002)
0.5	26.665(11.058)	11.974(5.586)	77.157(72.651)	7.790(6.439)	72.504(75.618)
1	21.610(10.790)	8.879(3.563)	60.993(46.164)	4.748(3.547)	53.304(27.982)
1.5	19.316(7.835)	7.689(2.015)	52.590(60.982)	3.288(1.918)	41.910(31.562)
2	18.016(7.382)	6.760(3.092)	46.411(34.592)	2.642(0.892)	32.793(15.249)
3	16.419(12.114)	6.102(1.439)	37.729(35.175)	1.965(0.759)	26.776(14.928)
4	14.938(9.931)	5.483(2.510)	31.955(26.162)	1.616(0.891)	22.822(10.765)
5	14.428(6.619)	5.171(1.701)	28.891(30.363)	1.380(7.211)	20.198(7.124)

Table B.5: ARL and sd(RL) values for $N_3(\mathbf{0}, \mathbb{I})$, $\alpha = 0.05$, $m_0 = 50$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ			ARL(sdRL)		
0	19.603(16.282)	20.068(11.174)	21.380(16.822)	19.988(14.872)	19.256(13.704)
0.5	8.080(4.188)	7.898(4.354)	17.771(13.124)	5.298(2.848)	19.197(22.651)
1	6.745(3.273)	6.078(3.331)	15.260(10.946)	3.283(1.919)	16.963(19.080)
1.5	6.167(1.781)	5.481(2.354)	13.431(7.976)	2.519(1.815)	15.315(10.311)
2	5.743(4.135)	4.838(1.590)	12.441(6.604)	2.108(0.641)	13.908(8.162)
3	5.110(2.314)	4.276(1.442)	10.192(5.098)	1.609(0.732)	11.421(5.488)
4	4.904(3.384)	3.831(0.981)	8.978(3.825)	1.345(0.548)	10.173(5.371)
5	4.664(2.873)	3.569(1.701)	7.615(2.459)	1.183(0.479)	9.560(4.247)

Table B.6: ARL and sd(RL) values for $N_3(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 50$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ			ARL(sdRL)		
0	88.015(73.597)	90.812(118.334)	88.710(116.658)	91.021(88.581)	93.492(47.237)
0.5	17.488(16.377)	15.373(7.889)	68.811(84.535)	9.238(6.524)	79.722(47.470)
1	13.172(3.154)	11.318(3.656)	61.510(77.810)	5.332(4.276)	53.669(37.374)
1.5	11.709(6.576)	9.748(2.116)	51.533(64.976)	3.829(1.426)	41.620(34.630)
2	10.659(3.720)	8.799(3.441)	42.175(49.717)	5.464(7.621)	35.953(27.653)
3	9.166(3.766)	7.637(2.367)	36.876(36.579)	2.217(0.886)	23.607(9.241)
4	8.409(3.391)	6.930(2.542)	28.727(24.763)	1.766(0.789)	18.506(5.672)
5	7.825(3.622)	6.456(2.198)	26.299(19.919)	1.507(1.237)	16.200(8.664)

Table B.7: ARL and sd(RL) values for $N_3(\mathbf{0}, \mathbb{I})$, $\alpha = 0.05$, $m_0 = 100$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ			ARL(sdRL)		
0	20.515(17.717)	19.612(15.533)	19.095(11.851)	18.987(15.170)	20.806(22.435)
0.5	11.272(7.412)	7.891(4.322)	16.318(9.864)	5.085(2.425)	18.663(15.541)
1	9.316(5.567)	6.252(3.368)	14.833(8.563)	3.260(2.199)	17.999(17.287)
1.5	8.896(5.452)	5.522(3.273)	13.239(7.361)	2.550(0.997)	17.019(17.764)
2	8.381(5.503)	4.897(1.758)	12.389(6.010)	2.073(1.107)	14.507(6.075)
3	8.006(5.842)	4.471(2.342)	10.607(5.388)	1.611(0.872)	12.776(7.683)
4	7.384(5.205)	4.087(1.661)	9.477(4.575)	1.334(0.556)	11.438(6.745)
5	7.043(5.626)	3.885(0.981)	8.737(2.944)	1.220(0.506)	10.912(4.627)

Table B.8: ARL and sd(RL) values for $N_3(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 100$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ	ARL(sdRL)				
0	91.542(104.056)	90.298(69.345)	98.057(182.264)	90.087(52.532)	90.638(71.074)
0.5	25.300(12.753)	13.666(6.556)	74.685(100.643)	8.417(3.751)	72.996(67.518)
1	20.482(13.079)	10.398(2.453)	67.226(92.808)	5.289(2.635)	52.120(25.786)
1.5	18.375(7.885)	9.014(4.502)	57.769(65.528)	3.811(1.703)	38.080(21.566)
2	16.994(9.477)	8.234(3.406)	50.587(74.494)	3.504(1.921)	34.135(18.533)
3	15.307(7.238)	7.194(2.371)	44.344(51.520)	2.229(1.367)	25.491(16.331)
4	13.786(6.445)	6.729(2.059)	38.553(38.615)	1.815(0.550)	20.590(10.013)
5	13.469(8.145)	6.343(1.929)	34.586(27.605)	1.550(0.724)	18.288(7.612)

Table B.9: ARL and sd(RL) values for $N_4(\mathbf{0}, \mathbb{I})$, $\alpha = 0.05$, $m_0 = 50$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ	ARL(sdRL)				
0	18.222(24.278)	19.589(19.671)	21.186(15.182)	19.944(19.146)	19.162(17.523)
0.5	7.939(4.018)	8.357(5.329)	17.838(13.329)	5.840(3.666)	18.541(16.163)
1	6.786(3.900)	6.903(2.385)	15.819(9.298)	3.782(2.622)	17.269(16.527)
1.5	6.341(5.707)	5.979(4.037)	14.293(8.924)	2.916(1.532)	14.646(15.634)
2	5.778(3.776)	5.560(2.221)	13.157(7.587)	2.360(0.804)	13.988(11.192)
3	4.922(3.727)	4.695(1.845)	10.928(5.643)	1.792(0.784)	11.499(6.472)
4	4.533(1.799)	4.287(2.281)	9.375(4.066)	1.508(0.539)	10.795(6.576)
5	4.396(2.554)	3.987(1.386)	8.528(3.546)	1.319(0.567)	9.112(6.850)

Table B.10: ARL and sd(RL) values for $N_4(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 50$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ	ARL(sdRL)				
0	90.712(59.491)	92.558(75.808)	108.901(167.811)	83.506(87.514)	95.224(132.864)
0.5	17.647(19.147)	17.316(6.028)	86.286(129.726)	10.198(6.579)	86.836(78.083)
1	13.409(5.966)	13.131(3.071)	65.277(76.162)	6.208(2.813)	65.325(62.615)
1.5	11.618(4.492)	11.208(5.087)	60.403(111.051)	4.327(2.648)	48.653(45.206)
2	10.917(4.386)	10.225(2.853)	52.511(56.081)	3.596(1.414)	37.072(29.952)
3	8.912(2.953)	8.956(2.511)	43.621(50.940)	2.470(1.028)	25.223(11.081)
4	8.296(5.118)	8.251(2.803)	35.226(22.043)	1.984(1.063)	19.847(5.751)
5	7.570(4.459)	7.663(1.480)	30.701(29.048)	1.683(0.756)	15.890(8.267)

Table B.11: ARL and sd(RL) values for $N_4(\mathbf{0}, \mathbb{I})$, $\alpha = 0.05$, $m_0 = 100$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ	ARL(sdRL)				
0	19.616(10.458)	19.594(16.355)	20.783(15.090)	20.358(15.841)	20.012(20.443)
0.5	11.039(5.455)	8.304(3.777)	18.341(10.253)	5.268(3.071)	17.936(21.133)
1	9.254(6.958)	6.943(3.423)	16.159(7.361)	3.461(1.415)	17.182(14.602)
1.5	8.806(5.991)	9.297(3.640)	15.322(9.843)	2.643(1.414)	16.456(10.258)
2	8.223(6.493)	5.769(2.573)	13.676(8.153)	2.124(1.091)	15.298(9.161)
3	7.547(3.933)	5.182(2.260)	12.024(4.405)	1.678(1.032)	13.022(8.691)
4	7.109(4.563)	4.779(2.930)	10.596(5.996)	1.383(0.400)	11.521(5.869)
5	6.815(4.887)	4.587(2.011)	9.849(4.862)	1.202(0.545)	10.651(9.209)

Table B.12: ARL and sd(RL) values for $N_4(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 100$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ	ARL(sdRL)				
0	90.804(82.614)	93.154(107.677)	104.065(155.655)	90.703(66.617)	94.497(71.426)
0.5	24.338(11.641)	16.237(9.823)	89.815(183.904)	10.122(4.242)	79.191(90.502)
1	19.780(7.036)	12.156(4.108)	75.774(108.597)	6.015(3.590)	54.006(41.533)
1.5	17.899(7.232)	10.290(4.156)	65.739(71.078)	4.339(2.681)	41.652(34.418)
2	15.607(5.306)	9.458(3.785)	57.185(63.986)	3.503(2.330)	35.348(21.022)
3	14.358(5.879)	8.350(2.478)	51.383(64.743)	2.511(1.407)	25.642(14.621)
4	13.124(3.789)	7.741(2.021)	43.099(45.340)	1.943(0.715)	20.829(6.926)
5	12.093(4.495)	7.333(0.936)	39.255(39.841)	1.604(0.824)	17.869(9.763)

Table B.13: ARL and sd(RL) values for $N_5(\mathbf{0}, \mathbb{I})$, $\alpha = 0.05$, $m_0 = 50$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ	ARL(sdRL)				
0	19.727(15.463)	20.036(17.719)	20.028(13.909)	19.543(9.869)	19.829(9.932)
0.5	8.637(4.581)	9.494(7.939)	17.327(11.462)	6.080(1.934)	19.542(19.572)
1	6.892(3.346)	7.745(4.381)	15.202(8.662)	3.873(2.201)	17.077(16.852)
1.5	6.541(5.597)	6.799(4.004)	13.862(7.184)	3.091(2.593)	15.944(15.321)
2	5.777(3.037)	6.098(3.944)	12.778(7.898)	2.491(1.014)	14.450(11.858)
3	5.156(3.252)	5.346(1.906)	10.585(5.332)	1.891(0.971)	12.403(4.594)
4	4.703(2.396)	4.967(2.339)	9.263(4.646)	1.530(0.501)	10.894(5.549)
5	4.304(2.668)	4.684(2.238)	8.378(3.505)	1.355(0.622)	9.804(3.780)

Table B.14: ARL and sd(RL) values for $N_5(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 50$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ	ARL(sdRL)				
0	90.820(100.124)	92.009(88.257)	102.350(190.268)	93.916(57.681)	100.254(58.187)
0.5	17.747(14.651)	20.465(5.643)	80.751(112.105)	11.066(6.426)	84.952(44.793)
1	13.464(6.383)	15.732(7.019)	69.249(99.653)	6.744(2.515)	63.025(55.669)
1.5	11.285(6.422)	14.299(5.448)	58.527(72.733)	4.736(2.181)	49.217(40.090)
2	10.189(5.554)	12.912(3.026)	53.973(65.726)	3.658(2.161)	38.313(25.404)
3	9.067(5.045)	11.636(4.665)	40.514(48.274)	2.702(1.275)	26.784(11.255)
4	8.218(4.852)	10.675(3.194)	35.055(32.124)	2.037(1.615)	20.525(8.389)
5	7.482(2.469)	9.941(2.195)	30.686(26.980)	1.720(0.760)	16.938(4.792)

Table B.15: ARL and sd(RL) values for $N_5(\mathbf{0}, \mathbb{I})$, $\alpha = 0.05$, $m_0 = 100$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ	ARL(sdRL)				
0	19.928(20.530)	20.264(18.183)	19.465(15.535)	19.448(14.791)	20.025(11.591)
0.5	10.879(10.060)	9.447(3.842)	17.081(9.991)	5.922(5.517)	19.472(13.986)
1	9.171(5.199)	7.837(5.905)	15.671(12.000)	3.928(1.461)	17.786(20.090)
1.5	8.774(9.391)	6.970(2.428)	14.416(9.115)	2.924(1.346)	16.381(14.070)
2	8.269(7.775)	6.369(1.889)	13.097(7.142)	2.399(1.358)	15.825(14.198)
3	7.487(5.486)	5.717(2.430)	11.452(5.042)	1.794(0.732)	13.723(8.374)
4	7.103(5.481)	5.320(2.912)	10.166(4.455)	1.466(0.854)	11.697(3.302)
5	6.502(5.253)	5.180(2.828)	9.098(4.110)	1.319(0.853)	10.877(7.900)

Table B.16: ARL and sd(RL) values for $N_5(\mathbf{0}, \mathbb{I})$, $\alpha = 0.01$, $m_0 = 100$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ	ARL(sdRL)				
0	88.990(86.422)	92.532(82.196)	99.370(158.413)	87.244(88.705)	92.816(41.133)
0.5	25.496(17.474)	18.625(11.752)	78.646(136.401)	9.738(2.962)	80.314(41.472)
1	20.497(7.662)	14.403(5.619)	71.937(96.195)	5.819(2.665)	66.359(35.892)
1.5	17.627(6.603)	12.510(2.944)	63.546(91.408)	4.459(1.401)	44.236(36.991)
2	15.705(9.581)	11.654(2.751)	55.891(61.285)	3.411(1.797)	35.512(15.500)
3	14.222(6.882)	10.183(1.510)	47.112(56.272)	2.422(1.002)	25.812(14.183)
4	12.948(8.736)	9.428(3.368)	39.312(34.372)	1.967(0.807)	21.542(6.949)
5	12.038(4.893)	8.852(2.509)	33.439(31.141)	1.648(0.935)	18.294(7.296)

Table B.17: ARL and sd(RL) values for $X_1 \sim N(0, 1)$, and $X_2 \sim \text{Uniform}(-\sqrt{3}, \sqrt{3})$, $\alpha = 0.05$, $m_0 = 100$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ	ARL(sdRL)				
0	19.317(13.320)	19.242(22.536)	32.428(32.598)	19.859(23.277)	19.934(21.893)
1	9.846(4.859)	5.350(2.437)	21.850(14.729)	3.140(1.775)	19.672(11.921)
2	8.508(3.354)	4.431(1.503)	17.274(9.758)	2.069(1.024)	16.736(21.742)
3	7.850(4.819)	3.971(2.009)	14.172(8.457)	1.594(1.084)	16.201(21.858)
4	7.704(3.270)	3.580(1.210)	12.343(7.142)	1.354(0.797)	14.025(10.431)
5	7.493(3.856)	3.351(0.955)	11.476(6.898)	1.209(0.370)	12.717(9.970)

Table B.18: ARL and sd(RL) values for $X_1 \sim N(0, 1)$, and $X_2 \sim \text{Uniform}(-\sqrt{3}, \sqrt{3})$, $\alpha = 0.01$, $m_0 = 100$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ	ARL(sdRL)				
0	96.790(59.131)	92.574(72.958)	104.741(181.506)	90.151(73.689)	102.836(114.370)
1	21.313(9.007)	9.294(3.309)	61.059(82.939)	5.027(2.882)	97.510(73.256)
2	17.469(9.157)	6.988(2.473)	48.505(46.352)	3.139(1.467)	59.031(37.914)
3	15.780(4.747)	6.006(2.684)	37.975(39.036)	2.259(0.713)	40.013(35.857)
4	14.578(9.208)	5.572(1.194)	34.327(33.984)	1.811(0.681)	29.967(21.034)
5	13.812(4.597)	5.083(1.586)	23.048(16.948)	1.559(0.897)	24.966(12.911)

Table B.19: ARL and sd(RL) values for $X_1, X_2, X_3 \sim N_3(\mathbf{0}, \mathbb{I})$; $X_4, X_5 \sim \text{Uniform}(-\sqrt{3}, \sqrt{3})$, $\alpha = 0.05$, $m_0 = 100$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ	ARL(sdRL)				
0	19.017(21.360)	18.965(25.978)	20.045(14.032)	19.663(22.611)	20.294(20.279)
1	8.231(6.336)	6.695(3.904)	15.667(9.044)	2.716(1.788)	17.764(12.583)
2	6.915(3.734)	5.506(1.059)	12.708(6.819)	1.745(0.815)	15.597(11.303)
3	6.642(3.770)	4.766(2.095)	10.953(6.144)	1.335(0.599)	13.811(7.932)
4	5.994(3.330)	4.458(2.328)	9.638(4.886)	1.176(0.363)	11.993(6.239)
5	5.735(4.400)	4.138(1.665)	8.712(4.287)	1.081(0.269)	10.794(7.780)

Table B.20: ARL and sd(RL) values for $X_1, X_2, X_3 \sim N_3(\mathbf{0}, \mathbb{I})$; $X_4, X_5 \sim \text{Uniform}(-\sqrt{3}, \sqrt{3})$, $\alpha = 0.01$, $m_0 = 100$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ	ARL(sdRL)				
0	93.148(101.201)	84.703(115.625)	114.401(219.248)	96.704(125.544)	92.877(79.490)
1	17.158(9.084)	11.145(2.934)	75.402(117.809)	4.658(2.084)	67.873(79.243)
2	13.981(6.572)	9.058(3.614)	58.746(91.159)	2.539(1.313)	44.356(25.338)
3	12.291(4.983)	8.185(2.864)	47.835(56.331)	1.858(0.830)	30.885(16.185)
4	11.404(6.688)	7.733(2.279)	40.255(39.977)	1.477(0.536)	24.103(9.144)
5	10.848(4.001)	7.254(2.890)	33.594(34.520)	1.272(0.603)	19.319(8.259)

Table B.21: ARL and sd(RL) values for $X_1 \sim N(0,1)$, and $X_2 \sim t_3$, $\alpha = 0.05$, $m_0 = 100$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ	ARL(sdRL)				
0	18.720(16.840)	19.537(25.073)	21.361(13.393)	19.322(18.504)	18.941(13.835)
1	9.313(3.776)	5.102(2.203)	14.457(11.462)	2.152(0.644)	18.093(12.021)
2	7.910(4.093)	4.118(1.923)	11.444(7.113)	1.460(0.333)	17.665(14.986)
3	7.350(3.422)	3.650(1.097)	9.963(5.091)	1.209(0.360)	17.330(13.368)
4	7.222(4.386)	3.382(1.565)	9.005(4.115)	1.153(0.362)	14.209(14.330)
5	6.873(4.045)	3.120(1.543)	8.391(3.056)	1.084(0.222)	13.729(9.969)

Table B.22: ARL and sd(RL) values for $X_1 \sim N(0,1)$, and $X_2 \sim t_3$, $\alpha = 0.01$, $m_0 = 100$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ	ARL(sdRL)				
0	84.917(87.200)	85.076(96.972)	114.899(213.108)	88.404(105.240)	91.044(61.496)
1	18.112(8.703)	8.042(4.248)	69.375(92.398)	3.336(2.519)	88.611(74.956)
2	15.179(10.802)	6.504(1.677)	51.682(55.197)	1.954(0.499)	57.344(70.725)
3	14.031(9.141)	5.691(2.080)	43.387(48.005)	1.511(0.470)	42.485(19.722)
4	13.318(6.138)	5.386(0.917)	37.777(34.295)	1.313(0.556)	31.272(15.381)
5	12.727(6.557)	4.935(1.500)	33.587(31.472)	1.169(0.285)	26.607(11.000)

Table B.23: ARL and sd(RL) values for $X_1, X_2, X_3 \sim N_3(\mathbf{0}, \mathbb{I})$; $X_4, X_5 \sim t_3$, $\alpha = 0.05$, $m_0 = 100$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ	ARL(sdRL)				
0	20.457(26.007)	18.422(16.365)	18.651(13.980)	20.373(26.404)	20.621(12.293)
1	8.748(7.393)	6.509(5.039)	15.287(11.908)	2.065(1.160)	19.237(16.112)
2	7.490(4.657)	5.513(1.747)	13.298(8.330)	1.343(0.862)	16.342(8.269)
3	6.827(6.768)	4.827(1.667)	11.175(4.851)	1.119(0.223)	15.089(10.277)
4	6.529(5.215)	4.377(2.251)	10.369(4.914)	1.052(0.244)	14.160(10.048)
5	6.310(3.489)	4.103(2.099)	9.489(4.756)	1.019(0.120)	13.558(6.218)

Table B.24: ARL and sd(RL) values for $X_1, X_2, X_3 \sim N_3(\mathbf{0}, \mathbb{I})$; $X_4, X_5 \sim t_3$, $\alpha = 0.01$, $m_0 = 100$

Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ	ARL(sdRL)				
0	92.906(133.224)	91.242(58.867)	99.889(148.683)	92.713(69.059)	111.522(95.305)
1	16.592(7.139)	13.950(6.324)	76.767(107.961)	2.878(2.021)	74.675(70.706)
2	13.549(6.728)	10.983(4.702)	65.113(90.050)	1.646(0.665)	56.282(36.254)
3	12.077(6.246)	9.749(2.677)	52.899(54.065)	1.272(0.410)	42.799(26.499)
4	11.233(5.398)	8.958(4.013)	45.782(50.558)	1.111(0.475)	29.555(11.245)
5	10.773(4.863)	8.238(3.008)	41.453(38.800)	1.051(0.307)	23.642(7.045)

Table B.25: ARL values for $X_1, X_2, X_3 \sim N_3(\mathbf{0}, \mathbb{I})$; $X_4, X_5 \sim t_3$, $\alpha = 0.05$, $m_0 = 100$

	Method	DFEWMA	EWAEL	SREWMA	MST	MSS
δ	Cut-offs	ARL(sdRL)				
0	t	20.457(26.007)	18.422(16.365)	18.651(13.980)	20.373(26.404)	20.621(12.293)
	normal	19.720(14.747)	19.700(15.753)	22.570(13.246)	16.189(8.212)	17.196(13.283)
1	t	8.748(7.393)	6.509(5.039)	15.287(11.908)	2.065(1.160)	19.237(16.112)
	normal	8.888(4.991)	7.002(3.292)	17.099(12.044)	3.647(2.254)	15.192(12.936)
2	t	7.490(4.657)	5.513(1.747)	13.298(8.330)	1.343(0.862)	16.342(8.269)
	normal	7.916(4.675)	5.529(1.467)	14.333(8.427)	2.288(1.794)	13.164(6.120)
3	t	6.827(6.768)	4.827(1.667)	11.175(4.851)	1.119(0.223)	15.089(10.277)
	normal	7.254(4.806)	4.949(2.270)	12.408(7.696)	1.646(1.171)	11.731(7.236)
4	t	6.529(5.215)	4.377(2.251)	10.369(4.914)	1.052(0.244)	14.160(10.048)
	normal	6.962(5.244)	4.480(0.800)	10.838(5.743)	1.382(0.607)	10.110(7.026)