

Signatures of spin in the  $\nu=1/3$  fractional quantum Hall effectA. F. Dethlefsen,<sup>1,\*</sup> E. Mariani,<sup>2,3</sup> H.-P. Tranitz,<sup>4</sup> W. Wegscheider,<sup>4</sup> and R. J. Haug<sup>1</sup><sup>1</sup>Institut für Festkörperphysik, Universität Hannover, 30167 Hannover, Germany<sup>2</sup>Department of Condensed Matter Physics, The Weizmann Institute of Science, 76100 Rehovot, Israel<sup>3</sup>Fachbereich Physik, Freie Universität Berlin, Arnimallee 14, D-14195 Berlin, Germany<sup>4</sup>Institut für Angewandte und Experimentelle Physik, Universität Regensburg, 93040 Regensburg, Germany

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The activation gap  $\Delta$  of the fractional quantum Hall state at constant filling  $\nu=1/3$  is measured in a *wide* range of perpendicular magnetic field  $B$ . Despite the full spin polarization of the incompressible ground state, we observe a sharp crossover between a low-field linear dependence of  $\Delta$  on  $B$  and a Coulomblike square root behavior at large  $B$ . From the global gap reduction we get information about the mobility edges in the fractional quantum Hall regime.

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In recent years the increased mobility of two-dimensional electron systems (2DES) has allowed for the experimental investigation of the fractional quantum Hall effect (FQHE)<sup>1,2</sup> at relatively small magnetic fields. In this regime the interplay between interactions and the electronic spin yields interesting properties of either the ground state or the excitation spectrum of the system. Quantum phase transitions between differently spin-polarized ground states have been predicted and experimentally observed for several FQH states while varying the perpendicular magnetic field  $B$ .<sup>3-12</sup>

In parallel, the theoretical understanding of the FQHE has been considerably deepened by the introduction of composite fermions (CF),<sup>13</sup> quasiparticles made of one electron and two magnetic flux quanta. The correlated many-electron problem in high magnetic fields can be interpreted in terms of weakly interacting CF in a smaller effective field, offering a unified theory of the compressible and incompressible electronic FQH states. The former are mapped onto CF Fermi liquids,<sup>14</sup> while the latter are viewed as the integer QHE of CF.

One of the main experimental signatures associated to the incompressible FQH states is a thermally activated longitudinal resistivity,  $\rho_{xx} \propto \exp[-\Delta/2T]$ , with the associated finite activation gap  $\Delta$ . An accurate determination of  $\Delta$  is therefore crucial in order to test the predictions of the CF theory on the spectrum of incompressible FQH states and to extract the quasiparticle effective parameters. Early measurements of  $\Delta$  vs  $B$  revealed the importance of disorder and finite thickness of the 2DES in reducing the gap with respect to the numerically calculated one.<sup>15</sup> However, the limited experimental range of variation of electron densities and mobilities allowed only a few data points for each incompressible state. Detailed measurements of the activation gaps in the low- $B$  regime were recently performed for filling factors  $\nu=2/3$  and  $2/5$  on high mobility samples.<sup>16</sup> The linearly vanishing gap close to their spin-polarization quantum phase transition highlighted the importance of the quasiparticle Zeeman energy in the spectrum at relatively small magnetic fields.

In this paper we present a detailed analysis of the activation gap for the paradigmatic FQH state at *fixed* filling factor  $\nu=1/3$  in a *very wide* range of *purely perpendicular* magnetic field. We clearly observe a sharp crossover between two regimes: a linear  $B$  dependence for small magnetic fields

and a  $\sqrt{B}$  dependence for higher fields. The linear part directly yields the CF  $g$  factor while the disorder-induced reduction of the activation gap with respect to the ideal clean case gives information about the mobility edges of the CF Landau levels (CFL).<sup>17</sup>

We now proceed to discuss the expectations of the free-CF theory on the activation gap and subsequently present our experimental measurements with the relative theoretical analysis.

In a FQH state at filling factor  $\nu \equiv n_e \Phi_0 / B$  ( $n_e$  the average electron density and  $\Phi_0 = h/e$  the magnetic flux quantum) the mismatch between the density of electrons and of flux quanta induces a huge ground-state degeneracy at the noninteracting level, making the many-body problem perturbatively untreatable. Within the Chern-Simons picture,<sup>17</sup> CF are created by attaching  $\phi$  additional flux quanta to each electron ( $\phi$  an even integer), generating an additional magnetic field  $b(\mathbf{r}) = \phi \Phi_0 n_e(\mathbf{r})$  [ $n_e(\mathbf{r})$  the local electron density] opposite to the external one, in order to partially compensate for it and thereby reduce the degeneracy. This gauge transformation depends only on the positions of the electrons and is uncoupled to the fermionic spin, thereby leaving the Zeeman splitting unchanged. CF's are then subject to an effective magnetic field  $B^*(\mathbf{r}) = B - b(\mathbf{r})$  that vanishes for  $\nu = 1/\phi$ , on the spatial average (mean field approximation). Near this filling factor, the cancellation is not exact and the residual  $B^* = B(1 - \phi\nu)$  yields CFL with an effective cyclotron energy  $\hbar\omega_c^* \equiv \hbar e B^* / m^*$  ( $m^*$  the CF effective mass). The CF filling factor  $p = n_e \Phi_0 / B^*$  is related to the electronic one by  $\nu^{-1} = p^{-1} + \phi$ , allowing the mean-field mapping of the principal sequence of the electronic FQH states into integer QHE of CF. In the following we will focus on the  $\nu=1/3$  state (i.e.,  $p=1$ ) and choose  $\phi=2$ , leading to  $B^* = B/3$ .

The relevant energy scale involved in electronic FQH states for GaAs systems is the Coulomb repulsion  $e^2/\epsilon\ell \propto \sqrt{B}$  (with  $\epsilon$  the dielectric constant and  $\ell \equiv \sqrt{\hbar/eB}$  the magnetic length), while the inter-LL cyclotron energy  $\hbar\omega_c \propto B$  is much larger for the interesting magnetic field range. Thus, as long as spin effects are not concerned (or in the fully polarized regime at high  $B$ ), the CF cyclotron gap  $\hbar\omega_c^*$  is expected to be proportional to  $e^2/\epsilon\ell$  (leading to a CF effective mass  $m^* \propto \sqrt{B}$ )<sup>14,18</sup> and describes the electronic FQH gap. When

spin is taken into account, each CFLL is split into two sub-levels separated by the Zeeman energy  $E_Z = g\mu_B B$ , with  $g$  the CF  $g$  factor. The spectrum of CFLL is therefore given by

$$E_{n,s}(B) = \left( n + \frac{1}{2} \right) \hbar\omega_c^* + sE_Z, \quad (1)$$

with  $n$  the CFLL index and  $s = \pm 1/2$ .<sup>19</sup>

Within the noninteracting CF picture, the  $\nu=1/3$  FQH state is mapped onto the integer QHE at  $p=1$ , meaning that the zero-temperature ground state is obtained by fully occupying the lowest CFLL  $E_{0,1/2}(B)$  (since the  $g$ -factor is negative in GaAs). In this way we easily recover the full spin polarization of the  $\nu=1/3$  state at  $T=0$ , independent of the value of  $B$ . However, due to the different  $B$  scaling of  $\hbar\omega_c^*$  and  $E_Z$ , the nature of the excitation gap  $\Delta_{id}$  (in the ideal clean case) is different for low/high magnetic fields. In particular,  $\Delta_{id} = \min[E_{0,-1/2}(B), E_{1,1/2}(B)] - E_{0,1/2}(B)$  leading to

$$\begin{aligned} \Delta_{id} &= E_Z \text{ for } B < B_c, \\ \Delta_{id} &= \hbar\omega_c^* \text{ for } B \geq B_c, \end{aligned} \quad (2)$$

with  $B_c$  such that  $E_{1,1/2}(B_c) = E_{0,-1/2}(B_c)$ . Notice that, in the low- $B$  regime, eventual deviations from the  $\sqrt{B}$  dependence of  $m^*$ , due to LL mixing, are immaterial for the expectations of the excitation gap, which is anyway linear in  $B$  and just dependent on  $g$ . Thus, a measurement of the slope  $\partial_B \Delta_{id}$  in the linear regime would directly yield the CF  $g$  factor. Similar arguments were used in Ref. 16 to extract  $g$  at  $\nu=2/3$  and  $2/5$ , but a linearization of the CFLL energy close to  $B_c$  was needed. The  $\nu=1/3$  case, in this sense, yields  $g$  without any corrections due to linearizations in the spectrum.

From the experimental point of view, the crossover from the linear to the  $\sqrt{B}$  scaling of the excitation gap was never clearly tested in a *single* sample until now. We succeeded in performing this experiment due to the high mobility of our 2DES and to the ability to modulate the density in a very wide range keeping the filling factor constant while varying the magnetic field.

Our 2DES is realized in an AlGaAs/GaAs heterostructure with a 70 nm thick spacer from the  $\delta$ -doping layer. A metallic topgate enables us to vary the electron density  $n_e$  between  $2.0$  and  $12.9 \times 10^{10} \text{ cm}^{-2}$  (see the inset of Fig. 1) with a zero field mobility reaching  $7 \times 10^6 \text{ cm}^2/\text{V s}$  at 40 mK. The filling factor  $\nu=1/3$  is then shifted from 2.5 T to 16.0 T with increasing density. In Fig. 1, the longitudinal resistivity  $\rho_{xx}$  vs  $B$  is shown for different densities at  $T=40$  mK, the lowest temperature in our experiment.

To obtain the activation energy for the different magnetic fields we investigate the temperature dependence of the resistivity minimum at  $\nu=1/3$ . We extract the gap  $\Delta$  out of the Arrhenius-plot data in Fig. 2, using the activated resistance behavior  $\rho_{xx} \propto \exp(-\Delta/2T)$ .

In Fig. 3 the measured activation energies  $\Delta$  are plotted versus the perpendicular magnetic field  $B$ . The error bars represent the standard deviation of the fits from the Arrhenius-plot data. Taking into account that the disorder potential in GaAs heterostructures is approximately 1–2 K

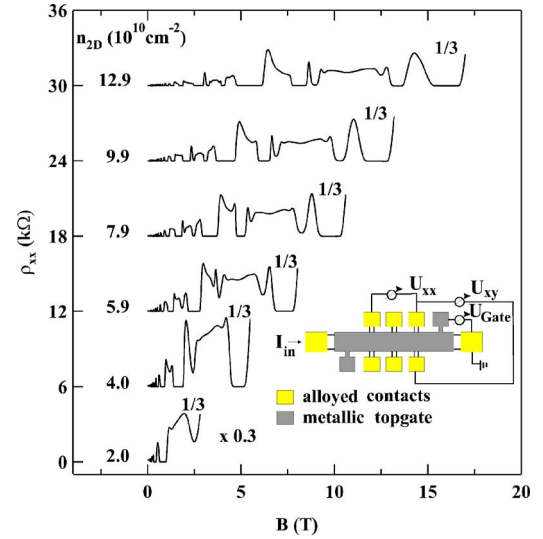


FIG. 1. (Color online) Variation of the longitudinal resistivity  $\rho_{xx}$  with the magnetic field at different electronic densities  $n_e$ . The different curves have been shifted for clarity. The inset shows the Hall bar geometry with a metallic topgate.

in high mobility specimen, the activation gaps below 2 K possess presumably larger error bars than shown.

The first thing we notice is a sharp crossover between two different scalings: a linear behavior at small fields and a  $\sqrt{B}$  dependence for  $B$  larger than  $B_c = 8.8$  T. This is in agreement with the *qualitative* expectations for independent CF.

A second important feature we observe is the disorder-induced reduction of  $\Delta$ , which vanishes at a finite  $B \approx 2$  T. However, the functional dependence of  $\Delta$  on  $B$  is the same as expected by the ideal theory, with no appreciable corrections for the overall magnetic field range, suggesting a *magnetic field-independent* reduction of  $\sim 1.7$  K. As a first test of the influence of disorder in our system we analyzed the low-field Shubnikov–de Haas oscillations for different densities, probing the role of scalar random potentials on the 2DES. We found an astonishingly weak dependence on density with a

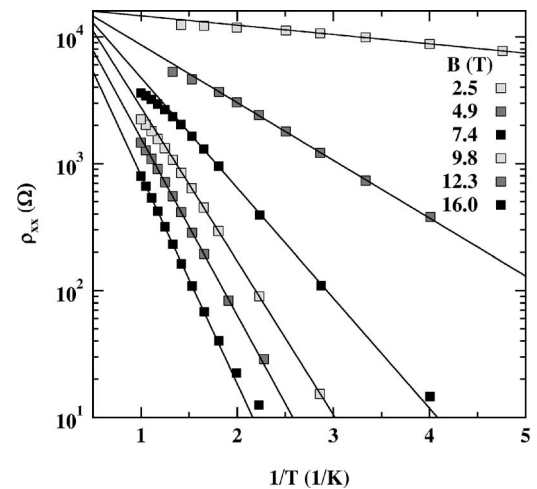


FIG. 2. Temperature dependence of the resistivity for filling factor  $\nu=1/3$  at different perpendicular magnetic fields. The solid lines are fits to the activated scaling  $\rho_{xx} \propto \exp(-\Delta/2T)$ .

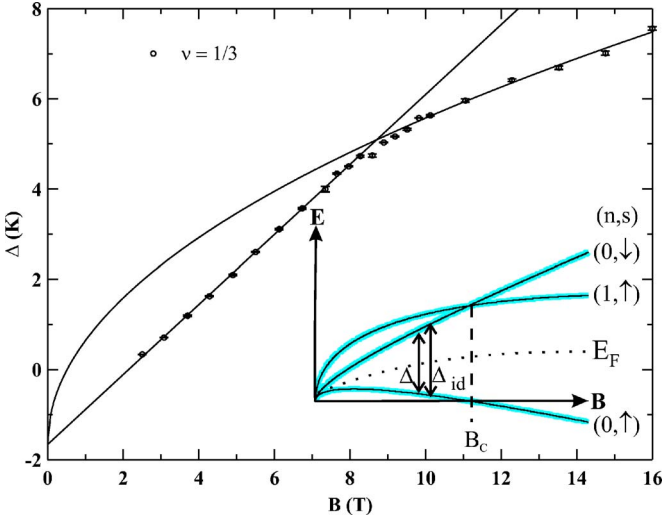


FIG. 3. (Color online) The activation gap  $\Delta$  at  $\nu=1/3$  versus  $B$ . The linear and square root scalings are clearly visible, as stressed by the two fit lines based on the free-CF model. A constant gap reduction of 1.7 K has been included, yielding the negative intercept of the curves at  $B=0$ . The relevant disorder-broadened CF-Landau bands are depicted in the inset.

scattering time varying from  $1.5 \times 10^{-12}$  s at  $2 \times 10^{10}$  cm $^{-2}$  to  $1.7 \times 10^{-12}$  s at  $6 \times 10^{10}$  cm $^{-2}$ . This consistency test supports the observed magnetic field-independent influence of the disorder in our results.

Finally, in Fig. 3 we observe an additional gap reduction of about 0.1 K, smoothen the sharp transition close to  $B_c$ .

Out of the slope  $\partial_B \Delta$  in the linear regime we directly extract an effective  $g$  factor at  $\nu=1/3$  of  $|g|=1.2$ . Previous measurements at fixed  $\nu=2/3$  and  $2/5$ ,<sup>16</sup> showed a strong dependence of  $g$  on the filling factor, due to interaction corrections,<sup>20</sup> and suggested an extrapolated value at  $\nu=1/3$  of  $|g| \approx 0.44$  associated to a *single* spin-flip process. These arguments, however, neglect the residual CF interactions, in the spirit of a quasiparticle picture of the original correlated problem. The  $\nu=1/3$  state is mapped onto a CF QH ferromagnet at  $p=1$  whose lowest energy charged excitations in the low- $B$  regime are expected to be smooth spin textures of the skyrmionic type.<sup>21–23</sup> Their size is determined by the interplay between the Zeeman energy (favoring small textures) and the exchange term of the residual CF interactions (favoring large skyrmions).

The linear part of the activation gap seems to suggest that the lowest charged excitations out of the CF QH ferromagnet are composite skyrmions, in agreement with the interpretation of previous data of Leadley *et al.*<sup>24</sup> The linear increase of the gap energies measured by Leadley *et al.* is in accordance with the one of our measurements. On the other hand, there is evidence that these charged excitations correspond to the creation of spin-reversed quasiparticles. Hints for this interpretation are given by the theoretical work of Chakraborty *et al.*<sup>5</sup> who used experimental data from Boebinger *et al.*<sup>15</sup> Again the slope of the measured and the calculated data, respectively, is roughly the same as in our data, which seems to indicate that for such high mobility samples the slope of the measured linear increase of the gap is the same.

In a sample with a much smaller mobility (reaching only  $8 \times 10^5$  cm $^2$ /V s) we observed a linear increase with magnetic field as well, but with a much smaller slope (corresponding to  $|g|=0.6$ ). This leads to the interpretation that the linearity of the activation gap in the low field regime of the FQH state  $\nu=1/3$  is intrinsic.

It is customary to define the CF mass as  $m^* = m_0 \alpha \sqrt{B(T)}$ , with  $m_0$  the free electron mass in vacuum. The parameter  $\alpha$  is sensitive to the finite thickness of the 2DES and to LL mixing which reduce the Coulomb interaction with respect to the exact 2D case. Using this definition we get  $B_c = 4/(3g\alpha)^2$  and, in the high- $B$  regime,  $\Delta(K) = -1.70 + 0.45 \sqrt{B(T)}/\alpha$  (having included the gap reduction). From both the measured values of the high-field gap and of  $B_c$  (which is not significantly affected by the disorder broadening of CFL), we *independently* get the *same* CF mass parameter  $\alpha=0.2$ , in agreement with previous analysis.<sup>11,19</sup> This estimate once again shows the importance of finite thickness corrections<sup>12</sup> to the quasiparticle effective parameters calculated from exactly 2D systems.<sup>7,18,25</sup>

In order to discuss the disorder-induced gap reduction, we point out that  $\Delta$  is rather the energy difference between the first unoccupied mobility edge and the last occupied one. In the integer QHE, mobility edges are the energies at which the localization length  $\xi(E)$  equals the typical sample size  $L$ . Within the scaling theory of the localization,<sup>26</sup>  $\xi(E)$  diverges close to a LL center as

$$\xi(E) = \xi_0 \left| \frac{E - E_c}{\Gamma_\sigma} \right|^{-\gamma}, \quad (3)$$

with  $E_c$  the energy of the center of the Landau band,  $\Gamma_\sigma$  the bandwidth (for a disorder potential with correlation length  $\sigma$ ) and  $\gamma$  the critical exponent. Finally,  $\xi_0$  can be extracted from the localized regime in the LL tails and is related with the correlation length  $\sigma$ , as a percolation picture suggests. Thus, the mobility edges are  $E_c \pm \Gamma_\sigma (\xi_0/L)^{1/\gamma}$ , yielding a gap reduction  $2\Gamma_\sigma (\xi_0/L)^{1/\gamma}$  whose  $B$  dependence is driven by  $\Gamma_\sigma$ .

In the case of CF, two disorder mechanisms broaden the CFL's into bands: a scalar random potential, mainly due to the  $\delta$  doping outside the 2DES, and a random magnetic field (RMF) scattering. The latter is due to fluctuations in the density which, via the flux attachment, produce fluctuations in the effective magnetic field felt by CF. It has been shown<sup>27</sup> that the scaling law (3) holds for CF as for electrons. The main difference is in the scaling of  $\Gamma_\sigma$  due to the two scattering mechanisms.

The contribution to  $\Gamma_\sigma$  due to the scalar disorder,  $\Gamma_\sigma^{\text{sc}}$ , can be estimated within the self-consistent Born approximation<sup>28</sup> as  $\Gamma_\sigma^{\text{sc}} = \Gamma^{\text{sc}} (1 + \sigma^2/\ell_{\text{CF}}^2)^{-1/2}$ , where  $\Gamma^{\text{sc}} = 2V_0/(\sqrt{2\pi}\ell_{\text{CF}})$  is the broadening due to a  $\delta$  correlated disorder potential  $V(\mathbf{r})$  with  $\langle V(\mathbf{r})V(\mathbf{r}') \rangle = V_0^2 \delta(\mathbf{r}-\mathbf{r}')$  and  $\ell_{\text{CF}} = \sqrt{3}\ell$  is the CF magnetic length. In our case, the dominant impurity scattering is due to the  $\delta$  doping at a distance  $d=70$  nm from the 2DES, inducing a disorder with a typical  $\sigma \approx d$ . For all the magnetic fields in our experiment ( $B > 2$  T),  $\ell_{\text{CF}} \ll \sigma$ , yielding  $\Gamma_\sigma^{\text{sc}} \approx \Gamma^{\text{sc}} \ell_{\text{CF}}/\sigma = 2V_0/(\sqrt{2\pi}\sigma)$ . Since  $V_0$  is an intrinsic property of the disorder potential independent of  $B$ , we deduce that

$\Gamma_{\sigma}^{\text{sc}}$  is magnetic field independent, leading to a *constant* reduction of the activation gap.

As far as the RMF contribution is concerned, the majority of results so far have been obtained close to  $\nu=1/\phi$ , where  $B^*$  is small (i.e. in the limit of high CFLL index  $n$ ).<sup>29</sup> Only recently, the tails of the Landau bands of fermions in a RMF with nonvanishing average have been studied.<sup>30</sup> In our case, where  $\ell_{\text{CF}} \ll \sigma$ , the fermionic states are essentially cyclotron orbits in presence of the local (smoothly varying) effective magnetic field  $B^*(\mathbf{r})=B^*+\delta B(\mathbf{r})$ , with  $\delta B(\mathbf{r})$  the RMF. The related typical energy shift  $\Gamma_{\sigma}^{\text{RMF}}$  is the cyclotron energy  $\hbar e \delta B(\mathbf{r})/m^*$  and the RMF acts pretty much like a scalar potential. Since  $\delta B(\mathbf{r})$  is proportional to the density fluctuations mainly driven by potential modulations, the relevant  $B$  dependence of the RMF broadening is through  $m^*$ , so that  $\Gamma_{\sigma}^{\text{RMF}} \propto B^{-1/2}$ .

A quantitative estimate of  $\Gamma_{\sigma}^{\text{RMF}}$  and  $\Gamma_{\sigma}^{\text{sc}}$  is extremely difficult. However, the observed  $B$ -independent gap reduction, particularly evident in the sharp linear low- $B$  regime, suggests that  $\Gamma_{\sigma}^{\text{sc}}$  dominates over  $\Gamma_{\sigma}^{\text{RMF}}$ . Such a behavior may be understood via the robust incompressibility of the  $\nu=1/3$

state which strongly suppresses the density fluctuations responsible for the RMF scattering.

The additional gap reduction of about 0.1 K close to  $B_c$  is probably due to an anticrossing between the ( $n=0, \downarrow$ ) and ( $n=1, \uparrow$ ) CFLL's, whose origin can be traced back to the spin-orbit coupling<sup>19,31</sup> and whose magnitude is consistent with the typical GaAs parameters.

In conclusion, we measured the activation gap of the  $\nu=1/3$  FQH ferromagnet in a wide range of the perpendicular magnetic field, observing a sharp crossover between two different regimes. The astonishing linearity of the measured activation gap in the low magnetic field range crosses over to a square-root dependence in high magnetic fields. Furthermore, the constant gap reduction seems to indicate a dominant scalar impurity scattering over the random magnetic field one.

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