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Endogenous agendas and seniority advantage*

Jon X. Eguia[†] Kenneth Shepsle[‡]

New York University Harvard University

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Abstract

We study a legislative assembly that chooses its agenda protocol endogenously. We generalize McKelvey and Riezman's (1992) seminal theory on seniority in legislatures, by allowing for a large class of ordinal agenda rules that assign different recognition probability to each legislator. We consider two stages – the selection of agenda rules, and the decision making that transpires under them. We predict that the agenda rules chosen in equilibrium preserve seniority distinctions, disproportionately favor more senior legislators, and generate an incumbency advantage to all legislators.

Keywords: Seniority, incumbency, endogenous agenda, recognition rule.

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[†]eguia@nyu.edu 19 West 4th St. 2nd fl. Dept. of Politics, NYU. New York, NY 10012, US.

[‡]kshepsle@iq.harvard.edu 1737 Cambridge St. CGIS-Knafel Bldg. Harvard University. Cambridge, MA 02138.

1 Introduction

Seniority is a characteristic feature of legislative institutions. Senior legislators typically take leadership roles, wield disproportionate clout in the selection of rules of procedure, and are also influential in determining the proposals that ultimately come to a vote before the assembly. The empirical literature in political science on the U.S. Congress, beginning with Abram and Cooper (1968) and Polsby, Gallagher, and Rundquist (1969), puts considerable emphasis on the importance of seniority as an organizational principle for the conduct of legislative business. In seeking to explain the legislative reliance on seniority, however, this literature emphasizes functionalist collective purposes without giving due consideration to the goals of individual legislators – career or policy.[1] An appeal to the functions served by a seniority rule – for example, the elevation of experienced legislator types to positions of authority, or the economizing on time and other resources that would otherwise be devoted to the contestation of authority – is insufficient to explain how these considerations affect the choices of individual members and decisive coalitions. It fails, that is, to tell us why it is in the interest of members of a self-governing group to select procedures that bestow differential advantage on senior members.[2]

In the more analytical literature on legislative organization, McKelvey and Riez-

[1]For exceptions to functionalist arguments, see the Epstein, Brady, Kawato, and O'Halloran (1997) and Krehbiel and Wiseman (2001).

[2]Functionalist explanations of seniority arrangements also plague the analysis of groups other than legislatures. On seniority rules in traditional tribal societies, see Simmons (1945). On the prominence given to seniority in labor contracts, see Mater (1940) and Burda (1990)

man (1992) were the first to tackle this issue formally (although a more informal development anticipating theirs is found in Holcombe 1989). Instead of offering functionalist reasons, they provide an explanation of the endogenous emergence of a seniority institution based on the benefit it provides each incumbent legislator in his or her pursuit of reelection. Granting differential power to senior legislators, allowing them in expectation to obtain a disproportionate share of resources for their districts, induces voters in every district to prefer reelecting their incumbent politician rather than a newly minted legislator. A seniority rule, in effect, begets an incumbency advantage. Self-interested legislators, caring only about policies that benefit their constituencies and thus that enhance their prospects of reelection, are inclined to support some form of seniority.

McKelvey and Riezman (1992) restrict attention to a binary notion of seniority in which legislators are either senior if they have been reelected at least once, or are junior if they have just been elected for the first time. They recognize that, in reality, seniority is ordinal, i.e., legislators are individually ordered from most to least senior. Muthoo and Shepsle (2010) extend their classic model by allowing for the endogenous choice of the number of terms that a legislator must serve in order to gain seniority, but they still work under the binary restriction of seniority. They too recognize that many seniority systems are ordinal not categorical.

In this paper we extend these models in several ways. We consider a game that possesses two legislative stages – a rules-selection stage and a policy-determination stage. The first stage, occurring in a "procedural state of nature" (Diermeier, Prato, and Vlaicu 2012), determines relevant procedural parameters. The second stage, operating under the procedures just determined, is the place where actual policy decisions are made. Our first extension is to derive an equilibrium in which a seniority institution is endogenously proposed by a selected legislator and approved by the group in the rules-selection stage, instead of being exogenously given as in the McKelvey-Riezman model. In particular, among all the ordinal rules that assign agenda power to legislators in accordance with their seniority status, we find an endogenous rule that emerges as an equilibrium in the rules-selection stage. This rule involves three levels of seniority: some legislators, including the one proposing the assignment of agenda power, are said to be "senior" and are recognized to make policy proposals with high probability; some others are said to be "semi-senior" and are recognized with low probability; and the rest are "juniors" and are not recognized at all.

A second extension of McKelvey-Riezman looks at a third stage of the game in which voters in each constituency determine whether to renew their incumbent's contract or replace her with a new legislator. McKelvey and Riezman restrict voters to a very simple class of strategies, allowing them to condition their actions on a limited set of factors. Their result, therefore, is more a partial equilibrium result. We show that their conclusions are robust to permitting voters a broader repertoire of strategies.

Third, we move beyond simple majority rule, developing our results in a q-majority setting of supermajority rules.

In our initial development of the rules-selection stage, someone is selected to propose a set of procedures as a take-it-or-leave-it proposal. This takes the form of a distribution of recognition probabilities for the first round of the bargaining stage. (Subsequent bargaining rounds, if necessary, are governed by equal recognition probabilities.) If this rules proposal is rejected, then an exogenous reversion set of procedures is imposed (equal recognition probabilities in *every* round of the bargaining game). As a final extension, we allow the rules proposal to specify a distinct recognition probability distribution for each bargaining round.

In the next section we provide the theoretical context. In section 3 we derive our major results. In section 4 we justify the equilibrium we have selected from among the multiplicity of equilibria that exist. Various extensions are taken up in section 5, followed by concluding remarks and additional comparisons to existing literature. All proofs of results are in an Appendix which follows.

2 Theoretical Framework

Consider an infinite horizon dynamic game played between a fixed set of N voters, one per district, and a set of legislators. We assume the number of districts is odd. An arbitrary period is denoted by t. Let Γ^t be the game played in period t. This period game is played by $2N$ agents: the N voters and N legislators. Legislators are strictly ordered by seniority. Seniority is defined as the number of period games that a legislator has already played, with ties broken randomly to create a strict order. Let N_t denote the set of legislators who serve in period t.[3]

The period t game has 3 stages, which we now describe:

1. Rules Stage

This stage contains three rounds. In the first round, Nature selects a legislator according to an exogenously given probability distribution[4]. Let $l(t)$ be the selected legislator. In the second round of this stage, $l(t)$ proposes an institutional arrangement, $a_t(i)$, in effect a recognition rule indicating the probability of each legislator i being recognized to make a proposal in the first round of the bargaining stage (see below). Formally, $a_t : [1, ..., N] \longrightarrow [0, 1]$ is a function such that $\sum_{i=1}^{N} a_t(i) = 1$. In the third round, each legislator votes either in favor of proposal a_t, or against it. If

[3] McKelvey and Riezman assume that the set of N legislators is fixed forever: $N_t = N$ for all t. If a legislator is not reelected, she goes to the assembly anyway, only she loses her seniority and pays a fine. Voters cannot get rid of their incumbent; they can only demote and fine her. We prefer to assume that voters can replace their representative. Note, therefore, that the cardinality of N_t is always N, but its composition may vary with t.

[4] We assume that this probability distribution either does not depend on seniority, or it is weakly increasing in seniority. If, strangely, the probability of being selected to make a rules proposal were decreasing in seniority, voters in a district may prefer to defeat their incumbent in some variants of our model. To be precise, we allow the possibility that a more junior legislator is recognized to make the rule proposal with higher probability than a senior, but it must be that the junior is recognized by virtue of her identity as coming from her specific district, and not by virtue of her seniority. For example, it can be that the legislator from district i is recognized in period $t + 1$ for any t that is a multiple of N, whether i is senior or junior at that time. This is the case of recognition rotating among the districts. Recognition of the most junior legislator, by virtue of being junior, is not permitted.

a simple majority of legislators vote in favor, the outcome of this round is recognition rule a_t.[5] Otherwise, the outcome is the reversion rule $a_t = \bar{a}$ which does not make seniority distinctions and recognizes each legislator with equal probability in the bargaining stage, that is, $\bar{a}(i) = \frac{1}{N}$ for each legislator i.[6]

2. Bargaining Stage

Legislators engage in Baron-Ferejohn (1989) style legislative bargaining in which a unit of wealth is divided. This stage has infinitely many rounds. For each legislator i, the probability that i is recognized to make a policy proposal in the first round $\rho = 1$ of bargaining is $a_t(i)$; the probability that i is recognized to make a policy proposal in round $\rho > 1$ (if bargaining reaches round ρ) is $\frac{1}{N}$ for any i. That is, we assume that the recognition rule approved at the rules stage can provide only a transitory advantage to some legislators in the bargaining stage, an advantage lasting for only one round of bargaining. If that first round of bargaining leads to failure, we assume all legislators are recognized to make policy proposals with equal probability in any subsequent round.

A policy proposal is a partition of the unit of wealth among the N legislators. Observing the proposal, legislators vote it up or down by simple majority rule. If a proposal is accepted in round ρ, the bargaining stage ends. If not, the stage moves to round $\rho + 1$. We assume there is discounting at the rate δ starting at the third

[5] Our results extend to supermajority acceptance rules as we show in section 4.1.

[6] In section 4.2 we extend the analysis to multi-round rules-recognition procedures that do not impose equal recognition probabilities after one round of bargaining failure.

round, so the total prize for each legislator is discounted by $\delta^{\rho-2}$ if the proposal is accepted in round $\rho > 2$.[7]

3. Election Stage

The voter in each district chooses whether to reelect her representative, or else to elect a new representative from an infinite pool of identical politicians. If the voter chooses a new politician, the new representative enters the assembly at the lowest level of seniority. Incumbents who are not reelected exit the game.

At the end of the election stage, the period ends, each legislator (reelected or not) keeps a fraction λ of the prize obtained by his district, and the voter in the district obtains $1 - \lambda$. The game advances to the next period, with discount $\pi \in (0, 1)$. The game Γ consists of the infinite sequence of period games Γ^t.

For each period t, let $\tau \in \{1, 2, 3\}$ denote a stage within the period, and let $\rho \in \{1, 2, 3, ...\}$ denote a round within a stage. A history $h(t, \tau, \rho)$ contains all the information about the actions played by Nature and all players in all periods through to $t - 1$, in all stages of period t through stage $\tau - 1$, and in all rounds of stage τ in period t through round $\rho - 1$. Given $h(t, \tau, \rho)$, let $h(t, \tau, \rho)|_{=t}$ denote the continuation history of play starting at the first stage of period t. Let H be the set of all histories.

We define a state variable θ_t, which is the strict order of seniority of all legislators, where $\theta_t^i = k$ means that legislator from district i is the $k - th$ most senior legislator

[7] It simplifies the expressions, without altering the intuition, to allow one round of bargaining to go through without discounting.

in period t. Let $\theta_t^i(h(t,1,1))$ be the seniority of the legislator from district i in period t, as a function of the history of play up to the end of period $t-1$.

A behavioral strategy s_i for an agent i is a sequence of mappings, one for each information set in which player i can be called upon to make a move. Each of these mappings is a function from the history of play at this information set to the set of feasible actions of agent i. We have already specified the set of feasible actions at each information set: Legislator $l(t)$ chooses a probability distribution (a recognition rule); all legislators make a binary choice approving or rejecting this probability distribution; then legislators engage in the standard Baron-Ferejohn bargaining game; finally voters make a binary choice.

We are interested in subgame perfect equilibria of the game Γ that are stationary as defined by McKelvey and Riezman (1992), so that each period game Γ^t is solved independently of the history of play in previous periods. We call this Stationarity I. That is, we seek equilibria made up of behavioral strategies that describe how to play each period game conditioning only on information available within the period game, as if at the end of each period all history were reduced to the state variable of seniority status and all other details of past play were forgotten. Furthermore, we are interested in the standard equilibrium strategies of the bargaining game that are stationary in the sense defined by Baron-Ferejohn; without this additional stationarity, the solution to the bargaining game is indeterminate, as almost any outcome could then be sustained in equilibrium (see Baron-Ferejohn). We call this Stationarity II.

Definition 1 *Given any player j, a strategy s_j satisfies stationarity I if for any period t, stage τ and round ρ, and for any two histories $h(t,\tau,\rho)$ and $h'(t,\tau,\rho)$ such that $\theta_t(h(t,1,1)) = \theta_t(h'(t,1,1))$ and $h(t,\tau,\rho)|_{=t} = h'(t,\tau,\rho)|_{=t}$, then $s_j(h(t,\tau,\rho)) = s_j(h'(t,\tau,\rho))$.*

Given any legislator i, a strategy s_i satisfies stationarity II if for any period t, any rounds ρ and ρ' and any history $(h(t,\tau,\max\{\rho,\rho'\}))$, $s_i(h(t,2,\rho)) = s_i(h(t,2,\rho'))$.

An equilibrium is stationary if every strategy satisfies stationarity I and every legislator's strategy satisfies stationarity II.

The intuition of stationarity I, borrowed from McKelvey and Riezman, is that if two histories lead to the same seniority ranking at the beginning of the period, then in a stationary strategy an agent does not dwell on details of previous play in other periods to decide how to play in the current period. Stationarity II is the standard stationarity in Baron-Ferejohn bargaining, adapted to the notation of our framework. It implies that looking only at the bargaining stage in a given period, given two structurally equivalent subgames (two subgames with identical continuation extended trees), agents play the same strategies in the two subgames; that is, if probabilities of recognition do not vary, agents play the same way in the subgame that starts after round 1 of bargaining, or after round $k > 1$ of bargaining.

As in most voting games, there exist many implausible equilibria in which all legislators vote in favor of any proposal: since no legislator is pivotal in this case, legislators are indifferent about the votes they cast. In a one-shot game, such equilibria are discarded assuming that agents never play weakly dominated strategies, and always vote as if they were pivotal. The analogous argument for dynamic games is to refine the set of equilibria by requiring each voter to eliminate any strategy that is weakly dominated in a given voting stage game considered in isolation while treating the equilibrium strategies of all players as fixed for all future stages and periods. These are "stage undominated strategies" (Baron and Kalai 1993). Eliminating strategies that violate stage weak dominance is equivalent to requiring each agent to vote as if she were pivotal in every subgame in which she is involved (Duggan and Fey 2006). We use this equivalence to define the refinement.

Definition 2 *An equilibrium strategy profile s satisfies stage weak dominance if for any period t, any legislator i and any history $h(t,\tau,\rho)$ such that a (rule or bargaining) proposal p is put to a vote, given s legislator i votes for p if the continuation value for i of passing p is strictly positive and votes against p if the continuation value for i of passing p is strictly negative.*

Stage weak dominance merely rules out equilibria in which voters vote against their strict interest because their votes do not count. Our solution concept is subgame perfect, stationary, stage weakly undominated Nash equilibrium. We refer to these equilibria merely as "equilibria."

3 Results

Let N_t^{-l} denote the set of legislators in period t excluding the rules proposer $l(t)$. In our first main result we show that a seniority-based recognition rule is an equilibrium proposal at the rules-selection stage. The rules proposer, $l(t)$, will assign most of the recognition probability to herself, but will distribute the remaining probability among senior legislators.

Proposition 1 *There exists an equilibrium in which*

i) In each period t, legislator $l(t)$ proposes recognition rule $a_t^*(i)$ such that $a_t^*(l(t)) = \frac{N+1}{2N}$ and $a_t^*(i) = \frac{1}{N}$ for any i among the $\frac{N-1}{2}$ most senior legislators in N_t^{-l}.

ii) In each period t, recognition rule $a_t^*(i)$ is approved by the assembly.

iii) In each period t, all legislators are reelected.

Other equilibria exist in which the rules proposer forms a minimal winning coalition with a different majority. We focus in Proposition 1 on this particular seniority equilibrium first because it is arguably the simplest equilibrium: it possesses a focal quality, with $l(t)$ choosing from among her most senior colleagues as coalition partners and endowing only them with the possibility of recognition in the bargaining stage. We elaborate on equilibrium selection more extensively in the next section, providing a political rationale. It should be noted about this equilibrium, that although districts with incumbents at least as senior as the median legislator have strict incentives to reelect them, those with an incumbent less senior than the median have only weak incentives to do so.[8]

We now provide another equilibrium in which, though slightly more complicated and less focal, has the property that each district has a *strict* incentive to reelect its incumbent legislator.

Example 1 *Consider an alternative equilibrium: If $l(t)$ is less senior than the median, she offers recognition probability $\frac{1}{N}$ to $\frac{N-1}{2}$ legislators randomly chosen from the set of legislators with greater seniority than $l(t)$; if $l(t)$ is more senior than the median, she offers recognition probability $\frac{1}{N}$ to the $\frac{N-1}{2}$ most senior legislators in N_t^{-l}. In this equilibrium, if any legislator i has a positive probability of being recognized to be the rules proposer, expected payoffs are strictly increasing in seniority for legislators less senior than the median. This follows because the prospect of having positive recognition probability in the bargaining stage depends upon whether a legislator is more senior than $l(t)$; the greater a legislator's seniority, the more likely she is more senior than $l(t)$. Thus, voters in every district have strict incentives to reelect their incumbent.*

Our model is richer than McKelvey and Riezman's in that our formulation does not restrict the strategies available to the players. In the McKelvey-Riezman model,

[8] As long as the minimal winning majority of legislators with positive probability of recognition does not include the most junior legislator, a voter in a district excluded from the cake is at worst indifferent between keeping her incumbent legislator or electing a new junior legislator who obtains nothing. Throughout we assume that if a pivotal voter in a district is indifferent between reelecting her incumbent or replacing him she reelects – hence the weak incentive referred to in the text.

citizens can only condition their vote at the electoral stage on the policy outcome in the current period.[9] Voters cannot condition on past history, on their legislator's seniority, on their legislator's vote in the assembly, or on any other action. Strategies that make votes contingent on these factors are not admissible in McKelvey and Riezman's theory. Given these restrictions, we interpret their results as partial equilibrium results: the equilibria that they identify are not shown to be robust against all possible deviations, but only against the very small set of deviations that are deemed admissible. Voters are forced to use very simple reelection strategies, without it being established that these reelection strategies are best responses among the set of all conceivable strategies.

We relax this restriction, allowing voters to condition on the whole history of the game. In this new framework, new equilibria arise that could not be imagined or constructed in the McKelvey-Riezman approach – the strategies comprising them are either impermissible or do not exist. Example 2 below illustrates that new, qualitatively different, equilibria emerge if we allow voters to use more sophisticated reelection strategies that condition on the actions of all legislators in the assembly. This, in turn, raises issues of equilibrium selection, something we elaborate on in the next section. Nevertheless, we show in our more general framework that the simple strategies identified by McKelvey and Riezman (legislators institutionalize seniority advantage and voters always reelect their incumbent) are robust against *all* possible

[9] See the definition of the voter game at McKelvey-Riezman (1992), p. 956.

deviations; thus, they constitute true best responses.

Example 2 *Suppose there are 3 districts, and the probability of recognition to make rules proposals is 1/2 for the most senior legislator and 1/2 for the second most senior. The standard equilibrium (Proposition 1) has the rules proposer proposing 2/3 probability for herself and 1/3 probability for the other senior legislator. In the subsequent bargaining game, the policy proposer – one of the two seniors – gets 2/3 of the cake and, in expectation, the other two legislators get 1/6 of the cake (ex post one gets 1/3 the other 0). Expected payoffs are 5/12 for the two senior legislators and 2/12 for the junior one. Everyone is reelected, the seniors strictly, the junior just weakly in the sense that the voters of the district are indifferent between reelecting and replacing.*

Now, however, suppose voters use a strategy that reelects their legislator if she is not the least senior, or if she is the least senior and the recognition rule for bargaining gives her exactly 1/6 of the probability of recognition. The junior legislator then is not reelected under an equal recognition rule, for example, because under this rule her probability of recognition is 1/3≠1/6. So she only votes in favor of a rule that grants exactly 1/6 recognition probability for herself, and against all other rules (since any other rule would mean her electoral defeat). That makes her a cheaper coalition partner at the rules stage. Thus, the sequence of stages plays out as follows:

- *With probability 1/2 a senior is recognized to propose a rule.*

• *He proposes 5/6 recognition probability for himself and 1/6 for the junior.*

• *In the bargaining game, with probability 5/6 he is recognized and proposes 2/3 to himself and 1/3 to one of the others; with probability 1/6 the junior is recognized and proposes 2/3 for herself and 1/3 to one of the others.*

•*Junior and rules proposer vote in favor of this rule, so it is approved.*

The expected payoff for each senior is $(1/2)\{(5/6)(2/3)+(1/6)(1/2)(1/3)\}+(1/2)\{(1/6)\} = \frac{3}{8}$. *The expected payoff for the junior is* $\frac{2}{8}$. *Everyone is reelected.*[10]

Equilibria of this kind, in which voters use sophisticated reelection rules, complicate their incumbent's optimization problem. An incumbent's objective no longer reduces to maximizing the expected share of the pie obtained in a given period. Equilibria with sophisticated reelection strategies do indeed exist, as the example just given illustrates. But we do not find them very plausible – in terms of the ability of a constituency either to commit to so exotic a strategy or to communicate this strategy to its legislator even if it could commit. If, instead, we select equilibria in which voters do not use such sophisticated rules, then legislators solve the legislative stages (rules stage and bargaining stage) myopically to maximize their expected share of the pie, and it then follows that the expected payoff must be $\frac{1}{N}$ for $\frac{N-1}{2}$ legislators (any $\frac{N-1}{2}$ legislators other than the most junior, who must obtain zero to guarantee that voters want to reelect their incumbent), and the rest for the rules proposer.

[10] *Note that the expected payoff to the district of the junior legislator improves with the novel strategy described above (expected payoff of 2/8 rather than 2/12).*

Proposition 1 and the examples thus far have allowed *any* recognition rule to be proposed. The next result constrains the set of available rules. We now assume that any proposed recognition rule governing the bargaining stage must satisfy a weak monotonicity constraint based on seniority. For any two legislators i and j with $\theta_t^i \leq \theta_t^j$, any admissible rule must assign recognition probabilities in the first round of bargaining so that legislators with greater seniority (lower θ_t^i) are at least as likely to be recognized as less senior legislators.

Definition 3 *A recognition rule a_t satisfies the Weak Seniority condition if $\theta_t^i \leq \theta_t^j$ implies $a_t(i) \geq a_t(j)$ for any legislators i and j.*

That is, we exclude those recognition rules that give a less senior legislator a strictly greater likelihood of being recognized than some of his or her more senior colleagues.

Proposition 2 *Assume any recognition rule governing the bargaining stage must satisfy the Weak Seniority condition. There exists an equilibrium in which*

i) In each period t, legislator $l(t)$ proposes recognition rule $a_t^*(i)$ such that $a_t^*(i) = \frac{1-\frac{1}{N}\max\{0,\frac{N+1}{2}-\theta_t^{l(t)}\}}{\theta_t^{l(t)}}$ for any legislator i such that $\theta_t^i \leq \theta_t^{l(t)}$, $a_t^*(i) = \frac{1}{N}$ for any i such that $\theta_t^i \in \left(\theta_t^{l(t)}, \frac{N+1}{2}\right]$ and $a_t^*(1,i) = 0$ for any i such that $\theta_t^i > \max\left\{\theta_t^{l(t)}, \frac{N+1}{2}\right\}$.

ii) In each period t, recognition rule $a_t^*(i)$ is approved by the assembly.

iii) In each period t, all legislators are reelected.

Furthermore, if all legislators face an equal probability of recognition at the rules stage, the expected payoff for each district in this equilibrium is strictly increasing in the seniority of the district's legislator.

To see what is happening, suppose the selected rules proposer were the most senior legislator – $\theta_t^{l(t)} = 1$. Then according to (i) above, she would give $\frac{1}{N}$ of recognition probability to the $\frac{N-1}{2}$ next-most-senior legislators, retain the residual for herself, and give zero to everyone else. This assignment is consistent with the Weak Seniority condition. If $\theta_t^{l(t)} = \frac{N+1}{2}$, i.e., $l(t)$ were the legislator with median seniority, then according to (i), she and each of her $\frac{N-1}{2}$ more senior colleagues would have a recognition probability of $\frac{2}{N+1}$ and zero for all others. If she were more senior than the median seniority, but not the most senior, she would give the number of legislators with less seniority than her but necessary to make up a majority $\frac{1}{N}$ of recognition probability, divide the remaining residual evenly among those, including her, with at least as much seniority as her, and give nothing to anyone else. Finally, if she had less seniority than the median, then she would allocate recognition probability evenly among all those, including her, with at least as much seniority as her and zero to all those below her.

All of these recognition-probability allocations are driven by the maximizing behavior of $l(t)$. Subject to the Weak Seniority condition (in all these cases recognition probability must be weakly monotonic in θ_t^i) she gives: (i) all those with at least as much seniority as her – including herself – the highest recognition probability; (ii) lower recognition probability to those below her but necessary to make up a majority, and (iii) zero to those unnecessary to a majority and not of higher seniority than her. This means that any vector θ_t maps into three seniority classes – seniors, semi-seniors, and juniors (the first two combine when $l(t)$ has less seniority than the median).

Finally, we underscore the finding that if the probability of selection to make a rules proposal in the "procedural state of nature" is the same for all legislators, then it follows that expected payoff is strictly monotonic in θ_t^i – the more experienced a legislator, the greater his or her expected payoff.

4 Justifying Equilibrium Selection

In our model there are a multiplicity of equilibria and thus an equilibrium-selection issue. We have refined away equilibria involving stage-dominated or non-stationary strategies on general principle. Other equilibria, as in Example 2, are implausible because of extraordinary commitment, coordination, and/or communications requirements. Nevertheless, among remaining equilibria we offer some compelling positive reasons to select the particular seniority equilibrium we have identified, in which recognition rules assign probability of recognition only to senior legislators and to the proposer of the rule. We argue that this equilibrium maximizes seniority advantage, and we provide two intuitions for why rational legislators want to maximize a seniority advantage.[11]

Somewhat casually, by "maximize a seniority advantage" we mean that those legislators we identify as senior or semi-senior – a majority in all – will seek arrangements which, ex ante, maximize the present discounted value of their payoff minus that of the most junior legislator. Because both constituency payoffs and legislator payoffs are monotonic in the share of wealth secured in the equilibrium outcome (no moral hazard), constituents of seniors and semi-seniors are provided the sharpest incentives to reelect their incumbent when this difference is maximized. Replacing their senior incumbent with a newly minted legislator when the seniority advantage is maximal imposes the largest costs on the constituency.

More formally, let $\phi_t^i(a^*)$ be the discounted value of the expected stream of future payoffs for the constituents of the legislator with seniority i evaluated at the beginning of period t, given equilibrium play with the sequence of recognition rules a^*. Then the seniority advantage of legislator i is $\phi_t^i(a^*) - \phi_t^N(a^*)$. The average seniority advantage, which takes into account the utility of every legislator, including junior ones, is $\sum_{i=1}^N \frac{\phi_t^i(a^*)}{N} - \phi_t^N(a^*) = \frac{1}{1-\pi} - \phi_t^N(a^*)$, where π is the discount factor across periods. The average or aggregate seniority advantage is maximized if $\phi_t^N(a^*)$ is minimized. We argue that if we introduce exogenous turnover, the equilibria that

[11] We should emphasize that our more general theoretical framework allows us to move beyond the binary option (whether to have a seniority system or not) in McKelvey and Riezman. We claim that, instead of an either-or choice, legislators will select a seniority system that provides them *maximal* value, and we provide reasons why. This issue cannot be addressed in the McKelvey-Riezman framework that permits only the binary options of having a seniority system or not having one.

we select are precisely those that maximize aggregate seniority advantage.

The first reason to seek to maximize seniority advantage is to increase the probability of reelection. Imagine the prospects of an incumbent seeking reelection are affected by stochastic factors. For a variety of reasons, explicable and inexplicable, a constituency may be "moody," possessing an anti-incumbent sentiment in period t, for example. Perhaps the most compelling stochastic factor is the nature of the period t challenger whose identity is not learned by the incumbent until the last stage of the game. The challenger selected for period t's election may constitute an unlucky draw for the incumbent – that is, he will possess electorally advantageous valence characteristics (youth, telegenic looks, reputation for competence or honesty).[12] If the stochastic effects – electoral mood, challenger valence – are large and unfavorable to the incumbent, then she may lose the election. Ex ante, to produce a cushion against adverse draws, incumbents will want to "stack the procedural deck" maximally in their favor in the bargaining game. In particular, in making the payoffs to junior legislators as small as possible in the bargaining game, the respective constituencies of all legislators will be maximally punished if they replace an incumbent. For sure in the seniority equilibrium we identify, that constituency will get a minimal expected

[12] Valence characteristics are those on which there is a constituency consensus that more is preferred to less (or less is preferred to more), in contrast to positional characteristics (left-wing versus right-wing issue positions) on which a constituency may be divided. Valence characteristics also tend to be fixed features of a candidate (e.g., youth) rather than endogenously chosen ones (e.g., issue positions) – what Spence (1974) called *indexes* rather than *signals*. The literature in political science on the importance of valence in electoral models and voting behavior is large. A sampling includes Ansolabehere and Snyder (2000), Aragones and Palfrey (2004), Enelow and Hinich (1982), Groseclose (2001), and Stone and Simas (2010).

payoff in the period $t+1$ bargaining. Moreover, by zeroing out the probability of recognition of *all* juniors, it means a constituency that replaces its incumbent will be required to wait the maximal amount of time, given exogenous turnover, before their newly minted legislator rises sufficiently on the seniority ladder to qualify for positive probability of recognition and a greater than minimal expectation of payments. In sum, an institutional arrangement that maximizes seniority advantage means that the stream of future payoffs for the constituency of a newly elected legislator remains very low for as many periods as possible, which is achieved by concentrating all the probability of recognition on senior legislators. This maximally deters constituencies from defeating incumbents.

There is a second intuition that rationalizes the selection of the equilibrium we identify. Suppose, in contrast to the first intuition, that there is no stochastic element in voter utility functions. Nevertheless, suppose there is stochastic turnover. With some (possibly small) probability, an incumbent legislator "dies" (e.g., death, elevation to high executive office, selection for a remunerative private-sector position, criminal conviction). Let us suppose further that the share of the bargaining outcome for his constituency that the legislator keeps, λ, is endogenously chosen by the incumbent.[13] In setting up seniority institutions in the "procedural state of nature," a senior legislator may extract (all of) the generated surplus for herself. Maximizing seniority advantage through procedural arrangements thus facilitates "corruption"

[13] In our results, and those of McKelvey-Riezman, λ is fixed exogenously.

(personal venality, directing funds or tax breaks or insider information to friends and family, etc.). Stochastic dying provides juniors the opportunity to move up on the seniority ladder while, at the same time, allows seniors to extract rents that are a function of the difference between senior and junior bargaining outcomes. Legislators will want to maximize this difference.

5 Generalizations

5.1 Supermajority Voting Rules

We have so far assumed that rules and policy proposals are selected by simple majority rule. However, decisions to adopt or change rules are often subject to supermajority requirements (Eraslan 2002, Polborn and Messner 2004, Barberà and Jackson 2004). In the U.S. House of Representatives, for example, the standing rules are adopted at the beginning of a new Congress by simple majority rule. If, however, during the course of considering a specific piece of legislation, proponents wish to cut through various procedural thickets dictated by the rules and move directly to a vote – that is, to "suspend the rules" in order to pass the particular bill – then a two-thirds majority is required. The U.S. Senate is nominally a simple majority rule legislative chamber – it only takes a simple majority to pass a bill or confirm a presidential nominee.[14] However, in order to proceed to a vote, debate must be brought to a close (cloture), and this requires the support of sixty out of the hundred members. Even more restrictive, a motion to proceed to vote on a *rules change* requires two-thirds of those present and voting – that is, 67 votes when all senators participate.

Suppose $q \in \left[\frac{N+3}{2}, N-1\right]$ votes are needed to approve $a_t(i)$ at the rules stage; otherwise, the equal recognition rule $\bar{a}$ is the default rule in the subsequent bargaining stage. The supermajority requirement forces the rules proposer to grant recognition probability to more agents.

Proposition 3 *Assume recognition rules are approved if at least* $q \in \left[\frac{N+3}{2}, N-1\right]$ *legislators vote in favor, otherwise policy bargaining occurs under an equal recognition rule. There exists an equilibrium in which*

i) In each period t, *legislator* $l(t)$ *proposes recognition rule* $a_t^*(i)$ *such that* $a_t^*(i) = \frac{2q-N-1}{(q-2)N+q} \equiv x$ *for any* i *in the subset of* $(q-1)$ *most senior legislators within* N_t^{-l} *and such that* $a_t^*(l(t)) = 1-(q-1)x$.

ii) In each period t, *recognition rule* $a_t^*(i)$ *is approved by the assembly.*

iii) In each period t, *all legislators are reelected.*

If we assume that feasible rules proposals are restricted to those that satisfy the Weak Seniority condition, then the following result obtains:

[14]Some matters, however, are explicitly noted in the Constitution as requiring a special majority. To expel a member, to convict an impeached executive or judicial officer, to ratify a treaty, or to override a presidential veto, for example, it is necessary for the concurrence of two-thirds of those present and voting.

Proposition 4 *Assume a recognition rule for the bargaining stage must satisfy the Weak Seniority condition, and is approved only if* $q \in [\frac{N+3}{2}, N-1]$ *legislators vote in favor. There exists an equilibrium in which*

i) In each period t*, legislator* $l(t)$ *proposes recognition rule* $a_t^*(i)$ *such that* $a_t^*(i) = \frac{1-\frac{1}{N}\max\{0,q-\theta_t^{l(t)}\}}{\theta_t^{l(t)}}$ *for any legislator* i *such that* $\theta_t^i \leq \theta_t^{l(t)}$, $a_t^*(i) = \frac{1}{N}$ *for any* i *such that* $\theta_t^i \in \left(\theta_t^{l(t)}, q\right]$ *and* $a_t^*(i) = 0$ *for any* i *such that* $\theta_t^i > \max\left\{\theta_t^{l(t)}, q\right\}$.

ii) In each period t*, recognition rule* $a_t^*(i)$ *is approved by the assembly.*

iii) In each period t*, all legislators are reelected.*

Furthermore, if all legislators face an equal probability of recognition at the rules stage, the expected payoff for each district in this equilibrium is strictly increasing in the seniority of the district's legislator.

Comparing the simple majority results of Propositions 1 and 2 to their q-majority counterparts, Propositions 3 and 4, it is evident that $l(t)$'s payoff declines with q: he or she must distribute recognition probability to a greater number of colleagues. Given two rules q and q' such that $q < q'$, in expectation (before the uncertainty over the identity of $l(t)$ is resolved), any legislator at least as senior as q and any legislator less senior than q' is strictly better off with rule q than rule q'.

5.2 Multi-Round Recognition Rules

The institutional arrangement $a_t(i)$ that we have considered so far involves the selection in the period t rules stage of a distribution of recognition probabilities for the first round of that period's bargaining stage. If the bargaining stage moves on to a second (or subsequent) round, we have assumed that all legislators are recognized with equal probability $\frac{1}{N}$ to make a proposal. That is, our results thus far have assumed that there is an exogenously imposed default distribution of recognition probabilities (equal recognition) for each bargaining round beyond the first if it is required. While this simplified the analysis, we now relax this assumption. The legislator selected to propose a distribution of recognition probabilities at the rules stage now is empowered to propose a distribution for each possible round of bargaining, not just the first. There now is no exogenously imposed default distribution of recognition probabilities in rounds after the first.

Formally, once selected in the first round of the rules-selection stage, $l(t)$ proposes an institutional arrangement $a_t(\rho, i)$, which consists of a proposal for recognition rules in the bargaining stage. A recognition rule indicates the probability of each legislator i being recognized to make a proposal in each round of bargaining; thus, the recognition rule contains a countable infinity of probability distributions over the N legislators, one distribution for each of the countable infinity of possible bargaining rounds. Specifically, $a_t : \mathbb{N} \times [1, ..., N] \longrightarrow [0, 1]$ is a function such that $\sum_{i=1}^{N} a_t(\rho, i) = 1$

for each positive integer ρ.[15]

As in the benchmark model, legislators vote on $a_t(\rho, i)$ and approve it or reject it by simple majority, and if they reject it, the outcome is the status quo rule $a_t = \bar{a}$ that assigns equal probability of recognition in all rounds of bargaining. Thus, there is still an exogenously imposed default if a rules proposal is rejected, but in the more general case considered here, the default option gives an equal probability of recognition in each required round of bargaining in period t; that is, $\bar{a}$ is a vector of probability distributions with each entry in each distribution equal to $\frac{1}{N}$.

With endogenous multi-round recognition rules, it is technically convenient to allow two rounds of bargaining after the first (and not just one) to proceed without discounting. It simplifies the expressions without altering the intuition.

The definition of Stationarity II must also be adjusted. Stationarity II requires that legislators play two structurally equivalent bargaining games in the same manner. With heterogeneous probability distributions in various rounds, two bargaining games are only structurally equivalent if the probability distributions in subsequent rounds are the same in the two games. Only in this case must agents play the same behavioral

[15] To keep the model tractable, we require that $a_t(\rho, i)$ be such that there exists some K such that $a_t(\rho, i) = a_t(K, i)$ for any $\rho > K$.

Eraslan (2002) shows that under stationarity II, there is a unique equilibrium of the bargaining game, even if agents have asymmetric probabilities of recognition, as long as these probabilities are constant. Our analysis allows asymmetric probabilities as in Eraslan, but does not require them to be constant for the first K rounds. Breitmoser (2011) shows that if the identity of the proposer is deterministic for a finite number of rounds, and it varies across rounds, there are multiple stationarity equilibria. Our analysis has the proposer chosen stochastically.

strategies in the two games.[16]

If multi-round recognition rules are feasible, the rules proposer $l(t)$ proposes a rule that distributes all the probability of recognition and all the expected payoff to a minimal winning majority of agents.

Proposition 5 *There exists an equilibrium in which*

i) In each period t, legislator $l(t)$ proposes recognition rule $a_t^*(\rho, i)$ such that for any bargaining round ρ, $a_t^*(\rho, l(t)) = \frac{N+1}{2N}$, $a_t^*(\rho, i) = \frac{1}{N}$ for any i in the set of the $\frac{N-1}{2}$ most senior legislators not including $l(t)$, and $a_t^*(\rho, i) = 0$ for any other legislator i.

ii) In each period t, recognition rule $a_t^*(\rho, i)$ is approved by the assembly.

iii) In each period t, all legislators are reelected.

This result is not exactly about seniority, but rather, about the endogenous emergence of asymmetric recognition rules that grant some legislators a greater probability of being recognized to make a proposal. There are other equilibria in which the legislator assigned to choose the recognition rules grants positive probability of recognition to $\frac{N-1}{2}$ other legislators, and, as long as in expectation seniors are at least as likely to get included in these coalitions as the most junior legislator, voters have an incentive to reelect their incumbents.

Suppose now that legislator $l(t)$ faces a more constrained set of options, so that the recognition rules must satisfy the Weak Seniority condition. This limits the ability of

[16]Formally: Given any legislator i, a strategy s_i satisfies stationarity II if for any period t, any rounds ρ and ρ' and any history $(h(t, \tau, \max\{\rho, \rho'\}))$ such that $a_t(\rho + k, m) = a_t(\rho' + k, m)$ for any legislator m and any $k \in \mathbb{N}$, $s_i(h(t, 2, \rho)) = s_i(h(t, 2, \rho'))$.

a junior legislator endowed with the power to propose the recognition rule to expect an extraordinarily disproportionate share of the unit of wealth that is subject to legislative bargaining (as she could in the unconstrained circumstance of the previous proposition). However, this legislator is still able to obtain a greater share of resources than any other.

Proposition 6 *Assume a_t must satisfy the Weak Seniority condition. There exists an equilibrium in which for some $z > x > \frac{1}{N}$,*

i) In each period t in which $\theta_t^l \leq \frac{N+1}{2}$, $l(t)$ proposes recognition rule $a_t^*(\rho, i)$ such that $a_t^*(1, i) = z$ for any legislator i with seniority ranking up to and including θ_t^l, $a_t^*(1, i) = x$ for any legislator i with seniority ranking from $\theta_t^l + 1$ to $\frac{N+1}{2}$, $a_t^*(1, i) = 0$ for any other legislator i.

ii) In each period t in which $\theta_t^l \geq \frac{N+3}{2}$, $l(t)$ proposes recognition rule $a_t^*(\rho, i)$ such that $a_t^*(1, i) = \frac{1}{\theta_t^l}$ for any legislator i with seniority ranking up to θ_t^l, $a_t^*(1, i) = 0$ for any other legislator i.

iii) In each period t, recognition rule $a_t^*(\rho, i)$ is approved by the assembly.

iv) In each period t, all legislators are reelected.

Furthermore, if all legislators face an equal probability of recognition at the rules stage, the expected payoff for each district in this equilibrium is strictly increasing in the seniority of the district's legislator.

The equilibrium recognition probabilities institutes a three-tiered seniority structure, with legislators with seniority up to and including that of $l(t)$ – seniors – recognized with a high probability z, semi-senior legislators with seniority rankings from $l(t)+1$ to the median recognized with probability x which is greater than $\frac{1}{N}$ but smaller than z, and juniors not recognized at all. In the proof of the proposition we find the exact value of z and x. As an example, if $N = 43$ and $l(t)$ is the third most senior legislator, the three seniors are recognized with probability .177 each, the next nineteen semi-seniors with probability .025 each (greater than $\frac{1}{N}$), and the twenty-one juniors are never recognized.

We have considered two restrictions on recognition rules: equal probability of recognition after the first round, and the Weak Seniority condition. We illustrate the relevance of each restriction on the ability of the proposer to extract additional surplus by the following numerical example. In the first two rows, endogeneity of recognition probabilities is restricted to one round; in rows two and four, the Weak Seniority condition is imposed. The top of the table (rows 1-4) reports the probability of recognition in the first round of bargaining, and the bottom of the table (rows 5-8) the expected payoff in the equilibria described in propositions 1-6.

Example 3 *Suppose $N = 15$, all decisions are taken by simple majority rule, and legislator 5 is selected as rules proposer. Thus, legislators $1-5$ are senior; $6-8$ are semi-senior; and $9-15$ are junior.*

		Prob. of 1st round recognition			
	Cases	*Proposer*	*Senior*	*Semi-Senior*	*Junior*
1	*1-Round (Prop 1)*	*0.533*	*0.067*	*0.067*	*0*
2	*1-Round, Weak Sen. (Prop 2)*	*0.160*	*0.160*	*0.067*	*0*
3	*Multi-round (Prop 5)*	*0.533*	*0.067*	*0.067*	*0*
4	*Multi-round, Weak Sen. (Prop 6)*	*0.159*	*0.159*	*0.068*	*0*

		Expected payoff			
	Cases	*Proposer*	*Senior*	*Semi-Senior*	*Junior*
5	*1-Round (Prop 1)*	*0.300*	*0.067*	*0.067*	*0.033*
6	*1-Round, Weak Sen. (Prop 2)*	*0.113*	*0.113*	*0.067*	*0.033*
7	*Multi-round (Prop 5)*	*0.533*	*0.067*	*0.067*	*0*
8	*Multi-round, Weak Sen. (Prop 6)*	*0.235*	*0.141*	*0.067*	*0*

The takeaway points from the tables in Example 3 are the following. First, the completely unconstrained proposer who need not observe weak seniority and whose proposal contains recognition probability distributions for each round of bargaining obtains the highest payoff (rows 3 and 7). The introduction of constraints reduces the rules proposer's advantage, redistributing expected payoff to seniors under the Weak Seniority condition (rows 4 and 8) and to juniors when unequal recognition rules are restricted to one round (rows 1 and 5). When both constraints are imposed, the distribution of expected payoffs is more equal (rows 2 and 6).

6 Discussion

Formal research on the origin of institutional rules that favor seniority in legislatures begins with the seminal paper by McKelvey and Riezman (1992). They establish the endogenous choice of a legislative seniority system, one that gives agenda recognition advantage to senior legislators, as an equilibrium feature of Baron-Ferejohn (1989)-style bargaining games in which participants make organizational choices at a prior stage of the game. This formalizes an informal argument of Holcombe (1989). Muthoo and Shepsle (2012) generalize their results, allowing for the endogenous determination of the definition of who is senior and who is junior.

The political imperative that drives these organizational choices is an *incumbency advantage.* Rather than taking the bargaining game as one in which each legislator has a fixed likelihood of being recognized to make a proposal (as in Baron-Ferejohn), legislative incumbents use their control of the rules and other organizational features to assign recognition likelihoods *differentially* with the aim of inducing constituencies to reelect their incumbents. They do this by selecting rules that make it very disadvantageous for a constituency to replace its incumbent representative with a newly minted legislator. However, these papers take a restrictive view of the menu of organizational options available (and in this sense are partial equilibrium results). For McKelvey and Riezman, the choice is between having a seniority system or not (where seniority is defined exogenously). For Muthoo and Shepsle, the choice is between having no seniority system or of adopting one and endogenously determining who qualifies as senior. In both of these, once the class of seniors is established, their members share recognition probability in a pre-assigned way.

The results of the present paper allow for a more nuanced set of organizational options. We characterize the organizational choice as one of selecting the rule by which an agenda setter will be chosen. An individual is randomly chosen to propose such a rule – a vector of probabilities giving for each legislator the likelihood of being selected to make a first bargaining proposal or, more generally, a sequence of probability vectors for making a proposal in each of the rounds of bargaining required to arrive at a decision. This rule proposer is not constrained in the rule proposal he or she makes (as is the case in the papers we identified above). We establish in our first two propositions that a "seniority equilibrium" exists, one in which the randomly selected proposer and other senior legislators share agenda power. Among the equilibria that exist in our formulation, after excluding those that fail to satisfy either stationarity or stage undominance, we defend the seniority equilibrium we identify as the one that uniquely maximizes the aggregate advantage of incumbent legislators. We extend our results to supermajority voting rules and to recognition rules that allow for differential recognition probabilities in each of the required bargaining rounds. Finally, an example illustrates the skewness of payoffs of the equilibria we identify.

In this concluding discussion we note other research papers that identify agenda power and the selection of voting rules as key aspects of majoritarian decision making in legislative bodies. Early papers by Romer and Rosenthal (1978), Banks and Gasmi (1987), and Harrington (1990) (along with the aforementioned Holcombe (1989) and Baron and Ferejohn (1989)) make clear that agenda power affects equilibrium outcomes, showing that payoffs are skewed toward those possessing the power to propose. They identify specific institutional features of agenda power that reduce the skew. Harrington, for example, shows that the skew is monotonically decreasing in the voting majority needed to make final decisions. Baron and Ferejohn demonstrate that allowing amendments to the proposal also reduces the skew. But in each of these papers the agenda institutions are taken as given rather than chosen.

A number of more recent papers provide insights about specific features of agenda institutions. Breitmoser (2011) examines the effects of limiting positive recognition-probability assignments to a restricted set of agents (e.g., members of a legislative committee). In his model a proposal is made from one of $T \subset N$ members, with T known in advance. Any of the members of T may seek initial recognition or offer amendments to an existing proposal and, if more than one seeks recognition, they are ordered in terms of seniority to initiate or amend. With these facts known in advance, he shows that the first proposer's payoff is no longer unique for $T > 1$ (as is the case in the standard Baron-Ferejohn setup), but his or her expected payoff is higher than in the standard case. That is, when proposal power is restricted, prioritized, and these constraints are commonly known, the expected advantage to the proposer is less determinate than in, but underestimated by, the standard model. In a similar spirit

Fan, Ali, and Bernheim (2010) examine a dynamic Baron-Ferejohn bargaining setting in which there is an informational connection between Γ^t and Γ^{t+k} – viz., at time t the agents know the subset of agents who will have positive recognition probability at time $t+k$. In this circumstance they find highly skewed expected payoffs – indeed, skewed in the extreme. However, in both of these papers, as in the earlier papers, the details of the agenda-power arrangement are given exogenously. Thus, while various agenda institutions can exaggerate or diminish the skew in expected payoffs, we have no sense of whether such arrangements would ever have been chosen by the legislature in a "procedural state of nature." Indeed, our results suggest that before procedures are established at all, each agent has an expectation of δ/N; so they would reject any agenda arrangement at the rules proposal stage that yielded them a lower expectation.[17]

A different strand of literature focuses on the selection of voting rules. Messner and Polborn (2004) explain which voting rule is selected (once and for all) in an overlapping generations context, when voters are aware that their preferences will change in known ways with age. In particular, older voters are less inclined to support policies with steep up-front costs and a distant payout than younger voters with a longer horizon. Ex ante, a voter anticipates this change in his or her preferences

[17] A number of other contributions have this same flavor, focusing attention on some specific features of agenda-setting rules. These include Gersbach (2004) on voting rules that depend on the motion on the floor, Cotton (2010) on proposal power that is retained across bargaining periods, and Diermeier and Fong (2011) that allows for reconsideration of an approved policy. The latter paper has an extensive bibliography of related papers.

and thus will seek to select a voting rule that maximizes the welfare of his or her "average future self." No explicit agenda-setting stage is modeled, so the origins of substantive proposals is left unspecified. Barbera and Jackson (2004) distinguish between an "ordinary business" decision rule, s, and a "constitutional" decision rule, S, for making changes in s. They seek an equilibrium pair (s, S) with the property that no $s' \neq s$ is preferred to s by at least S voters. They refer to a pair with this property as a *self-stable constitution*. Once a constitution is in place, ordinary business consists of a binary choice between the status quo and a change. As in Messner and Polborn (2004), agenda setting does not figure in this constitution – a status quo is in place and an (unmodeled) alternative to it emerges in each period.

Agenda setting, and the endogeneity of agenda institutions, is central to our paper and to a recent paper by Diermeier, Prato and Vlaicu (2012). They consider a "procedural state of nature" (their term) in which a set of legislators – or any self-governing group for that matter – selects the procedures by which it will conduct its business. As applied to legislatures they note two stylized facts and regard them as puzzles to be explained: (1) Why are procedures restrictive, granting asymmetric agenda advantage to some legislators? and (2) Why are these procedures persistent, that is, not (often) revoked by a majority? To address these questions, they build a model combining features of Baron-Ferejohn bargaining and single-peaked legislator policy preferences based on a one-dimensional spatial model. They account for the first question with an appeal to risk aversion – symmetric recognition rules provide the *expectation* of centrist policy outcomes preferred by majorities (i.e., the median voter), but is accompanied by undesirable variance. For the second question they appeal to opportunity costs – time spent on revisiting procedural decisions is time taken away from substantive policy making. These conceptual developments require additional theoretical machinery (one-dimensional policy space, concave utility functions, costly decision making) that specializes the argument, but nevertheless is well within the spirit of the legislative modeling literature.

A major innovation of Diermeier, Prato and Vlaicu's paper, and where it may be differentiated from our own and the rest of the literature, lies in its ability to address procedural commitment. In the play of the Diermier et al. game, after a standing procedure is agreed to (an assignment of proposal probabilities to legislators) which, in principle, may apply to all future policy decisions, a policy problem arises exogenously but unknown at the time the standing procedure is established. Once known, a majority may choose to "suspend" the standing proposal probabilities and devise an ad hoc procedure just for this policy, but at a cost. The possibility of suspension puts bounds on the standing procedure the legislators will select ex ante. Diermeier et al. find that equilibrium procedures are asymmetric and persistent, thus providing an equilibrium explanation for the stylized regularities mentioned above. These asymmetric and persistent recognition probabilities are "majoritarian" in the sense that the median voter is part of the majority coalition supporting them. Their very persistence – that is, the ex post unwillingness of a majority to replace a standing procedure with an ad hoc alternative – is a form of procedural self-enforcement. In contrast, our model does not shroud the policy bargaining problem ex ante in uncertainty, and while in our model an approved standing procedure cannot be revoked within a period, the equilibrium agenda rule in period t in no way constrains legislators in their choice of an agenda rule in period $t+1$. Stationarity, rather, implies that they will select the same agenda rule.

Diermeier, Prato and Vlaicu identify the majoritarian procedural rules that emerge in equilibrium in a legislature that bargains over ideological policies. We identify the majoritarian procedural rules that emerge in equilibrium in a legislature that bargains over distributive policies, and we show that these rules favor seniority.

7 Appendix

We begin with a useful lemma:

Lemma 1 *Any equilibrium in which voters always reelect incumbents is such that in any bargaining stage the policy proposer in the first round of bargaining keeps $\frac{N+1}{2N}$ of the cake and offers $\frac{1}{N}$ to any $\frac{N-1}{2}$ other agents, and this policy is approved.*

Proof. In an equilibrium in which incumbents are reelected no matter what, the actions in one period have no effect over expected payoffs in future periods, hence each legislator seeks to maximize her period payoff, conditional on reelection, which means that each legislator plays the bargaining game myopically as if there were no continuation game. The probability of recognition is symmetric across agents and constant over all rounds after the first. Hence, the subgame that starts with a legislator recognized to make a proposal in round $k \in \mathbb{N}$, including $k = 1$, is identical to the subgame that starts with a legislator being recognized in the standard Baron-Ferejohn bargaining game, and the unique stationary II class of equilibria of that game is such that the proposer keeps $\frac{N+1}{2N}$ and offers $\frac{1}{N}$ to any $\frac{N-1}{2}$ other agents (there is no discount rate in this expression because discounting does not occur for the first period). ■

Proof of Proposition 1

Proof. Assume first that each legislator is reelected in every period (we later show that this holds in equilibrium). Since the equilibrium is by definition stationary, assuming that all legislators are assured reelection on and off the equilibrium path, the actions in period t have no consequences in future periods. Thus legislators seek to myopically maximize their payoff in this period. By lemma 1, the unique equilibrium of the bargaining game in this case is such that the policy proposer obtains $\frac{N+1}{2N}$ of the cake, and $\frac{N-1}{2}$ other legislators obtain $\frac{1}{N}$; let these legislators be randomly chosen. Thus, the expected period payoff for each legislator i is

$$a_t(i)\frac{N+1}{2N} + [1 - a_t(i)]\frac{1}{2N}. \quad (1)$$

Given that the equal recognition default rule $\bar{a}(i)$ grants an expected payoff in the bargaining game of $\frac{1}{N}$ to each legislator, in order for rule $a_t(i)$ to be approved, the expected payoff (1) must be at least $\frac{1}{N}$ for at least $\frac{N+1}{2}$ agents, hence $l(t)$ solves

$$\begin{aligned} a_t(i)\frac{N+1}{2N} + [1 - a_t(i)]\frac{1}{2N} &= \frac{1}{N} \\ a_t^*(i) &= \frac{1}{N} \end{aligned}$$

for $\frac{N-1}{2}$ agents and keeps the rest of the probability of recognition to herself. This is the optimal rule for $l(t)$ among those that can be approved in equilibrium. Thus, $l(t)$ best responds by proposing it, and the $\frac{N-1}{2}$ agents who get probability of recognition $\frac{1}{N}$ best respond by voting to approve it. In particular, assume that $l(t)$ offers $a_t^*(i) = \frac{1}{N}$ for any i among the $\frac{N-1}{2}$ most senior legislators in N_t^{-l}.

Given these legislators' strategies, the expected payoff for a voter is weakly greater in the seniority of the voter's legislator. Hence, voters best respond by reelecting their legislators.

We complete the characterization of the equilibrium by describing the actions dictated by the equilibrium strategies off the equilibrium path. First, any deviations in any period prior to t is ignored at period t; play returns to equilibrium play as if the play had stayed along the equilibrium path. Suppose $l(t)$ deviates to propose recognition rule $a_t'(i) \neq a_t^*(i)$. At $\rho = 3$, any legislator $i \in N_t$ for whom $a_t^*(i) \geq \frac{1}{N}$ votes in favor of the proposal and any legislator such that $a_t^*(i) < \frac{1}{N}$ votes against it. At the bargaining stage, the policy proposer ignores deviations at the rules stage and plays as if along the equilibrium path, while following a deviation by the policy proposer, any legislator $i \in N_t$ who obtains at least $\frac{1}{N}$ votes in favor of the proposal and any legislator who obtains less than $\frac{1}{N}$ votes against it in the first round of bargaining (in subsequent rounds, the cutoffs to vote in favor are discounted by δ). At the election stage, voters reelect their legislators, even off the equilibrium path. ■

Proof of Proposition 2

Proof. Assume first that each legislator is reelected in every period (we later show that this assumption holds in equilibrium). Since the equilibrium is by definition stationary, assuming that all legislators are assured reelection on and off the equilibrium path, the actions in period t have no consequences in future periods. Thus legislators seek to myopically maximize their payoff in this period. By lemma 1, the unique equilibrium of the bargaining game in this case is such that the policy proposer obtains $\frac{N+1}{2N}$ of the cake, and $\frac{N-1}{2}$ other legislators obtain $\frac{1}{N}$ (let these legislators be randomly chosen). Thus, the expected period payoff for each legislator i is

$$a_t(1,i)\frac{N+1}{2N} + [1 - a_t(1,i)]\frac{1}{2N} = \frac{a_t(1,i)N + 1}{2N}.$$

Suppose $\theta_t^{l(t)} < \frac{N+1}{2}$. At the rules stage, agent $l(t)$ maximizes $\frac{a_t(1,l(t))N+1}{2N}$ subject to two restrictions: (i) $a_t(1,i) \geq a_t(1,l(t))$ for the $\theta_t^{l(t)-1}$ most senior legislators, and

(ii) $a_t(1,i) \geq \frac{1}{N}$ for at least $\frac{N+1}{2}$ legislators. The two constraints are binding, so

$$\begin{aligned} a_t(1,i)\frac{N+1}{2N} + [1 - a_t(1,i)]\frac{1}{2N} &= \frac{1}{N} \\ a_t^*(1,i) &= \frac{1}{N}, \end{aligned}$$

and the unique solution is to assign probability of recognition $\frac{1}{N}$ to $\frac{N+1}{2} - \theta_t^{l(t)}$ agents with seniority less than $l(t)$, and $\frac{1-\frac{1}{N}\left(\max\{0,\frac{N+1}{2}-\theta_t^{l(t)}\}\right)}{\theta_t^{l(t)}}$ to legislators at least as senior as $l(t)$.The suggested equilibrium is one particular instance of this solution class, in which the $\frac{N+1}{2} - \theta_t^{l(t)}$ agents assigned probability $\frac{1}{N}$ are those with seniority $\theta_t^i \in (\theta_t^{l(t)}, \frac{N+1}{2}]$.

Voters are best responding by reelecting their incumbents because under these strategies, senior legislators have a greater expected payoff in each period than junior ones, so there is no gain for a voter in making a legislator less senior, and the voter thus best responds by always reelecting her representative. In fact, if every agent faces an equal probability of recognition at the rules stage, the expected payoff for a legislator i with seniority ranking $\theta_t^i \leq \frac{N+1}{2}$ is

$$\begin{aligned} &\sum_{k=1}^{\theta_t^i - 1} \frac{1}{2N^2} + \sum_{k=\theta_t^i}^{\frac{N+1}{2}} \frac{1}{N} \frac{\frac{1-\frac{1}{N}\left(\frac{N+1}{2}-k\right)}{k} N + 1}{2N} + \sum_{k=\frac{N+3}{2}}^{N} \frac{1}{N} \frac{\frac{1}{k} N + 1}{2N} \\ = \;& \frac{\theta_t^i - 1}{2N^2} + \frac{1}{2N^2} \sum_{k=\theta_t^i}^{\frac{N+1}{2}} \frac{N + 4k - 1}{2k} + \frac{1}{2N^2} \sum_{k=\frac{N+3}{2}}^{N} \frac{N+k}{k}, \end{aligned}$$

which is strictly decreasing in θ_t^i, specifically, it changes by $\frac{1}{2N^2} - \frac{1}{N^2} - \frac{N-1}{4N^2\theta_t^i} - \frac{1}{2N^2}\frac{N+\theta_t^i}{\theta_t^i} = -\frac{3N+4\theta_t^i-1}{4N^2\theta_t^i}$ with an increase of one unit in θ_t^i, and the expected payoff for a legislator with seniority ranking $\theta > \frac{N+1}{2}$ is

$$\sum_{k=1}^{\theta_t^i-1} \frac{1}{2N^2} + \sum_{k=\theta_t^i}^{N} \frac{\frac{1}{k}N+1}{2N^2} = \frac{1}{2N^2}\left(N + \sum_{k=\theta_t^i}^{N} \frac{N}{k}\right) = \frac{1}{2N}\left(1 + \sum_{k=\theta_t^i}^{N} \frac{1}{k}\right),$$

which is strictly decreasing in θ_t^i, specifically, it drops by $\frac{1}{2N\theta_t^i}$ with an increase of one unit in θ_t^i. ■

Proof of Proposition 3

Proof. Let N_t be the set of all legislators serving in the assembly in period t. Let A be the set of legislators composed of $l(t)$ and the $q-1$ most senior legislators in N_t^{-l}.

Assume incumbents are always reelected. Since the conjectured equilibrium is stationary and all legislators are reelected, the actions in this period have no consequences in future periods. Thus legislators seek to myopically maximize their payoff in this period. Given the recognition rule $a_t^*(\rho, i)$, construct an equilibrium of the bargaining game such that in the first round of bargaining over the cake, the proposer j gets $\frac{N+1}{2N}$ of the cake, and $\frac{N-1}{2}$ randomly selected members of set A, each get $\frac{1}{N}$, which is every agent's continuation value in the bargaining game.

At the recognition rules stage, the status quo rule $\bar{a}(\rho, i)$ grants an expected payoff in the bargaining game of $\frac{1}{N}$ to each legislator. Thus, in order for a recognition rule to be approved, it must grant an expected utility of at least $\frac{1}{N}$ to at least $q-1$ agents other than the rule proposer. Subject to this constraint, the best that legislator $l(t)$ can aspire to is thus a recognition rule that lets $q-1$ agents have an expected payoff of exactly $\frac{1}{N}$ and lets $l(t)$ enjoy all the remaining surplus. Rule $a_t^*(\rho, i)$ delivers utility

$$\frac{2q-N-1}{(q-2)N+q}\frac{N+1}{2N}+\left(1-\frac{2q-N-1}{(q-2)N+q}\right)\frac{N-1}{2(q-1)}\frac{1}{N}=\frac{1}{N}$$

to exactly $q-1$ agents and the surplus $N-q+1$ to $l(t)$ hence it is approved with the votes of all these agents, and it maximizes the expected payoff of $l(t)$.

We may now relax the assumption that incumbents are reelected off the equilibrium path; if they are not, legislators are even more worse-off so they do not deviate from the equilibrium path in which they are reelected.

Thus, given the voters' behavior, and given stationarity, legislators are best responding at every stage and round. It remains to be shown that voters are best responding by reelecting their incumbents along the equilibrium path. They are because under these strategies, senior legislators have a greater expected payoff in each period than junior ones, so there is no gain for a voter in making a legislator less senior, and the voter thus best responds by always reelecting her representative. ■

Proof of Proposition 4

Proof. Assume first that each legislator was assured reelection. Since the equilibrium is by definition stationary, assuming that all legislators are assured reelection on an off the equilibrium path, the actions in period t has no consequences in future periods.

Thus legislators seek to myopically maximize their payoff in this period. By lemma 1, the unique equilibrium of the bargaining game in this case is such that the policy proposer obtains $\frac{N+1}{2N}$ of the cake, and $\frac{N-1}{2}$ other legislators obtain $\frac{1}{N}$. Let the set of legislators who receive $\frac{1}{N}$ be randomly determined. Thus, the expected period payoff for each legislator i is

$$a_t(1,i)\frac{N+1}{2N} + [1 - a_t(1,i)]\frac{1}{2N} = \frac{a_t(1,i)N+1}{2N}.$$

Suppose $\theta_t^{l(t)} < q$. At the rules stage, agent $l(t)$ maximizes $\frac{a_t(1,l(t))N+1}{2N}$ subject to two restrictions: (i) $a_t(1,i) \geq a_t(1,l(t))$ for the $\theta_t^{l(t)-1}$ most senior legislators, and (ii) $a_t(1,i) \geq \frac{1}{N}$ for at least q legislators. The two constraints are binding, so the unique solution is to assign probability of recognition $\frac{1}{N}$ to $q - \theta_t^{l(t)}$ agents with less seniority than $l(t)$, and $\frac{1-\frac{1}{N}\left(\max\{0,q-\theta_t^{l(t)}\}\right)}{\theta_t^{l(t)}}$ to legislators at least as senior as $l(t)$.

$$\begin{aligned} a_t(1,i)\frac{N+1}{2N} + [1 - a_t(1,i)]\frac{1}{2N} &= \frac{1}{N} \\ a_t^*(1,i) &= \frac{1}{N}. \end{aligned}$$

The suggested equilibrium is one particular instance of this solution class, in which the $q - \theta_t^{l(t)}$ agents assigned probability $\frac{1}{N}$ are those with seniority $\theta_t^i \in (\theta_t^{l(t)}, q]$.

Hence, if each legislator is assured reelection on and off the equilibrium path, legislators would follow the equilibrium strategies described above. Suppose that legislators are not assured reelection off the equilibrium path; deviating from the equilibrium path is then weakly less attractive to legislators, hence staying on it remains a best response. Therefore, it suffices to sustain these strategies that legislators be reelected along the equilibrium path, not necessarily off it.

Voters are best responding by reelecting their incumbents because under these strategies, senior legislators have a greater expected payoff in each period than junior ones, so there is no gain for a voter in making a legislator less senior, and the voter thus best responds by always reelecting her representative along the equilibrium path. In fact, if every agent faces an equal probability of recognition at the rules stage, the expected payoff for a legislator i with seniority ranking $\theta_t^i \leq q$ is

$$\begin{aligned} & \sum_{k=1}^{\theta_t^i-1} \frac{1}{2N^2} + \sum_{k=\theta_t^i}^{q} \frac{1}{N} \frac{\frac{1-\frac{1}{N}\left(\frac{N+1}{2}-k\right)}{k}N+1}{2N} + \sum_{k=q+1}^{N} \frac{1}{N} \frac{\frac{1}{k}N+1}{2N} \\ = & \frac{1}{2N^2}\left(\theta_t^i - 1 + \sum_{k=\theta_t^i}^{q} \frac{N+4k-1}{2k} + \sum_{k=q+1}^{N} \frac{N+k}{k}\right), \end{aligned}$$

which is strictly decreasing in θ_t^i, specifically, it changes by $\frac{1}{2N^2} - \frac{1}{N^2} - \frac{N-1}{4N^2\theta_t^i} - \frac{1}{2N^2}\frac{N+\theta_t^i}{\theta_t^i} = -\frac{3N+4\theta_t^i-1}{4N^2\theta_t^i}$ with an increase of one unit in θ_t^i. The expected payoff for a legislator with seniority ranking $\theta > q$ is

$$\sum_{k=1}^{\theta_t^i-1} \frac{1}{2N^2} + \sum_{k=\theta_t^i}^{N} \frac{\frac{1}{k}N+1}{2N^2} = \frac{1}{2N^2}\left(N + \sum_{k=\theta_t^i}^{N} \frac{N}{k}\right) = \frac{1}{2N}\left(1 + \sum_{k=\theta_t^i}^{N} \frac{1}{k}\right),$$

which is strictly decreasing in θ_t^i, specifically, it drops by $\frac{1}{2N\theta_t^i}$ with an increase of one unit in θ_t^i. ■

Proof of Proposition 5

Proof. Take the voters' behavior as given. Since the conjectured equilibrium is stationary and all legislators are reelected, the actions in this period have no consequences in future periods. Thus legislators seek to myopically maximize their payoff in this period. Given the recognition rule $a_t^*(\rho, i)$, equilibrium play of the bargaining game involves forming a coalition at the first round between the proposer who gets the whole unit of wealth, and the $\frac{N-1}{2}$ junior legislators other than $l(t)$ who get nothing. Expected payoffs thus correspond exactly to the recognition probabilities.

At the recognition rules stage, the status quo $\bar{a}(\rho, i)$ grants an expected payoff in the bargaining game of $\frac{1}{N}$ to each legislator. Thus, in order to have recognition rule $a_t^*(\rho, i)$ approved, legislator l must grant an expected payoff in the bargaining game of at least $\frac{1}{N}$ to at least $\frac{N-1}{2}$ other legislators. The best that legislator $l(t)$ can aspire to is thus a recognition rule that lets $l(t)$ have an expected payoff of $1 - \frac{1}{N}\frac{N-1}{2} = \frac{N+1}{2N}$. Rule $a_t^*(\rho, i)$ achieves just that, and is approved with the favorable votes of exactly $\frac{N+1}{2}$ agents.

Thus, given the voters' behavior, and given stationarity, legislators are best responding at every stage and substage. It remains to be shown that voters are best responding by reelecting their incumbents. Which they are, because under these strategies, senior legislators have a greater expected payoff in each period than junior ones, so there is no gain for a voter in making a legislator less senior, and the voter thus best responds by always reelecting her representative. ■

Proof of Proposition 6

Proof. We complete the description of the equilibrium recognition rule $a_t^*(\rho, i)$ as follows. In each period t in which $\theta_t^l \leq \frac{N+1}{2}$, $l(t)$ proposes $a_t^*(2, i) = \frac{2}{N+3}$ for any legislator i with seniority ranking up to $\frac{N+3}{2}$; $a_t^*(2, i) = 0$ for any other legislator i; and $a_t^*(\rho, k) = 1$ for some $k \neq l(t)$ and $a_t^*(\rho, i) = 0$ for any $i \neq k$, for any $\rho \geq 3$. In each period t in which $\theta_t^l \geq \frac{N+3}{2}$, $l(t)$ proposes $a_t^*(2, i) = \frac{2}{N+3}$ for $i = l(t)$ and any legislator i with seniority ranking up to $\frac{N+1}{2}$, $a_t^*(2, i) = 0$ for any other legislator i; and $a_t^*(\rho, k) = 1$ for some $k \neq l(t)$ and $a_t^*(\rho, i) = 0$ for any $i \neq k$, for any $\rho \geq 3$.

Take the voters' behavior as given. Since the conjectured equilibrium is stationary and all legislators are reelected, the actions in this period have no consequences in future periods. Thus legislators seek to myopically maximize their payoff in this period.

i) Consider first the periods in which $\theta_t^l \leq \frac{N+1}{2}$.

Given the recognition rules $a_t^*(\rho, i)$, if the bargaining stage reaches the third round of bargaining, legislator k gets all the cake. That means the continuation value in round two for every other agent is zero, so the proposer in round two gets to keep all the cake. So in round one, the continuation value of every agent is equal to the agent's recognition probability in round two. There are $\frac{N-3}{2}$ with continuation value zero at round one, and they always vote in favor of any proposal. The policy proposer needs one more agent supporting her proposal, and she needs to offer $\frac{N+3}{2}$ to this agent. In the equilibrium we construct, it is always $l(t)$ who is selected by other proposers; this may be seen as the limit case in which $l(t)$ granted herself an epsilon smaller recognition probability in the second round so that she would always be included in the first round. Thus in the first round, the proposer offers their expected value of the continuation game to exactly $\frac{N-1}{2}$ agents, namely $l(t)$ who gets $\frac{2}{N+3}$ and to $\frac{N-3}{2}$ legislators who get nothing. If $l(t)$ is the proposer, she randomly chooses one of the agents with seniority from $\theta_t^l + 1$ to $\frac{N+1}{2}$ and offers her $\frac{2}{N+3}$. Thus proposals are approved with the votes of these agents and the proposer who keeps $\frac{N+1}{N+3}$ for herself.

At the rules proposal stage:

Let x be the probability of recognition of any legislator with seniority ranking from θ_t^l+1 to $\frac{N+1}{2}$. Let z be the probability of recognition of legislators with seniority ranking up to θ_t^l. Legislators with seniority ranking from $\theta_t^l + 1$ through $\frac{N+1}{2}$ collectively have a probability of recognition $x(\frac{N+1}{2} - \theta_t^l)$, leaving legislators with ranking up to θ_t^l with $1 - x(\frac{N+1}{2} - \theta_t^l)$ so that $z = \frac{1}{\theta_t^l} + x\left(\frac{2\theta_t^l - N - 1}{2\theta_t^l}\right) = \frac{2+2\theta_t^l x-(N+1)x}{2\theta_t^l}$.

Then the expected payoff in the first round of bargaining for a legislator with seniority ranking from $\theta_t^l + 1$ to $\frac{N+1}{2}$ is $x\frac{N+1}{N+3} + \frac{2-(N+1)x+2\theta_t^l x}{2\theta_t^l}\frac{1}{\frac{N+1}{2}-\theta_t^l}\frac{2}{N+3}$, where the first term corresponds to the probability that i is a proposer, and the second to the probability that $l(t)$ is a proposer and chooses i as a coalition partner. This must be equal to $\frac{1}{N}$ to make the legislator support the recognition rule.

$$
\begin{aligned}
x\frac{N+1}{N+3}+\frac{2-(N+1)x+2\theta_t^l x}{2\theta_t^l}\frac{2}{N+1-2\theta_t^l}\frac{2}{N+3} &= \frac{1}{N} \\
x(N+1)+\frac{4-2(N+1)x+4\theta_t^l x}{\theta_t^l(N+1-2\theta_t^l)} &= \frac{N+3}{N} \\
x(N+1)+\frac{4\theta_t^l x-2(N+1)x}{\theta_t^l(N+1-2\theta_t^l)}+\frac{4}{\theta_t^l(N+1-2\theta_t^l)} &= \frac{N+3}{N} \\
x\left[N+1+\frac{4\theta_t^l-2(N+1)}{\theta_t^l(N+1-2\theta_t^l)}\right] &= \frac{N+3}{N}-\frac{4}{\theta_t^l(N+1-2\theta_t^l)} \\
x &= \frac{\frac{N+3}{N}-\frac{4}{\theta_t^l(N+1-2\theta_t^l)}}{N+1+\frac{4\theta_t^l-2(N+1)}{\theta_t^l(N+1-2\theta_t^l)}} =
\end{aligned}
$$

$$
\frac{\frac{(N+3)\theta_t^l(N+1-2\theta_t^l)-4N}{\theta_t^l(N+1-2\theta_t^l)N}}{\frac{4\theta_t^l-2(N+1)+(N+1)\theta_t^l(N+1-2\theta_t^l)}{\theta_t^l(N+1-2\theta_t^l)}} = \frac{(N+3)\theta_t^l(N+1-2\theta_t^l)-4N}{4N\theta_t^l-2N(N+1)+N(N+1)\theta_t^l(N+1-2\theta_t^l)}
$$

Thus

$$
z=\frac{1}{\theta_t^l}+\frac{(N+3)\theta_t^l(N+1-2\theta_t^l)-4N}{4N\theta_t^l-2N(N+1)+N(N+1)\theta_t^l(N+1-2\theta_t^l)}\left(\frac{2\theta_t^l-N-1}{2\theta_t^l}\right)
$$

and $z-x=\frac{1}{\theta_t^l}-x(\frac{N+1}{2\theta_t^l})$ so $z-x\geq 0$ if $x\leq\frac{2}{N+1}$, much must hold, because if $x\geq\frac{2}{N+1}$, then the utility for the agents assigned probability of recognition x would be greater than $\frac{2}{N+1}\frac{N+1}{N+3}=\frac{2}{N+3}>\frac{1}{N}$ for any $N>3$, a contradiction.

These probabilities of recognition imply that legislators with seniority ranking up to θ_t^l-1 enter the bargaining game with an expected payoff of $z\frac{N+1}{N+3}$, legislators with seniority ranking from θ_t^l+1 to $\frac{N+1}{2}$ with an expected payoff of $\frac{1}{N}$ and legislator $l(t)$

with an expected payoff of $z\frac{N+1}{N+3} + (1-z)\frac{2}{N+3}$. The aggregate expected payoff for legislators with seniority ranking 1 through $\theta_t^l - 1$ is

$$1 - z\frac{N+1}{N+3} - (1-z)\frac{2}{N+3} - \left(\frac{N+1}{2} - \theta_t^l\right)\frac{1}{N}$$

We need to show that this aggregate is at least $\frac{\theta_t^l - 1}{N}$.

$$\begin{aligned}
1 - z\frac{N+1}{N+3} - (1-z)\frac{2}{N+3} - \left(\frac{N+1}{2} - \theta_t^l\right)\frac{1}{N} &\geq \frac{\theta_t^l - 1}{N} \\
\frac{2N(N+3) - 2N(N+1)z - 2N + 2Nz - (N+1)(N+3) + 2(N+3)\theta_t^l}{2N(N+3)} &\geq \frac{2(N+3)\theta_t^l - 2(N+3)}{2N(N+3)} \\
(N+1)(N+3) - 2N(N)z - 2N &\geq 0 \\
N^2 + 2N + 4 &\geq 2N^2 z \\
z &\leq \frac{N^2 + 2N + 4}{2N^2}.
\end{aligned}$$

The right hand side is always above $\frac{1}{2}$, and z must be below $\frac{1}{2}$ if $\theta_t^l > 1$. Thus, agents 1 through $\theta_t^l - 1$ have an expected payoff of at least $\frac{1}{N}$ as desired.

The status quo $\bar{a}(\rho, i)$ grants an expected payoff in the bargaining game of $\frac{1}{N}$ to each legislator. Thus, in order to have recognition rule $a_t^*(\rho, i)$ approved, legislator $l(t)$ must grant an expected payoff in the bargaining game of at least $\frac{1}{N}$ to at least $\frac{N-1}{2}$ other legislators. Legislator $l(t)$ successfully minimizes the expected share of the unit of wealth obtained by legislators with seniority ranking $\theta_t^l + 1$ to $\frac{N+1}{2}$ to $\frac{1}{N}$. Legislator $l(t)$ cannot minimize the share obtained by her seniors so much, because they are protected by the constraint that their proposal power in the first stage be at least as high as that of $l(t)$. Thus, the most that $l(t)$ can hope for is to maximize her expected continuation value subject to always being included in the coalition put together by the agent that is chosen to make the proposal at the first round of bargaining. This is achieved by maximizing the continuation value of the agent with $\frac{N-1}{2}$ -th lowest continuation value, and making $l(t)$ be that agent, which is exactly what a_t^* does by minimizing the continuation value of $\frac{N-3}{2}$ to zero, and equalizing the continuation value of all other agents, and assuming that all proposers break indifference by choosing to offer her continuation value to $l(t)$ in the first bargaining round, and not to any of the other agents.

Thus, recognition rule a_t^* is the rule that maximizes the utility of agent $l(t)$ with seniority ranking $\theta_t^l \leq \frac{N+1}{2}$ among the set of recognition rules that can be approved by the assembly.

ii) Consider the periods in which $\theta_t^l > \frac{N+1}{2}$.

As before, if the bargaining game enters the third round, k gets all the cake, so if it enters the second round, whoever is the policy proposer in this round gets all the cake, so in the first round the continuation value for each of the $\frac{N+3}{2}$ agents with equal recognition probability is $\frac{2}{N+3}$. The agent who makes a policy proposal in the first round must offer $\frac{2}{N+3}$ to one of those $\frac{N+3}{2}$ agents. In the equilibrium we construct, each agent other than $l(t)$ chooses $l(t)$ with probability $1 - \frac{N+3}{2}p$ and each of the other agents with probability p, while $l(t)$ randomizes among all agents up to when she is the proposer. Then, the expected payoff in the bargaining game of each legislator with seniority up to $\frac{N+1}{2}$ is $\frac{1}{\theta_t^l}\frac{N+1}{N+3}+\frac{1}{\theta_t^l}\frac{1}{N+1}\frac{2}{N+3}+p(1-\frac{2}{\theta_t^l})\frac{2}{N+3}$, where the first term corresponds to the case when the agent is the proposer, the second to the case that $l(t)$ is the proposer, and the third to the case where any other legislator is the proposer. We assume that the equilibrium played is such that it minimizes p subject to $\frac{1}{\theta_t^l}\frac{N+1}{N+3}+\frac{1}{\theta_t^l}\frac{1}{N+1}\frac{2}{N+3}+p(1-\frac{2}{\theta_t^l})\frac{2}{N+3}\geq\frac{1}{N}$.

Legislators with seniority ranking from $\frac{N+3}{2}$ to θ_t^l obtain nothing if they are not the proposer; their expected payoff is $\frac{1}{\theta_t^l}\frac{N+1}{N+3}$. Legislator $l(t)$ obtains the rest of the share.

Since at least $\frac{N+1}{2}$ obtain an expected payoff in the bargaining game of at least $\frac{1}{N}$ under recognition rule $a_t^*(\rho,i)$, the recognition rule is approved. By the same logic as in the previous case, we note that $a_t^*(\rho,i)$ grants legislator $l(t)$ the highest possible expected payoff in the bargaining game subject to the double constraint that the recognition rule must be approved by the assembly (which binds if $\theta_t^l\geq N-1$ in which case $p>0$ and the expected payoff of the most senior legislators is $\frac{1}{N}$) and the constraint that recognition rules in the first round must be weakly increasing in seniority (which is always binding).

Thus, given the voters' behavior, and given stationarity, legislators are best responding at every stage and substage. It remains to be shown that voters are best responding by reelecting their incumbents. They are, because under these strategies senior legislators have a greater expected payoff in each period than junior ones, so there is no gain for a voter in making a legislator less senior, and the voter thus best responds by always reelecting her representative. ■

Calculations for Example 3

Calculations for the example.

Row 1: The rules proposer is recognized as policy proposer with probability $\frac{N+1}{2N} = \frac{16}{30} = 0.533$. Her payoff is

$$\frac{8}{15}\frac{N+1}{2N} + \frac{7}{15}\frac{1}{2}\frac{1}{N} = \frac{8}{15}\frac{8}{15} + \frac{7}{450} = 0.3.$$

Senior and semi-senior legislators are recognized with probability $\frac{1}{15}$. Their payoff is $\frac{1}{15}\frac{16}{30} + \frac{14}{15}\frac{1}{2}\frac{1}{15} = 0.067$.

Junior legislators are never recognized. Their payoff is $\frac{1}{2}\frac{1}{15} = 0.03$.

Row 2: The rules proposer and senior legislators are recognized with probability $\frac{1-\frac{3}{15}}{5} = \frac{4}{25} = 0.16$. Their payoff is $\frac{4}{25}\frac{8}{15} + \frac{21}{25}\frac{1}{2}\frac{1}{15} = 0.113$. Semi-senior legislators are recognized with probability $\frac{1}{15}$ and their payoff is as in row one; junior legislators are never recognized and their payoff is as in row 1.

Row 3: Recognition probabilities are as in row one, but expected payoffs now coincide with these probabilities.

Row 4: The probability x of recognition for semi-senior legislators is

$$\frac{(N+3)\theta_t^l(N+1-2\theta_t^l)-4N}{4N\theta_t^l-2N(N+1)+N(N+1)\theta_t^l(N+1-2\theta_t^l)} = \frac{18(5)(6)-60}{300-30(16)+15(16)5(6)} = 0.068.$$

The probability of recognition z for senior legislators is:

$$z = \frac{1}{\theta_t^l} + x\left(\frac{2\theta_t^l - N - 1}{2\theta_t^l}\right) = \frac{1}{5} + 6.8376 \times 10^{-2}(\frac{10-16}{10}) = 0.159.$$

Expected payoffs for senior legislators are:

$$(1 - 0.159(\frac{16}{18}) - (1 - 0.159)\frac{2}{18} - (\frac{16}{2} - 5)\frac{1}{15})/4 = 0.141\,.$$

Expected payoff for the proposer is

$$1 - 4 * 0.141\; - 0.2 = 0.235\,.$$

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