Equivalence Principle's Test with Improved Accuracy using a Cryogenic Differential Accelerometer Installed on a Pendulum.

EQUIVALENCE PRINCIPLE'S TEST WITH IMPROVED ACCURACY USING A CRYOGENIC DIFFERENTIAL ACCELEROMETER INSTALLED ON A PENDULUM.

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Summary

We present here a concept for a new experimental test of the Weak Equivalence Principle (WEP) carried out in the gravity field of the Sun. Two test masses of different materials are the central elements of a differential accelerometer with zero baseline. The differential accelerometer is placed on a pendulum, in such a way as to make the common center of mass coincident with the center of mass of the pendulum. Ensuring a very precise centering, such a system should provide a high degree of attenuation of the local seismic noise, which together with an integration time of the order of tens of days would allow verification of the WEP with an accuracy improved by at least an order of magnitude with respect to the state of the art. One of the strengths of this experiment is the know-how acquired from a previous study and technology development (GREAT: General Relativity Accuracy Test) that involved a test of the WEP in the gravity field of the Earth, in free fall inside a co-moving capsule released from a stratospheric balloon. The description of the experiment will be followed by a critical analysis of the challenges associated with its implementation.

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I. Introduction

The universality of free fall, postulated by G. Galilei and expressed in terms of equality between inertial and gravitational mass by I. Newton, is at the basis of Einstein's General Relativity, in terms of the local identity of the gravitational field with a non-inertial reference frame; it is part of the Weak Equivalence Principle (WEP), to be distinguished from the Strong Equivalence Principle, which states the covariance of all the laws of nature (including gravitation itself) with respect to all continuous coordinate transformations. The Schiff conjecture (Schiff 1960, Thorne et al. 1973) implies that every theory that satisfies WEP is a metric theory, and vice-versa, i.e. every metric theory satisfies WEP. On the basis of such a conjecture, a WEP experiment allows to test the viability of the metric theories as opposed to the non-metric ones. No experimental deviation from equivalence has as yet been found, even by very high sensitivity experiments. Every WEP experiment is essentially the test of the universality of the free fall of bodies, as it was described in the famous experiment performed by G. Galilei.

The precision of a WEP experiment can be expressed in terms of the Eötvös parameter η which denotes the ratio between the differential acceleration that acts on two test masses of different material, divided by the mean acceleration acting on them: $\Delta a = \eta \cdot a$; see Will 2006.

The difficulty of such an experiment, as already expressed by G. Galilei itself, is due to the drag acceleration acting on the free-fall bodies.

A better precision can be achieved by comparing the oscillation of two pendulums and by considering that the two masses are in free fall along the tangent to the trajectory of their respective oscillation. For such a kind of experiments (G. Galilei, I. Newton, F. Bessel) a precision $\eta < 10^{-3}$ can be achieved. The use of a torsion balance (R. von Eötvös) allows obtaining $\eta < 10^{-8}$ (Eötvös et al. 1922). In such an experiment the inertia is determined by the centrifugal force originated by the rotation of the Earth, and the gravitation is the *g* component necessary for compensating it. A possible WEP violation causes a rotation of the torsion balance. The difficulty of such an experiment relies on the fact that the violation signal occurs at zero frequency. The effect can be detected by exchanging the position of the two masses. By using a torsion balance, and by locking it on the gravitational field of Sun at equilibrium with the inertia of the Earth that rotates around it, the violation signal turns out to be modulated by the rotation of the Earth around its axis with a 24 hours period. The precision of such an experiment is (R.H. Dicke, V.B. Braginsky) $\eta = 10^{-11}$(Roll et al. 1964, Braginskii and Panov 1972). Better results can be achieved by means of space experiments, whenever a satellite is in "free-fall" around the Earth. The conceptual arrangement is always the same: a very sensitive system detects the position of the two test masses, and, in order to avoid the effects of the gravitational gradient, the two centers of mass have to be coincident. The sensitivity of the system is increased by holding the two masses by a very soft spring, by which a small acceleration acting on them causes a large displacement that can be easily detected (under ideal conditions they ought to be free). In the last decades several experiments were set up to test the validity of WEP (for a review see Will 2006). Apart from Lunar Laser Ranging and free fall experiments, the majority of them were performed on-ground, where several sources of noise (among them seismic noise) ultimately limit their accuracy. For this reason the environment offered by space is suitable for substantial improvements. According to the international proposal Satellite Test of the Equivalence Principle (STEP), it seems possible to attain a precision $\eta < 10^{-17}$ (Overduin et al. 2012). In such a case, the system works at liquid He temperature (4.2 K), and uses a drag-free satellite. An Italian non-cryogenic satellite named Galileo Galilei (GG) is under study, with a target precision $\eta < 10^{-16}$ (Nobili et al. 2012) and a French experiment named MICROSCOPE is under development to be flown in the next years (Touboul et al. 2012). Of course space experiments are very costly and need a rather long time to be developed. The GReAT (General Relativity Accuracy Test) experiment that will be described in the next section has the advantage of combining the relative low-noise environment given by free fall with repeatability, resulting in a competitive performance with respect to other ongoing projects. It aims at reaching an accuracy of five parts in 10^{15} (Iafolla et al. 2008).

We present here a concept for a new experiment carried out in the gravity field of the Sun. Two test masses of different materials are the central elements of two harmonic oscillators that form a differential accelerometer with zero baseline, i.e. their centers of mass are made coincident as far as possible. The differential accelerometer is placed on a pendulum, in such a way as to make the common center of mass coincident with the center of mass of the pendulum. Ensuring a very precise centering, such a system should provide a high degree of attenuation of the local seismic noise, which – together with an integration time of the order of tens of days – would allow a verification of the WEP with an accuracy improved by at least an order of magnitude with respect to the state of the art. One of the strengths of this experiment is the know-how acquired from a previous study and technology development (GREAT: General Relativity Accuracy Test) that involved a test of the WEP in the

gravity field of the Earth, in free fall inside a co-moving capsule released from a stratospheric balloon. Specifically, high vacuum and cryogenic techniques already tested in laboratory will be utilized for obtaining high quality factors of the mechanical oscillators, together with techniques for signal pick-up and processing that provide high levels of common mode rejection.
The paper is organized as follows. In section 2 is presented in its general structure the experiment, while in section 3 are described the differential accelerometer that represents the hearth of this experiment. In section 4 it will be described the experiment itself, in which the differential accelerometer is installed inside a pendulum mass.

II. GReAT (General Relativity Accuracy Test)

On-ground and in-space tests had already verified the WEP validity to a precision of $5 \cdot 10^{-13}$. In spite of a so high accuracy, grand unification and quantum gravity theories indicate a violation of this symmetry at higher level, demanding for more accurate experiments. GReAT aims to obtain a precision better than 5 parts in 10^{15}; this precision is intermediate between the precision obtained with the past experiments and what is predicted for the future space experiments (STEP: 10^{-18}, GG: 10^{-17}, MICROSCOPE: 10^{-15}).
The accuracy of ground-based tests of the WEP is limited by the Earth's seismic noise and by the small strength of the gravitational signal source. Increase of signal strength and isolation from external noise are essential ingredients to improve the experimental accuracy. By performing the experiment in free fall under the action of the Earth's gravity acceleration, the signal is three orders of magnitude stronger than that of the Sun, used by ground-based experiments (Eötvös type), and by using the drag-shielded free-fall technique of GiZero, the vibrational acceleration noise can be reduced at a level under $10^{-12} g$. The vertical free-fall, compared with the orbital free fall ("drag-free" satellite), has the drawback of having a much shorter duration – that means short integration time – but it has the advantages of being much less expensive and offers the possibility to be repeated at intervals of a few months, allowing to adjust the experimental set-up, if necessary. The experiment will be repeated with different material pairs for the test masses of the detector; the first flight will be carried out with test masses of the same material in order to exclude any possible spurious signal that can mimic a WEP violation.

General description of the experiment

A system of two masses of different material represents the main part of a differential accelerometer which is cooled down to the temperature of liquid helium (4.2K) and spun up to a maximum rate of 0.5 Hz about the horizontal axis during a free fall. The free fall is performed inside the GiZero evacuated capsule, released from an altitude of about 40km. The possible WEP violation signal will be detected at the spin frequency, while the gravity gradients (due to the capsule mass and to the Earth's mass) produce components at twice the spin frequency. The strength of the gravity gradient components is proportional to the distance between the centers of mass of the two proof masses. The components at the spin frequency can be further canceled out by keeping the spin axis of the detector as close as possible to the horizontal plane.
When compared to future proposed satellite experiments (which could reach higher accuracy) and to classic on-ground experiments, GReAT results as a good compromise, potentially able to improve by two orders of magnitude the accuracy of the test.
In table 1 it is reported a list of the main parts constituting the GiZero facility.

Table 1 *List of the main parts constituting the facility for the free-fall experiment.*

Helium balloon
Gondola attached to the balloon with the mechanism to release the capsule and other house-keeping equipment
Shielding capsule with large vacuum chamber
Liquid-helium, evacuated cryostat
Instrument package which houses the detector inside a high-vacuum chamber
Spin/release mechanism for spinning the instrument package and releasing it into the capsule at the start of the fall
Differential acceleration detector
Two video cameras at the top of the capsule for monitoring the motion of the instrument package falling inside the capsule for post-flight processing
Telemetry system for the downlink from the capsule to the ground

Transonic parachute for decelerating the capsule at the end of the fall

To perform the experiment it is necessary that the balloon reaches its floating altitude so to move at the speed of the local wind, i.e. the capsule will be at zero relative speed with respect to the local wind; if its vertical profile is constant, the capsule and the instrument package in free fall would move laterally during the fall with the same initial lateral velocity and hence maintain the same lateral distance with respect to one another. We have to notice that also Coriolis accelerations on the capsule and on the falling package are the same and, consequently, they do not alter their relative position. However, if the wind vertical profile changes, the capsule will experience a lateral force that will change its lateral speed while the instrument package will not experience such a force. A diameter of 1.5 m is large enough to guarantee that the instrument package does not touch the walls of the capsule, with a lateral wind gradient up to 50 knots over the 4.3 km of vertical drop. If the balloon is launched during the periodically-occurring wind reversal times the vertical wind gradient is much smaller than the value indicated above.

Due to the low relative velocity of the experimental package and to the high level of vacuum ($<10^{-6} mbar$) present inside the capsule, a disturbance from the capsule wall to the free-floating detector produces on it a residual acceleration less than $10^{-12} g/\sqrt{Hz}$. The free-falling capsule reduces the acceleration noise to values unmatched by any other Earth-based drop facility and comparable to values achieved on board drag-free satellites. This acceleration noise acts in the same mode on the two test masses (common-mode noise) and it can be rejected by the common-mode rejection factor (CMRF) of the differential accelerometer. For a conservative value of 10^4 for the CMRF, the influence of these accelerations on the differential measurement is made negligible. It is also important to note that the acceleration noise components produced by the residual gas are proportional to the pressure inside the chamber. This means that the pressure can be reduced in subsequent flights if, for any unanticipated reasons, their influence on the measurement proves to be greater than estimated. It is, in fact, well within the state-of-the-art to obtain pressures at room temperature as low as $10^{-8} mbar$ in large volumes.

Figure 1 shows a numerical evaluation of the spectral density of the gravity gradient Γ_{zx} (z and x are the vertical and horizontal axis, respectively) from two point 1 μm apart along the central axis of the GiZero capsule and originated by the capsule mass distribution (green line). The red line represents a numerical evaluation of the gravity gradient that acts on the differential accelerometer when it falls along the capsule axis, while rotating at 1 Hz around the y axis. The peak at 2 Hz is the hypothetical signal due to a WEP violation at the level of $\eta = 10^{-15}$. We stress the fact that the rotation clearly allows to separate the gravity gradient effects and the hypothetical WEP violation effect.

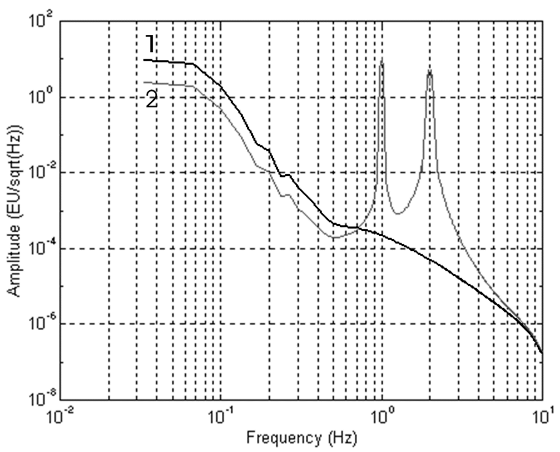


Figure 1 - numerical evaluation of the gravity gradient due to the mass of the capsule (line 1) and of the signal on the rotating accelerometer during the free-fall plus a possible signal of WEP violation with $\eta \approx 10^{-15}$ (line 2).

Several prototypes of accelerometers have been implemented and tested in laboratory conditions; the experimental activities were focused in the development of a mechanical oscillator with an high quality factor, low noise electronics and very high stability vs temperature variations. A prototype of accelerometer with sensitivity equal to $3.3 \cdot 10^{-12} g/\sqrt{Hz}$, evaluated by measuring separately its parameters, was implemented and tested in laboratory for a long time period. A value close to $10^{-10} g/\sqrt{Hz}$ due to the seismic noise was obtained in a quite environmental place. A sensitivity of $4 \cdot 10^{-14} g/\sqrt{Hz}$ can be obtained for the same prototype, increasing the mechanical quality factor and using a low noise temperature preamplifier matched to the capacitive transducer. A cryogenic version could reach a sensitivity equal to $5.7 \cdot 10^{-15} g/\sqrt{Hz}$. In the rest of this section we want focus our attention on the description of the experimental activities required to obtain the experiment goal.

In figure 2 it is shown an outline of the mechanical structure of a prototype of differential accelerometer. Two sensing masses of different materials are connected to a rigid frame by means of a couple of flexural elements so that their two centers of mass are made coincident. An acceleration acting on them can give a rotation around the common rotation axis passing through the center of the flexural elements. Such a rotation can be detected by means of two capacitive bridge transducers. Each bridge is composed of two sensing capacitors, the faces of each sensing mass and two mechanically fixed faces, and two external fixed capacitors. The capacitive bridges are biased at 10 kHz in order to transfer the signal from low to high frequency and to allow the preamplifier get rid of its 1/*f* noise. The differential value is attained by subtracting the two signals. A precise mechanical machining determines the coincidence of the two centers of mass. A possible difference in the position of the two centers of mass, and the gravity gradient between two such points, produce a signal at two times the rotation frequency, as previously explained.

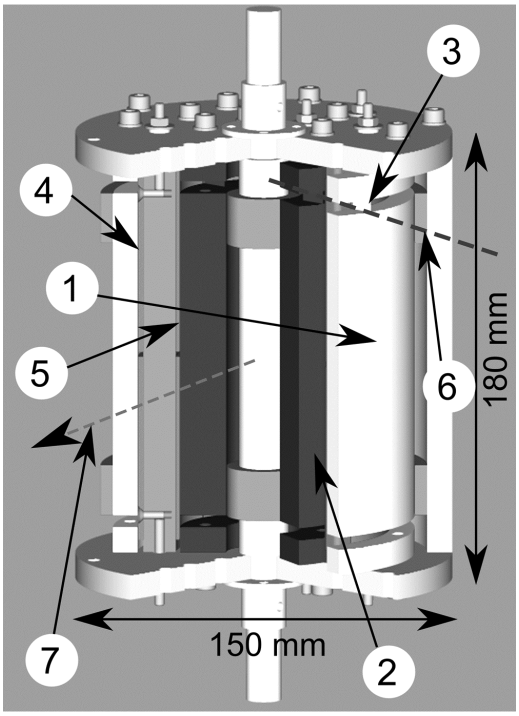


Figure 2 - Outline of a differential accelerometer prototype built at IAPS/INAF.
1 Outer sensing mass.
2.Inner sensing mass.
3.Flexural suspension
4.Capacitive pickup outer sensing mass.
5.Capacitive pickup inner sensing mass.
6.Pivot axis.
7.Sensing axis direction.

The design of the differential acceleration detectors capitalized on the experience gained in our laboratory and on several numerical simulations carried out by our partners at the Harvard-Smithsonian Center for Astrophysics (CfA). The two sensing masses consist of solid hollow right cylinders with spherical ellipsoids of inertia so as to cancel the second-order gravity-gradient torques (quadrupole moments). The design of the proposed capacitive detector can accommodate a variety of sensing masses with different dimensions and materials. The differential detector must be designed as much as possible in a way that allows modifications from one flight to the next based on the experience gained from the previous flight. The centers of mass of the sensing masses are made as close as technically permitted, in order to minimize the effect of gravity-gradient forces, rotational motion, and linear accelerations upon the differential output signal. The two sensing masses are made of different materials. The prototype implemented at IAPS/INAF has the two sensing masses of the same material (e.g., aluminum-aluminum). This prototype could be flown in a test balloon flight so to characterize the noise environment during the free fall.

The two sensing masses are constrained by the flexural springs to rotate about a common axis and their resonant frequencies are electrostatically controlled for frequency matching. The lower is the resonant frequency of the two oscillators, the more sensitive is the detector, but with a smaller dynamic range. On the other hand, the higher the resonant frequency, the larger the dynamic range and the shorter the time constant of the transient oscillations. The value of the resonant frequency stems from a trade-off between sensitivity on one side and fast transient response and large dynamic range (and also tolerance of centrifugal forces) on the other side. A value of the resonant frequency in the range 2-5 Hz strikes a balance between the above competing requirements. Once the instrument is built with a specific mechanical resonant frequency, this frequency can be lowered by supplying a constant voltage to the feedback capacitor fixed plates. All the other modal frequencies of the instrument are at least two orders of magnitude higher than the controlled flexural frequency. This wide frequency separation allows most of the signal energy to excite the degree of freedom of interest. The sensing masses of this detector are not subject to electrostatic charging because they are grounded to the instrument case through the flexural springs. A high *Q*-factor is obtained by means of liquid helium refrigeration and by eliminating dissipation sources, i.e., the flexural springs, the instrument case and the capacitor moving plates are machined from the same block of material.

Intrinsic noise of the differential accelerometer

For a differential accelerometer using bridge capacitive transducers and in the case of the matched preamplifier, the formula to evaluate the spectral density of acceleration noise for one single accelerometer, in the case where $\omega_S < \omega_0$, can be written as:

$$S_a = \left(\frac{\omega_0 k}{m_{eff}} \left(\frac{4T}{Q} + 2T_A \frac{\omega_0}{\omega_S + \omega_P} \right) \right)^{1/2} \quad \mathrm{ms^{-2}/\sqrt{Hz}} \qquad 1)$$

The two terms in inner parentheses correspond to the Brownian noise and to the preamplifier noise, respectively; ω_0 is the detector resonant frequency; ω_S the signal frequency; ω_P the pumping frequency of the bridge; k the Boltzmann's constant; T the temperature of the sensing masses; T_A the preamplifier noise temperature; Q the quality factor and m_{eff} the effective mass of the sensing element. The effective mass is linked to the real mass m and it simply converts a translational into a rotational degree of freedom. For this detector prototype, $m_{eff} \approx 1.8$ m. Clearly, from Eq.1 the sensitivity of the detector increases by decreasing the resonant frequency and the temperature, and by increasing the effective mass of the sensing mass and the Q-factor. Liquid helium refrigeration will be used to provide low Brownian noise, high thermal stability, low thermal gradients, and a high Q-factor which are necessary to attain the desired instrument sensitivity.

Common-Mode Rejection Factor

During the free-fall the differential accelerometer is under the action of residual accelerations that act as common mode. As an example we indicate the noise induced by the vibrations of the capsule wall that propagate trough the residual gas present in it or the sinusoidal noise due to the precession of the experiment platform. The CMRF quantifies the ability of the device to perform such a rejection. A theoretical evaluation indicated in 10^4 the required value for this parameter. In figure 3 are reported the results of an experimental activity to demonstrate the capability of the experimental apparatus to obtain the right values for the CMRF. In the figures are shown respectively the two acceleration output in the time and frequency domain of the two separate accelerometers and their difference, when subject to a sinusoidal common-mode acceleration with amplitude of $10^{-3}g$; it is possible to see that at the excitation frequency the attenuation, or CMRF, is equal to 10^4. This value is obtained adjusting the amplitude and phase of one of the two output. The adjustment of the phase is at the level of $4.4 \cdot 10^{-3}\ s$.

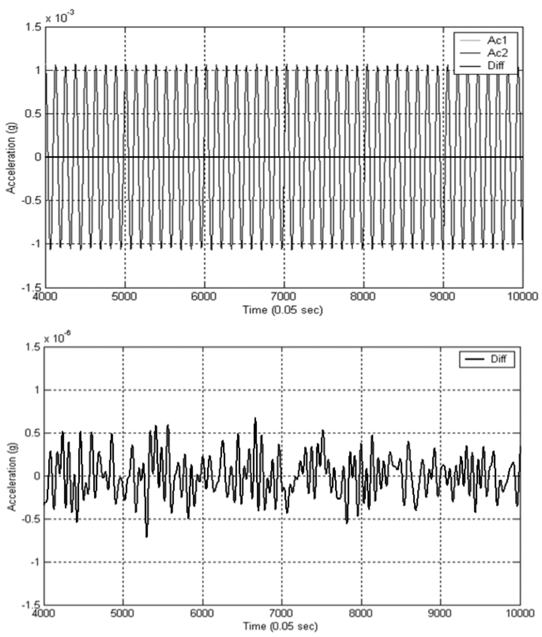


Figure 3(a)- Sinusoidal output of the two accelerometers in the time domain and their difference.

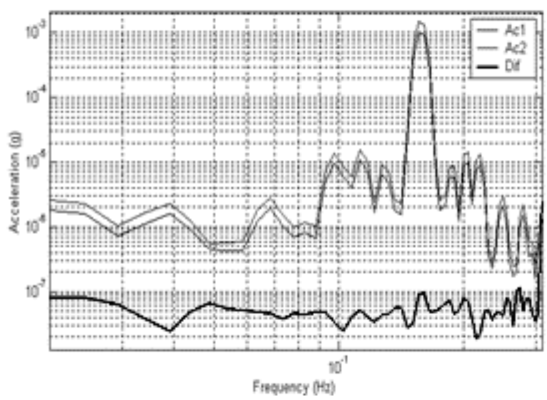


Figure 3(b) - Spectra of the signal plotted in the previous plot.

In table 2 are compared the characteristics of the precursor accelerometers already implemented with the required values to be obtained for the proposed WEP test.

Table 2 Characteristics of IFSI precursor accelerometers and WEP detector requirements.

Item	Characteristics of precursor prototype accelerometers	Requirements for WEP detector
Temperature	Ambient	Liquid He
Q-factor	1000*	$> 10^5$
Resonant frequency, ω_0	4 Hz	2 ÷ 5 Hz

Sensing mass	0.2 kg	> 1 kg
Amplifier noise temperature	10 K	≤ 100 mK
Frequency range	Wide band $10^{-5} \div 1$ Hz	Monochromatic signal at 0.5 Hz
External acceleration noise	seismic ($10^{-8} \div 10^{-10}$ g/$\sqrt{\text{Hz}}$)	≤ 10^{-12} g/$\sqrt{\text{Hz}}$
Common-mode rejection, χ	10^{-4}	≤ 10^{-4}
Linearity range	10^{6}	≥ 10^{6}
Acceleration noise (differential) spectral density	10^{-10} g/$\sqrt{\text{Hz}}$	≤ 10^{-14} g/$\sqrt{\text{Hz}}$

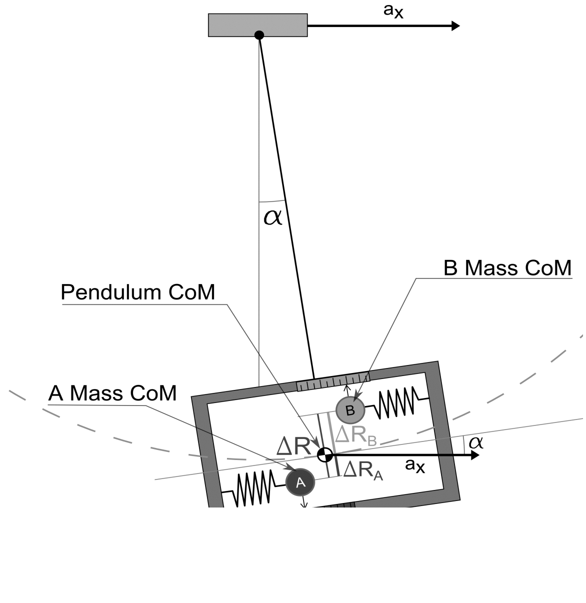
*The *Q*-factor is artificially kept at low level to prevent saturations by using a low vacuum inside the instrument.

III. WEP test using a pendulum: GREAT_G

Hereafter it is described the new idea about the possibility to perform the WEP test by installing the differential accelerometer directly inside the pendulum mass. The basic difference with respect to GReAt is that in the last one the free fall is performed in the Earth gravity field and the violation signal (otherwise in DC) is modulated at the rotation frequency (equal to 1 Hz) of the differential accelerometer during its free fall; in this new concept of experiment, the free fall is in the field of the Sun, following the fall of Earth itself (to which the experiment is solidly fixed), and the signal is modulated at the Earth rotation frequency ($1.157\cdot10^{-5}$ Hz). For GReAT the reduction of noise limiting the measurement precision is determined from the free-falling capsule (vacuum and low-temperature conditions inside it), while in the proposed new experiment this task is given to the pendulum (together with vacuum and cryogenics). It is to be stressed that the pendulum, in the new experiment, has the mere task of attenuating the vibrational noise to which the attachment point is subject, avoiding its perturbing the measurement. The general idea followed in the new experiment proposal is to employ the know-how obtained in the development of GReAT and in particular the basic principles used to implement the differential accelerometer; the proposed sensitivity for GReAT will be the starting point for an extrapolation of the possible value obtainable on ground. In table 3 it is shown a comparison between the values of the two experiments.

Table 3 Comparison between the main characteristics of GReAT and the proposed pendulum experiment.

Parameter	GReAT	GReAT_G	Note
a [m/s^2]	9.8	$6\cdot10^{-3}$	Gravitational acceleration
T [sec]	30	10^6	Integration time
a_n [$\frac{g}{\sqrt{Hz}}$]	$2.5\ \cdot 10^{-14}$	$2.5\ \cdot 10^{-14}$	Intrinsic noise of the differential accelerometer
CMRF			Common mode rejection factor
f_s [Hz]	1	$1.157\cdot10^{-5}$	Frequency of the modulated violation signal
Δa [g]	$2.5\ \cdot 10^{-14}$	$2.5\ \cdot 10^{-14}$	Differential acceleration to be detected
$\eta = \frac{a}{\Delta a}$	$5\cdot10^{-15}$	10^{-13}	Eötvös parameter: attainable value



In the following the main experiments characteristics necessary to evaluate the precision attainable with GReAT_G are discussed.

Pendulum Attenuation

With respect to the pendulum ability of attenuating the noise also at low frequency, it is necessary to remember that the *COM* of a pendulum is in "free fall" along the direction tangent to its trajectory; this implies that a test mass with *com* coincident with it is not subject to disturbances due to the noise acceleration applied at the point of attachment of the pendulum itself. From another point of view the mass of the pendulum is not an inertial reference frame; if not perfectly in the center of mass, it is subject to a differential acceleration. Referring to figure 4 we would like to remember the basic formulas for the pendulum and for its attenuations of the lateral acceleration of its pivot point:

$$\omega^2\alpha + \frac{\omega_0}{Q}\omega\alpha + \omega_0{}^2\alpha = \ddot{\alpha}_F \qquad \frac{\alpha}{\ddot{\alpha}_F} = \frac{1}{\omega^2 + \frac{\omega_0}{Q}\omega + \omega_0{}^2} \qquad \frac{a_x}{L} = \frac{\ddot{x}}{L} = \ddot{\alpha}_F$$

From these formulas it is easy to evaluate the acceleration that will be transferred in the two single accelerometers and the expected attenuation in case the length of the pendulum is 1 m and the radial distance between the two com to the pendulum COM is about 10 μm :

$$a_{1\,DIFF} = \ddot{\alpha} \cdot \Delta R_1 = a_x \cdot \frac{\Delta R_1}{L}; \qquad a_{2\,DIFF} = \ddot{\alpha} \cdot \Delta R_2 = a_x \cdot \frac{\Delta R_2}{L}; \qquad Att_p = \frac{\Delta R_1}{L} \cong \frac{\Delta R_2}{L} \cong 10^{-5}$$

Maximum attenuation on the pendulum due to its dissipations
The presence of the pendulum dissipations gives a limit on the attenuation; in some sense we can think that subject to one acceleration the pendulum starts to swing and the deceleration due to its dissipation acting on it is directly and entirely detected by the differential accelerometer. It is easy to show that this limit, connected to the pendulum mechanical Q, is given by the following formula:

$$\mathrm{Att}_{\mathrm{max}} = \frac{\omega_s}{Q_p\,\omega_p}$$

In our case, where $L = 1\,m\ and\ Q_p = 100, Att_{max} = 2 \cdot 10^{-7}$.

Pendulum Brownian Noise
Another limit in the possible precision attainable in a pendulum measurement is due to the Brownian noise that acts on the mass of the pendulum and so directly on the test masses of the differential accelerometer; this acceleration is given by the following formula:

$$a_b = \ddot{\alpha}_B \cdot L = \sqrt{\frac{4\,k\,T\,\omega_p}{Q_p M L}} = 2 \cdot 10^{-12}\ \frac{m}{s^2}/\sqrt{Hz}$$

The indicated value for the acceleration is obtained for the following values: $Q_p = 100;\ T = 300k;\ L = 1m; M = 100kg$.

Common Mode Rejection Factor
Apart from the attenuation attainable by installing the differential accelerometer on the pendulum, as we have seen before, the differential accelerometer is able to perform an extra attenuation of the vibrational noise acting on it; in some sense it is like to take into account the two differences in the distances of the two com of the test mass from the COM of the pendulum, as well as the differences in the two single accelerometers (mechanical and electrical part). By means of a calibration of this factor at low frequencies, we hope to obtain a value equal to $CMRF = 10^3$.

Integration time
As we said, the free fall of the experiments will be performed in the field of gravity of the Sun and in order to ensure a large reduction of the stochastic noise, the signal will be read-out for about $10^6\ s$ (about 12 days), with a consequent attenuation of a factor 10^3. It is clear that this attenuation works only for the stochastic noise and not for the deterministic one.

Intrinsic Noise of the Differential Accelerometer
As already said, in this new experiment we will use the differential accelerometer developed for GReAT whose sensitivity is equal to:

$$5.7 \cdot 10^{-14}\ \frac{m}{s^2}/\sqrt{Hz}$$

Horizontal noise acceleration acting on the pendulum pivot
The experiment will be performed in a very quiet laboratory, making sure that the horizontal acceleration is close to the minimum as indicated in the geophysical literature for the New Low-Noise Model (NLNM). The values at frequencies of 1/(24·3600), where we like to see the violation signal, taken from the NLNM is equal to $a_x = 10^{-5}\ \frac{m}{s^2}/\sqrt{Hz}$.

Estimate of the precision attainable with GReAT_G

The $a_x = 10^{-5}\ \frac{m}{s^2}/\sqrt{Hz}$ acceleration acting on the pivot of the pendulum is attenuated by the factor of 10^5 by the pendulum, so that every element of the differential accelerometer detects an acceleration equal to:

$$a_1 = \ddot{\alpha} \cdot \Delta R_1 = a_x \cdot \frac{\Delta R_1}{L} = 10^{-5} \cdot 10^{-5} = 10^{-10}\ \frac{m}{s^2}/\sqrt{Hz}\ ;$$

$$a_2 = \ddot{\alpha} \cdot \Delta R_2 = a_x \cdot \frac{\Delta R_2}{L} = 10^{-5} \cdot 10^{-5} = 10^{-10}\ \frac{m}{s^2}/\sqrt{Hz}$$

Taking into account the $CMRF = 10^3$ and the integration time $T_I = 10^6\ s \equiv 11.57\ g$ the resulting differential noise acceleration is:

$$a_2 - a_1 = \ddot{\alpha} \cdot L \cdot \frac{\Delta R_1}{L} \cdot \frac{1}{CMRF \cdot \sqrt{T_I}} = 10^{-10} \cdot 10^{-6} \cdot = 10^{-16}\ \frac{m}{s^2}$$

Remembering that the gravitational acceleration of the Sun is about $6 \cdot 10^{-3}\ m/s^2$, it results

$$\eta = \frac{\Delta a}{a_s} = \frac{10^{-16}}{6 \cdot 10^{-3}} = 1.6 \cdot 10^{-14}$$

In this evaluation we neglect the noise terms introduced by the intrinsic noise of the differential accelerometer, equal to $5.7 \cdot 10^{-14}\ \frac{m}{s^2}/\sqrt{Hz}$ and integrated for $10^6\ s$.

In figure 5 are shown the renderings of the cryostat inside which it is installed the pendulum, with a detail of the pivot that will be used.

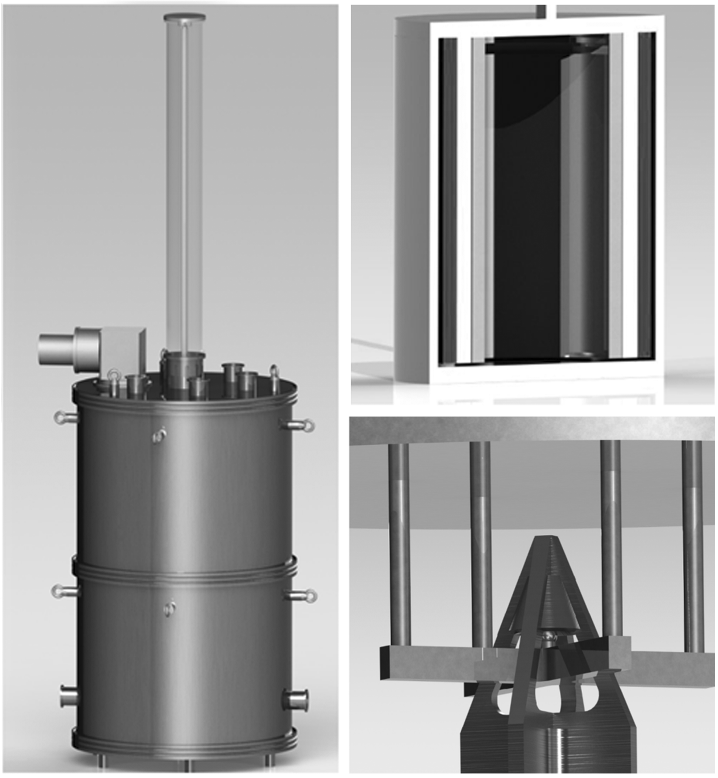


Figure 5 - Drawing of the cryostat inside which is installed the pendulum, with a particular of the pivot that will be used.

IV. Conclusions

The reported evaluation indicate the possibility of performing a test of the Weak Equivalence Principle on ground and in the Sun gravity field with a precision of a part on 10^{13}; this result will not improve on what has been obtained so far with the classical experiments employing torsion balances and Lunar Laser Ranging; its importance lies in the fact that it employs the differential accelerometer developed for the Weak Equivalence Principle test via a free fall from a stratospheric balloon, without the necessity of bringing in substantial modifications, instead simplifying its operations (its rotation being not necessary). Preliminary evaluations point to the possibility of an order of magnitude precision improving, consequently gaining by an order of magnitude the overall test precision with respect to the state of the art.

Of not secondary importance is the development of a technique for testing a high-sensitivity differential accelerometer, which can be used in a subsequent, GReAT-kind, experiment; not disregarding the possibility of exporting the same technology in a possible space experiment, in which the differential accelerometer is installed on-board an Earth-orbiting spacecraft, bringing back in this case at the value of g the acceleration of the field in which the experimental package falls, modulating the possible violation signal at the spacecraft orbital frequency and still having an integration time of tens of days. In this last condition the spacecraft will have to be of the drag-free type, that is capable of reducing the noise acting on it by non-gravitational accelerations. The noise reduction task with drag-free techniques, common with every in-orbit Weak Equivalence Principle experiment, is simply delegated to the pendulum in the GReAT_G experiment.

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