From deterministic chaos to noise in optical delay systems

Du chaos deterministe au bruit dans des systèmes optiques avec retard

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abstract and key words

An optical device with a variational structure driven by a delayed feedback, $F(x(t-d)) = \sin Ax(t-d)$ is shown to display high dimensional chaos with dimension increasing linearly with two parameters, the delay d and the feedback frequency A. For large delay d and large frequency A, the system is shown to display Gaussian-Markovian statistics like a system driven by a white noise. Decreasing the frequency A leads to effects very similar to those of a colored noise, and decreasing the delay leads to quite special phenomena like phase transitions, giving rise to new peaks in the probability distribution. An analytical description using the tools of stochastic equations agrees with the numerical results.

Delay differential equations, Langevin equation, Lyapunov dimension, Gaussian Markovian statistics, phase transition.

résumé et mots clés

Un système optique possédant une structure variationnelle, et forcé par une rétro-injection retardée, $F(x(t-d)) = \sin Ax(t-d)$, émet un signal lumineux chaotique dont la dimension croit linéairement avec deux paramètres, le retard d et la fréquence A. Pour de grandes valeurs du retard et de la fréquence, le signal émis a une statistique Gaussienne et Markovienne, comme la solution d'une équation de Langevin forcée par un bruit blanc. Lorsqu'on diminue la valeur du paramètre A, la statistique est modifiée comme celle d'une équation de Langevin forcée par un bruit coloré. Lorsqu'on diminue le retard, de nouveaux pics apparaissent dans la distribution de probabilité, comme dans les transitions de phase induites par du bruit coloré. Une description analytique utilisant les méthodes des signaux aléatoires permet d'interpréter les résultats numériques.

Equation différentielle à retard, équation de Langevin, dimension de Lyapunov, statistique Gaussienne et Markovienne, transition de phase.

1. Introduction

The present study concerns deterministic processes, however it is noteworthy that a large part of this study uses the tools of signal theory initiated and largely developped by B. Picinbono. Signal theory is most often dealing with stochastic equations including a noise source term. On the contrary deterministic equations by definition lack any *a priori* noise term. Nevertheless chaotic signals are observed in high dimension deterministic systems, like partial differential equations (PDE), leading to turbulence, as well as in low dimensional ones, for example in a set of three ordinary differential equations, like the famous Lorenz equations[1], which displays temporal chaos of low fractal dimension. Here we consider a system described by a deterministic delay differential equation, displaying high dimensional temporal chaos. How much differ the properties of deterministic chaotic regimes from those of the random chaos? We limit ourselves to a class of equations of type

$$\dot{x}(t) + \frac{\partial V(x)}{\partial x} = F(x(t-d)) \qquad (1)$$

which has a well-known mechanical analogy, as it describes the motion of a particle inside a potential $V(x)$ driven by a force F. Usually the random motion is supposed to be generated either by an additional noise source leading to a stochastic equation of type (3), or by a diffusion process, leading to a PDE. In Eq. (1), the motion is driven by a delayed force, all the times being scaled to the response time of the system. Actually the time delay comes into play in various evolution equations[2], in physiology[3], economics[4], astrophysics[5], combustion[6], geophysics[7], hydrodynamics[8], and optoelectronics circuitry[9]. In such systems the dynamics are generally characterized by two different time scales, for example a short damping rate, and a long feedback time.

The differential delay equations have been shown to display chaotic solutions with a Lyapunov dimension linearly increasing with the delay[10]. Therefore, Eq. (1) can generate chaos of dimension as large as desired. That seems paradoxical, since it implies that a deterministic equation may generate chaos with all characteristics of complete randomness. Is it really random chaos? Any answer should generally be very difficult to state, because the mathematical definition of a random function requires multitime probability distributions for any set of time. There are some exceptions, for example the Gaussian noise, which is completely defined by the two-time probabilities. For this reason, it is especially interesting to find a determistic system giving something very similar to Gaussian random noise. The optoelectronic, or hybrid device[11], with linear damping and periodic feedback

$$\dot{x}(t) + x(t) = \sin Ax(t-d) \tag{2}$$

offers this opportunity. The role of the feedback memory is emphasized in section 2, leading to a physical understanding of the Lyapunov dimension. The rich temporal dynamical behavior of this system is described in section 3. By increasing the feedback parameter A, equation (2) is shown to asymptotically behave as a linear Langevin equation driven by a white noise $\xi(t)$

$$\dot{x}(t) + x(t) = \xi(t) \tag{3}$$

An approximate Fokker-Planck equation is presented in section 4 for intermediate values of A. The delay is shown to induce new peaks in the probability distribution. Similar phenomenon occurs in phase transition problems due to a non white noise.

2. Dimension proportional to the delay

The delay differential equations are infinite dimensional in the sense that they need an infinite number of initial conditions to be solved, since one must know the value of the variable x on the whole time interval $(0, d)$. This property may also be understood by writing formally the delay term as an infinite differential operator acting on the function $x(t)$, *i.e.* $x(t-d) = \sum_n \frac{(-1)^n}{n!} \frac{d^n}{dt^n} x(t)$. Then a delay differential equation may correspond to an infinite system of ODEs. They were also compared[12] to one-dimensional PDEs, when setting $t = \sigma + nd$, with a space-like variable σ defined on the domain $(0, d)$ and a discrete time variable n. These two different pictures point out that the delay creates the high dimensionality of delay differential equations.

2.1. historical view of the delay role in optics

Indeed, taking the delay as control parameter, one may observe successive bifurcations from the steady state solution until chaos. This property was emphasized by Ikeda[13], who first predicted optical chaos in the transmitted intensity $I(t) = |E(t)|^2$ of a passive ring cavity with two-level atoms, illuminated by a continuous-wave (CW) laser. In the dispersive limit, the two equations for the total energy w stored by the atoms, and the complex electric field amplitude $E(t)$ are

$$\dot{w} + w = -1 - w\,|E|^2 \tag{4}$$

$$E(t+d) = E_0 + RE(t)e^{i\theta w(t)} \tag{5}$$

where the time is scaled to the atomic relaxation time, θ is the linear refractive index, E_0 the CW input field amplitude, and R the reflectivity of the mirrors.

Earlier studies of this problem were performed by using the time mapping model

$$E_{n+1} = E_0 + RE_n e^{i\theta(1-|E_n|^2)} \tag{6}$$

assumed to be a fair representation of solutions of Eqs. (4-5) for large delays. However, as shown in Fig. 1, the numerical simulations of eqs. (4-5) invalidates the mapping model for the description of chaos. The mapping approximation is valid for transients only, which displays successive square signals of duration d and various amplitudes. Progressively the damped oscillations formed at the beginning of each interval $(nd, \varepsilon t(n+1)d)$ fill the whole delay interval, leading to a noisy signal with small correlation time[14],[15]. We showed[14] the inadequacy of the discrete mapping for describing the chaotic signals, by computing the dimension D_L of the chaotic solutions, using the Kaplan and Yorke conjecture[16] which relates the dimension to the Lyapunov exponents set in decreasing order, $D_L = j + \sum_i \Lambda_j / |\Lambda_{j+1}|$, where j is the largest integer for which $\Lambda_1 + .. + \Lambda_j \geq 0$. The Kaplan and Yorke conjecture was verified by Farmer[10] for the Mackey-Glass differential delay equation. The dimension was shown to increase linearly with the delay for Eqs. (4-5), while the dimension of the mapping solution is always smaller than 2. For example D_L is equal to 1.6 in the

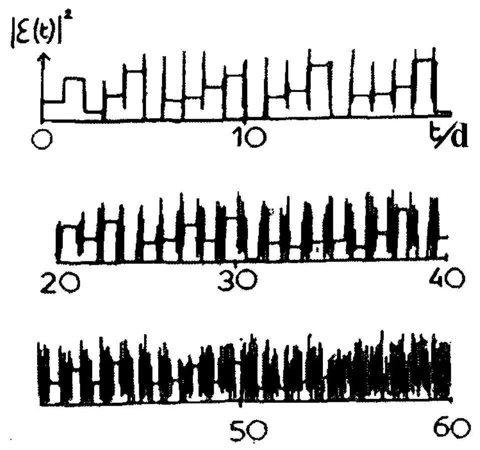

Figure 1. – Temporal evolution of the output intensity with the ring cavity model including absorption, Eqs.(2-4) of Ref. 15, for d = 100, $\theta = 6\pi$, $\alpha\ell = 4$, $E_0 = 1.7$.

transients of Fig. 1 (mapping solution), while it is about 420 after transients. Contrary to what could be guessed, the larger is the delay, the larger is the difference between the results borrowed from the mapping model (6), and the actual solution of the delay differential system (4-5).

The confusion about the validity of the mapping model lasted several years among the specialists of optical chaos. It was supported by the first experimental observation of period doubling route to chaos in the hybrid device[11], interpreted thanks to a mapping deduced from Eq. (2).

2.2. interpretation of the dimension.

In order to understand the role of the delay, we studied the chaotic regime of the hybrid (Eq. 2) and ring cavity (Eqs. 4-5) devices, and compared with Farmer's study on the model proposed by Mackey and Glass[3] to describe the white blood cell production by the bone marrow

$$\dot{x}(t) + x(t) = \frac{Ax(t-d)}{1 + x^{10}(t-d)} \tag{7}$$

These investigations were performed through ergodic quantities like the spectrum of Lyapunov exponents, $\{\Lambda_i\}$, the dimension D_L, which is approximately twice the number of positive Λ_i, and the metric entropy h which is the sum of positive Λ_i. In the three systems the dimension was found to linearly increase with the delay, while the entropy h was nearly constant. Farmer already foresaw that the behavior of both h and D_L should be related to a sampling time other than the delay. We showed that this other time is the correlation time δ of the feedback force $F(x(t))$. In all three cases we have found [15] that h and δ are independent of and much smaller than the delay, and that the dimension obeys approximately the rule

$$D_L \approx \frac{d}{\delta}, \tag{8}$$

leading to an intuitive understanding of this large dimentional chaos. The interaction between the signal and its feedback can be seen as a set of kicks, of mean duration δ. Inside the n^{th} interval, $(nd, (n+1)d)$ the $\frac{d}{\delta}$ kicks are uncorrelated events, while each of them is correlated to another kick in the $(n-1)^{th}$ interval, as confirmed by the secondary extrema of the feedback correlation functions at approximately $d, 2d, 3d$.

Finally the number of kicks is the effective number of degrees of freedom, and the entropy is the average amount of information stored during each kick.

3. Gaussian statistics for the hybrid device

The hybrid and the Mackey-Glass equations (2, 7), are systems of type (1) with *linear damping*. They have the formal asymptotic solution

$$x(t+d) = \int_0^{t+d} du e^{-u} F(x(t-u)) \tag{9}$$

where the feedback depends on a parameter A. While both systems display high dimensional chaos (for large delay), they display very different statistics, because of different expressions for the feedback $F(x)$. One may understand the essential role of the feedback shape $F(x)$ from the following argument. The function $F(x) = \frac{Ax}{1+x^{10}}$ is a bell shaped function with width of order unity. On the contrary, as A increases $F(x) = \sin Ax$ oscillates more and more rapidly, with respect to the variable x .

3.1. correlation functions

Let us focus on the case of the periodic feedback, eq. (2). When $x(t)$ slightly increases until $x + 2\pi/A$, the strength of the feedback force oscillates from -1 to $+1$. Therefore the characteristic times of $x(t)$ and $F(x(t))$ are drastically different. Analytical expressions of the correlation functions of the signal, $\Gamma_x(\tau)$, and of the feedback $\Gamma_F(\tau)$ have been derived[17]. The shortest time scale appears in the expression of $\Gamma_F(\tau)$, which has an area $D = \int_0^\infty \Gamma_F(\tau) d\tau$ nearly equal to $1/2A$, and a width at mid-height of the central peak,

$$\delta = 2/A \tag{10}$$

in good agrement with the numerical curves (Fig. 2).

Three time scales appear in $\Gamma_x(\tau)$, it is quadratic for τ of order $\sqrt{\delta}$, then decreases exponentially as

$$\Gamma_x(\tau) = e^{-|\tau|} \qquad (11)$$

and finally displays secondary peaks at $\tau = d, 2d, 3d..$, as illustrated in Figs. 7 of Ref. 17. These time scales are also visible on the solution $x(t)$ shown in Fig. 3, which displays a complex dynamical behavior.

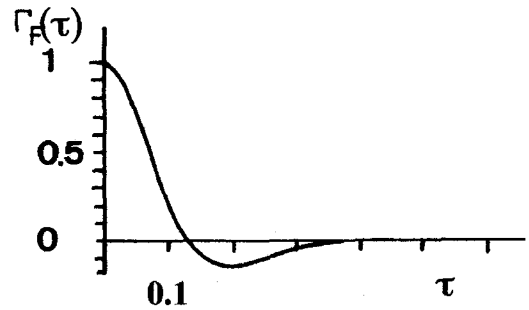

Figure 2. – Correlation function $\Gamma_F(\tau)$ of the sine feedback sin[Ax(t)], for Eq.(2), with A = 24, d = 5.The secondary peaks at τ = nd vanish for this large value of A, they are visible for A smaller or equal to 10.

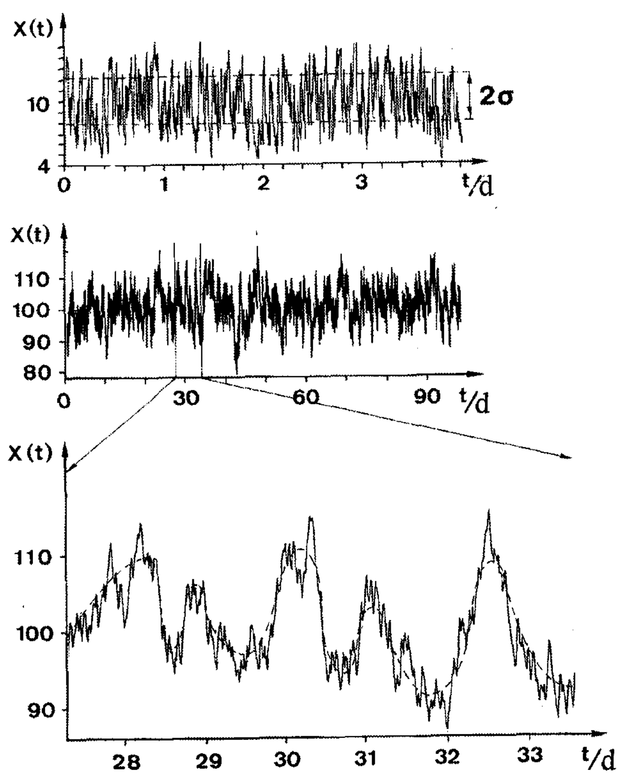

Figure 3. – Temporal evolution of the scaled variable X(t) = Ax(t) + X_0 for the hybrid device, with d = 1. Curve (a) corresponding to A = 6.7, X_0 = 10, it is a low dimensional chaos, with $D_L \simeq 7$, and a variance σ reported on the right. The lower curves (b-c) corresponding to A = 67, .X_0 = 100, are high dimensional chaos, with $D_L \simeq 70$. In curve (c) which is an enlarged part of curve (b), the two characteristic time scales $\sqrt{\delta} \simeq 0.2$, and 1 are visible.

3.2. probability distribution

In the limit of large A, Eq. (9) shows that the quantity $x(t + d)$, as given by the right hand side, is a sum of an infinite number of independent contributions, with decreasing weights. A generalization of the central limit theorem allows to conclude that the variable $x(t)$ tends to a Gaussian one, as confirmed by the numerical investigations[17]. Therefore generic solutions of Eq. (2) behave like solutions of a linear stochastic Langevin equation driven by a white noise, Eq. (3). For large but finite values of A, the function $x(t)$ is a Gaussian process for a sampling time larger than δ.

Let us note that this differential delay system contains two parameters A and d working towards large dimensionality of chaos. The Gaussian character of the solution is due to the periodic feedback, which oscillates very rapidly for large A. While any differential delay equation displays high dimensional chaos for large delay, only those with short memory feedback lead to Gaussian statistics. Indeed the probability distribution of the chaos for the ring cavity model (4-5), or the Mackey-Glass model (7), or the Bernoulli model[19] is not Gaussian whatever the value of A[18].

4. Fokker-Planck description

We shall now use another description of the solution $x(t)$, very conveniently used in the case of stochastic equations. It consists in deriving the evolution equation for the probability distribution $P(x, t)$, bypassing the difficulty of defining the random functions. One starts from the Kramers-Moyal expansion[20]

$$\frac{\partial P}{\partial t} = \sum_n \frac{(-1)}{n!} \frac{\partial^n (B_n P)}{\partial x^n} \qquad (12)$$

where the coefficients B_n are defined as conditional momenta of small increments, for a given value $x(t) = x$,

$$B_n = \lim_{\tau -> 0} < [x(t + \tau) - x(t)]^n >_x \qquad (13)$$

For Markov processes the B_n's are independent of the past. Moreover when only the drift and diffusion coefficients B_1 and B_2 are nonzero, the expansion in Eq. (12) reduces to the well-known Fokker-Planck equation. This is the case of the linear Langevin equation (3) driven by an added white noise.

4.1. linear damping, quadratic potential

For our delay system with small memory time δ, the coefficients B_n are shown [18] to be of order δ^n. In the limit of large A, one has

$B_1 = -\frac{\partial V}{\partial x}$ and $B_2 = D$. The stationary probability distribution is then

$$P(x) = Ne^{\frac{-x^2}{2D}} \tag{14}$$

as for a stochastic Langevin equation, in agreement with the central limit theorem derivation carried in section 3.

What is the probability distribution for decreasing values of A? An accurate analysis showed that, in the range $15 \leq A \leq 70$, the probability remains Gaussian, it obeys Eq. (14) with $D \rightarrow D_{eff} = D(1 - \tau_F)$, where $\tau_F = 2\pi/A$ defines the total width of the correlation function Γ_F (*cf.* Fig. 2). This result agrees with recent studies[21] on Langevin equations driven by a narrow coloured noise of correlation function $\Gamma_\xi(\tau) = \frac{D}{\tau_F} e^{-\left|\frac{\tau}{\tau_F}\right|}$.

In the parameter range $A \simeq 10$, the memory time of the feedback force $\sin Ax(t)$ becomes of the order of the response time of the system (*cf.* Fig. 1 of Ref. (18)). The correlation function $\Gamma_F(\tau)$ widdens its central part, falls to zero for τ_F of order unity, and gets secondary peaks at $\tau = d, 2d...$, showing a non-Markovian behavior. However the knowledge of $\Gamma_F(\tau)$ allows to estimate the coefficients B_n, and to derive an approximate Fokker-Planck equation, with a diffusion coefficient D_{eff}, and a drift term

$$B_1 = -\frac{\partial V}{\partial x} + < \sin\left[Ax(t-d)\right] >_x \tag{15}$$

which is responsible for the change in the probability distributions in agreement with the numerics (Figs. 4). For large delay the function $P(x)$ is still bell-shaped, the long range memory of the feedback just flattens the maximum. But for delay of order unity, the central peak of the distribution splits, giving rise to a double-hump function. In the case of system driven by noise, such a phenomenon was also observed, and named "noise-induced transition". In our delay system, new peaks in $P(x)$ occur when the memory time of the feedback becomes of the order of the response time. They also occur in the case of low dimensional chaos (for $A = 4$, *cf.* Fig. 13-b of Ref. 18, with D_L typically of order 5). Fig. 4 shows that as d decreases, the noise-like chaotic solution bifurcates into another one with different statistics. Therefore the delay acts as a bifurcation parameter inside the chaotic regime. In order to differentiate this phenomenon from the phase transition induced by noise, we call our observation "delay-induced transition".

4.2. nonlinear damping, quartic potential

The above study was generalized[18] to the case of nonlinear damping, for quartic single and double-well potentials corresponding respectively to $V(x) = \frac{1}{4}x^4$, and

$$V(x) = \frac{1}{4}(x^2 - B^2)^2 \tag{16}$$

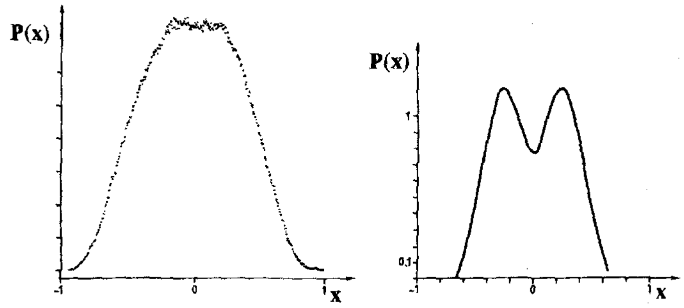

Figure 4. – Numerical probability distribution P(x) for the solution of Eq. (2), with $A = 10, d = 10$ (left curve), and $d = 1$ (right curve).

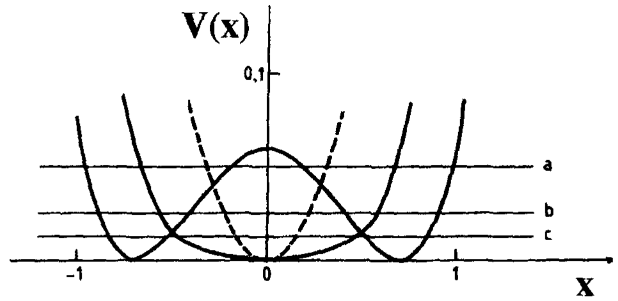

Figure 5. – The single and double-well quartic potential are drawn in full lines, the quadratic potential $V(x) = x^2/2$ in dashed line. The horizontal levels a, b, c correspond respectively to the values of $D \simeq 1/2A$ for $A = 10, 20, 40$.

drawn in Fig. 5. A striking feature is the invariance of $\Gamma_F(\tau)$ for very different shapes of the potential. Everything works as if the feedback $F(x(t)) = \sin Ax(t)$, has its own dynamics, independently of the left-hand side. This explains why in this case, Eq. (1) may be compared to a Langevin equation driven by noise, even in the case of finite values of A. Actually the statistics of the sine feedback force are neither Gaussian, nor exponentially correlated (*cf.* Fig. 2), that is noticeably different from most of the studies on Langevin equations dealing with exponentially correlated Gaussian forces.

The solution $x(t)$ of Eqs. (1-16) describing the chaotic motion of a particle inside the double-well potential, is drawn in Fig. 6. The particle stays a much longer time in one of the wells for $A = 20$ than for $A = 10$. The mean-first passage time τ_x, which is the decay time of the correlation function of $x(t), \Gamma_x(\tau) \simeq (-\left|\frac{\tau}{\tau_x}\right|)$, fits very well the Kramers relation for stochastic double-well systems

$$\tau_x = \frac{\pi}{\sqrt{2}B^2}(\frac{B^4}{4D}) \tag{17}$$

where $B^4/4$ is the height of the barrier in the potential $V(x)$.

The probability distribution, reported in Figs. 6 of Ref. (18) agrees with the one of the Langevin equation solution,

$$P(x) = Ne^{\frac{-V(x)}{D}} \tag{18}$$

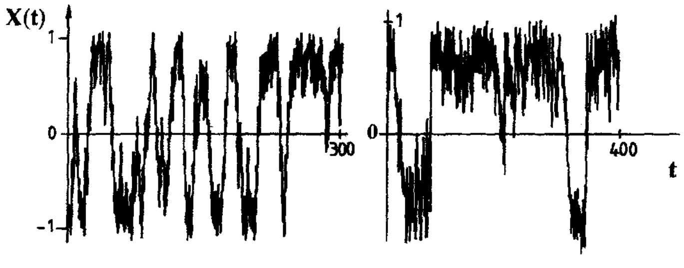

Figure 6. – Time traces of the double-well chaotic solutions for B = 0.7, d = 1, A = 20 (right curve) and A = 10 (left curve).

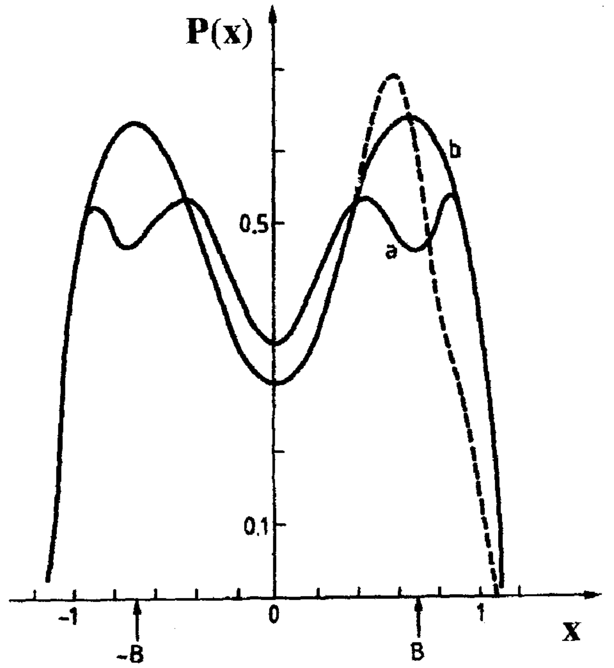

Figure 7. – Numerical probability distribution for the chaotic double-well potential solutions, for B = 0.7. Curve (a) shows the delay induced transition phenomenon, for A = 10, d = 1. Curve (b), for A = 10, d = 20, approximately obeys the expression in Eq. (18). The dashed line (only the right part is reported for clarity), corresponds to A = −10, d = 1. In this case the quantity changes sign, $< \sin[Ax(t-d)] >_x$ leading to an enhanced peak.

For $A \simeq 10$, the probability distribution is approximately given by $P(x) = Ne^{\frac{-V_{eff}(x)}{D}}$, with the effective potential $V_{eff}(x) = \int_0^x B_1(u)du$, as illustrated in Fig. 7. The feedback memory effects are similar to those described in the case of linear damping. For delay of order unity, new peaks are formed on the extrema of the probability distribution. The delay induced transition phenomenon is then independent of the shape of the potential.

In the parameter space (A, d), a fuzzy border line may be defined, between randomlike and deterministic chaos. Along the boundary the delay structure of the equation is visible as a kind of phase transition. Beyond the boundary, one goes asymptotically towards random chaos.

In summary we have presented a deterministic system which continously evolves from a purely deterministic towards a stochastic behavior, by increasing two parameters A and d. By adjusting the parameter A, it is easy to control the feedback memory, and by adjusting the delay, to control the dimension.The stochastic behavior occurs for large delay and large A. High dimensional chaos for intermediate value of A also presents analogy with the solutions of a *Langevin* equation driven by a coloured noise, while here the correlation function of the feedback force is not exponential. The above study definitely proves the inadequacy of the mapping approximation for describing the dynamics of the high dimensional solutions. Surprisingly, the low-dimensional chaotic solutions obtained for intermediate values of A and delay of order unity, present some analogy with a stochastic system driven by coloured noise.

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