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OPTIMIZATION PROBLEM OF ALLOCATING LIMITED PROJECT RESOURCES WITH SEPARABLE CONSTRAINTS

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Abstract. *The authors consider the mathematical model and solution method for the optimization problem of the allocation of limited resources of a project as a problem of the arrangement of rectangular objects, where objects being placed have variable metric characteristics that are subject to functional dependences. The partial quality criteria and the constraints of the feasible domain of the problem are formalized.*

Keywords: *optimization, allocation of limited resources, arrangement of geometrical objects.*

INTRODUCTION

Problems of the optimal allocation of various limited resources such as financial, time, personnel, material, etc., arise in many practical fields. Two types of problems are distinguished:

— optimal enterprise resource planning [1] in batch production, where the time resource is not limited or this limit is not critical, i.e., continuous planning;

— investment project resource management [2], where a finite set of operations have a unique set of properties and subject to performance in strict time frame.

The importance of these problems increases because of the increasing pressure on economic entities caused by competition, stringent requirements of investors and ultimate customers, and other objective factors.

This range of problems is the focus of constant attention of experts [3–6]. Academician Glushkov initiated investigations in this field in our country, made a huge contribution to their organization, participated and supported the formation of the research areas such as network planning, enterprise resource management, and scheduling theory.

Academician Mikhalevich and his scientific school obtained the fundamental scientific results [7, 8], which underlie exact and approximate methods of the solution of optimization problems of industrial scheduling, including determination of shortest network paths.

Solution tools for limited-resource distribution problems based on fuzzy logic, soft computing, interval mathematics, etc., are presented in [9, 10].

An important class of problems of the second type are optimization problems of the resource management of construction–investment projects [3, 11], including engineering services reconstruction. The analysis of their practical formulations shows that such projects engage considerable capital investments, use a great amount of fixed and floating assets, and are long-term, which increases the risk of project failure under conditions of the dynamically varying environment.

The solution of these problems suggests continuing the studies on their modeling as multidimentional multicriteria problems of operations research theory [12, 13], including the development of a technique for the allowance for technological constraints and preferences of a decision-maker (DM) in forming the feasibility region of the optimization problem.

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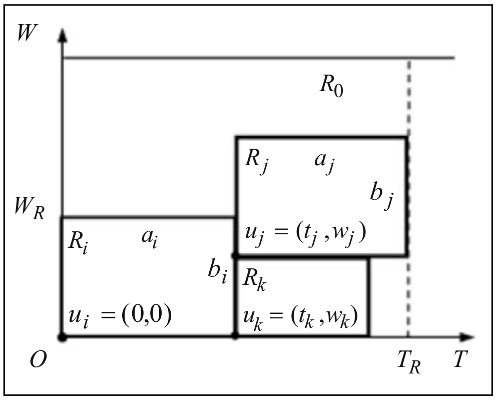

Fig. 1. Schematic arrangement of jobs $\{R_i, R_j, R_k\}$ in resource space.

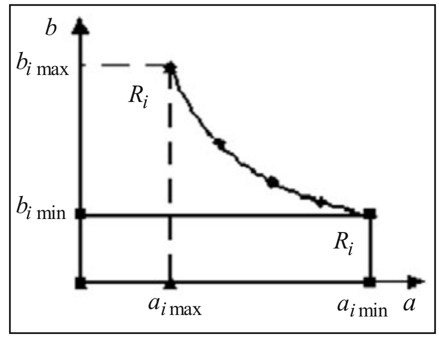

Fig. 2. Graphic representation of job R_i for different metric characteristics.

In the paper, we will develop the approach proposed in [14, 15] and based on the use of results of the field of optimization geometrical design theory [16] such as the arrangement of geometrical objects with variable metric characteristics and spatial shape in a bounded domain [17].

The purpose of the paper is to determine and formalize the partial criteria of solution quality and the set of constraints for the feasibility region of the optimization problem of the allocation of limited resources of the project as the problem of the arrangement of rectangular objects with variable metric characteristics, related by functional dependences, development of optimization methods of problem solution, and carrying out numerical experiments.

PROBLEM STATEMENT AND CONSTRUCTION OF THE OPERATION MODEL

Let us consider the practical resource management problems arising at different stages of the construction–investment project.

Let there be a project R consisting of N jobs (operations), $R = \{R_i\}$, $i = 1, \dots, N$. The partial order condition $R_i \prec R_j$, $i, j \in \{1, \dots, N\}$, $i \neq j$, defined by the specific sequence of jobs (job R_j immediately follows R_i) is specified on the set of jobs R. Note that determining this sequence involves the DM (project manager) since in practice there can be several such sequences.

For each job R_i its amount S_i expressed in man $\cdot$ hours, $S_i = \text{const}$ is known. No more than W_i immediate executors are allocated for the project in the large at every instant of time.

It is necessary to make a schedule of project jobs, optimal in the required resources.

Let us consider this problem as a 2*D* problem of the theory of optimizational geometrical design, within which the properties of the objects under study are interpreted as geometrical characteristics. Then project resources in the large can be represented as a domain R_0 of the two-dimensional space of resources *OTW*, where *OT* is the time axis (project runtime), *OW* is the axis of the work force whose scale is coordinated with measure units of *T*.

Each job R_i of the project R can be represented as a layout object R_i, of rectangular shape $R_i(a_i, b_i)$. The metric characteristics a_i and b_i denote the job run time R_i and the number of its executors, respectively, at each instant of time. The time of the beginning of job execution R_i and its association with the necessary amount of work force are determined by the parameters $v_i = (t_i, w_i)$ of job arrangement in the resource space *OTW* (Fig. 1).

Remark 1. On the assumption that metric characteristics of the objects R_i, $i = 1, 2, \dots, N$, are variables: a_i, b_i −var, and the volume S_i, $S_i = a_i \times b_i = \text{const}$, the relation $b_i = S_i / a_i$ holds.

Remark 2. Based on the allowance for the technical characteristics of the project for executing each job R_i, the maximum and minimum admissible values of resources are chosen. In other words, metric characteristics of object R_i are elements of sets A and B (Fig. 2):

$$a_i \in A_i, \; b_i \in B_i, \tag{1}$$

where $A_i = [a_{i\,\min}, a_{i\,\max}]$, $B_i = [b_{i\,\min}, b_{i\,\max}]$, $a_{i\,\min} > 0$, and $b_{i\,\min} > 0$.

Depending on the formulation of a specific problem, the sets A and B can be both continuous and discrete. If resource representation is discrete,

$$b_i = \lceil S_i / a_i \rceil. \tag{2}$$

Note that the relations $S_i \le a_{i\min} \times b_{i\max}$ and $S_i \le a_{i\max} \times b_{i\min}$ hold for $\lceil S_i / a_i \rceil \ne S_i / a_i$. Hence, in such formulation parameters $u_i = (v_i, a_i) = (t_i, w_i, a_i)$ are endogenic for job R_i. Next we assume that the notation R_i is equivalent to $R_i(u_i)$.

The set of jobs R_j immediately following R_i can consist of more than one element $j \in \{1, 2, \dots, J\}$, $1 \le j < N$, $i \ne j$. Denote by I_i^j the set of indices of such jobs.

Thus, the optimal resource allocation problem can be formulated as the problem of the arrangement of N rectangles $\{R_i\}$ without overlapping in a rectangle R_0 of the form

$$W_R \times T_R \to \min_{\mu \in D \subset E^{3N+2}}, \tag{3}$$

where W_R and T_R are metric characteristics of the rectangular layout domain R_0, $u = (u_1, u_2, \dots, u_N)$ and D is the feasibility domain. It has the following system of constraints:

— layout of the set of objects R in R_0 (the condition of the presence of each job as a part of the project)

$$R_i \subset R_0; \tag{4}$$

— the condition of mutual pairwise non-intersection of the objects (ensuring the impossibility of using one resource by two jobs simultaneously)

$$\operatorname{int} R_i \cap \operatorname{int} R_j = \varnothing; \tag{5}$$

— the condition of partial order of the jobs

$$R_j \succ R_i, \; j \in I_i^j; \tag{6}$$

— constraints for the value of job resources

$$a_i \in A_i, b_i \in B_i; \tag{7}$$

— the condition of retaining the amount of job of the project

$$b_i = S_i / a_i, \; i, j = 1, 2, \dots, N, \; i \ne j. \tag{8}$$

In view of Remark 1, the problem under study belongs to the class of problems of the arrangement of geometric objects with variable metric characteristics provided that the latter are related by functional dependences.

As a matter of fact (and by the standard solution technique [13]), the problem is two-criteria and can be represented as a sequence of two one-criterion problems:

$$T_R \to \min_{u \in D_1 \subset E^{2N+1}}, \tag{9}$$

$$W_R \to \min_{u \in D_2 \subset E^{2K+1}}. \tag{10}$$

The subdomain D_1 of the feasible domain D of problem (9) is defined without constraints for W_R (or W_R is assumed sufficiently large).

Solving problem (9) yields the duration T_R^* of the critical path [13] of the project R, the set of critical operations R_k, $k = 1, 2, \dots, K_1$, whose total duration constitutes the duration T_R^* of the critical path, and total time reserves for noncritical operations $\overline{R}_k$, $k = 1, 2, \dots, K$, $K + K_1 = N$.

The domain D_2 of the feasible solutions of problem (10) is defined by the constraints (4)–(8) provided that $T_R = T_R^*$ and the parameters of the arrangement of objects modeling critical operations R_k, $k = 1, 2, \dots, K_1$, are constant.

In solving problems (9), (10), parameters a_i, $i = 1, 2, \dots, N$, of the objects are fixed (generally speaking, for problem (10) this condition is obligatory only for critical operations). Criterion (10) can be represented as

$$\Delta W \to \min_{u \in D_2 \subset E^{2K+1}}, \tag{11}$$

where

$$\Delta W = \max_{t \in \{1,2,\dots,T_R\}} W_t - W_{t-1}, \tag{12}$$

which is an equivalent representation in terms of the basic problem.

Thus, the ultimate goal of the solution of problem (3)–(8) is to construct the optimal schedule $G(T_R, \Delta W)$ of the project R.

If the parameters of the project obtained as a result of the solution of problems (9), (11), and (12) are such that additional resources can be allocated for its prompt execution, then the third problem occurs, of bringing the project to a more intensive level of performance [13] of the form

$$T_R \to \min_{u \in D_3 \subset E^Z}, \tag{13}$$

where $Z = 3K_1 + 2K + 1$. The metric characteristics of the objects modeling the critical operations R_k, $k = 1, 2, \dots, K_1$, are variable.

ANALYTIC DESCRIPTION OF CONSTRAINTS (4)–(6)

The condition of the arrangement (4) of the set of objects R_i in the domain R_0 with allowance for Remarks 1 and 2 and expression (7) is specified by the system of linear inequalities

$$\begin{cases} t_i \ge 0, \\ T_R - t_i - a_i \ge 0, \\ w_i \ge 0, \\ W - w_i - \dfrac{S_i}{a_i} \ge 0, \\ -a_i + a_{i\max} \ge 0, \\ a_i - a_{i\min} \ge 0, \end{cases} \quad i = 1, \dots, N. \tag{14}$$

The mutual non-intersection condition (5) is specified by the set of linear inequalities

$$\left\langle \begin{array}{l} t_j - t_i - a_i \ge 0, \\ w_j - w_i - S_i / a_i \ge 0, \\ t_i - t_j - a_j \ge 0, \\ w_i - w_j - S_j / a_j \ge 0, \end{array} \right. \quad i, j = 1, \dots, N, \ i \ne j. \tag{15}$$

The partial ordering condition (6) is represented as a system of linear inequalities

$$t_j - t_i - a_i \ge 0, \ j \in I_i^j, \ i, j = 1, 2, \dots, N, \ i \ne j. \tag{16}$$

Two approaches are possible in this case (proceeding from the scheduling practice). If the condition that job R_j follows immediately after job R_i is obligatory, then the respective constraint in (16) is a strict equality. Otherwise condition (16) means that job R_j should be executed not earlier than job R_i is completed. Next it is shown that the presence or absence of condition (16) and of its more stringent analog in the form of equality considerably influences the estimate of the computation complexity of the problem solution algorithm.

Let constructing the schedule $G(T_R, \Delta W)$ of project R necessarily imply the solution of problem (13). Considering this condition, we will present the main properties of the optimization problem (3)–(8).

Property *D*_1. The domain D is a nonconvex disconnected bounded point set with piecewise smooth boundary $\Psi = \mathrm{Fr}\, D$. Each connectivity component of the feasible domain D is multiply connected.

Property *D*_2. The number of constraints for the feasible domain D of problem (3)–(8) quadratically depends on the number of the objects.

Property *D*_3. Domain D is representable as a union of a finite number of subdomains D_g of the form

$$D=\bigcup_{g=1}^{G} D_g,\ G=O(4^{N(N-1)/2}). \tag{17}$$

The subdomain D_g is described by the system $F_g(u)\ge 0$, which includes N systems of nonlinear inequalities (14), $N(N-1)/2$ inequalities (one from each set of inequalities (15) for each pair of objects), and the partial ordering condition (16).

Property *D*_4. Functions of the form $f(t_i,t_j,a_j)=t_i-t_j-\dfrac{S_j}{a_j}$ and $f(t_i,t_j,a_j)=t_j-t_i-\dfrac{S_i}{a_i}$ are convex and belong to the class of separable functions [18].

Property *D*_5. Objective functions of the considered partial problems (9), (10), (13) are linear. Hence, the optimal solution of these problems is reached on the boundary of the appropriate sets D_1, D_2, D_3.

Property *D*_6. In the general case, the optimal solution u^* of each partial problem is determined by the system $F^*(u)=0$ of linear (and nonlinear for problem (13)) equations from $F_g(u)\ge 0$. The rank I of system $F^*(u)=0$ is equal to the dimension of the space where the problem is considered. The system constraints $F^*(u)=0$ are called job list [18].

This property is not obvious for problem (13), but it holds since the convex functions of the nonlinear constraints are single-valued by formulation.

Property *D*_7. For any point u^* the relation $u^*\in\bigcap_{q>1} D_{gq}$ holds.

The constraints of the problem that are transformed to equalities at the current point u are called active set [18]. From the property D_7 it follows that for the considered problems the dimension J of the active set is greater than I.

SOLUTION METHODS FOR PROBLEM (3)–(8)

In view of the properties D_1 and D_3 of the feasible domain, the optimization problem under study as well as its partial subproblems (9), (10), and (13) are a multidimentional multiextremum combinatorial optimization problem with disconnected feasible domain belonging to the class of NP-hard problems [20].

The tools of the solution of partial problems (9), (10) (provided that the metric characteristics of the objects are constant) as rectangular arrangement problems with a linear objective function and linear constraints are adequately represented in the scientific literature, for example in [21, 22]. Therefore, in the present paper we consider the modifications of exact and approximate methods of local and global optimization for the solution of problem (13).

Exact Solution Methods for Problem (13). According to [18], it is theoretically possible to determine the global minimum of the objective function of problem (13).

Based on the characteristics of the constraint functions of the problem and property D_3 of the feasible domain, problem (13) belongs to the class of nonlinear combinatorial optimization problems. The general ideology of the solution of such problems is to construct a solution tree (we call it A) and use it to order the exhaustive search of subsets of the feasible domain of the problem, with a more simple structure, and to determine the locally optimal solution on each such subset.

Two implementations of the solution tree A are possible: the first is a tree A^1 constructed based on the exhaustive search of the convex subsets D_g of the feasible domain of the problem (property D_3), the second is a tree A^2 constructed based on the set of systems of equations containing the system $F^*(\mu)=0$ (property D_6).

Let us consider the special features of the modifications of both approaches to the formation of the structure of the solution tree of problem (13) taking into account variable metric characteristics of the objects and the partial ordering condition on the set of objects.

Implementation 1. The root of the solution tree A_0^1 corresponds to the system of inequalities (14), one of the four constraints from the set (15) can be added to node $A_{p(i,j)}^1$ at each next level, where $p(i,j)$ is the number of the pair (i,j),

$i < j$. At the last, $(N(N-1)/2)$th level of the tree A^1, all the convex subsets D_g will be constructed (property D_3). Thus, the problem of searching for the global extremum reduces to the truncated enumeration and solution of a finite set of convex programming problems with linear objective function.

The upper-bound estimate of the number of nodes at the last level of the solution tree A^1 is equal to G from (17), and the variability of the metric characteristics of the rectangles R_i, which model critical operations, does not influence the number of tree nodes. However, (17) is an overestimate since allowance for constraint (16), especially in the form of an equality, reduces the number of the considered nodes of the solution tree for each specific implementation of the problem.

Assume that the partial order condition has the form of the equality

$$t_j - t_i - a_i = 0,\ j \in I_i^j,\ i, j = 1, 2, \dots, N,\ i < j. \tag{18}$$

Denote $\aleph = \sum_{i=1}^{N-1} I_i^j$. Then with allowance for (18) tree A^1 will have $(N(N-1)/2 - \aleph)$ levels. Moreover, the choice of the first or third node at each $p(h, j)$th level of the solution tree is uniquely determined by a similar choice at the $p(h, i)$th level $h < (i, j)$. The optimal problem solution on the system of inequalities $F_g(\mu) \geq 0$ is a local minimum of problem (13).

Implementation 2. If the metric characteristics of the rectangles (project jobs) are constant (problem (10)), the solution tree A^2 is based on the possibility of constructing a bijection Ψ on the system of equations $F^*(v) = 0$ (property D_6) of the form

$$\Psi: t_i \leftrightarrow F_k^*(v) = 0,\ F_k^*(v) \in \{t_i - t_l - a_l, t_i\}; \tag{19}$$

$$\Psi: w_i \leftrightarrow F_k^*(v) = 0,\ \ F_k^* \in \{w_i - w_l - b_l, w_i\};$$

$$\Psi: T_R \leftrightarrow F_k^*(v) = 0,\ F_k^*(v) = T_R - t_l - a_l,$$

$$k \in \{1, 2, \dots, 2N+1\};\ i, l = 1, 2, \dots, N;\ i \neq l.$$

The solution tree A^2 is used to implement all the possible systems of equations with property (19), including the system $F^*(v) = 0$. There are $2N$ levels of the tree A^2, each corresponding to some variable arrangement parameter t_i or w_i, $i = 1, 2, \dots, N$. At each next level, node $A^2_{r(i)}$, $r(i) = \{i*2, i*2+1\}$, can be supplemented with one of the two appropriate constraints from set (15), which becomes active; there are $(N-1)$ such constraints. The upper estimate of the number of nodes at the last, $2N$th, tree level A^2 for constant metric characteristics of the objects is equal to $(2N)^N$.

If the metric characteristics (a_i, b_i) (1), (2) of the object R_i are variable, it becomes impossible to construct a bijection (19). In other words, this means that not only the number of levels of the tree A^2 increases to the $(Z+1)$ since it is now necessary to consider the variables a_i and T_R, but also the numbers of tree nodes that can be added at the current level. The upper-bound estimate of the number of nodes is equal to Z^Z. Allowance for constraint (16) reduces the number of the considered nodes of the solution tree for each specific implementation of the problem; however, the total complexity of the problem remains nonpolynimial.

Approximate Problem Solution Methods. The approach based on the optimization with respect to groups of variables is one of the most interesting and includes two stages.

Stage 1. Determining a locally optimal solution based on the modified method of the optimization based on groups of variables.

Stage 2. The enumeration of local extrema based on redetermining the sequence of object arrangement.

In the present paper, we assume that the object allocation order is given, the more so that for problems (10), (13) the sequence of object allocation is defined by the solution of problem (9).

Let us present the framework of Stage 1 (optimization with respect to groups of variables [16]):

— the objects are allocated one by one according to a given sequence of numbers, the objects allocated earlier are considered fixed, the objects subject to the arrangement at the later iterations of the method are not taken into account;

— the current object R_i is allocated with regard for the requirement of the minimization of the current objective function Ξ.

Thus, in the general case, for the set of the considered problems at each ith iteration of the method of optimization with respect to groups of variables, a problem $\Xi \to \min_{(t_i, w_i, a_i)\in\Theta_i}$ is solved, where the domain Θ_i is a three-dimensional section of the subdomain $\hat{\Theta} \subset E^{3i}$ of feasible solutions D of the main problem, $t_l, w_l, a_l =$ const, $l = 1, \ldots, i-1$, and t_i, w_i, a_i are var, the domain $\hat{\Theta}$ is generated by constraints (4)–(6) for the set of objects $\{R_l\}$, $l = \overline{1, i}$.

A PRIORI LINEARIZATION OF THE CONSTRAINTS

A technique for the global linearization of nonlinear constraints of the problem is proposed in [18]. It allows linear approximation of the problem with any prescribed accuracy without increasing the dimension of the space of parameters to which the feasible domain of the problem belongs, on the assumption that variable metric characteristics of jobs are continuous.

The considered transformation $D \xrightarrow{\Im} D^L$ replaces the nonlinear constraint functions from (14) and (15) with the appropriate linear functions (one or several).

The number of approximation elements depends on its accuracy $\varepsilon = \max(d^n - d_k^n)$ specified a priori, where n is the number of approximation nodes, d^n and d_k^n, respectively, are the coefficients of the equation of the secant connecting the adjacent nodes of the approximation, and of the equation of a tangent to the current segment of the nonlinear function.

The convexity of the nonlinear constraint function (property D_4) means the following: if the approximation accuracy ε is such that more than one linear approximating function is necessary, then the linearized description of the corresponding boundary section $\Psi = \mathrm{Fr} D$ is specified by the system of the constructed linear constraints.

Thus, the mapping $\Im$ for $n = 1$ has the form

$$\Im\,(a_i (w_j - w_i) + S_i) = \alpha_i \times a_i + \beta_i \times (w_j - w_i) - d_i,$$

where $\alpha_i = A_i / \delta_i$, $\beta_i = B_i / \delta_i$, $\delta_i = \sqrt{A_i^2 + B_i^2}$, $A_i = b_{i\,\max} - b_{i\,\min}$, $B_i = a_{i\,\max} - a_{i\,\min}$, and $d_{ij}^n = b_{i\,\min} B - a_{i\,\max} A$.

The approximation $\Im\,(W - w_i + S_i / a_i)$ of the nonlinear constraint function from (14) is carried out similarly.

Let us consider the main properties of the mapping $\Im$.

Property $\Im$_1. Due to the transformation $\Im$ the polyhedral approximation set D^L belongs to the space of the same dimension as the original domain D. In turn, due to the transformation $D_g \xrightarrow{\Im} D_g^L$ the convex linear subdomain D_g^L belongs to the same space as the original subdomain D_g does.

Property $\Im$_2. A single application of this transformation ($n = 1$) does not increase the number of problem constraints. The linear approximation of the set (15) of nonlinear constraints becomes

$$\left\langle \begin{array}{l} t_j - t_i - a_i \geq 0, \\ \alpha_i a_i + \beta_i \times (w_j - w_i) - d_{ij} \geq 0, \\ t_i - t_j - a_j \geq 0, \\ \alpha_j a_i + \beta_j (w_i - w_j) - d_{ij} \geq 0, \end{array} \right. \quad i, j = 1, \ldots, N, \;\; i \neq j. \tag{20}$$

Property $\Im$_3. Multiple application of the approximating procedures increases the number of constraints (20) but does not change the number G of subsets D_g. Hence, such characteristics of the solution tree A^1 applied to order the subsets D_g in order to obtain the globally optimal solution also remain invariable, as the number of levels of the solution tree and the number of nodes of the tree added at each intermediate level.

MODIFICATION OF THE METHOD OF LOCAL OPTIMIZATION OF LINEARIZED PROBLEM

A linearized problem on the subset D_g^L has the form

$$\Xi \to \min_{D_g^L \subset E^Z}, \tag{21}$$

where the domain D_g^L is specified by the system $F_g^L(u) \geq \delta^n$ of linear constraints, components of the vector δ^n are zero, and the values $d_{ij}^{\eta}, d_{ji}^{\eta}$, $\eta = 1, \dots, n$.

Problem (21) is solved based on the active set method [18]. Let us consider the implementation of the scheme of active set, whose $(m+1)$th iteration has the form $u^{m+1} = u^m + \Delta u^m \cdot p^m$, where u^m are the values of the variables at previous iteration, Δu^m is step, and p^m is descent direction.

At the point u^m the signs of components of the vector of Lagrange multipliers λ are defined as solutions of the nondegenerate system of linear equations $H^T \lambda = c$, where H is the matrix of the coefficients of job list constraints at the considered point, c is the vector of coefficients of the objective function, in this case it has the form $(0, 0, \dots, 0, 1)$.

In view of the features of vector c, the vector of Lagrange multipliers λ is the Zth column of the matrix H^T.

If all the components $\lambda_s \geq 0$, $s = 1, \dots, Z$, then u^m is a solution of problem (21). If some $\lambda_s < 0$, then the direction p^m is determined as a solution of the linear system of equations $Hp^m = e_s$, where e_s denotes the sth column of the unit matrix, i.e., p is the sth column of the matrix H^{-1} (the sth row of the matrix $(H^T)^{-1}$).

For the set V of all constraints of problem (21) out of the job list, the upper estimate $\overline{\alpha}$

$$\overline{\alpha} = \begin{cases} \min\limits_{v} \dfrac{d_v^n - h_v^T u^m}{h_v^T p^m} & \text{if } h_v^T p^m < 0, \ v \in V, \\ +\infty, & \text{if } h_v^T p^m \geq 0 \text{ for all } v \in V, \end{cases}$$

of the step Δu^m implementing the equality $h_v^T(u^m + \overline{\alpha}^m p^m) = d_v^n$ is calculated at the mth iteration of the method, where ν is the index of inactive constraint.

The presence of property D_7 means that vector u^m can simultaneously satisfy two inequalities, for example, the first and the second, from set (20). In that case the inactive constraint is that with the greater estimate of the step Δu^m. Thus, the passage from one convex subdomain D_g^L of the linearized feasible domain D^L to the other, D_{g+1}^L, is carried out.

In other words, the constraint is selected, which will be included in the job list at the next iteration of the solution algorithm with simultaneous passage to the other convex subdomain, i.e., the motion generally takes place in a nonconvex domain.

The program was coded in Object Pascal 6.0 in the Borland Delphi 7.0 environment. The program includes (i) a data input module that is capable of setting and correcting work characteristics and sequence; (ii) an analytic module that has procedures of constructing approximation functions with a prescribed accuracy and implementing exact and approximate methods; and (iii) a data display and storage module.

The proposed tools for modeling and solution of the limited resource allocation problem were applied to solve the following practical problem. The object under study was attached warehouses with an automatic system for collecting and palletizing boxes of finished goods of the “Philip Morris Ukraine” Joint-Stock Company Kharkov. The project implied installation of an automatic fire alarm system.

Without going into details, the system is mounted and adjusted as follows:

— preparatory work (scaffolding, preparation of cables, equipment, and workplaces);

— laying of vinyl pipes and cables;

— testing of joints (for reliability of fixation of pipes on supports);

— installation of equipment and instruments.

The fire alarm system is commissioned in compliance with appropriate regulations.

TABLE 1

Work number	Work type and materials, units	Amount of work and materials	Number of workers		Labor input, man h	
			min	max	per work unit	total
1	Preparatory work	4	2	4	20	80
2	Installation of base hub, piece	3	3	6	73	219
3	Installation of control instrument on loop 1, piece	19	4	6	11.52	218.88
4	Installation of final control unit, piece	5	4	6	50	250
5	Installation of control panels, piece	1	1	2	34.56	34.56
6	Installation of power switchers, piece	1	1	2	23.04	23.04
7	Installation of panel or indicator for up to 10 signals, piece	2	1	2	13.44	26.88
8	Installation of control instrument for 4 loops, piece	20	4	10	21.12	422.4
9	Installation of control instrument for 8 loops, piece	8	2	8	21.12	168.96
10	Installation of interior tubular scaffold, piece	1	2	5	70	70
11	Punching of circular 25 mm holes, piece	100	2	5	0.191	191
12	Laying cables in pipes, m	98	6	10	9.60	940.8
13	Installation of grounding bridge, piece	5	1	1	0.6	3
14	Laying of cables in trays, m	57.7	3	8	6.05	349
15	Laying of vinyl pipes, m	98	6	20	27.84	2728
16	Penetration of wall for 27 mm pipes, piece	10	1	2	1.07	10.7
17	Installation of cable boxes, piece	10	1	2	0.96	9.6
18	Connection of single-pair cable, piece	5	2	4	9.60	48
19	Brining out of single-pair cable, piece	2	1	2	9.60	19.2
20	Installation of automatic alarm indicators, piece	564	10	20	4.22	2380
21	Installation of explosion-protected alarm indicators, piece	17	3	6	4.22	71.74
22	Installation of manual fire alarms, piece	41	3	6	1.92	78.72
23	Mounting of light indication units, piece	2	1	1	1.92	3.84
24	Installation of buttons on control panels, piece	20	1	2	1.92	38.4
25	Installation of wall cabinet, piece	19	2	4	3.84	72.96
26	Installation of 1 kW horn, piece	53	2	5	2.88	152.64
27	Installation of wall light pointers, piece	100	3	5	0.83	83
28	Routing of cables into cable boxes, piece	172	1	2	0.10	17.2
29	Disassembling of internal tubular scaffold, piece	1	2	5	70	70
30	Testing of alarm circuit, piece	1	2	4	151.68	151.68
31	Commissioning of system	1	1	1	24	24

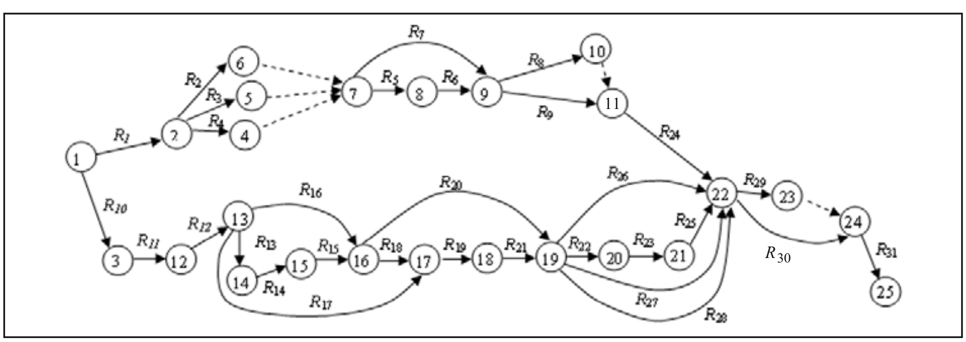

Fig. 3. Network model of works done to mount and adjustment the fire alarm system.

The total estimated cost of the project is 584.925 thousand UAH. Estimated labor input is 9.036 thousand man·h. Estimated salary is 147.613 thousand UAH. Average job class is 3.3.

The project involves 31 works characterized in Table 1.

Figure 3 shows a network (graph) project work ordering model where arrows indicate works, circles indicate the beginning and end of work, dashed arrows show dummy operations needed to demonstrate the relations among works.

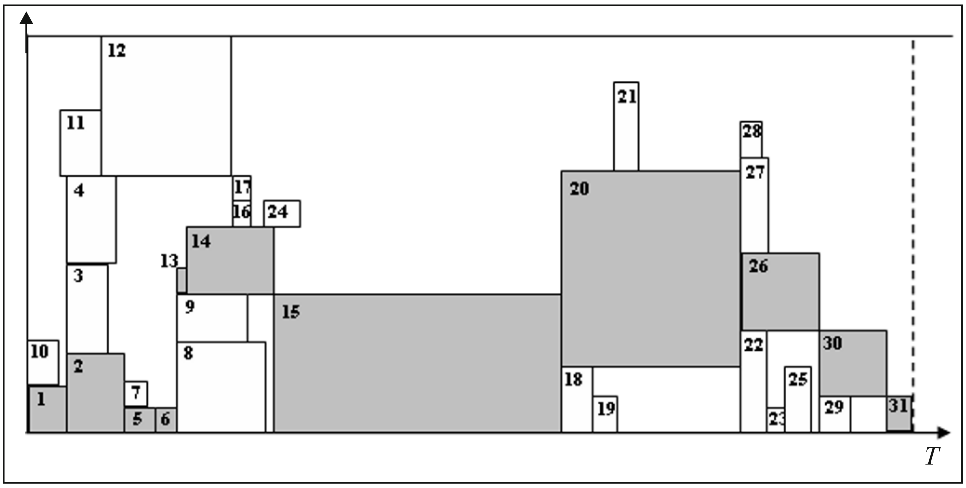

Fig. 4. Determining the critical path of the project
(critical operations are grayed).

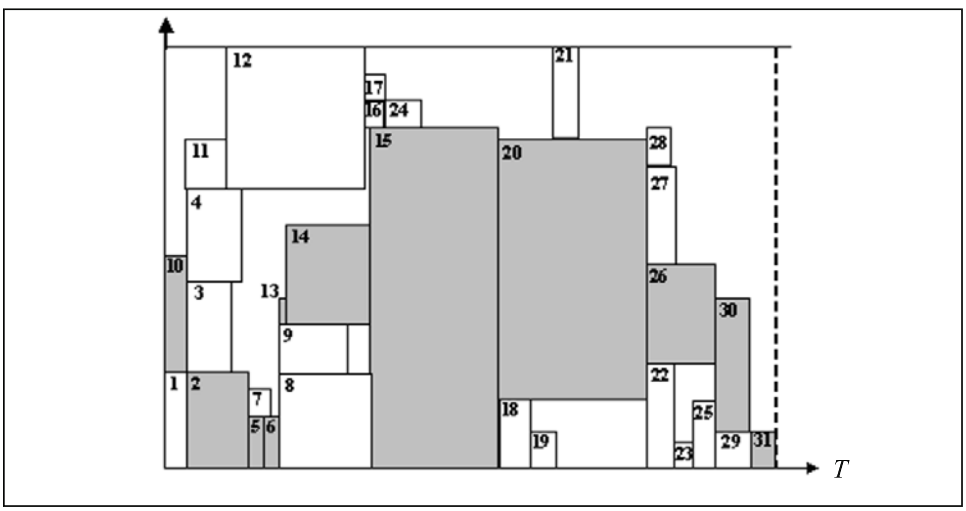

Fig. 5. Optimization of characteristics of critical operations.

By solving problem (9), we have estimated the necessary number of technical experts (18) engaged in the project and the critical path of the project (145 days; Fig. 4). Solving problem (13) made it possible, by intensifying the critical works without affecting the characteristics of noncritical operations and increasing the number of workers, to reduce the project time to 81 days (Fig. 5).

CONCLUSIONS

We have constructed a mathematical model of the multicriterion problem of the allocation of limited resources of a project as a set of separable optimization problems of the allocation of a finite set of geometrical objects with variable metric characteristics. We have emphasized the main properties of the problem and considered the exact and approximate methods of its solution. We have also proposed an approach to constructing a linear approximation of separable functions of problem constraints, which differs from the well-known approaches in the possibility of specifying the approximation accuracy as an exogenous parameter, and also ensuring the membership of the approximation polyhedral set in the original space of independent variables of the problem. We have performed an algorithmic and program implementation of the proposed methods and solved the practical problem of determining the parameters of the project of an automatic fire alarm system. The further studies imply the construction of the computational algorithms of the solution of 3D packing problems for objects with variable metric characteristics.

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