Stability Analysis of Human–Adaptive Controller Interactions

Tansel Yucelen,* Yildiray Yildiz,† Rifat Sipahi,‡ Ehsan Yousefi,§ and Nhan Nguyen¶

In this paper, stability of human in the loop model reference adaptive control architectures is analyzed. For a general class of linear human models with time-delay, a fundamental stability limit of these architectures is established, which depends on the parameters of this human model as well as the reference model parameters of the adaptive controller. It is shown that when the given set of human model and reference model parameters satisfy this stability limit, the closed-loop system trajectories are guaranteed to be stable.

I.   Introduction

Adaptive control design approaches[1–4] are important candidates for uncertain dynamical systems, since they can effectively cope with the effects of system uncertainties online and require less modeling information than fixed-gain robust control design approaches[5,6]. Motivated from this fact, the results of this paper builds on a well-known and important class of adaptive controllers; namely, model reference adaptive controllers[7,8], where their architecture includes a reference model, a parameter adjustment mechanism, and a controller.

Specifically, a desired closed-loop dynamical system behavior is captured by the reference model, where its output (respectively, state) is compared with the output (respectively, state) of the uncertain dynamical system. This comparison yields to a system error signal, which is used to drive the parameter adjustment mechanism online. Then, the controller adapts feedback gains to minimize this error signal using the information received from the parameter adjustment mechanism. As a consequence, the output (respectively, state) of the uncertain dynamical system behaves as the output (respectively, state) of the reference model asymptotically or approximately in time, and hence, guarantees system stability and achieves a level of desired closed-loop dynamical system behavior.

From this standpoint, model reference adaptive controllers offer mathematical tools to effectively cope with system uncertainties arising from ideal assumptions, linearization, model order reduction, exogenous disturbances, and degraded modes of operations. However, in certain applications when humans are in the loop[9–15], they can lead to unstable system trajectories. The contribution of this paper is to analyze stability of human in the loop model reference adaptive control architectures. For a general class of linear human

---

*T. Yucelen is an Assistant Professor of the Department of Mechanical Engineering, University of South Florida, Tampa, Florida, USA (E-mail: `yucelen@lacis.team`; Website: `http://www.lacis.team/`).

†Y. Yildiz is an Assistant Professor of the Mechanical Engineering Department and the Director of the Systems Laboratory at the Bilkent University, Ankara, Turkey (Email: `yyildiz@bilkent.edu.tr`; Website: `http://www.syslab.bilkent.edu.tr/`).

‡R. Sipahi is an Associate Professor of the Mechanical and Industrial Engineering Department and the Director of the Complex Dynamic Systems and Control Laboratory at the Northeastern University, Boston, Massachusetts, United States of America (Email: `rifat@coe.neu.edu`; Website: `http://www1.coe.neu.edu/∼rifat/`).

§E. Yousefi is a graduate student of the Mechanical Engineering Department and a member of the Systems Laboratory at the Bilkent University, Ankara, Turkey (Email: `ehsan.yousefi@bilkent.edu.tr`).

¶N. Nguyen is the Technical Group Lead of the Advanced Control and Evolvable Systems Group in the Intelligent Systems Division at the Ames Research Center of the National Aeronautics and Space Administration, Moffett Field, California, United States of America (Email: `nhan.t.nguyen@nasa.gov`; Website: `http://ti.arc.nasa.gov/profile/ntnguye3/`).

American Institute of Aeronautics and Astronautics

models with time-delay, a fundamental stability limit of these architectures is established, which depends on the parameters of this human model as well as the reference model parameters of the adaptive controller. It is shown that when the given set of human model and reference model parameters satisfy this stability limit, the closed-loop system trajectories are guaranteed to be stable [a].

II.   Problem Formulation

For representing human in the loop model reference adaptive controllers, we consider the block diagram configuration given by Figure 1, where the outer loop architecture includes the reference that is fed into the human dynamics to generate a command for the inner loop architecture in response to the variations resulting from the uncertain dynamical system and the inner loop architecture includes the uncertain dynamical system and the model reference adaptive controller components (i.e., the reference model, the parameter adjustment mechanism, and the controller). Specifically, at the other loop architecture, we consider a class of human models with time-delay given by

$$
\begin{aligned}
\dot{\xi}(t) &= A_\mathrm{h}\xi(t) + B_\mathrm{h}\theta(t-\tau), \quad \xi(0) = \xi_0, & (1)\\
c(t) &= C_\mathrm{h}\xi(t) + D_\mathrm{h}\theta(t-\tau), & (2)
\end{aligned}
$$

where $\xi(t) \in \mathbb{R}^{n_\xi}$ is the internal human state vector, $c(t) \in \mathbb{R}^{n_c}$ is the command representing the output of the outer loop architecture in Figure 1,

$$
\theta(t) \triangleq r(t) - E_\mathrm{h}x(t), \tag{3}
$$

$\theta(t) \in \mathbb{R}^{n_r}$, with $r(t) \in \mathbb{R}^{n_r}$ being the bounded reference and $x(t) \in \mathbb{R}^n$ being the state vector to be defined that is received from the inner loop architecture, $\tau \in \mathbb{R}_+$ is the internal human time-delay, and $A_\mathrm{h} \in \mathbb{R}^{n_\xi \times n_\xi}$, $B_\mathrm{h} \in \mathbb{R}^{n_\xi \times n_r}$, $C_\mathrm{h} \in \mathbb{R}^{n_c \times n_\xi}$, $D_\mathrm{h} \in \mathbb{R}^{n_c \times n_r}$, and $E_\mathrm{h} \in \mathbb{R}^{n_r \times n}$. Note that the dynamics given by (1), (2), and (3) is general enough to capture, for example, widely studied linear time-invariant human models with time-delay including Neal-Smith model and its extensions[9, 10, 17–19].

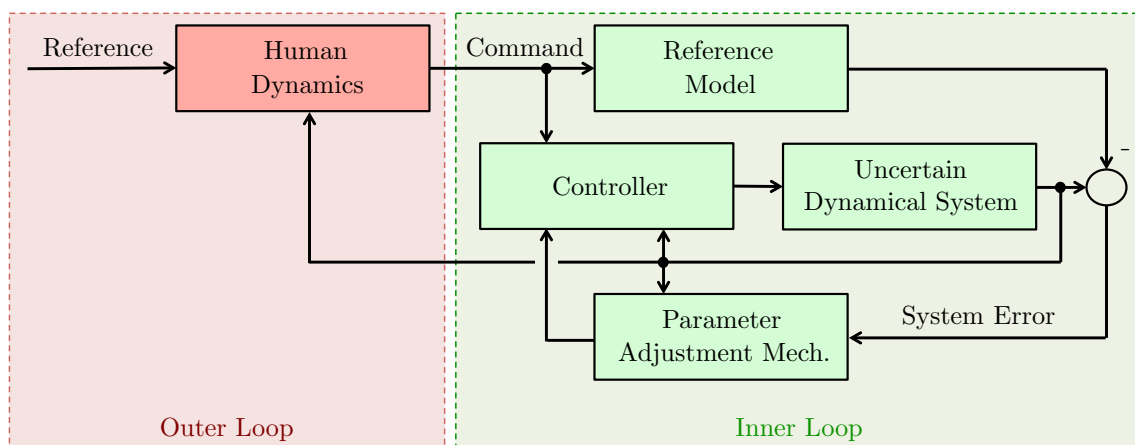[a]Extended version of this paper is currently under review [16].

Figure 1. Block diagram of the human in the loop model reference adaptive control architecture.

American Institute of Aeronautics and Astronautics

Next, at the inner loop architecture, we consider the uncertain dynamical system given by

$$\dot{x}_{\mathrm{p}}(t) \;\;=\;\; A_{\mathrm{p}}x_{\mathrm{p}}(t) + B_{\mathrm{p}}\Lambda u(t) + B_{\mathrm{p}}\delta_{\mathrm{p}}\big(x_{\mathrm{p}}(t)\big), \quad x_{\mathrm{p}}(0) = x_{\mathrm{p}_0}, \tag{4}$$

where $x_{\mathrm{p}}(t) \in \mathbb{R}^{n_{\mathrm{p}}}$ is the accessible state vector, $u(t) \in \mathbb{R}^m$ is the control input, $\delta_{\mathrm{p}} : \mathbb{R}^{n_{\mathrm{p}}} \to \mathbb{R}^m$ is an uncertainty, $A_{\mathrm{p}} \in \mathbb{R}^{n_{\mathrm{p}} \times n_{\mathrm{p}}}$ is a known system matrix, $B_{\mathrm{p}} \in \mathbb{R}^{n_{\mathrm{p}} \times m}$ is a known control input matrix, and $\Lambda \in \mathbb{R}_+^{m \times m} \cap \mathbb{D}^{m \times m}$ is an unknown control effectiveness matrix. Furthermore, we assume that the pair $(A_{\mathrm{p}}, B_{\mathrm{p}})$ is controllable and the uncertainty is parameterized as

$$\delta_{\mathrm{p}}\big(x_{\mathrm{p}}\big) \;\;=\;\; W_{\mathrm{p}}^{\mathrm{T}}\sigma_{\mathrm{p}}\big(x_{\mathrm{p}}\big), \quad x_{\mathrm{p}} \in \mathbb{R}^{n_{\mathrm{p}}}, \tag{5}$$

where $W_{\mathrm{p}} \in \mathbb{R}^{s \times m}$ is an unknown weight matrix and $\sigma_{\mathrm{p}} : \mathbb{R}^{n_{\mathrm{p}}} \to \mathbb{R}^s$ is a known basis function of the form $\sigma_{\mathrm{p}}\big(x_{\mathrm{p}}\big) = \big[\sigma_{\mathrm{p}_1}\big(x_{\mathrm{p}}\big), \sigma_{\mathrm{p}_2}\big(x_{\mathrm{p}}\big), \ldots, \sigma_{\mathrm{p}_s}\big(x_{\mathrm{p}}\big)\big]^{\mathrm{T}}$. Note for the case where the basis function $\sigma_{\mathrm{p}}\big(x_{\mathrm{p}}\big)$ is unknown that the parameterization in (5) can be relaxed without significantly changing the results of this paper, for example, by considering[20, 21]

$$\delta_{\mathrm{p}}\big(x_{\mathrm{p}}\big) \;\;=\;\; W_{\mathrm{p}}^{\mathrm{T}}\sigma_{\mathrm{p}}^{\mathrm{nn}}\big(V_{\mathrm{p}}^{\mathrm{T}}x_{\mathrm{p}}\big) + \varepsilon_{\mathrm{p}}^{\mathrm{nn}}\big(x_{\mathrm{p}}\big), \quad x_{\mathrm{p}} \in \mathcal{D}_{x_{\mathrm{p}}}, \tag{6}$$

where $W_{\mathrm{p}} \in \mathbb{R}^{s \times m}$ and $V_{\mathrm{p}} \in \mathbb{R}^{n_{\mathrm{p}} \times s}$ are unknown weight matrices, $\sigma_{\mathrm{p}}^{\mathrm{nn}} : \mathcal{D}_{x_{\mathrm{p}}} \to \mathbb{R}^s$ is a known basis composed of neural networks function approximators, $\varepsilon_{\mathrm{p}}^{\mathrm{nn}} : \mathcal{D}_{x_{\mathrm{p}}} \to \mathbb{R}^m$ is an unknown residual error, and $\mathcal{D}_{x_{\mathrm{p}}}$ is a compact subset of $\mathbb{R}^{n_{\mathrm{p}}}$.

To address command following at the inner loop architecture, let $x_{\mathrm{c}}(t) \in \mathbb{R}^{n_{\mathrm{c}}}$ be the integrator state satisfying

$$\dot{x}_{\mathrm{c}}(t) \;\;=\;\; E_{\mathrm{p}}x_{\mathrm{p}}(t) - c(t), \quad x_{\mathrm{c}}(0) = x_{\mathrm{c}_0}, \tag{7}$$

where $E_{\mathrm{p}} \in \mathbb{R}^{n_{\mathrm{c}} \times n_{\mathrm{p}}}$ allows to choose a subset of $x_{\mathrm{p}}(t)$ to be followed by $c(t)$. Now, (4) can be augmented with (7) as

$$\dot{x}(t) \;\;=\;\; Ax(t) + B\Lambda u(t) + BW_{\mathrm{p}}^{\mathrm{T}}\sigma_{\mathrm{p}}\big(x_{\mathrm{p}}(t)\big) + B_{\mathrm{r}}c(t), \quad x(0) = x_0, \tag{8}$$

where $x(t) \triangleq \big[x_{\mathrm{p}}^{\mathrm{T}}(t), x_{\mathrm{c}}^{\mathrm{T}}(t)\big]^{\mathrm{T}} \in \mathbb{R}^n$, $n = n_{\mathrm{p}} + n_{\mathrm{c}}$, is the (augmented) state vector, $x_0 \triangleq \big[x_{\mathrm{p}_0}^{\mathrm{T}}, x_{\mathrm{c}_0}^{\mathrm{T}}\big]^{\mathrm{T}} \in \mathbb{R}^n$,

$$A \;\;\triangleq\;\; \begin{bmatrix} A_{\mathrm{p}} & 0_{n_{\mathrm{p}} \times n_{\mathrm{c}}} \\ E_{\mathrm{p}} & 0_{n_{\mathrm{c}} \times n_{\mathrm{c}}} \end{bmatrix} \in \mathbb{R}^{n \times n}, \tag{9}$$

$$B \;\;\triangleq\;\; \big[B_{\mathrm{p}}^{\mathrm{T}}, 0_{n_{\mathrm{c}} \times m}^{\mathrm{T}}\big]^{\mathrm{T}} \in \mathbb{R}^{n \times m}, \tag{10}$$

$$B_{\mathrm{r}} \;\;\triangleq\;\; \big[0_{n_{\mathrm{p}} \times n_{\mathrm{c}}}^{\mathrm{T}}, -I_{n_{\mathrm{c}} \times n_{\mathrm{c}}}\big]^{\mathrm{T}} \in \mathbb{R}^{n \times n_{\mathrm{c}}}. \tag{11}$$

In this inner loop architecture setting, it is reasonable to set $E_{\mathrm{h}} = \big[E_{\mathrm{h}_{\mathrm{p}}}, 0_{n_{\mathrm{r}} \times n_{\mathrm{c}}}\big]$, $E_{\mathrm{h}_{\mathrm{p}}} \in \mathbb{R}^{n_{\mathrm{r}} \times n_{\mathrm{p}}}$, in (3) without loss of generality since a subset of the accessible state vector is usually available and/or sensed by the human at the outer loop (not the states of the integrator).

American Institute of Aeronautics and Astronautics

Finally, consider the feedback control law at the inner loop architecture given by

$$u(t) = u_{\mathrm{n}}(t) + u_{\mathrm{a}}(t), \tag{12}$$

where $u_{\mathrm{n}}(t) \in \mathbb{R}^m$ and $u_{\mathrm{a}}(t) \in \mathbb{R}^m$ are the nominal and adaptive control laws, respectively. Furthermore, let the nominal control law be

$$u_{\mathrm{n}}(t) = -Kx(t), \tag{13}$$

$K \in \mathbb{R}^{m \times n}$, such that $A_{\mathrm{r}} \triangleq A - BK$ is Hurwitz. Using (12) and (13) in (8) yields

$$\dot{x}(t) = A_{\mathrm{r}}x(t) + B_{\mathrm{r}}c(t) + B\Lambda\big[u_{\mathrm{a}}(t) + W^{\mathrm{T}}\sigma\big(x(t)\big)\big], \tag{14}$$

where $W^{\mathrm{T}} \triangleq \big[\Lambda^{-1}W_{\mathrm{p}}^{\mathrm{T}}, (\Lambda^{-1} - I_{m \times m})K\big] \in \mathbb{R}^{(s+n) \times m}$ is an unknown (aggregated) weight matrix and $\sigma^{\mathrm{T}}\big(x(t)\big) \triangleq \big[\sigma_{\mathrm{p}}^{\mathrm{T}}\big(x_{\mathrm{p}}(t)\big), x^{\mathrm{T}}(t)\big] \in \mathbb{R}^{s+n}$ is a known (aggregated) basis function. Considering (14), let the adaptive control law be

$$u_{\mathrm{a}}(t) = -\hat{W}^{\mathrm{T}}(t)\sigma\big(x(t)\big), \tag{15}$$

where $\hat{W}(t) \in \mathbb{R}^{(s+n) \times m}$ be the estimate of W satisfying the parameter adjustment mechanism

$$\dot{\hat{W}}(t) = \gamma\sigma\big(x(t)\big)e^{\mathrm{T}}(t)PB, \quad \hat{W}(0) = \hat{W}_0, \tag{16}$$

where $\gamma \in \mathbb{R}_+$ is the learning rate, $e(t) \triangleq x(t) - x_{\mathrm{r}}(t)$ is the system error with $x_{\mathrm{r}}(t) \in \mathbb{R}^n$ being the reference state vector satisfying the reference system

$$\dot{x}_{\mathrm{r}}(t) = A_{\mathrm{r}}x_{\mathrm{r}}(t) + B_{\mathrm{r}}c(t), \quad x_{\mathrm{r}}(0) = x_{\mathrm{r}_0}, \tag{17}$$

and $P \in \mathbb{R}_+^{n \times n} \cap \mathbb{S}^{n \times n}$ is a solution of the Lyapunov equation $0 = A_{\mathrm{r}}^{\mathrm{T}}P + PA_{\mathrm{r}} + R$ with $R \in \mathbb{R}_+^{n \times n} \cap \mathbb{S}^{n \times n}$. Although we consider a specific yet widely studied parameter adjustment mechanism given by (16), one can also consider other types of parameter adjustment mechanisms[22–35] and still use the approach presented in this paper.

Based on the given problem formulation, the next section analyzes the stability of the coupled inner and outer loop architectures depicted in Figure 1 in order to establish a fundamental stability limit (to guarantee the closed-loop system stability when this limit is satisfied by the given human model at the outer loop and the given adaptive controller at the inner loop).

III.   Stability Analysis

For stability analysis purposes, we now write the system error dynamics using (14), (15), and (17) as

$$\dot{e}(t) = A_{\mathrm{r}}e(t) - B\Lambda\tilde{W}^{\mathrm{T}}(t)\sigma\big(x(t)\big), \quad e(0) = e_0, \tag{18}$$

where $\tilde{W}(t) \triangleq \hat{W}(t) - W \in \mathbb{R}^{(s+n)\times m}$ is the weight error and $e_0 \triangleq x_0 - x_{r_0}$. In addition, we write the weight error dynamics using (16) as

$$\dot{\tilde{W}}(t) = \gamma\sigma\big(x(t)\big)e^{\mathrm{T}}(t)PB, \quad \tilde{W}(0) = \tilde{W}_0, \tag{19}$$

where $\tilde{W}_0 \triangleq \hat{W}(0) - W$. The following lemma is now immediate.

Lemma 1.[16] Consider the uncertain dynamical system given by (4) subject to (5), the reference model given by (17), and the feedback control law given by (12), (13), (15), and (16). Then, the solution $\big(e(t), \tilde{W}(t)\big)$ is Lyapunov stable for all $\big(e_0, \tilde{W}_0\big) \in \mathbb{R}^n \times \mathbb{R}^{(s+n)\times m}$ and $t \in \overline{\mathbb{R}}_+$.

Since the solution $\big(e(t), \tilde{W}(t)\big)$ is Lyapunov stable for all $\big(e_0, \tilde{W}_0\big) \in \mathbb{R}^n \times \mathbb{R}^{(s+n)\times m}$ and $t \in \overline{\mathbb{R}}_+$ from Lemma 1, this trivially implies that $e(t) \in \mathcal{L}_\infty$ and $\tilde{W}(t) \in \mathcal{L}_\infty$. At this stage, it should be noted that one cannot use the Barbalat's lemma[36] to conclude $\lim_{t\to\infty} e(t) = 0$, since the boundedness of the reference model needs to be assured first. From this standpoint, we next show the boundedness of the reference model, which also reveals the fundamental stability limit for guaranteeing the closed-loop system stability.

Using (2) in (17), we first write

$$\begin{aligned}
\dot{x}_{\mathrm{r}}(t) &= A_{\mathrm{r}}x_{\mathrm{r}}(t) + B_{\mathrm{r}}\big(C_{\mathrm{h}}\xi(t) + D_{\mathrm{h}}\theta(t-\tau)\big), \\
&= A_{\mathrm{r}}x_{\mathrm{r}}(t) - B_{\mathrm{r}}D_{\mathrm{h}}E_{\mathrm{h}}x_{\mathrm{r}}(t-\tau) + B_{\mathrm{r}}C_{\mathrm{h}}\xi(t) - B_{\mathrm{r}}D_{\mathrm{h}}E_{\mathrm{h}}e(t-\tau) + B_{\mathrm{r}}D_{\mathrm{h}}r(t-\tau).
\end{aligned} \tag{20}$$

Next, it follows from (1) that

$$\dot{\xi}(t) = A_{\mathrm{h}}\xi(t) - B_{\mathrm{h}}E_{\mathrm{h}}x_{\mathrm{r}}(t-\tau) - B_{\mathrm{h}}E_{\mathrm{h}}e(t-\tau) + B_{\mathrm{h}}r(t-\tau). \tag{21}$$

Finally, by letting $\phi(t) \triangleq [x_{\mathrm{r}}^{\mathrm{T}}(t), \xi^{\mathrm{T}}(t)]^{\mathrm{T}}$, one can write

$$\dot{\phi}(t) = \mathcal{A}_0\phi(t) + \mathcal{A}_\tau\phi(t-\tau) + \varphi(\cdot), \quad \phi(0) = \phi_0, \tag{22}$$

using (20) and (21), where

$$\mathcal{A}_0 \triangleq \begin{bmatrix} A_{\mathrm{r}} & B_{\mathrm{r}}C_{\mathrm{h}} \\ 0_{n_\xi \times n} & A_{\mathrm{h}} \end{bmatrix} \in \mathbb{R}^{(n+n_\xi)\times(n+n_\xi)}, \tag{23}$$

$$\mathcal{A}_\tau \triangleq \begin{bmatrix} -B_{\mathrm{r}}D_{\mathrm{h}}E_{\mathrm{h}} & 0_{n \times n_\xi} \\ -B_{\mathrm{h}}E_{\mathrm{h}} & 0_{n_\xi \times n_\xi} \end{bmatrix} \in \mathbb{R}^{(n+n_\xi)\times(n+n_\xi)}, \tag{24}$$

$$\varphi(\cdot) \triangleq \begin{bmatrix} -B_{\mathrm{r}}D_{\mathrm{h}}E_{\mathrm{h}}e(t-\tau) + B_{\mathrm{r}}D_{\mathrm{h}}r(t-\tau) \\ -B_{\mathrm{h}}E_{\mathrm{h}}e(t-\tau) + B_{\mathrm{h}}r(t-\tau) \end{bmatrix} \in \mathbb{R}^{n+n_\xi}. \tag{25}$$

As a consequence of Lemma 1 and the boundedness of the reference, one can conclude that $\varphi(\cdot) \in \mathcal{L}_\infty$.

To reveal the fundamental stability limit, an approach would be to employ frequency domain tools, where one studies the eigenvalues of the corresponding linear time invariant system with time delay.[37] Moreover, since the time delay in human dynamics can in general be known in practice for certain applications, at least within a certain range, it is possible to utilize the delay information in the stability analysis as stated in the following theorem.

American Institute of Aeronautics and Astronautics

Theorem 2.[16] Consider the uncertain dynamical system given by (4) subject to (5), the reference model given by (17), the feedback control law given by (12), (13), (15), and (16), and the human dynamics given by (1), (2), and (3). Then, $e(t) \in \mathcal{L}_\infty$ and $\tilde{W}(t) \in \mathcal{L}_\infty$. If, in addition, the real parts of all the infinitely many roots of the following characteristic equation

$$\det\left(sI - \left(\mathcal{A}_0 + \mathcal{A}_\tau e^{-\tau s}\right)\right) = 0, \tag{26}$$

where $I \in \mathbb{R}^{(n+n_\xi) \times (n+n_\xi)}$ is the identity matrix, have strictly negative real parts, then $x_{\mathrm{r}}(t) \in \mathcal{L}_\infty$, $\xi(t) \in \mathcal{L}_\infty$, and $\lim_{t \to \infty} e(t) = 0$.

IV.   Numerical Example

In order to demonstrate the stability criterion stated in Theorem 2, an illustrative numerical example is provided here to show the effect of various incorporated parameters. Specifically, consider the human transfer function given by

$$\frac{k_{\mathrm{p}}(\tau_{\mathrm{z}}s + 1)}{\tau_{\mathrm{p}}s + 1} e^{-\tau s}. \tag{27}$$

Table 1 shows numerical values used in this section[9]. By definition, A_{r} (of the reference model) is $A - BK$. The optimal K is found using the linear quadratic regulator (LQR) method, which is achieved by minimizing the performance index

$$J(x, u) = \int_0^\infty (x^{\mathrm{T}} Q x + u^{\mathrm{T}} R u) \mathrm{d}t, \tag{28}$$

(see, for example, Ref. 38). Here, R matrix is considered to be of the form μI, where I is the identity matrix, and the stability is checked for various values of μ. In the following subsections, the effect of various time delays and various Q matrices on the stability of the system are analyzed.

A.   Effect of various time delays

In this section, the effect of time delay τ on the stability is analyzed. Figure 2 depicts the real part of the right most pole (RMP) of the system versus μ for various time delays.[39] From this figure and for the selected range of μ, we have the following information:

- for $0 \leq \tau < 0.386$, the system is always stable as indicated with $RMP < 0$;

- for $0.386 \leqslant \tau < 0.551$, the system is stable for smaller values of μ; unstable for larger μ, and stable once again for even larger μ values;

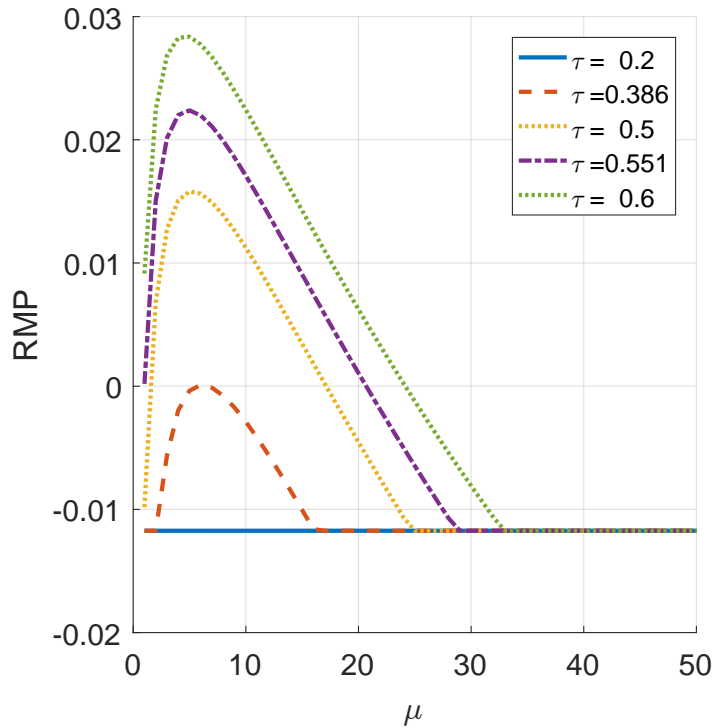| A_{p}	$[-0.003,\ 0.039,\ 0,\ -0.322;\ -0.065,\ -0.319,\ 7.740,\ 0;\ 0.020,\ -0.101,\ -0.429,\ 0;\ 0,\ 0,\ 1,\ 0]$
B_{p}	$[0.010;\ -0.180;\ -1.160;\ 0]$
E_{p}	$[0,\ 0,\ 0,\ 1]$
E_{h}	$[0,\ 0,\ 0,\ 1,\ 0]$
(27)	$k_{\mathrm{p}} = 5,\ \ \tau_{\mathrm{z}} = 1,\ \ \tau_{\mathrm{p}} = 5,\ \ \tau = 0.5$

Table 1. Numerical data

Figure 2. Right Most Pole (RMP) vs μ for various time delays τ

- for $\tau \geqslant 0.551$, the system is unstable for smaller values of μ and stable for larger μ,

B. Effect of various Q matrices

Next, the effect of various Q matrices on the performance index of LQR method is analyzed. In order to satisfy the criteria of the LQR method, the Q matrix is considered to be a diagonal positive definite matrix, $Q = diag\{Q_{1,1}, \ldots, Q_{k,k}, \ldots, Q_{5,5}\}$. Since we focus on the pitch attitude tracking task (similar to Ref. 9) in this example, the considered state for the performance index is pitch angle $\theta(t)$. Therefore, $Q_{1,1}$, $Q_{2,2}$, and $Q_{3,3}$, which are associated with the out of purpose states of the plant are considered to be zero, and $Q_{4,4}$ and $Q_{5,5}$, which are associated with fourth state of the plant, i.e., $\theta(t)$, and the state of the integrator $x_c(t)$, are important as they relate to the coupling effects of inner and outer loops and are considered to be 1 and 2.5, respectively.

Figure 3 depicts RMP of the system versus μ for various fourth diagonal element of the Q matrix ($Q_{4,4}$), which penalizes the fourth state of the plant. Entry $Q_{5,5}$ is kept the same as its original value 2.5. Accordingly, for the selected range of μ it could be noted that

- for $Q_{4,4} < 0.0.817$, the system is unstable for smaller μ and stable for larger μ;

- for $0.817 \leqslant Q_{4,4} \leqslant 1.697$, the system is stable for smaller μ then becomes unstable and stable once again as μ is increased;

- for $Q_{4,4} > 1.697$, the system is always stable.

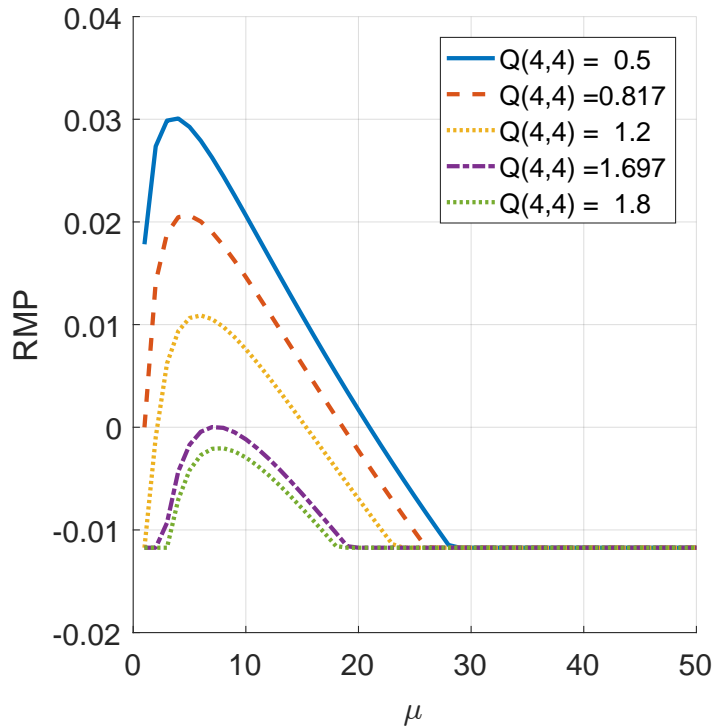American Institute of Aeronautics and Astronautics

Figure 3. Right Most Pole (RMP) vs μ for various $Q_{4,4}$ elements. Here $Q_{5,5} = 2.5$

Next the effects of $Q_{5,5}$ is analyzed. In Figure 4, RMP of the system is depicted with respect to μ while keeping $Q_{4,4} = 1$. Accordingly, it could be noted that

- for $Q_{5,5} \leqslant 1.475$, the system is always stable;

- for $1.475 \leqslant Q_{5,5} < 4.182$, the system is stable for smaller μ but then switches to unstable and then to stable configuration as μ is increased;

- for $Q_{5,5} \geqslant 4.182$, the system is unstable for smaller μ values but then recovers stabiilty for larger μ values.

V.   Conclusion

Human in the loop model reference adaptive control architectures were analyzed and a fundamental stability limit was presented in this paper. This limit resulted from the coupling between outer and inner loop architectures, where the outer loop portion includes the human dynamics modeled as linear time-invariant systems with time delay and the inner loop portion includes the uncertain dynamical system, the reference model, the parameter adjustment mechanism, and the controller. A case study based on a pilot model is presented next to demonstrate the approach.

References

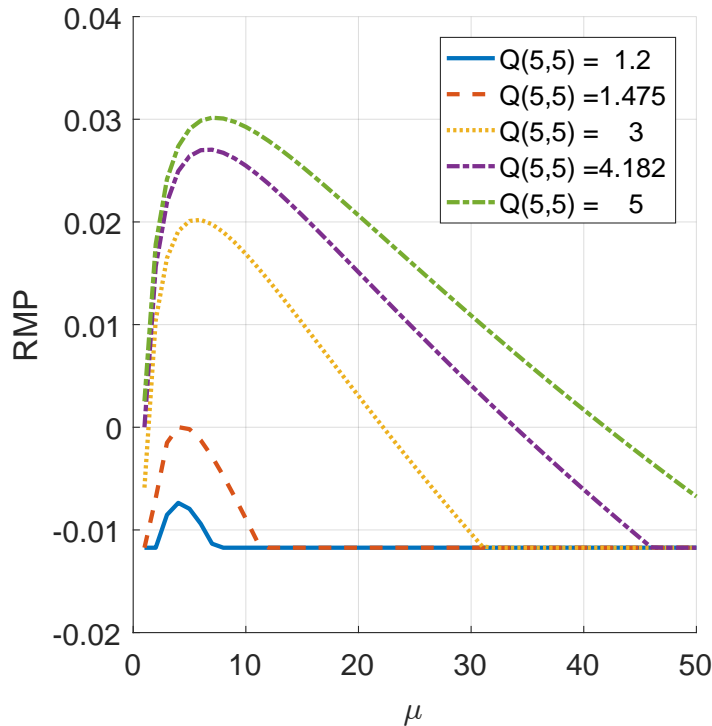[1] P. A. Ioannou and J. Sun, *Robust adaptive control.* Courier Corporation, 2012.

Figure 4. Right Most Pole (RMP) vs μ for various $Q_{5,5}$ elements. Here $Q_{4,4} = 1$.

[2] K. S. Narendra and A. M. Annaswamy, *Stable adaptive systems*. Courier Corporation, 2012.

[3] E. Lavretsky and K. Wise, *Robust and adaptive control: with aerospace applications*. Springer Science & Business Media, 2012.

[4] K. J. Åström and B. Wittenmark, *Adaptive control*. Courier Corporation, 2013.

[5] T. Yucelen and W. M. Haddad, "A robust adaptive control architecture for disturbance rejection and uncertainty suppression with $\mathcal{L}_\infty$ transient and steady-state performance guarantees," *International Journal of Adaptive Control and Signal Processing*, vol. 26, no. 11, pp. 1024–1055, 2012.

[6] T. Yucelen, G. De La Torre, and E. N. Johnson, "Improving transient performance of adaptive control architectures using frequency-limited system error dynamics," *International Journal of Control*, vol. 87, no. 11, pp. 2383–2397, 2014.

[7] H. P. Whitaker, J. Yamron, and A. Kezer, "Design of model reference control systems for aircraft," *Cambridge, MA: Instrumentation Laboratory, Massachusetts Institute of Technology*, 1958.

[8] P. V. Osburn, H. P. Whitaker, and A. Kezer, "New developments in the design of adaptive control systems," *Institute of Aeronautical Sciences*, 1961.

[9] S. Ryu and D. Andrisani, "Longitudinal flying qualities prediction for nonlinear aircraft," *Journal of guidance, control, and dynamics*, vol. 26, no. 3, pp. 474–482, 2003.

[10] C. J. Miller, "Nonlinear dynamic inversion baseline control law: Architecture and performance predictions," *AIAA Guidance, Navigation, and Control Conference*, 2011.

[11] A. Trujillo and I. Gregory, "Adaptive controller adaptation time and available control authority effects on piloting," *NASA Technical Reports Server*, 2013.

[12] A. Trujillo and I. M. Gregory, "Wetware, hardware, or software incapacitation: Observational methods to determine when autonomy should assume control," in *AIAA Aviation Technology, Integration, and Operations Conference*, 2014.

[13] A. C. Trujillo, I. M. Gregory, and L. E. Hempley, "Adaptive state predictor based human operator modeling on longitudinal and lateral control," in *AIAA Modeling and Simulation Technologies Conference*, 2015.

[14] Y. Yildiz and I. Kolmanovsky, "Stability properties and cross-coupling performance of the control allocation scheme capio," *Journal of Guidance, Control, and Dynamics*, vol. 34, no. 4, pp. 1190–1196, 2011.

[15] D. Acosta, Y. Yildiz, R. Craun, S. Beard, M. Leonard, G. Hardy, and M. Weinstein, "Piloted evaluation of a control allocation technique to recover from pilot-induced oscillations," *Journal of Aircraft*, vol. 52, no. 1, pp. 130–140, 2014.

[16] T. Yucelen, Y. Yildiz, R. Sipahi, E. Yousefi, and N. T. Nguyen, "Stability analysis of human-adaptive controller interactions," *International Journal of Control, in review*, 2016.

[17] D. Schmidt and B. Bacon, "An optimal control approach to pilot/vehicle analysis and the neal-smith criteria," *Journal of Guidance, Control, and Dynamics*, vol. 6, no. 5, pp. 339–347, 1983.

[18] A. J. Thurling, "Improving uav handling qualities using time delay compensation," DTIC Document, Tech. Rep., 2000.

[19] J. B. Witte, "An investigation relating longitudinal pilot-induced oscillation tendency rating to describing function predictions for rate-limited actuators," DTIC Document, Tech. Rep., 2004.

[20] F. L. Lewis, A. Yesildirek, and K. Liu, "Multilayer neural-net robot controller with guaranteed tracking performance," *IEEE Trans. Neural Networks*, vol. 7, pp. 388–399, 1996.

[21] F. L. Lewis, K. Liu, and A. Yesildirek, "Neural net robot controller with guaranteed tracking performance," *IEEE Trans. Neural Networks*, vol. 6, pp. 703–715, 1995.

[22] K. S. Narendra and A. M. Annaswamy, "A new adaptive law for robust adaptation without persistent excitation," *IEEE Trans. Autom. Control*, vol. 32, no. 2, pp. 134–145, 1987.

[23] P. Ioannou and P. Kokotovic, "Instability analysis and improvement of robustness of adaptive control," *Automatica*, vol. 20, no. 5, pp. 583–594, 1984.

[24] J. B. Pomet and L. Praly, "Adaptive nonlinear regulation: Estimation from Lyapunov equation," *IEEE Trans. on Autom. Ctrl.*, vol. 37, pp. 729–740, 1992.

[25] T. Yucelen and A. J. Calise, "Kalman filter modification in adaptive control," *Journal of Guidance, Control, and Dynamics*, vol. 33, no. 2, pp. 426–439, 2010.

[26] N. Nguyen, K. Krishnakumar, and J. Boskovic, "An optimal control modification to model-reference adaptive control for fast adaptation," *Proc. AIAA Guid., Navig., and Contr. Conf.*, Honolulu, Hawaii, 2008.

[27] N. Nguyen, M. Bakhtiari-Nejad, and A. Ishihira, "Robust adaptive optimal control with large adaptive gain," *Proc. Amer. Contr. Conf.*, St. Louis, MO, 2010.

[28] T. Yucelen, A. J. Calise, and N. T. Nguyen, "Evaluation of derivative-free adaptive controller with optimal control modification," in *Proceedings of the 2011 AIAA Guidance, Navigation, and Control Conference. Portland, Oregon*, 2011.

[29] A. J. Calise and T. Yucelen, "Adaptive loop transfer recovery," *Journal of Guidance, Control, and Dynamics*, vol. 35, no. 3, pp. 807–815, 2012.

[30] G. Chowdhary and E. N. Johnson, "Theory and flight test validation of a concurrent learning adaptive controller," *AIAA J. Guid. Contr. Dyn.*, vol. 34, pp. 592–607, 2010.

[31] G. Chowdhary, T. Yucelen, M. Mühlegg, and E. N. Johnson, "Concurrent learning adaptive control of linear systems with exponentially convergent bounds," *International Journal of Adaptive Control and Signal Processing*, vol. 27, no. 4, pp. 280–301, 2013.

[32] T. Yucelen and A. J. Calise, "Derivative-free model reference adaptive control," *Journal of Guidance, Control, and Dynamics*, vol. 34, no. 4, pp. 933–950, 2011.

[33] T. Yucelen and W. M. Haddad, "Low-frequency learning and fast adaptation in model reference adaptive control," *Automatic Control, IEEE Transactions on*, vol. 58, no. 4, pp. 1080–1085, 2013.

[34] T. Yucelen, B. Gruenwald, and J. A. Muse, "A direct uncertainty minimization framework in model reference adaptive control," in *AIAA Guidance, Navigation, and Control Conference*, 2015.

[35] B. Gruenwald and T. Yucelen, "On transient performance improvement of adaptive control architectures," *International Journal of Control*, vol. 88, no. 11, pp. 2305–2315, 2015.

[36] H. K. Khalil, *Nonlinear Systems*. Upper Saddle River, NJ: Prentice Hall, 1996.

[37] R. Sipahi, S. I. Niculescu, C. T. Abdallah, W. Michiels, and K. Gu, "Stability and stabilization of systems with time delay," *IEEE Control Systems*, vol. 31, no. 1, pp. 38–65, Feb 2011.

[38] K. Ogata, *Modern Control Engineering*. Upper Saddle River, NJ: Pearson, 2010.

[39] D. Breda, S. Maset, and R. Vermiglio, "Pseudospectral differencing methods for characteristic roots of delay differential equations."

American Institute of Aeronautics and Astronautics