Dynamic risk spillovers between gold, oil prices and conventional, sustainability and Islamic equity aggregates and sectors with portfolio implications

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ABSTRACT

This paper investigates the time-varying equicorrelations and risk spillovers between crude oil, gold and the Dow Jones conventional, sustainability and Islamic stock index aggregates and 10 associated disaggregated Islamic sector stock indexes (basic materials, consumer services, consumer goods, energy, financials, health care, technology, industrials, telecommunications and utilities), using the multivariate DECO-FIAPARCH model and the spillover index of Diebold and Yilmaz (2012). We also conduct a risk management analysis at the sector level for commodity-Islamic stock sector index portfolios, using different risk exposure measures. For comparison purposes, we add the aggregate conventional Dow Jones global index and the Dow Jones sustainability world index. The results show evidence of time-varying risk spillovers between these markets. Moreover, there are increases in the correlations among the markets in the aftermath of the 2008–2009 GFC. Further, the oil, gold, energy, financial, technology and telecommunications sectors are net receivers of risk spillovers, while the sustainability and conventional aggregate DJIM indexes as well as the remaining Islamic stock sectors are net contributors of risk spillovers. Finally, we provide evidence that gold offers better portfolio diversification benefits and downside risk reductions than oil.

1. Introduction

The global financial system has increasingly become more complex due to ongoing structural changes including technological improvements and innovative financial products, which have affected the world's economy and the global financial architecture. In particular, the last two decades have been an era of global financial integration connecting several asset classes together and leading to amplified correlations between them. In the eyes of asset managers and policymakers, this complex situation is a tough challenge that requires hedging through diversification and other protection measures to provide financial stability.

Consecutively, the Islamic finance industry has become an alternative business model to its conventional counterparts in the world of financial intermediation and a promise for more diversification opportunities and financial stability. Over the past two decades, the Shariah-compliant industry has experienced significant growth due to growing interest from the Western world and faith-oriented investors, particularly after the 2008 financial crisis, and as a result of accumulation of oil wealth in faith-supporting countries and a strong participation from faithful investors, combined with a keen willingness of regulators to give more room for this industry. Current estimates of the size of the global Islamic finance assets under management range between USD 1.7 and 2.1 trillion in 2016 and the Islamic Finance industry is expected to grow further in the future.

The Islamic-listed stock securities, which are part of the Islamic finance industry, are a subset of the broader global Islamic securities universe that meets defined screening criteria to assess their compliance with the Shariah principles, and hence their suitability to be considered as Shariah compliant securities. Therefore, the volatility and pricing movements in global stock markets also have an effect on the Shariah-compliant securities (IFSI Stability Report, 2016). This fact leads one to search for more viable alternative asset classes to foster

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the diversification sought by global investors who prefer to include Islamic securities in their investments. Commodities, gold and oil, in particular, are the very first candidates that come to mind in this process. From a traditional perspective, the commodity and equity markets (whether Sharia-compliant or not) are normally inversely related, and therefore commodities are considered to be good portfolio diversifiers (Kang, 2012). For example, gold has traditionally been used as a hedge against inflation and crises. As the US dollar weakens and inflation creeps up, investors would prefer to invest in gold in order to take advantage of potentially higher inflation (Baur and McDermott, 2010). Similarly, in times of substantial price decreases in the stock market, not only gold but also oil may tend to increase in price (Dorsman et al., 2012).[1] If goal is to reduce the risk of Islamic equity portfolios, the literature has not paid much attention to combinations that include gold, oil and Islamic equity indexes.

Examples of those studies that have paid some attention to this asset mixture include Abdullah et al. (2016), Mensi et al. (2015a, 2015b), and Nagayev et al. (2016). However, the problem with these examples is that the first two studies focus on a very limited set of countries (namely Singapore, Malaysia, Philippines and Saudi Arabia), while the latter two consider only a global Islamic equity market index. This shows that a much deeper research is required to analyze the nexus between the most popular commodities, gold and oil, and Islamic equity markets at the aggregate and sector levels. In addition, the obvious missing point is that when we consider Islamic equities at the aggregate level, we miss the opportunity of diversification at the sectoral level. To the best of our knowledge, this study is one of the first that analyzes the interactions between vital commodities (gold and oil) and the Islamic equities at the sectoral level, employs downside risk reduction measures and investigate portfolio diversification benefits. We also consider the aggregate conventional Dow Jones global index (W1DOW) and Dow Jones sustainability world index (W1SGI) for comparison purposes.[2] It is clear that such an analysis would be invaluable in risk management and portfolio construction. Therefore, in this study, we focus on the relationship between gold and oil, and both the Dow Jones aggregate conventional, sustainability, Islamic equity indexes as well as the 10 disaggregate Islamic equity sectors at a global scale. This comprehensive analysis should make this study the first in the literature in this regard.

In particular, using daily data covering almost 20 years, we first implement the multivariate dynamic equicorrelation-fractionally integrated asymmetric power autoregressive conditional heteroskedasticity (DECO-FIAPARCH) model to measure dynamic correlations among conventional stock index, sustainability stock index, the aggregate Islamic stock market, their 10 Islamic sectors and the two commodity markets (oil and gold). Then, we apply the generalized spillover index of Diebold and Yilmaz (2012, 2014, 2016) to examine the directional spillovers and net spillovers across the commodities and the Islamic sector indices. Further, we use the approach of Kroner and Ng (1998) and variance-minimizing hedging strategies to find the time-varying optimal portfolio weights, using the commodities and the Islamic sectoral indexes together to investigate the usefulness of the gold and oil markets for risk management in the Islamic sectoral stock markets. Finally, we estimate the corresponding Value-at-Risk (VaR),

Semivariance (SV) and the Regret (Re) measures to help guide portfolio managers planning to use Islamic equities in designing their strategies.

Our contributions to the literature are three-fold. First, as indicated above, this is the first study that investigates the diversification benefits and the interactions between commodities such as gold and oil, and the Islamic equities at the sectoral level in a global environment. In fact, the aggregate DJIM is sector oriented as the Shariah-compliant firms are heavily concentrated in some sectors like basic materials, technology and industrials. These dissimilarities motivate us to address these interactions and diversification with Islamic indexes. Second, the study covers almost 20 years of daily data which includes crucial events such as the 2001 dotcom bubble, the 2008 global financial crisis (GFC) and the 2012 European sovereign debt crisis (ESDC). Such events allow us to examine the dynamics of equicorrelations, volatility spillovers, optimal portfolio structures and hedging strategies during crisis periods. Third, we use the state of the art methodologies such as the DECO-FIAPARCH, the generalized spillover index of Diebold and the Yilmaz (DY) (2012), optimal portfolio weighting by Kroner and Ng (1998) in our analysis. Further, the findings are strengthened by various risk effectiveness measures.

Our results show a time-varying equicorrelations for the commodity and Islamic stock markets. Furthermore, the spillover index analysis reveals that gold has a lower impact on the Islamic stock markets than the crude oil market, and the latter is a greater receiver of shocks than gold. The results indicate that oil and gold are net receivers of volatility, while surprisingly the aggregate Islamic stock (DJIM) index is a net contributor to volatility spillovers. Furthermore, the recent GFC and the ESDC intensify the total volatility spillovers across the considered markets. Among the 10 Islamic sectors, the consumer goods and the industrials are the highest net volatility contributors, while the finance, technology and telecommunication sectors are the lowest contributors of volatility spillovers, and in fact they are net receivers with the finance sector being the most vulnerable. Further, the risk spillovers between the Islamic stock sectoral markets are globally weak. We find that the optimally weighted portfolio offers both the best risk reductions and the largest downside risk reduction for all oil-Islamic stock sector pairs, the gold-DJIM, gold-Islamic consumer services, the gold-Islamic consumer goods, the gold-Islamic health care, and the gold-Islamic industrials. For the rest of the gold-Islamic sector stock pairs, the hedged portfolio provides the best risk reductions. Finally, gold offers better diversification benefits, risk reductions and the largest downside risk reductions than oil.
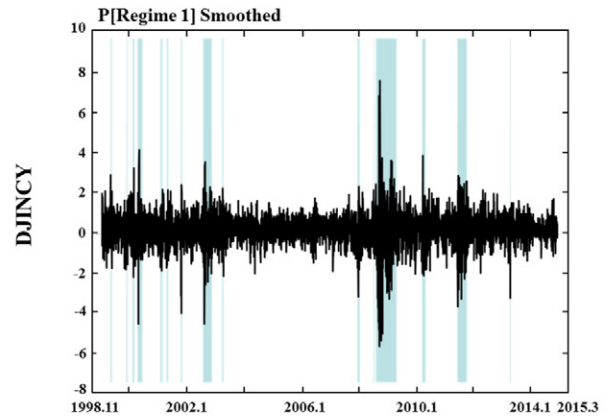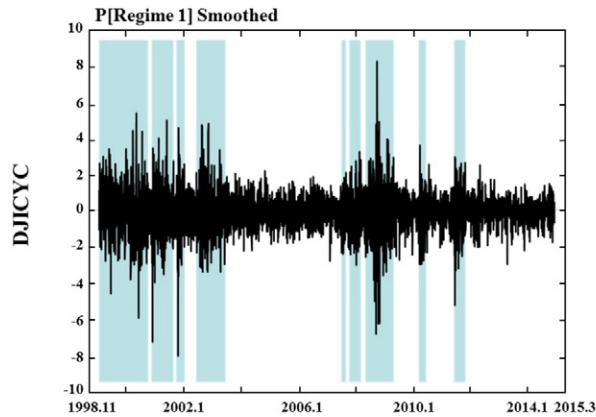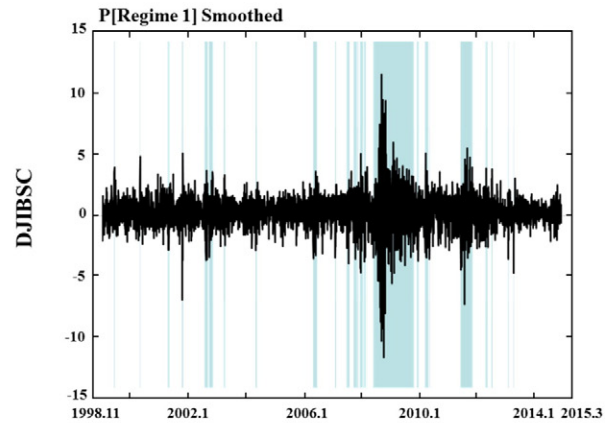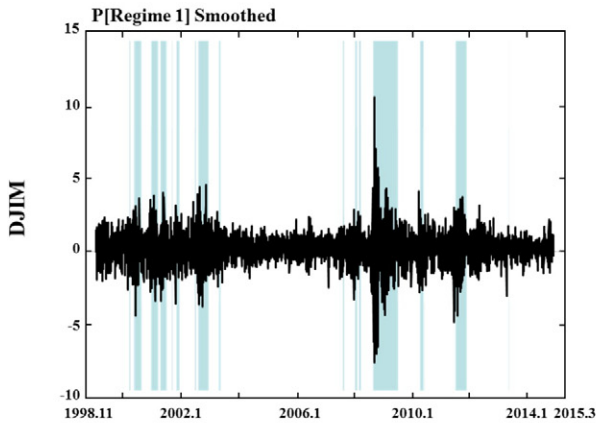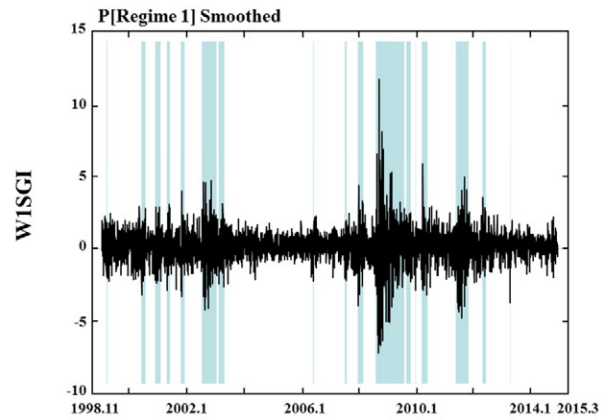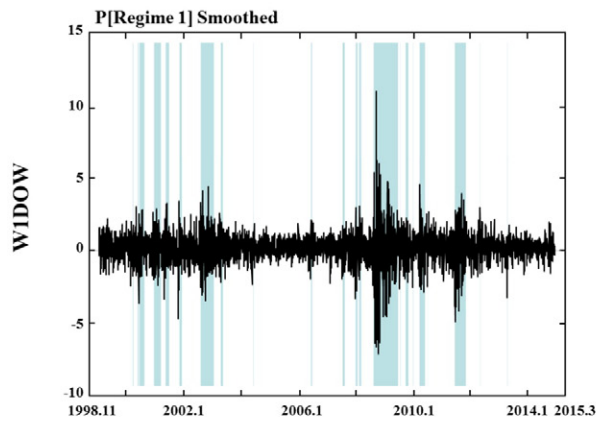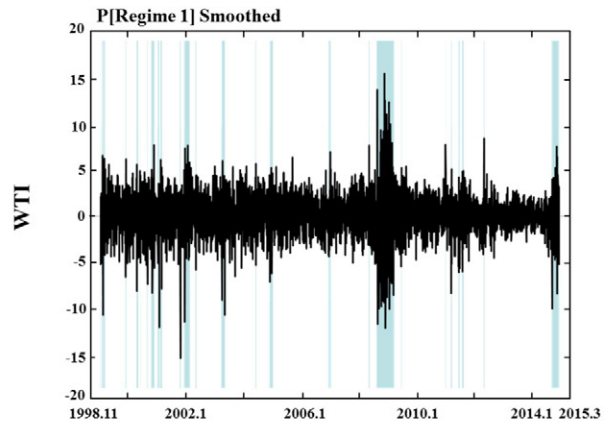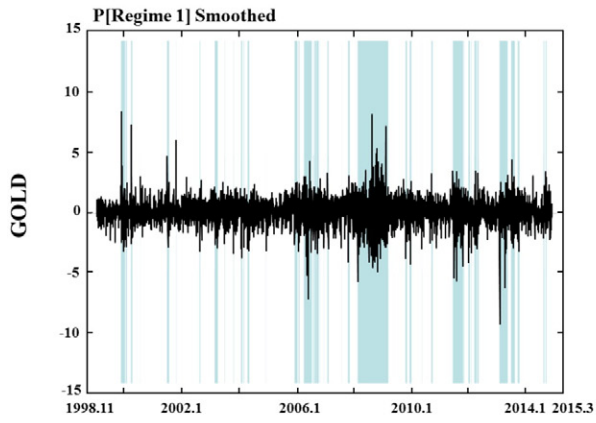
The remainder of this study is organized as follows. Section 2 presents the literature review. Section 3 discusses the methodology. Section 4 describes the data and conducts some preliminary analysis. Section 5 reports and discusses the empirical results. Section 6 provides concluding remarks.

2. Literature review

The early empirical literature has focused on the relative performance of the Islamic finance industry in comparison to its conventional counterparts (e.g., Hayat and Kraussl, 2011; Milly and Sultan, 2012; Beck et al., 2013; Jawadi et al., 2014; Al-Khazali et al., 2014). However, with the occurrence of the recent global financial crisis and the following eurozone debt crisis, researchers and policy makers redirected their attention to the potential risk management applications of Islamic financial assets. The major part of those recent studies focuses on analyzing spillovers between Islamic and conventional assets, and shows how to use those assets in portfolio constructions. The earlier studies in the literature consider combining conventional equities with their Islamic counterparts. For example, in a theoretical framework, Umar (2017) considers two types of stock investors: faith-based and conventional-based. The faith-based investors invest in Shariah complaint equities only and exclude conventional equities from the asset menu. On the other hand, the conventional investors' asset

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[1] However, with the easy access of commodities through financialization in the recent years, commodity prices are not only determined by their primary supply and demand anymore but also by this process. Therefore, the traditional interpretations of the relation between equities and commodities have been questioned lately (Silvennoinen and Thorp, 2013).

[2] The Dow Jones Sustainability Indices are a family of best-in-class benchmarks for investors who have recognized that sustainable business practices are critical to generating long-term shareholder values and who wish to reflect their sustainability convictions in their investment portfolios. The family was launched in 1999 as the first global sustainability benchmark and tracks the stock performance of the world's leading companies in terms of economic, environmental and social criteria. For further information the reader can refer to the following link http://webcache.googleusercontent.com/search?q=cache:http://www.sustainability-indices.com/index-family-overview/djsi-family-overview/
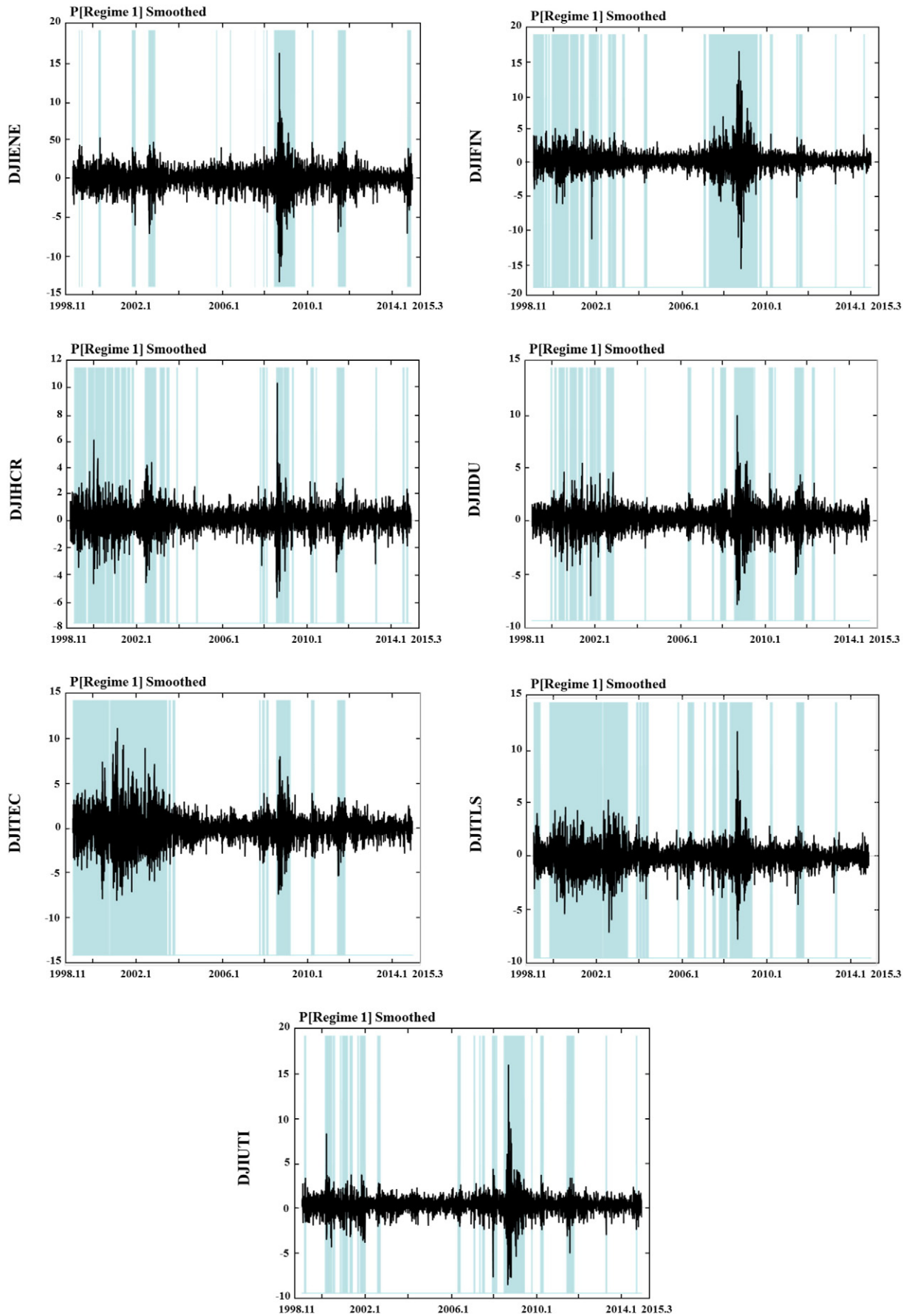
Fig. 1. Time variation of all sample returns. Note: The shaded areas highlight regimes of excess volatility according to the two regime Markov-switching dynamic regression (MS-DR).[6]
[6] For further details on the MS-DR model, see Hamilton (1988) and Hamilton and Susmel (1994). The MS-DR method is conducted by Ox metrics.

Table 1
Descriptive statistics of gold, WTI oil, Dow Jones sustainability world index, conventional Dow Jones global index, DJIM index and the Islamic sector stock index returns.

	Mean	Max.	Min.	Std. dev.	Skewness	Kurtosis	JB	Q(30)	Q²(30)	ADF	PP	KPSS	ARCH-LM(10)
GOLD	0.0345	8.8303	−9.8105	1.1784	−0.143	9.045	6206***	51.76***	713.8***	−63.62***	−63.62***	0.1589	20.63***
WTI	0.0327	16.407	−16.544	2.3971	−0.255	7.344	3242***	56.09***	3289***	−66.02***	−66.08***	0.2827	57.81***
W1DOW	0.0153	11.470	−7.6568	1.0536	−0.280	15.448	15,177***	100.7***	7744***	−56.26***	−55.96***	0.0763	162.92***
W1SGI	0.0085	12.221	−7.7746	1.1699	−0.072	11.50	12,264***	119.9***	5880***	−57.99***	−57.74***	0.0669	112.83***
DJIM	0.0169	10.984	−8.1854	1.0993	−0.283	10.67	10,041***	47.83***	6345***	−41.91***	−70.21***	0.0900	182.84***
DJIBSC	0.0224	12.062	−12.551	1.4338	−0.546	12.62	15,887***	264.7***	8711***	−41.02***	−64.62***	0.0831	207.92***
DJICYC	0.0269	8.7624	−8.2846	1.1479	−0.167	7.677	3725***	200.5***	2950***	−43.57***	−73.36***	0.1106	71.678***
DJINCY	0.0201	7.8587	−6.1352	0.8502	−0.325	10.96	10,812.***	352.5***	4610***	−43.89***	−71.57***	0.1242	136.84***
DJIENE	0.0191	16.879	−13.905	1.5287	−0.456	14.08	48,114***	122.6***	7227***	−42.93***	−73.14***	0.0814	201.95***
DJIFIN	0.0111	17.277	−16.981	1.5741	−0.023	20.18	50,026***	199.0***	6533***	−43.86***	−1814***	0.0450	154.12***
DJIHCR	0.0219	10.747	−6.1933	0.9841	−0.112	10.35	9178***	139.6***	3010***	−44.12***	−69.80***	0.2114	94.99***
DJIIDU	0.0215	10.231	−8.3919	1.1872	−0.332	9.251	6695***	192.9***	6136***	−40.64***	−66.94***	0.0876	153.9***
DJITEC	0.0125	11.719	−8.2984	1.6886	0.120	7.442	3353.***	60.74***	4438.***	−43.19***	−73.84***	0.1316	87.41***
DJITLS	0.0069	12.426	−8.0750	1.1992	0.088	9.457	7069***	106.1***	3949***	−43.59***	−70.37***	0.0826	106.8***
DJIUTI	0.0024	16.509	−9.3299	1.1455	0.097	24.30	76,931***	166.1***	4559***	−44.29***	−71.55***	0.1227	203.2***

Notes: J-B denotes the empirical statistics of the Jarque-Bera test for normality, the Ljung-Box Q(30) and Q²(30) tests for no autocorrelation of the residuals and the square residuals, respectively. ADF, PP and KPSS are the empirical statistics of the Augmented Dickey and Fuller (1979), and the Phillips and Perron (1988) unit root tests and the Kwiatkowski et al. (1992) stationarity test, respectively. The ARCH-LM(10) test of Engle (1982) checks the presence of ARCH effects. The asterisk *** denotes the rejection of the null hypotheses of normality, no autocorrelation, unit root, stationarity, and conditional homoscedasticity at the 1% significance level.

menu comprises both Islamic and conventional equities. Accordingly, the inclusion of Islamic equities in the asset menu of faith-based investors results in substantial welfare gains; and this result has been empirically verified by more recent studies (Hammoudeh et al., 2014; Rahim and Masih, 2016; Dewandaru et al., 2016; Charfeddine et al., 2016; Majdoub et al., 2016).[3] Less attention has been given on combining Islamic equities and commodities in the literature. Among those few, Abdullah et al. (2016) find that the Philippine Islamic stock index is less correlated with crude oil in the short run (as evidenced by a continuous wavelet transform analysis) and that an investor holding crude oil can gain by including the Malaysian Islamic stock index in the portfolio (as evidenced by a dynamic conditional correlation analysis).

Mensi et al. (2015a) examine the time-varying linkages of the Saudi stock market with major commodity futures markets including WTI oil, gold, silver, wheat, corn and rice, using the bivariate DCC–FIAPARCH model with and without structural breaks. For mixed commodity–stock portfolios, those authors find strong evidence of diversification benefits, hedging effectiveness and downside risk reductions. Using the MGARCH-DCC and wavelet coherence framework, Nagayev et al. (2016) reveal that the correlations between commodity markets and the Dow Jones Islamic market world index are time-varying and highly volatile throughout the January 1999–April 2015 period. A substantial and persistent increase is observed in the return correlations between commodities and Islamic equity at the onset of the 2008 GFC. However, the trends in the recent two years suggest that this association is heading toward its pre-crisis levels, offering again diversification benefits for Islamic equity holders. These benefits vary across different commodities in various time scales. Overall, gold, natural gas, soft commodities, grains and livestock are better portfolio diversifiers than oil and other metals. Mensi et al. (2015b) examine whether the Sharia-compliant stocks measured by the Dow Jones Islamic world emerging market index (DJIWEM) and gold can serve as a hedge and/or a safe-haven asset in the six GCC stock markets, by using a vine copula approach. The results show that GCC and global investors can realize both risk diversification benefits and downside risk reductions during tranquil and downturn periods by including gold or DJIWEM in their portfolios.

The problem with the abovementioned studies is that they all consider Islamic equities at the aggregate level, and therefore they miss the opportunities of examining diversification at the sectoral level. Indeed, among the whole literature, the studies that take into account the interactions of such sector indexes can be counted on the fingers of one hand. For example, Balcilar et al. (2015) assess the risk exposures of major Islamic sector indexes with respect to shocks in global conventional markets but without including refuge assets and find positive risk exposures of Islamic equity sectors with respect to developed market shocks. Those authors find that both the in- and out-of-sample results suggest that portfolios supplemented with positions in the Islamic equity sectors yield much improved risk-adjusted returns, thereby implying significant international diversification benefits. In particular, the Financials, Healthcare, Telecommunication and Utilities sectors are found to have greater significance in global diversification strategies due to their higher weights allocated in the optimal portfolios. A recent study by Mensi et al. (2016) takes a different perspective of Islamic sector investing from the one undertaken by Balcilar et al. (2015). Accordingly, Mensi et al. (2016) analyze the dynamic spillovers across 10 Dow Jones Islamic and conventional sector index pairs. Using four different MVGARCH-cDCC models, they find evidence of the claim that the conditional correlations for all the sector pairs (except those of the Telecommunication and Utilities sectors) increase after the onset of the global financial crisis, suggesting non-subsiding risks, contagion effects and gradual greater financial linkages. Accordingly, the Islamic sectors' risk exposure can be effectively hedged over time in diversified portfolios containing conventional sector stocks.[4]

However, to the best of our knowledge, there is no study that analyzes the interaction between commodities such as gold and oil, and the Islamic equities at the sectoral level as explained in the introduction section. The extensive literature reviewed above suggests that such an analysis would be invaluable in risk management, in particular for portfolio construction.

3. Empirical method

This section discusses the empirical methods used in this study. First, we present a multivariate DECO-FIAPARCH model, which measures the dynamic conditional correlations between the markets under consideration as explained earlier. Second, we present the spillover index of Diebold and Yilmaz (2012), which identifies the dynamics of directional volatility spillovers across the commodity and stock markets under consideration.

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[3] Alternatively, some studies suggest that combining the conventional U.S. markets with emerging Islamic equity markets leads to improved portfolio performance (see Majdoub and Mansour, 2014; Saiti et al., 2014).

[4] Both results are indirectly supported by Yilmaz et al. (2015) and Sensoy (2016).

Table 2

Historical unconditional correlations of sample returns between gold, WTI oil, Dow Jones sustainability world index, conventional Dow Jones global index, DJIM index and the Islamic sector stock indices.

	GOLD	WTI	W1DOW	W1SGI	DJIM	DJBSC	DJICYC	DJNCY	DJIENE	DJIFIN	DJIHCR	DJIIDU	DJITEC	DJITLS	DJIUTI
GOLD	1.0000														
WTI	0.2195	1.0000													
W1DOW	0.1032	0.2681	1.0000												
W1SGI	0.1261	0.2614	0.9457	1.0000											
DJIM	0.1118	0.2813	0.9758	0.9114	1.0000										
DJIBSC	0.3202	0.3173	0.8245	0.8167	0.7863	1.0000									
DJICYC	−0.0222	0.1076	0.8101	0.7145	0.8106	0.5587	1.0000								
DJINCY	0.0685	0.1698	0.8368	0.7864	0.7945	0.7077	0.7145	1.0000							
DJIENE	0.1829	0.4452	0.7467	0.7223	0.7482	0.7499	0.5144	0.6290	1.0000						
DJIFIN	0.0174	0.0983	0.5763	0.4797	0.5474	0.4415	0.5517	0.5091	0.4143	1.0000					
DJIHCR	0.0281	0.1112	0.7433	0.7074	0.7489	0.5396	0.6476	0.7138	0.5631	0.3828	1.0000				
DJIIDU	0.0868	0.2419	0.9447	0.8755	0.9307	0.7860	0.7732	0.7800	0.6556	0.5790	0.6269	1.0000			
DJITEC	−0.0077	0.1475	0.7833	0.6913	0.8518	0.4960	0.7054	0.5380	0.4307	0.4352	0.5111	0.7747	1.0000		
DJITLS	0.0834	0.1749	0.7667	0.7606	0.7538	0.5952	0.6021	0.6066	0.4971	0.4393	0.5265	0.7123	0.5922	1.0000	
DJIUTI	0.1568	0.2781	0.6826	0.7019	0.6459	0.6716	0.4435	0.5700	0.6449	0.3519	0.5133	0.6137	0.3689	0.5593	1.0000

3.1. The DECO-FIAPARCH model

Engle (2002) develops the dynamic conditional correlation (DCC)-GARCH model, which offers flexibility to simultaneously model the multivariate conditional volatility of stock returns and their time-varying correlations. Despite its flexibility, the DCC estimation involves computing the correlation of too many pairs sampled $n(n-1)/2$ times, which produces results that are difficult to interpret (Aboura and Chevallier, 2014). To overcome this limitation, Engle and Kelly (2012) propose to use the DECO-GARCH model, which can eliminate the computational and presentational difficulties of high-dimension systems (Pan et al., 2016). The DECO model is a special version of the DCC model in which the correlations across all pairs of assets are equal but the common equicorrelation is time-varying. Another advantage of equicorrelation is that it provides superior forecasting ability during the crisis periods across the various portfolios (Clements et al., 2014).

The AR(1) model in which the dynamics of current stock returns are explained by their lagged returns is defined as follows:

$$r_t = \mu + \xi r_{t-1} + \varepsilon_t, t \in T \text{ with } \varepsilon_t = z_t \sqrt{h_t} \tag{1}$$

where $|\mu| \in [0 \infty), |\xi| < 1$ and the innovations $\{z_t\}$ follow the Student's t-distribution ($z_t \sim ST(0,1,\nu)$). The Student's t-distribution is estimated with the parameter (ν), which represents the number of degrees of freedom (df) and measures the degree of leptokurtosis displayed by the density (Fiorentini et al., 2003). The conditional variance h_t is positive with probability one and is a measurable function of the variance-covariance matrix Σ_{t-1}.

The FIAPARCH (p,d,q) model of Tse (1998) is formally expressed as follows:

$$h_t^{\delta/2} = \omega[1-\beta(L)]^{-1} + \left[1-[1-\beta(L)]^{-1}\phi(L)(1-L)^d\right](|\varepsilon_t| - \lambda\varepsilon_t)^{\delta}, \tag{2}$$

where ω, β, ϕ, and d are the parameters to be estimated. The parameter d where $0 \le d \le 1$ measures the long-range memory in the conditional volatility, L denotes the lag operator, δ is the power term of returns for the predictable structure in the volatility persistence, and $\lambda > 0$ represents the asymmetry parameter indicating that negative shocks give rise to higher volatility than positive shocks of equal size.

We follow Engle (2002) to obtain the dynamic conditional correlations. We assume that $E_{t-1}[\varepsilon_t] = 0$ and, $E_{t-1}[\varepsilon_t \varepsilon_t'] = H_t$, where $E_t[\cdot]$ is the conditional expectation on using the information set available at time t. The conditional variance-covariance matrix, H_t, can be written as:

$$H_t = D_t R_t D_t \tag{3}$$

where $D_t = \text{diag}(h_{11,t}^{1/2}, \cdots, h_{NN,t}^{1/2})$ is the $N \times N$ diagonal matrix of conditional standard deviations of the residuals, which are obtained from taking the square root of the conditional variance modelled by an univariate AR(1)-FIAPARCH(1,d,1) model. Moreover, R_t is a matrix of time-varying conditional correlations, which is given by:

$$R_t = \{Q_t^*\}^{-1/2} Q_t (Q_t^*)^{-1/2}, \tag{4}$$

$$Q_t^* = \text{diag}[Q_t], \tag{5}$$

$$Q_t = [q_{ij,t}] = (1-a-b)S + au_{t-1}u_{t-1}' + bQ_{t-1}, \tag{6}$$

where $u_t = [u_{1,t}, \cdots, u_{n,t}]'$ is the standardized residuals (i.e. $u_{i,t} = \varepsilon_{i,t}/h_{i,t}$), $S \equiv [s_{i,j}] = E[u_t u_t']$ is the $n \times n$ unconditional covariance matrix of u_t, and a and b are non-negative scalars satisfying $a+b < 1$. The resulting model is called the DCC model.

In this context, Aielli (2013) proves that the estimation of the covariance matrix Q_t in this way is inconsistent because $E[R_t] \neq E[Q_t]$, and suggests the following consistent model (cDCC model) for the correlation-driving process:

$$Q_t = (1-a-b)S^* + a\left(Q_{t-1}^{*}{}^{1/2}u_{t-1}u_{t-1}'Q_{t-1}^{*}{}^{1/2}\right) + bQ_{t-1}, \tag{7}$$

where S^* is the unconditional covariance matrix of $Q_t^{*1/2}u_t$.

Engle and Kelly (2012) suggest that we model ρ_t by using the cDCC process to obtain conditional correlation matrix Q_t and then taking the mean of its off-diagonal elements. This approach, which reduces estimation time, is called the dynamic equicorrelation (DECO) model. The scalar equicorrelation is defined as:

$$\rho_t^{DECO} = \frac{1}{n(n-1)}\left(J_n'R_t^{cDCC}J_n - n\right) = \frac{2}{n(n-1)}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}}, \tag{8}$$

where $q_{ij,t} = \rho_t^{DECO} + a^{DECO}(u_{i,t-1}u_{j,t-1} - \rho_t^{DECO}) + b^{DECO}(q_{ij,t} - \rho_t^{DECO})$, which is the $(i,j)^{\text{th}}$ element of the matrix Q_t from the cDCC model. We then use this scalar equicorrelation to estimate the conditional correlation matrix:

$$R_t = (1-\rho_t)I_n + \rho_t J_n, \tag{9}$$

where J_n is the $n \times n$ matrix of ones and I_n is the n-dimensional identity matrix.

Table 3
Estimation of the multivariate AR(1)-FIAPARCH(1,d,1)-DECO model.

	Gold	WTI	W1DOW	W1SGI	DJIM	DJIBSC	DJICYC	DJINCY	DJIENE	DJIFIN	DJIHCR	DJIIIDU	DJITEC	DJITLS	DJIUTI
Panel A: estimates of the AR(1)-FIAPARCH model															
Const.(μ)	0.0346**	0.0328***	0.0051	−0.0007	0.0066	0.0219	0.0137	0.0138	0.0307	0.0264	0.0114	0.0161	0.0200	0.0146	0.0310**
	(0.0170)	(0.0293)	(0.0134)	(0.0145)	(0.0133)	(0.0202)	(0.0157)	(0.0109)	(0.0191)	(0.0160)	(0.0123)	(0.0160)	(0.0173)	(0.0154)	(0.0134)
AR(1)	−0.0112	−0.0239	0.1627***	0.1096***	0.1281***	0.2034***	0.0805 ***	0.1033***	0.0949***	0.0796***	0.0708***	0.1887***	0.0916***	0.1242***	0.0674***
	(0.0170)	(0.0173)	(0.0151)	(0.0155)	(0.0141)	(0.0159)	(0.0173)	(0.0160)	(0.0165)	(0.0161)	(0.0163)	(0.0157)	(0.0154)	(0.0161)	(0.0174)
Const.(ω)	0.8582***	0.0959***	0.0129***	0.0160***	0.0189***	0.0200***	0.0265***	0.0118***	0.0328***	0.0920***	0.0182***	0.0172***	0.0456***	0.0356***	0.0239***
	(0.3247)	(0.0262)	(0.0023)	(0.0033)	(0.0033)	(0.0044)	(0.0393)	(0.0021)	(0.0068)	(0.0349)	(0.0036)	(0.0031)	(0.7333)	(0.0077)	(0.0064)
d-Figarch	0.3201***	0.4487***	0.3276***	0.3642***	0.3642***	0.4114***	0.3788***	0.3545***	0.4452***	0.6062***	0.3723***	0.3695***	0.4394***	0.4747***	0.4491***
	(0.0629)	(0.0573)	(0.0292)	(0.0342)	(0.0342)	(0.0544)	(0.0327)	(0.0359)	(0.0507)	(0.0730)	(0.0381)	(0.0323)	(0.0331)	(0.0477)	(0.0541)
Arch	0.3774***	0.3664***	0.2133***	0.2031***	0.2031***	0.3798***	0.2713***	0.2465***	0.3046***	0.1985***	0.2766***	0.2245***	0.2248***	0.2513***	0.2593***
	(0.1051)	(0.0544)	(0.0558)	(0.0507)	(0.0507)	(0.0446)	(0.0542)	(0.0533)	(0.0402)	(0.0478)	(0.0553)	(0.0462)	(0.0368)	(0.0486)	(0.0810)
Garch	0.6246***	0.7071***	0.4917***	0.5185***	0.5185***	0.6811***	0.5840***	0.5479***	0.6683***	0.7177***	0.5727***	0.5381***	0.6108***	0.6529***	0.5779***
	(0.0928)	(0.0696)	(0.0624)	(0.0533)	(0.0533)	(0.0517)	(0.0546)	(0.0563)	(0.0509)	(0.0750)	(0.0636)	(0.0523)	(0.0412)	(0.0585)	(0.0894)
APARCH(λ)	−0.1067	0.4065***	0.9711***	0.8446***	0.8446***	0.4492***	0.8914***	0.8728***	0.4646***	0.3846***	0.7037***	0.7833***	0.7611***	0.3719***	0.2899***
	(0.1095)	(0.1514)	(0.1384)	(0.1568)	(0.1568)	(0.1137)	(0.1233)	(0.1727)	(0.1233)	(0.0858)	(0.1280)	(0.1366)	(0.1280)	(0.1031)	(0.0818)
APARCH(δ)	2.0973***	1.4211***	1.2258***	1.2750***	1.2292***	1.5749***	1.1548***	1.2579***	1.4591***	1.4973***	1.3049 ***	1.2968***	1.1884***	1.4572***	1.5847***
	(0.1319)	(0.1716)	(0.0641)	(0.0771)	(0.0728)	(0.1056)	(0.0838)	(0.0845)	(0.1185)	(0.1294)	(0.0912)	(0.0772)	(0.0882)	(0.1291)	(0.1235)
Panel B: estimates of the DECO model															
CORij															0.6368***
															(0.0972)
a_{DECO}															0.0382***
															(0.0070)
b_{DECO}															0.9606***
															(0.0077)
df															7.6079***
															(0.2171)
Panel C: diagnostic tests															
Q(30)	44.895	15.501	35.279	16.655	29.105	40.455	39.891	21.585	38.583	27.568	36.105	38.367	33.052	27.907	29.939
	[0.0301]	[0.9865]	[0.2326]	[0.9764]	[0.4595]	[0.0767]	[0.1069]	[0.8687]	[0.1353]	[0.5410]	[0.1705]	[0.1143]	[0.3202]	[0.5228]	[0.4170]
Q²(30)	39.345	30.007	31.981	34.153	37.112	18.769	31.461	30.347	43.973	22.361	24.547	23.498	28.658	33.937	26.778
	[0.0755]	[0.4652]	[0.3684]	[0.2747]	[0.1163]	[0.9051]	[0.3929]	[0.4479]	[0.0279]	[0.8643]	[0.6523]	[0.7077]	[0.5355]	[0.2833]	[0.6348]

Notes: Q(30) and Q²(30) are the Ljung-Box test statistic applied to the standard residuals and the squared standardized residuals, respectively. The *p*-values are in brackets while the standard error values are reported in parentheses. *, **, and *** indicate significance at the 10%, 5% and 1% levels, respectively.
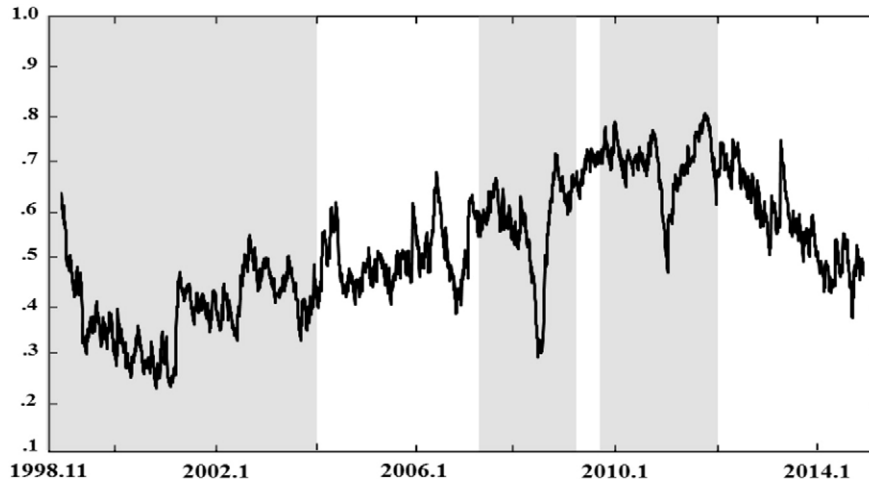
Fig. 2. Dynamic equicorrelation for the group.

Note that the estimation of the DECO model is carried out using a two-step maximum likelihood of the probability density function of a multivariate Student's t-distribution expressed as:

$$l_t(\Theta) = \log\left\{\left(\Gamma\left(\frac{v+2}{2}\right)v\right)\Big/\left((v\pi)\Gamma\left(\frac{v}{2}\right)(v-1)\right)\right\} - (1/2)\log(|H_t|) - (1/2)$$
$$\times (v+2)\log\left[1 + \left(\varepsilon_t' H_t^{-1}\varepsilon_t\right)\big/(v-2)\right], \tag{10}$$

where $\Gamma(\cdot)$ is the Gama function, v is the degree of freedom for the Student's t-distribution, H_t is a conditional variance-covariance matrix. Θ is a parameter vector with all of the coefficients of the DECO-FIAPARCH model.

3.2. Spillover index framework

We apply the generalized VAR methodology, variance decomposition, and the generalized spillover index of Diebold and Yilmaz (2012) to examine the directional spillovers and net spillovers across the two commodity futures prices (Gold and WTI) and Islamic sector indices. Following Diebold and Yilmaz (2012), we assume a covariance stationary n-variable VAR(p):

$$y_t = \sum_{i=1}^{p}\Phi_i y_{t-1} + \varepsilon_t, \tag{11}$$

where y_t is the $n \times 1$ vector of endogenous variables, Φ_i are $n \times n$ autoregressive coefficient matrices, and ε_t is a vector of error terms that are assumed to be serially uncorrelated. If the VAR system above is a covariance stationary, then a moving average representation is written as $y_t = \sum_{j=0}^{\infty} A_j \varepsilon_t$, where the $n \times n$ coefficient matrix A_j obeys a recursion of the form $A_j = \Phi_1 A_{j-1} + \Phi_2 A_{j-2} + \ldots + \Phi_p A_{j-p}$, with A_0 being the $n \times n$ identity matrix and $A_j = 0$ for $j < 0$. The total, directional, and net spillovers are generated by generalized forecast-error variance decompositions of the moving average representation of the VAR model. The framework of generalized variance decompositions eliminates any dependence of the results on the ordering of the variables.

Koop et al. (1996) and Pesaran and Shin (1998) propose the following H-step-ahead generalized forecast-error variance decomposition:

$$\theta_{ij}(H) = \frac{\sigma_{jj}^{-1}\sum_{h=0}^{H-1}\left(e_i' A_h \Sigma e_j\right)^2}{\sum_{h=0}^{H-1}\left(e_i' A_h \Sigma A_h' e_i\right)} \tag{12}$$

where Σ is the variance matrix of the vector of errors ε, and σ_{jj} is the standard deviation of the error term of the j th equation. Finally, e_i is a selection vector with a value of one for the i th element, and zero otherwise. The spillover index yields a $n \times n$ matrix $\theta(H) = [\theta_{ij}(H)]$, where each entry gives the contribution of variable j to the forecast error variance of variable i. Own-variable and cross-variable contributions are contained in the main diagonal and the off-diagonal elements of $\theta(H)$ matrix, respectively.

Because the own- and cross-variable variance contribution shares do not sum to one under the generalized decomposition (i.e., $\sum_{j=1}^{n}\theta_{ij}(H) \neq 1$), each entry of the variance decomposition matrix is normalized by its row sum as follows:

$$\theta_{ij}(H) = \frac{\theta_{ij}(H)}{\sum_{j=1}^{n}\theta_{ij}(H)}, \tag{13}$$

with $\sum_{j=1}^{n}\tilde{\theta}_{ij}(H) = 1$ and $\sum_{i,j=1}^{n}\tilde{\theta}_{ij}(H) = n$ by construction.

This allows us to define a total spillover index as:

$$TS(H) = \frac{\sum_{i,j=1,i\neq j}^{n}\tilde{\theta}_{ij}(H)}{\sum_{i,j=1}^{n}\tilde{\theta}(H)} \times 100 = \frac{\sum_{i,j=1,i\neq j}^{n}\tilde{\theta}_{ij}(H)}{n} \times 100 \tag{14}$$

This index measures the average contribution of spillovers from shocks in all (other) markets to the total forecast error variance. Additionally, this index is flexible and enables the identification of the directional spillovers among all markets. Specifically, the directional spillovers received by market i from all other markets j are defined as:

$$DS_{i\leftarrow j}(H) = \frac{\sum_{i,j=1,i\neq j}^{n}\tilde{\theta}_{ij}(H)}{\sum_{i,j=1}^{n}\tilde{\theta}_{ij}(H)} \times 100 = \frac{\sum_{i,j=1,i\neq j}^{n}\tilde{\theta}_{ij}(H)}{n} \times 100 \tag{15}$$

Similarly, the directional spillovers transmitted by market i to all other markets j are defined as:

$$DS_{i\rightarrow j}(H) = \frac{\sum_{i,j=1,i\neq j}^{n}\tilde{\theta}_{ji}(H)}{\sum_{i,j=1}^{n}\tilde{\theta}_{ji}(H)} \times 100 = \frac{\sum_{i,j=1,i\neq j}^{n}\tilde{\theta}_{ji}(H)}{n} \times 100 \tag{16}$$

The set of directional spillovers provides a decomposition of total spillovers into those coming from (or to) a particular market. For instance, in the present application this means that this spillover matrix $\theta(H)$ consists of the main diagonal elements reflecting
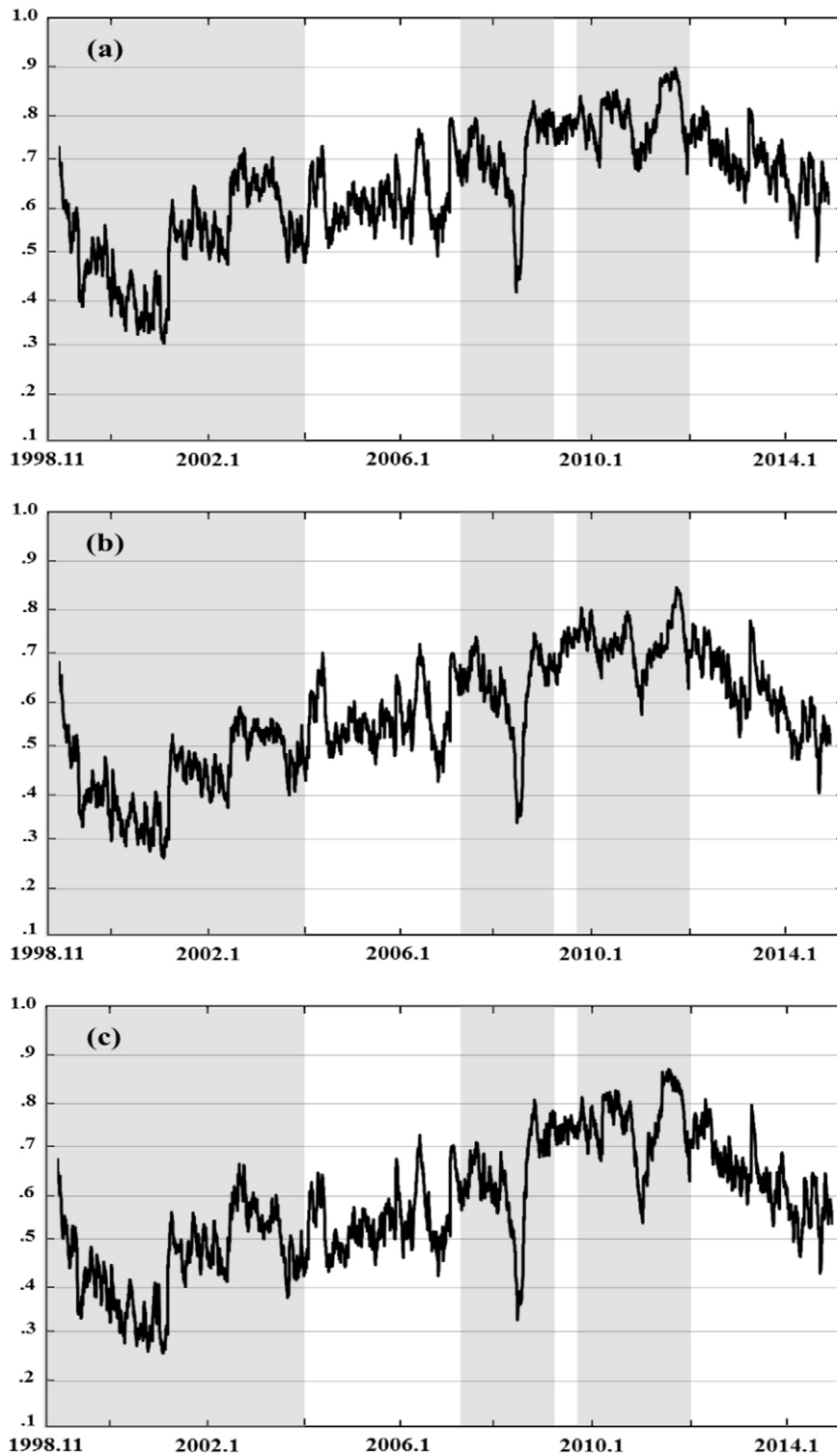
Fig. 3. Dynamic equicorrelation for stock indices and commodity futures; (a) within stocks; (b) gold-stocks; (c) WTI-stocks.

own-market spillovers, and the off-diagonal elements reflecting cross-market spillovers.

Finally, subtracting Eq. (16) from Eq. (15), we compute the net volatility spillovers from each market to all other markets as:

$$NS_i(H) = DS_{i \to j}(H) - DS_{i \leftarrow j}(H) \tag{17}$$

The net spillovers demonstrate whether a market is a receiver or transmitter of spillovers in net terms. It is also our interest to examine

the net pairwise spillovers (NPS) as following:

$$NPS_{ij}(H) = \left[\frac{\tilde{\theta}_{ji}(H)}{\sum_{i,k=1}^{n} \tilde{\theta}_{jk}(H)} - \frac{\tilde{\theta}_{ij}(H)}{\sum_{j,k=1}^{n} \tilde{\theta}_{jk}(H)} \right] \times 100 \tag{18}$$

The net pairwise spillover between markets i and j is simply the difference between the gross shocks transmitted from market i to market j and those transmitted from j to i.

Table 4
Total volatility spillovers.

To (i)	GOLD	WTI	W1DOW	W1SGI	DJIM	DJIBSC	DJICYC	DJINCY	From(j) DJIENE	DJIFIN	DJIHCR	DJIIDU	DJITEC	DJITLS	DJIUTI	**From others**
GOLD	65.95	0.58	3.67	3.12	2.78	4.07	2.89	2.72	2.73	2.78	1.21	3.74	0.61	1.16	2	34
WTI	1.03	41.67	4.78	4.59	4.24	3.13	3.99	4.33	7.17	9.24	3.16	4.51	2.49	2.46	3.23	58.3
W1DOW	0.72	1.71	12.06	10.58	11.11	8.11	6.61	7.8	6.96	2.38	7.02	10.19	3.84	4.19	7.71	88.9
W1SGI	0.65	1.87	11.97	12.29	10.89	7.44	6.16	8.12	6.56	1.88	7.7	9.49	3.48	4.55	7.95	88.7
DJIM	0.58	1.67	11.8	9.93	11.92	7.67	6.59	7.56	6.64	2.04	7.44	10.03	4.97	4.43	7.72	89.1
DJIBSC	1.05	1.51	10.26	9.25	9.41	11.95	5.29	7.4	9.85	3.7	6.08	9.01	2.09	3.9	9.26	88.1
DJICYC	0.88	1.13	9.84	7.65	9.74	5	16.79	7.32	4.71	4.42	7.36	9.82	6.55	4.87	3.9	83.2
DJINCY	0.76	1.36	11.43	9.89	10.3	6.94	7.22	11.73	6.51	2.76	8.18	9.57	3.12	3.79	6.42	88.3
DJIENE	0.75	2.24	10.19	9.07	9.68	8.58	5.54	8	11.05	3.27	7.86	8	2.28	4	10.5	90
DJIFIN	1.68	1	7.98	5.56	7.16	6.91	8.33	6.81	7.5	21.03	5.47	7.84	2.89	3.41	7.43	80
DJIHCR	0.5	1.12	9.91	8.69	9.87	5.25	7.69	9.85	6.18	2.32	17.26	7.32	3.97	3.98	6.07	82.7
DJIIDU	0.78	1.77	11.61	9.89	11.03	7.73	7.08	7.24	6.47	2.66	5.84	12.37	4.87	4.58	6.09	87.6
DJITEC	0.21	1.76	9.23	6.94	12.69	2.57	8.72	3.9	2.45	1.59	4.91	10.28	26.26	6.81	1.67	73.7
DJITLS	0.6	1.66	9.02	9.86	9.89	4.29	5.44	6.76	4.91	1.39	8.61	6.4	5.1	18.2	7.88	81.8
DJIUTI	0.73	1.69	10.19	9.08	9.37	8.45	4.64	7.78	7.59	2.43	7.29	7.46	2.33	3.5	17.46	82.5
To others	10.9	21.1	131.9	114.1	128.1	86.2	86.2	95.6	86.2	42.9	88.1	113.7	48.6	55.6	87.8	1196.9
All	76.9	62.8	143.9	126.4	140.1	98.1	103	107.3	97.3	63.9	105.4	126	74.9	73.8	105.3	**Total**: 79.79%

Notes: The underlying variance decomposition is based on a daily VAR of order 4 (as determined by the Schwarz information criterion), identified using a generalized VAR spillover framework by Diebold and Yilmaz (2012). The (i,j)th element of the table shows the estimated contribution to the variance of the 10-day-ahead forecast error of i coming from innovations to variable j. The diagonal elements (i = j) are the own variance shares estimates, which show the fraction of the forecast error variance of market i that is due to its own shocks. The last column "**From others**" shows the total spillovers received by a particular market from all other markets, while the row "**To others**" shows the spillover effect directed by a particular market to all other markets. The lower right corner "**Total**" indicates the level of total spillovers.

4. Data and preliminary analysis

4.1. Data

We use daily closing spot price data for gold, WTI crude oil and the aggregate and disaggregate Dow Jones Islamic sectoral indexes. The WTI futures crude oil benchmark, the reference crude for the United States, is traded on NYMEX. Concerning the Islamic equities, they include the aggregate Dow Jones Islamic Market (DJIM) Index and the associated ten disaggregate sectors including Basic Materials, the Consumer Services, the Consumer Goods, the Energy, the Financials, the Health Care, the Industrials Index, the Technology, the Telecommunications, the Utilities.[5] For comparison purposes, we consider the conventional Dow Jones global index (W1DOW) and Dow Jones sustainability world index (W1SGI).

The study period runs from November 9, 1998 through March 5, 2015. This period is marked by several episodes of wide instabilities for commodity prices (e.g., spectacular increases (plunges) in oil prices throughout 2007 and early 2008 (2014–2015) and food price surges during 2007–2008), major events (e.g., the Gulf wars and the 9/11/2001 terrorist attack) and three severe crises (e.g., the 2008–2009 GFC and the 2010–2012 ESDC).

The closing oil prices are expressed in USD/barrel and complied from the US Energy Information Administration (EIA) website (www.eia.gov), while the gold time series is extracted from the World Gold Council website (https://www.gold.org/). The data for the Islamic stock index series are extracted from Bloomberg.

We select oil and gold because they have extensive economic impacts on financial activities and since their roles are crucial in affecting international stock markets. In the flight-to-quality literature, for example, gold is well known for its role as a safe haven asset (Baur and Lucey, 2010; Baur and McDermott, 2010). This valuable yellow metal is highly liquid and is used as a good vehicle of investment. Oil because of its extensive forward and backward linkages with economic sectors is strongly related to stock markets. Oil prices instability may draw unexpected shifts in returns and volatility of stock markets. Hence, the relationship between these markets is quite complex and deserves a further analysis, particularly at the sectoral level.
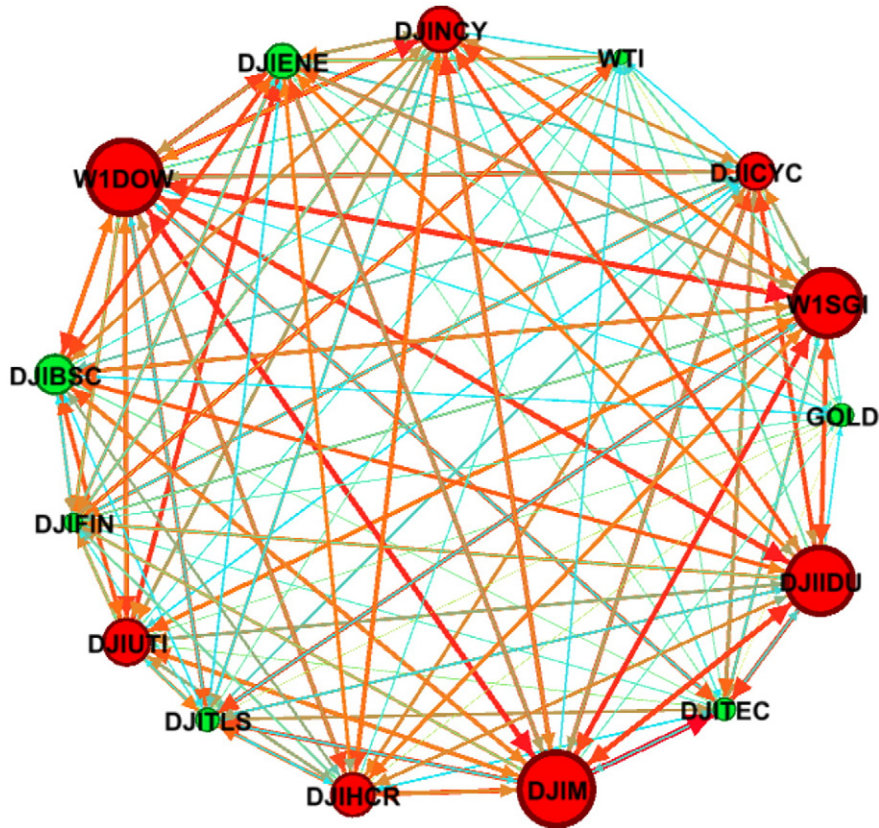
As an illustration, Fig. 1 shows the dynamics of return series under consideration during the sample period. We see that the commodity and stock returns are especially volatile after the mid-2008. The volatility clustering of oil market is more pronounced than gold market.

4.2. Preliminary analysis

We calculate the continuously compounded daily returns by taking the difference in the logarithms of two consecutive prices. Table 1 presents the descriptive statistics of the daily commodity, conventional Dow Jones global, Dow Jones sustainability equity index and Dow Jones Islamic (aggregate and sectors) stock return series. As shown in this table, the average daily returns are positive for all return series. Additionally, gold reveals the highest average returns, while the utilities present the lowest average returns. The unconditional volatility as measured by the standard deviation ranges is the highest for oil, followed by the technology index and the financial index. The gold price, the health care index and the consumer goods index are the least volatile among all indices. Note that the Dow Jones sustainability equity index is more volatile than both the conventional and Islamic (aggregate) equity indexes. The skewness coefficients are negative for the majority of the return series with the exception of the telecommunications, technology and utilities index return. The kurtosis coefficients are above three for all the return series which is the value for the Gaussian distributions. These findings show that the probability distributions of all return series are skewed and leptokurtic, thus rejects the normal distribution which is also confirmed by the Jarque-Bera statistic (JB). Further, we apply the conventional augmented Dickey and Fuller (1979) and the Phillips and Perron (1988) unit root statistics, and the stationarity property under the null using the Kwiatkowski et al. (1992) test. The results indicate that all return series are stationary.

Further, we examine the existence of the ARCH effects, which shows that all return series exhibit the ARCH behavior, underscoring that some stylized facts such as fat-tails, clustering volatility and persistence characterize the commodity and Islamic stock sector returns. Table 2 presents the results of the unconditional correlation levels between the conventional Dow Jones stock index, Dow Jones sustainability stock index, aggregate Islamic stock index, Islamic stock sectoral indices, oil and gold market returns. The results show that the correlations between gold and Islamic stock sector index returns are close to zero or negative. In contrast, the correlations between oil and the Islamic index returns are

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[5] Table A1 in the Appendix summarizes the notations of these Islamic sectoral indexes, and the DJ aggregate conventional and sustainability indexes.

Fig. 4. Risk spillover - network diagram. Note: This figure shows the risk spillovers between oil, gold, DJIM index and the ten Islamic sectors. The size of a node highlights that the magnitude of a net transmission/reception TO or FROM other variables. The red (green) color of a node shows that a variable is a net transmitter (receiver) in the system. The edge size underscores the magnitude of the pair-wise spillover, while the magnitude is also reflected through the color type (light blue (weak), orange (medium), red (strong)).
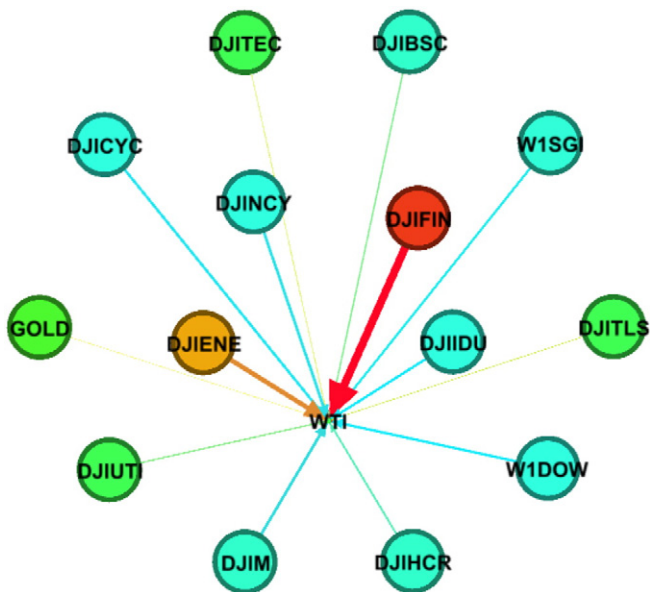
Fig. 5. Net pair-wise directional spillovers between oil, the conventional index, the sustainability index and the DJIM index and their corresponding ten sectors- network diagram. Note: The figure shows the net pair-wise directional spillovers between oil and both the aggregate and the disaggregate Islamic stock markets. The colors of the nodes indicate the magnitude of net transmitters (red (strong), orange (medium), light blue (weak) and green (very weak). The edge size shows the magnitude of the pair-wise spillovers. The edge arrow indicates pairwise directional connectedness.

positive and weak for all pairs. Similar results are found for correlations of the conventional stock index and the Dow Jones sustainability stock index with commodity markets as these correlations are positive and weak. More precisely, for the gold market, the highest correlation is with the energy sector, followed by the basic materials, while the lowest correlation is for the technology-gold pair followed by the consumer services-gold pair. Looking at the WTI crude, this market presents the highest (lowest) correlation with energy and basic materials (financial and consumer services) sectors. These results may be explained by the differences in the market concentration of some sectors. In fact, the basic materials sector is characterized by high market concentration. More interestingly, the Islamic stock sectoral indices are less correlated with gold than oil, implying the presence of more portfolio diversification benefits using the yellow metal than the black gold.

Finally, the long memory test results reveal the presence of long memory behavior for all squared return series (as a proxy variable of volatility), which clearly supports our decision to use the fractionally integrated APARCH-based approach to examine the issue of time-varying correlations among the markets under consideration. The results are available upon request.

5. Empirical results and policy implications

5.1. Marginal model results

To select the best marginal model, we examine different GARCH models (standard GARCH, FIGARCH, FIEGARCH and FIEGARCH) by considering different combinations of the parameters p, q, r and m for values ranging from zero to a maximum lag of 2. Table 3 presents the estimated results of the marginal model. The estimates of the univariate FIAPARCH model (Panel A) show that the one-lagged returns of the mean equation

Fig. 6. Net pair-wise directional spillovers between gold, DJIM and both conventional, sustainability and the ten Islamic sectors- network diagram. Note: The figure shows the net pair-wise directional spillovers between gold and both the aggregate and disaggregate Islamic stock markets. The size of a node shows the magnitude of a net transmission/reception TO/FROM gold. The colors of the nodes indicate the magnitude of net transmitters (red (strong), orange (medium), light blue (weak) and green (very weak). The edge size underscores the magnitude of the pair-wise spillovers. The edge arrow indicates pairwise directional connectedness.

are positive and statistically significant at the 1% level for all stock returns (but not for gold and oil), indicating that the historical returns are instantaneously and rapidly embodied in the current returns for these stock markets. Moreover, the fractionally integrated coefficient (d) is significant for all the markets considered, revealing a high level of persistence. Among the commodity markets, the highest (d) parameter is bestowed on the WTI crude oil. Concerning the Islamic stock sectoral market indexes, the lowest long memory parameter is for the industrial sector, while the financial sector is the highest persistent sector. These results reflect the relative dissimilarity of weights between the sectors in the aggregate DJIM index.

Looking at the aggregate level, the conventional Dow Jones stock index is less persistent than both the Dow Jones sustainability and DJIM indexes. Moreover, the degree of freedom (df) of the Student t-distributions are significant at the 1% level, suggesting that the tails of the error terms are heavier than those of the normal distribution. This result thus indicates that using the Student t-distribution to deal with these properties is appropriate.

Panel B of Table 3 presents the estimates of the DECO process. The a_{DECO} and b_{DECO} coefficients are positive and significant at the 1% level. This finding emphasizes the importance of shocks between the commodity and the Islamic stock sectoral markets. Furthermore, the b_{DECO} parameter is significant and very close to one, revealing a higher persistence of volatility across the considered markets. It is worth noting that the significance of the parameters a_{DECO} and b_{DECO} indicates the appropriateness of the DECO-FIAPARCH model in modeling the time-varying equicorrelations between the considered markets. Moreover, the sums of a_{DECO} and b_{DECO} coefficients are <1, indicating that the estimated DECO parameters lie within the range of typical estimates from the GARCH model. However, the dynamic equicorrelation is statistically significant at the 1% level. It is positive and less than one, suggesting the presence of diversification benefits. Investors can thus have the opportunity to allocate their portfolio in distinctive sectors.

The diagnostic tests summarized in Panel C show no evidence of misspecification in our marginal model. In fact, the Ljung-Box test statistics for the standardized residuals and the squared standardized residuals do not reject the null hypothesis of no serial correlation for most cases.

Fig. 2 displays the dynamic equicorrelation for the group of the commodity, conventional, sustainability and both aggregate and disaggregate Islamic stock markets. As shown in this figure, we observe positive time-varying equicorrelations for the commodity and Islamic stock markets over the sample period. This result reveals that investors frequently change their portfolio structure by rebalancing their portfolios. More importantly, we identify three regimes for which the correlations between the markets change significantly. The highest level of the three correlation regimes are observed at the end of the 1997–1998 Asian financial crisis and the resulting 2000 default of the Long Term Capital Management Fund, the 2008–2009 GFC and the 2010–2012 ESDC, showing decreased opportunities for diversification benefits during those turmoil periods. During the burst of the dot-com bubble of 2001, we observe a little bit increase in the correlations
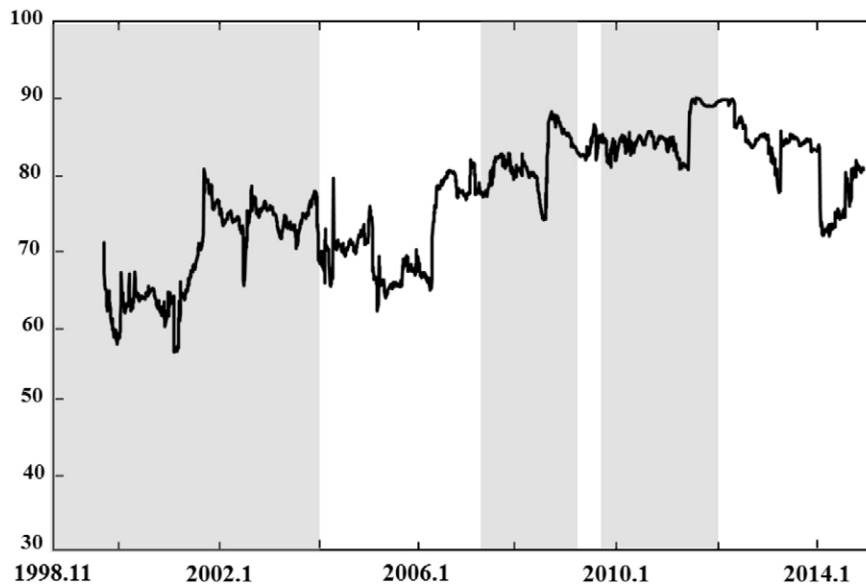
Fig. 7. The dynamics of the total volatility spillover index. Notes: The dynamic total return and volatility spillovers are calculated from the forecast error variance decompositions on 10-step-ahead forecasts. The total spillover indices are estimated using 200-day rolling windows.

Table 5
Net directional pairwise index.

	GOLD	WTI	DJIM	W1DOW	W1SGI	DJIBSC	DJICYC	DJINCY	DJIENE	DJIFIN	DJIHCR	DJIIDU	DJITEC	DJITLS	DJIUTI
GOLD	0	−0.45	2.95	2.47	2.2	3.02	2.01	1.96	1.98	1.1	0.71	2.96	0.4	0.56	1.27
WTI	0.45	0	3.07	2.72	2.57	1.62	2.86	2.97	4.93	8.24	2.04	2.74	0.73	0.8	1.54
DJIM	−2.95	−3.07	0	−1.39	−0.69	−2.15	−3.23	−3.63	−3.23	−5.6	−2.89	−1.42	−5.39	−4.8	−2.48
W1DOW	−2.47	−2.72	1.39	0	0.96	−1.81	−1.49	−1.77	−2.51	−3.68	−0.99	−0.4	−3.46	−5.3	−1.13
W1SGI	−2.2	−2.57	0.69	−0.96	0	−1.74	−3.15	−2.74	−3.04	−5.12	−2.43	−1	−7.72	−5.5	−1.65
DJIBSC	−3.02	−1.62	2.15	1.81	1.74	0	0.29	0.46	1.27	−3.21	0.83	1.28	−0.48	−0.4	0.81
DJICYC	−2.01	−2.86	3.23	1.49	3.15	−0.29	0	0.1	−0.83	−3.91	−0.33	2.74	−2.17	−0.6	−0.74
DJINCY	−1.96	−2.97	3.63	1.77	2.74	−0.46	−0.1	0	−1.49	−4.05	−1.67	2.33	−0.78	−3	−1.36
DJIENE	−1.98	−4.93	3.23	2.51	3.04	−1.27	0.83	1.49	0	−4.23	1.68	1.53	−0.17	−0.9	2.91
DJIFIN	−1.1	−8.24	5.6	3.68	5.12	3.21	3.91	4.05	4.23	0	3.15	5.18	1.3	2.02	5
DJIHCR	−0.71	−2.04	2.89	0.99	2.43	−0.83	0.33	1.67	−1.68	−3.15	0	1.48	−0.94	−4.6	−1.22
DJIIDU	−2.96	−2.74	1.42	0.4	1	−1.28	−2.74	−2.33	−1.53	−5.18	−1.48	0	−5.41	−1.8	−1.37
DJITEC	−0.4	−0.73	5.39	3.46	7.72	0.48	2.17	0.78	0.17	−1.3	0.94	5.41	0	1.71	−0.66
DJITLS	−0.56	−0.8	4.83	5.31	5.46	0.39	0.57	2.97	0.91	−2.02	4.63	1.82	−1.71	0	4.38
DJIUTI	−1.27	−1.54	2.48	1.13	1.65	−0.81	0.74	1.36	−2.91	−5	1.22	1.37	0.66	−4.4	0
Net	−23.14	−37.3	42.95	25.39	39.09	−1.92	3	7.34	−3.73	−37.11	5.41	26.02	−25.14	−26	5.3
Conclusion	Net-recipient	Net-recipient	Net-contributor	Net-contributor	Net-contributor	Net-recipient	Net-contributor	Net-contributor	Net-recipient	Net-recipient	Net-contributor	Net-contributor	Net-recipient	Net-recipient	Net-contributor

between the considered markets, confirming the recoupling hypothesis. This significant increase in the cross-market correlations after a shock hits indicates a pure contagion or herding and may reflect a shift in investors' appetite for or aversion to risk. It is worth noting that this is a minor crisis comparing to the 1997–1998 Asian crisis and the 2007–2008 GFC episodes. More interestingly, we show that the correlations are positive along the sample period and reflect phases of decreases and increases. This means that changes in the volatility transmission imply changes in diversification opportunities. In fact, contagion decreases the role of oil and gold as potential vehicles for diversification benefits. Looking at the sample after 2012, we view a decrease in the correlations for all cases, which is an indication of the presence of diversification benefits. The dynamic equicorrelation varies approximately between 0.2 and 0.7 but with elevated levels during the 2008–2009 GFC and the 2010–2012 ESDC, supporting the contagion effects.[6]

Fig. 3 plots the time-varying equicorrelation for the stock indices, the oil-stock and the gold-stock blocks. Similarly to Fig. 2, we observe a variation in the correlations. Also, the trajectories of all blocks are similar but with differences in magnitude. Graphically, we see that the correlations between gold and stock are lower than those of only stock or oil-stock. To sum up, investors investing in Islamic stock companies may see more benefits in diversifying and investing in the gold and oil markets. The presence of positive and increasing correlations underscores an increasing integration between commodity and Islamic stock markets during the last years.

5.2. Total volatility spillover index and rolling-sample spillover analysis

Table 4 summarizes the estimates of the total volatility spillover matrix. The (i, j)th entry in each panel is the estimated contribution to the forecast-error variance of variable i coming from innovations of market j. The row sums excluding the main diagonal elements (termed 'From others') and the column sums (termed 'To others') report the total spillovers to (received by) and from (transmitted by) each volatility.

The total volatility spillovers reach 79.79%. Looking at the directional spillovers transmitted 'To others', gold has a much lower impact on the Islamic stock markets than the crude oil market does. In fact, gold contributes only 0.58% to the forecast-error variance of the DJIM index, 1.03% to the forecast-error variance of oil and 7.94% to those of the associated Islamic sectors, 0.72% to the conventional DJ global index, 0.65% to the DJ sustainability index, while oil contributes 1.67% to that of the DJIM index, 15.24% to those of the ten sectors and the remaining to both conventional and sustainability Dow Jones index.

Islamic stock indexes also contribute to the forecasting error variance of the gold metal and oil markets. In fact, the DJIM index contributes 128% to the remaining markets (oil, gold, conventional index, sustainability index and the ten sector indexes). This index also contributes 2.78% to the forecasting-error variance of gold and 4.24% to that of oil. Oil and gold respectively contribute 01.67% and 0.58% to the forecasting variance of the DJIM index. This result indicates that gold provides greater diversification benefits than oil market. On the other hand, oil acts as a price discovery tool for the DJIM index. Conventional DJ global index and sustainability index contributes significantly to the aggregate and disaggregate Islamic indexes.

Among the Islamic sectors, the consumer goods and the industrials are the highest net volatility contributors, while the finance, the technology and the telecommunication are the lowest contributors to

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[6] Forbes and Rigobon (2002) define the contagion as a significant increase in cross-market linkages after a shock to one country (or a group of countries). Thus, contagion does not occur if two markets show a high degree of comovement during both stability and crisis periods. The interdependence is used instead if strong linkages between the two economies exist in all states of the world. Ahmad et al. (2013) define the contagion as significant increases in cross market correlations during the turmoil period, while any continued increase in cross market correlation at high levels is referred to as interdependence.
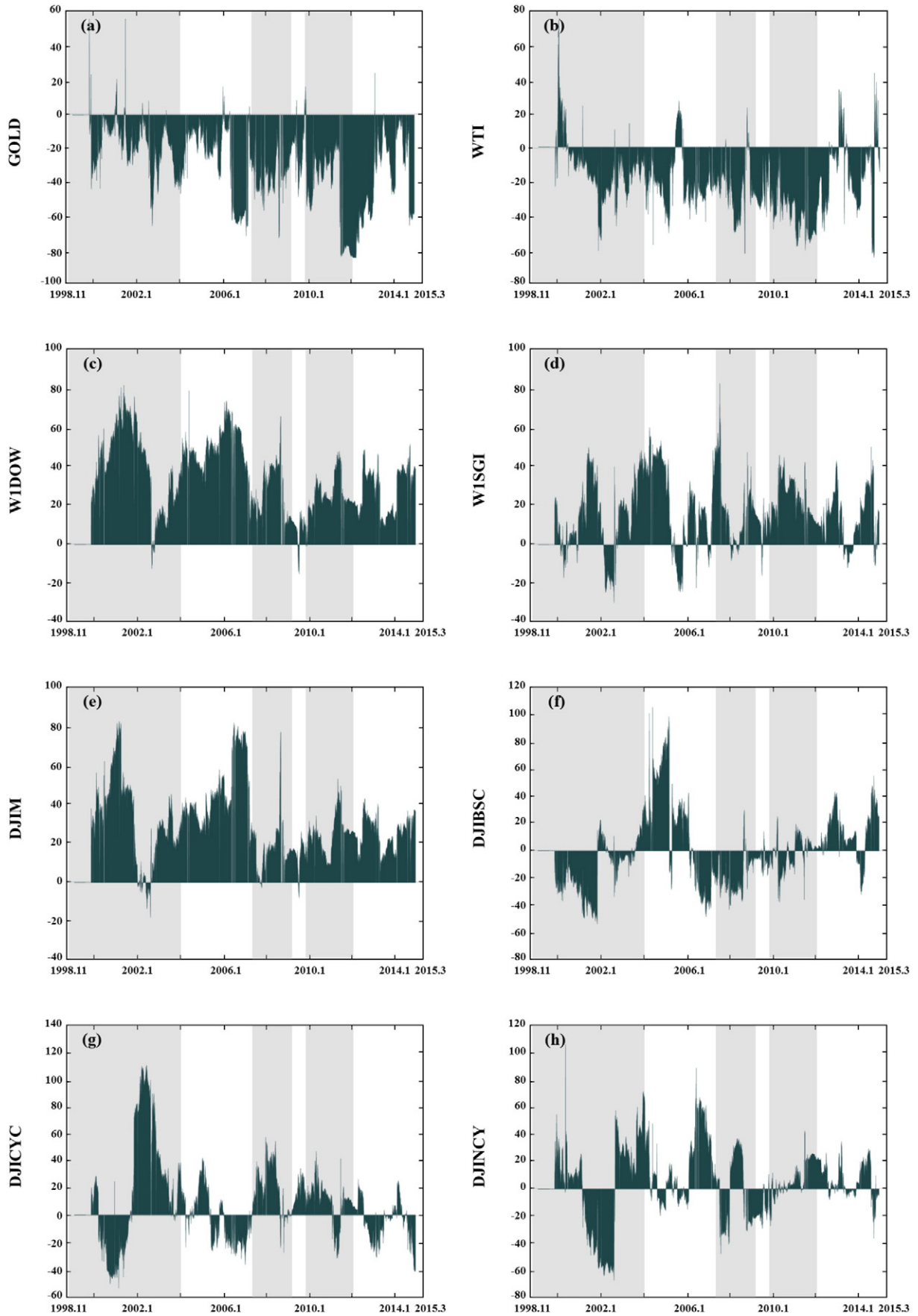
Fig. 8. Net volatility spillover index. Notes: The notations are defined in the appendix.
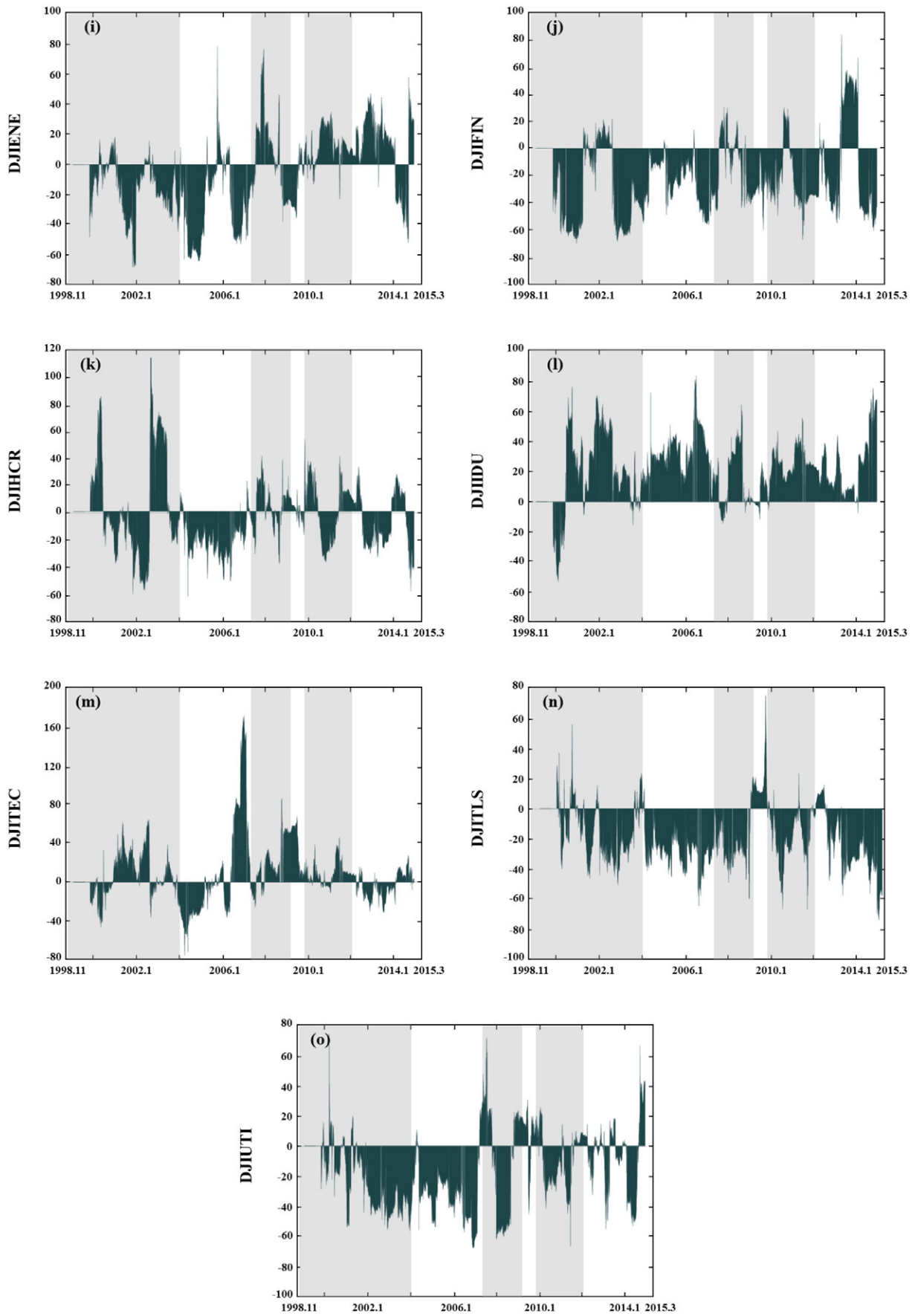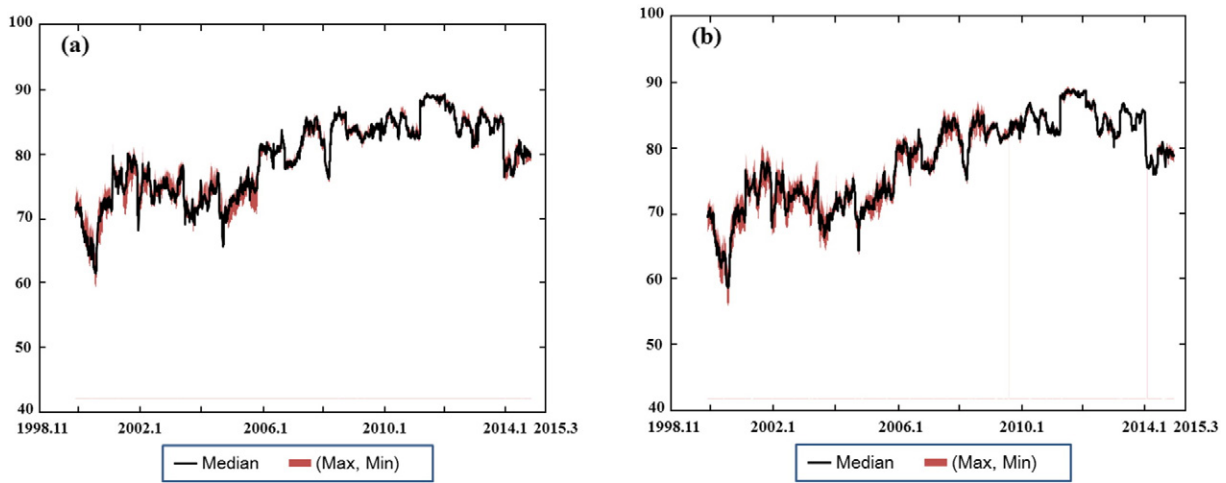
Fig. 8 (continued).

Fig. 9. Robustness tests. Note: (a) Sensitivity of the index to the VAR lag structure (max, min, and median values of the index for the VAR orders 2–6); (b) Sensitivity of the index to forecast horizon (max, min, and median values over 5- to 10-day horizons).

volatility spillovers. Further, the risk spillovers between the Islamic stock sectoral markets are globally weak. Taking for example the industrial sector, the risk spillover coefficient varies between 6.4% for the telecommunications sector and 10.28% for the technology sector. For the utilities, health care and financial sectors, these sectors receive similar risk spillovers from the industrials. Looking at the telecommunications sector, one can see that this sector has similar spillovers to the rest of sectors, with the exception of technology. The interpretations of the remaining sectors are similar.

Fig. 4 plots the risk spillover network diagram and shows that conventional stock index, sustainability stock index, DJIM and the industrials sector are the most net contributors of risk spillovers, while the financials, basic materials, energy, technology, telecommunication, oil and gold sectors are among the largest net receivers of shocks from the rest of the markets. Figs. 5 and 6 display respectively the net pair-wise directional spillovers between the oil- stock and the gold- stock pairs. These figures synthetically display the main results for our dynamic analysis of the net pairwise directional connectedness. They provide a visualization of the complex network of innovation among 15 markets. In fact, they synthetically illustrate the main results of our analysis of the net directional connectedness using the DY's (2014) graphical methodology. We show that gold imports volatility from financial sectors, while oil imports volatility from the aggregate DJIM index and the cyclical basic materials and industrials sectors (see Figs. 5 & 6).

Fig. 7 illustrates the evolution of the volatility spillover index. This figure shows that the total volatility spillovers increase and decrease over time. This result suggests that investors should modify their portfolio's structure accordingly. However, we provide evidence that the crises intensify the total volatility spillovers across the markets.

Table 6
Optimal portfolio weights and hedge ratios for the commodity futures and stock indices.

Pairs	Optimal portfolio weights				Hedge ratios			
	Mean	St. Dev	Max	Min	Mean	St. dev	Max	Min
W1DOW/Gold	0.2793	0.3027	1.2199	−0.2755	0.4220	0.1970	1.4105	0.0781
W1SGI/Gold	0.3789	0.3168	1.2284	−0.2589	0.4772	0.2283	1.4427	0.0908
DJIM/Gold	0.3272	0.3032	1.1774	−0.2352	0.3986	0.1886	1.3613	0.0733
DJIBSC/Gold	0.5642	0.2644	1.2309	−0.0373	0.5253	0.2668	2.0846	0.0965
DJICYC/Gold	0.3682	0.1060	1.1652	−0.2801	0.4232	0.1771	1.2900	0.0856
DJINCY/Gold	0.0286	0.0753	0.5556	−0.7481	0.1029	0.0829	0.7994	0.0049
DJIENE/Gold	0.6086	0.2726	1.2083	−0.1439	0.5606	0.2608	1.7918	0.1266
DJIFIN/Gold	0.4819	0.3663	1.1941	−0.2541	0.5169	0.3035	2.3506	0.1170
DJIHCR/Gold	0.2672	0.3049	1.1430	−0.2870	0.3658	0.1541	1.2167	0.0925
DJIIDU/Gold	0.3832	0.2964	1.1945	−0.2097	0.4235	0.2020	1.4379	0.0723
DJITEC/Gold	0.5419	0.3492	1.1822	−0.2353	0.5810	0.2882	2.2740	0.1226
DJITLS/Gold	0.3726	0.3284	1.0777	−0.2884	0.4343	0.1842	1.3179	0.0977
DJIUTI/Gold	0.3195	0.2896	1.1558	−0.2263	0.3962	0.1935	1.6526	0.0817
W1DOW/WTI	−0.0164	0.1312	0.7803	−0.3227	0.2540	0.1377	0.8443	0.0479
W1SGI/WTI	−0.0501	0.1081	0.5589	−0.3350	0.2240	0.1170	0.7074	0.0449
DJIM/WTI	−0.0093	0.1214	0.6219	−0.2821	0.2101	0.1089	0.6810	0.0401
DJIBSC/WTI	0.1006	0.1520	0.9770	−0.1790	0.2879	0.1706	0.9800	0.0444
DJICYC/WTI	0.0131	0.1489	0.8128	−0.3036	0.2201	0.0952	0.6099	0.0345
DJINCY/WTI	−0.0510	0.0856	0.5700	−0.3035	0.1691	0.0902	0.6515	0.0279
DJIENE/WTI	0.1169	0.1529	0.8685	−0.2323	0.2969	0.1498	0.9201	0.0534
DJIFIN/WTI	0.0903	0.2192	0.9944	−0.2981	0.2671	0.1485	0.9933	0.0386
DJIHCR/WTI	−0.0208	0.1247	0.7789	−0.2945	0.1921	0.0877	0.6330	0.0376
DJIIDU/WTI	0.0125	0.1288	0.6726	−0.2851	0.2280	0.1225	0.7521	0.0436
DJITEC/WTI	0.1371	0.2527	0.9360	−0.2864	0.2910	0.1225	0.8299	0.0442
DJITLS/WTI	0.0229	0.1646	0.7351	−0.2964	0.2249	0.0095	0.6172	0.0454
DJIUTI/WTI	−0.0126	0.1243	0.9574	−0.3037	0.2098	0.1104	0.8959	0.0399

Note: This table reports the optimal weight of a commodity (w_t^C) at time t in a commodity-Islamic sector (or conventional or sustainability) stock portfolio, while the corresponding remaining portions are for equities. It also summarizes the hedge ratio consisting of a long position of one USD in the (Islamic, conventional or sustainability) sectoral stock market hedged by a short position of β USD in the commodity market.
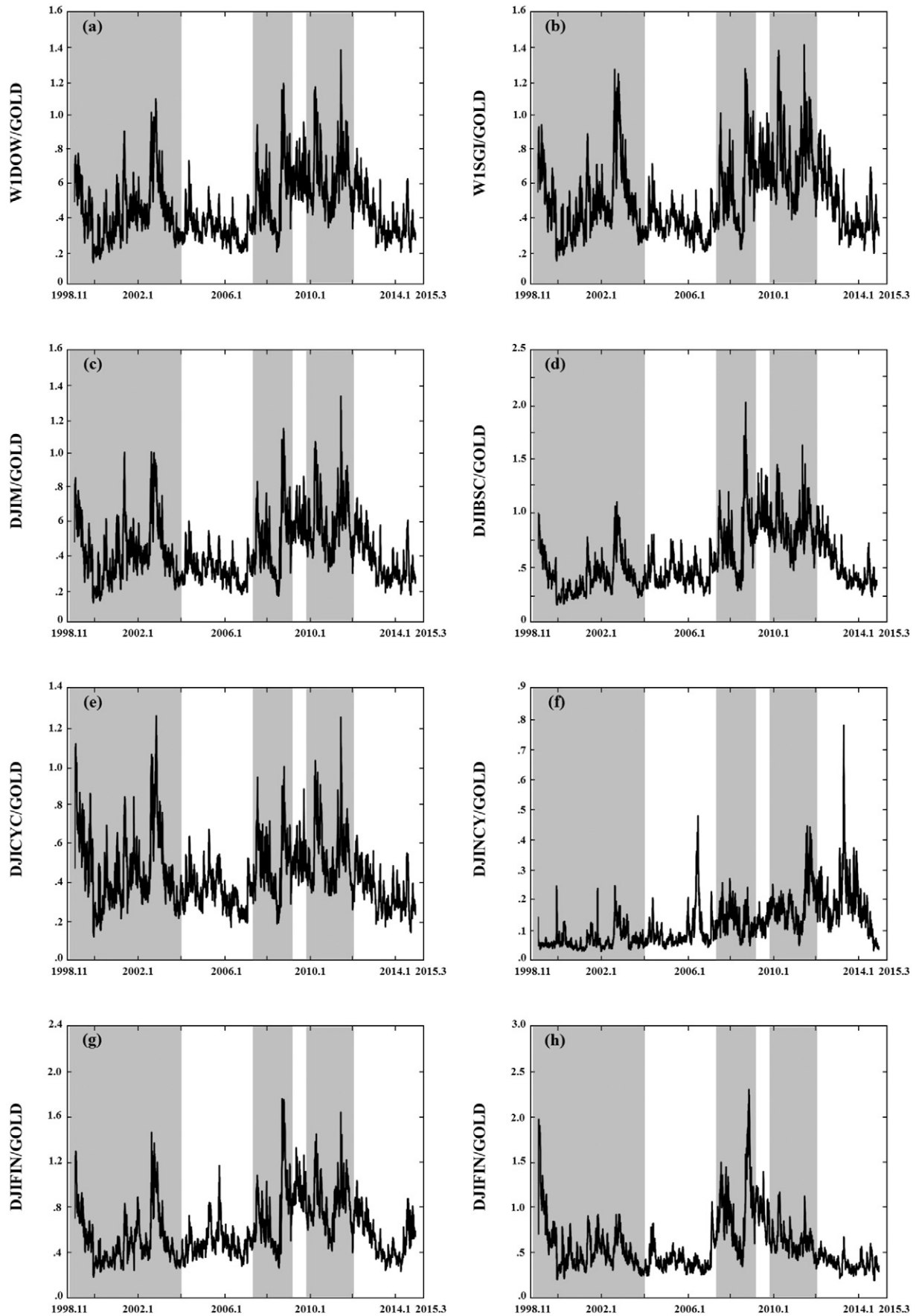
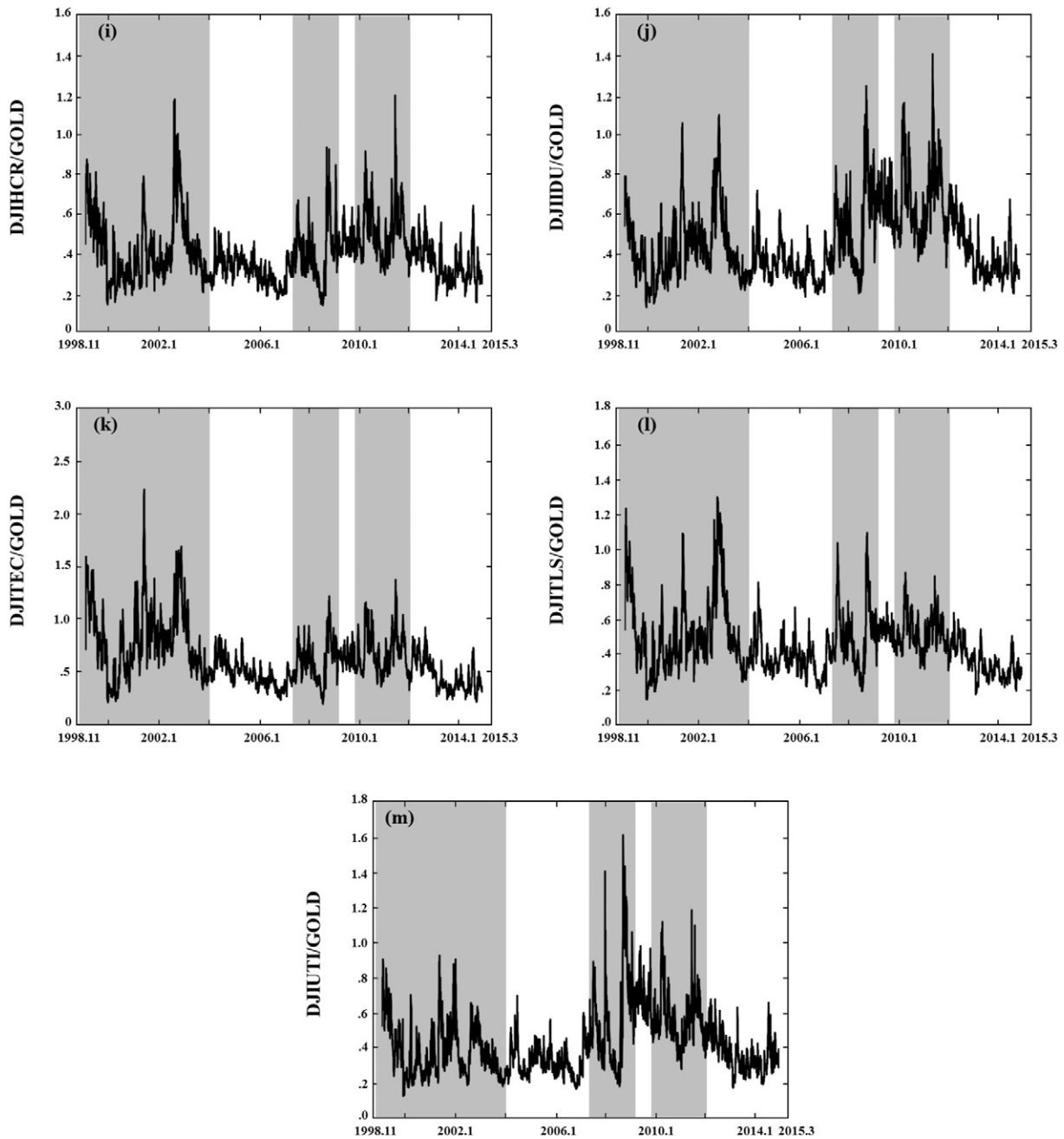**Fig. 10.** Time-varying hedge ratios between the GOLD and stock markets.

Fig. 10 (*continued*).

More precisely, the volatility spillovers attain their maximum level during the turbulent years 2008–2009 and 2010–2012, which correspond to the GFC and ESDC periods. In addition, we can conclude that the time-varying volatility spillovers can be affected by other major economic events like the high oil price instability in summer 2008 and January 2014, the 2003 gulf war and the 2007–2008 commodity crisis. These major events increase the spillovers between these markets, and thereby decrease the investment diversification opportunities.

5.3. Net volatility spillover and robustness tests

We deepen our study by determining the directional volatility spillovers among the conventional, sustainability, Islamic sector and commodity markets. In fact, we determine the net receivers and net contributors to volatility spillovers. Specifically, we decompose the total volatility spillover index into two directional spillovers as illustrated in Table 5: (i) the receiver of volatility spillovers, termed directionally as 'from', and (ii) the transmitter of volatility spillovers, termed directionally as 'to'. The dynamic net volatility spillover index is then quantified by subtracting directional 'to' spillovers from directional 'from' spillovers. Then positive (negative) values indicate a source (recipient) of return and volatility to (from) others. The results (shown in Table 5) indicate that oil and gold are net receivers of volatility, while the conventional index, the sustainability index and the DJIM index are net contributors to volatility spillovers. Regarding the Islamic sectors, five out of the ten sectors are net contributors to volatility. These sectors are the consumer services, consumer goods, health care, industrials and utilities sectors. The remaining sectors are net receivers of volatility. Among the ten sectors, the highly cyclical industrial sector is the most contributor of risk to the other markets, while the financial

Table 7
Risk evaluation for the different GOLD-stock portfolios.

		Portfolio II	Portfolio III	Portfolio IV
W1DOW	RiskRed.	**0.98733**	0.51436	0.93144
	VaRRed.	**0.02508**	0.01869	0.01155
	SV Red.	**0.25440**	0.00048	0.00641
	ReRed.	**0.05070**	0.00721	0.02535
W1SGI	RiskRed.	**0.95483**	0.69804	0.89216
	VaRRed.	**0.02065**	0.01770	0.01992
	SV Red.	**0.09759**	0.00456	0.04287
	ReRed.	**0.03128**	0.00709	0.02082
DJIM	RiskRed.	**0.97418**	0.39922	0.92898
	VaRRed.	**0.01574**	0.01278	0.01549
	SV Red.	**0.01831**	0.00045	0.00705
	ReRed.	**0.04274**	0.00669	0.02660
DJIBSC	RiskRed.	0.97721	0.03202	**0.98293**
	VaRRed.	0.00541	0.01279	**0.02263**
	SV Red.	**0.09043**	0.01652	0.03280
	ReRed.	0.04694	0.00833	**0.04909**
DJICYC	RiskRed.	**0.97416**	0.38721	0.96517
	VaRRed.	0.00688	0.01328	**0.01746**
	SV Red.	**0.01591**	0.00045	0.01304
	ReRed.	**0.03995**	0.00670	0.03612
DJINCY	RiskRed.	**0.98011**	0.62005	0.53293
	VaRRed.	0.00934	**0.01353**	0.01057
	SV Red.	**0.08520**	0.00028	0.01217
	ReRed.	**0.02933**	0.00528	0.01103
DJIENE	RiskRed.	0.98283	0.17965	**0.99526**
	VaRRed.	0.01082	0.01377	**0.01869**
	SV Red.	0.02843	0.00087	**0.04504**
	ReRed.	0.05334	0.00944	**0.07252**
DJIFIN	RiskRed.	0.92924	0.18388	**0.95941**
	VaRRed.	0.01082	0.01107	**0.01303**
	SV Red.	**0.06983**	0.00056	0.05263
	ReRed.	**0.02645**	0.00748	0.02379
DJIHCR	RiskRed.	**0.98662**	0.48202	0.73273
	VaRRed.	0.00737	0.01377	**0.01770**
	SV Red.	**0.29729**	0.00039	0.11322
	ReRed.	**0.06491**	0.00944	0.03713
DJIIDU	RiskRed.	**0.95513**	0.38058	0.88437
	VaRRed.	0.01008	0.01328	**0.01598**
	SV Red.	**0.09236**	0.00048	0.04839
	ReRed.	0.03042	0.00690	0.21956
DJITEC	RiskRed.	0.97291	0.50729	**0.99600**
	VaRRed.	0.01229	0.01180	**0.01377**
	SV Red.	0.18212	0.01051	**0.37358**
	ReRed.	0.04272	0.01050	**0.06922**
DJITLS	RiskRed.	0.97676	0.62733	**0.99314**
	VaRRed.	0.01082	0.01057	**0.01451**
	SV Red.	0.18215	0.00056	**0.21135**
	ReRed.	0.04283	0.00751	**0.04629**
DJIUTI	RiskRed.	0.98362	0.33088	**0.98843**
	VaRRed.	0.00590	**0.01352**	0.00418
	SV Red.	0.23549	0.00047	**0.39498**
	ReRed.	**0.04855**	0.00692	0.03847

Notes: This table reports the results of risk evaluation for portfolios composed of commodity and stocks, compared to a pure Islamic stock portfolio. Portfolio II and IV's weights are given by Eqs. (14) and (15), respectively. On the other hand, Portfolio III has equal weights. "Risk Red." indicates the risk effectiveness ratio in Eq. (16). "VaR. Red." is the reduction in the value-at-risk portfolio with respect to Portfolio I (positive values indicate a VaR reduction). Similarly, "SV Red." and "Re Red." indicate the reduction using the Semivariance and Regret risk measures, respectively. The bold values refer to the portfolio that has the best risk reduction among the three portfolios for each of the commodity–Islamic stock pairs.

Table 8
Risk evaluations for different WTI-stock portfolios.

		Portfolio II	Portfolio III	Portfolio IV
W1DOW	RiskRed.	**0.87829**	0.21279	0.79561
	VaRRed.	0.01254	0.01770	**0.02065**
	SV Red.	**0.13703**	0.00598	0.03566
	ReRed.	**0.03713**	0.00774	0.01893
W1SGI	RiskRed.	**0.98989**	0.76921	0.73732
	VaRRed.	0.01352	0.01820	**0.02065**
	SV Red.	**0.13332**	0.00067	0.00909
	ReRed.	**0.11509**	0.00818	0.02996
DJIM	RiskRed.	**0.99729**	0.79209	0.59349
	VaRRed.	**0.02336**	0.01278	0.01328
	SV Red.	**0.01191**	0.00063	0.00619
	ReRed.	**0.38625**	0.00794	0.02487
DJIBSC	RiskRed.	**0.99818**	0.65232	0.93166
	VaRRed.	**0.06271**	0.01352	0.01672
	SV Red.	**0.07387**	0.00098	0.00222
	ReRed.	**0.15132**	0.00820	0.02668
DJICYC	RiskRed.	**0.99319**	0.77917	0.62547
	VaRRed.	0.01155	0.01180	**0.01401**
	SV Red.	**0.02279**	0.00067	0.00705
	ReRed.	**0.02874**	0.01318	0.01992
DJINCY	RiskRed.	**0.98630**	0.87632	0.95302
	VaRRed.	0.00737	**0.01278**	0.01254
	SV Red.	**0.07643**	0.00037	0.06418
	ReRed.	**0.08769**	0.00607	0.08015
DJIENE	RiskRed.	**0.99587**	0.59769	0.85106
	VaRRed.	**0.01746**	0.01426	0.01377
	SV Red.	**0.03581**	0.00012	0.00142
	ReRed.	**0.25094**	0.01097	0.03252
DJIFIN	RiskRed.	**0.98931**	0.61079	0.89322
	VaRRed.	**0.01451**	0.00909	0.01155
	SV Red.	**0.01557**	0.00094	0.00128
	ReRed.	**0.12517**	0.00973	0.03595
DJIHCR	RiskRed.	**0.99325**	0.83483	0.50718
	VaRRed.	0.01205	**0.01451**	0.01254
	SV Red.	**0.01648**	0.00051	0.00548
	ReRed.	**0.12892**	0.00714	0.02340
DJIIDU	RiskRed.	**0.99154**	0.76165	0.66965
	VaRRed.	**0.01426**	0.01278	0.01180
	SV Red.	**0.01767**	0.00073	0.00736
	ReRed.	**0.13408**	0.00855	0.02726
DJITEC	RiskRed.	**0.99507**	0.51553	0.83061
	VaRRed.	**0.01893**	0.01205	0.01057
	SV Red.	**0.03274**	0.00147	0.01377
	ReRed.	**0.18105**	0.01213	0.03727
DJITLS	RiskRed.	**0.99250**	0.75502	0.58973
	VaRRed.	0.01033	**0.01082**	0.00983
	SV Red.	**0.02024**	0.00073	0.00622
	ReRed.	**0.14139**	0.00850	0.02491
DJIUTI	RiskRed.	**0.99400**	0.77519	0.81151
	VaRRed.	0.00811	0.01352	**0.01598**
	SV Red.	**0.20121**	0.00006	0.00904
	ReRed.	**0.14317**	0.00779	0.03009

Notes: see the notes of Table 7. Numbers in bold indicate the best portfolio performance.

sector is the most receiver of risk from the other markets. Financial sectors are marked by their volatility. For the two commodities, oil is a more receiver of shocks than gold is, which is significantly used in central banks' international reserves and is a store of value. Oil is a cyclical commodity, while gold is a safe haven.

The graphical evidence shown in Fig. 8 confirms the results of Tables 4–5. The figure plots the time-variations of the net volatility spillover index for each Islamic stock sector, the conventional index, the sustainability index, gold and oil, and highlights that the magnitude of volatility spillovers has often changed during the GFC.

To do the robustness analysis, we have conducted two tests to check the sensitivity of the spillover results. First, we check the choice of the order of the VAR. For this purpose, we compute the spillover index for orders 2 to 6 and plot the minimum, maximum, and the median values in Fig. 9(a). Second, we plot the spillover index for the forecast horizons varying from 5 to 10 days in Fig. 9(b). These figures show that the spillover indexes appear to follow similar patterns whatever the choice of the order of the VAR or the choice of the forecast horizon, suggesting that the total spillover plot is not sensitive to the choice of the order of the VAR or the choice of the forecast horizon. Similar alternative values as robustness tests are also adopted by previous studies in the literature (Diebold and Yilmaz, 2009, 2012, 2014; Chau and Deesomsak, 2014; Antonakakis and Kizys, 2015 among others).

5.4. Portfolio risk implications

The empirical evidence reported above has important implications in terms of asset allocations and portfolio risk management. To help individual and institutional investors make more informed decisions, we thus analyze the usefulness of the oil and gold commodity markets in constructing better portfolios. Specifically, we follow Mensi et al. (2015a) to investigate the usefulness of these commodity markets in having better portfolio risk management of the conventional index, the sustainability index and the Islamic stock sectoral markets. We compare the risks for three different portfolios with that of a benchmark portfolio (Portfolio I) composed exclusively of the stock indexes in this case. We could thus assess the potential reduction in the portfolio risk generated by the inclusion of commodities in a more diversified portfolio.

First, we consider the risk-minimizing commodity-stock (sector) portfolio (Portfolio II) without reducing the expected return. According to Kroner and Ng (1998), at time t the optimal weight of a commodity in this portfolio (w_t^C) is given by:

$$w_t^C = \frac{h_t^S - h_t^{CS}}{h_t^C - 2h_t^{CS} + h_t^S}, \text{ with } w_t^C = \begin{cases} 0 & w_t^C < 0 \\ w_t^C & 0 \le w_t^C \le 1 \\ 1 & w_t^C > 1 \end{cases}, \qquad (19)$$

where (h_t^C),(h_t^S), and (h_t^{CS}) are the conditional volatility of a commodity market, the conditional volatility of the stock market and the conditional covariance between the commodity and the stock markets at time t, respectively. From the budget constraint, the optimal weight of the (conventional or sustainability or Islamic) sector market is equal to $(1 - w_t^C)$. For each commodity-(conventional, sustainability or Islamic) stock pair, all information needed to compute w_t^C is obtained from the bivariate DECO-FIAPARCH model with Student-t distributions.

Second, we consider another portfolio where the weights are exogenously determined (Portfolio III), i.e., an equally weighted portfolio, which usually has a good out-of-sample performance (DeMiguel et al., 2009). Finally, we consider a fourth portfolio where the weights are determined according to a variance minimization-hedging strategy (Portfolio IV), consisting of a long position of one USD in the (conventional, sustainability or Islamic) aggregate or sectoral stock market hedged by a short position of β USD in the commodity market, where β at time t is given by:

$$\beta_t = \frac{h_t^{CS}}{h_t^C}, \qquad (20)$$

The risk-reduction effectiveness of each portfolio is evaluated by comparing the percentage reduction in the variance of any portfolio with respect to that of Portfolio I, expressed as:

$$RE_{Var} = 1 - \frac{Var(P_j)}{Var(P_I)}, \qquad (21)$$

where $Var(P_j)$ and $Var(P_I)$ are the variances of Portfolio j and Portfolio I, respectively, and $j = $ II, III, IV for the portfolios. RE_{Var} takes values in [0,1], where a higher value indicates a greater variance reduction.

Further, we assess the attractiveness of the commodity markets (oil and gold) in providing downside risk protection, using three different downside risk measures. For each portfolio, we estimate the Value-at-Risk (VaR), the Semivariance (SV) and the Regret (Re). The VaR provides the maximum loss in a portfolio value for a given time period and a given confidence level.[7]

The downside risk gains are evaluated by considering the downside risk reductions for Portfolios II, III and IV with respect to Portfolio I, using the risk-reduction ratio in Eq. (21) for each downside risk measure.

Table 6 shows the optimal portfolio weights and the hedge ratios for the conventional index, the sustainability index, Islamic sector indices and the commodity futures (oil and gold). Let us take the gold asset first. We find that investors should hold 32.72% of this yellow metal and 67.28% of the budget in the DJIM index. For the gold-Islamic sector pairs, the optimal weight for gold varies between 2.86% for the consumer goods sector and 60.86% for the energy sector. This result means that for the consumer goods sector the optimal weight of the gold holding in a one-dollar gold stock portfolio should be 2.86%, while the remaining 97.14% should be invested in the consumer goods. Similar interpretations hold for the other sectors and both conventional and sustainability indexes. On the whole, we conclude that investors should hold more stocks than gold in their portfolios in order to minimize the risk while keeping the expected return unchanged. More interestingly, the net receiver sectors (energy, finance, telecommunication and technology) require more commodity assets than the net contributor sectors.

As for the WTI crude oil, we find that investors should also hold more (Islamic, conventional and sustainability) stocks than oil in their portfolios. By comparing the oil and gold markets, an investor should hold a greater proportion of her wealth in a gold- stock pair than in an oil-stock pair in order to minimize risk. This result is explained by the better diversification benefits bestowed by gold on portfolios, compared to the high instability of the oil market. The value of the hedge ratio is higher when the investor uses gold than oil to do hedging. Taking for example the gold-DJIM pair, a hedge ratio of 0.3986 implies that one USD long in gold should be shorted by about 39.86% of the DJIM index. Similar interpretations are valid for the other cases.

Fig. 10 displays the dynamic hedge ratios across the gold and stock markets. The paths suggest a variability is in order for the estimated hedge ratio. In fact, we observe significant time-varying hedge ratios over the sample data, suggesting a dissimilarity between the Islamic sectors, conventional and sustainability indexes and also a switching behavior of investors toward risk. Therefore, investors adjust their portfolio structure and hedging positions frequently according to the (Islamic, conventional and sustainability) stock and commodity market conditions (bear, normal and bull markets).

To deepen our analysis, we assess the risk and downside risk-reductions of the different pairs. The risk evaluation results for the gold-stock and the oil-stock pairs at the 99% confidence level are presented in Tables 7–8. Viewing Table 7, the optimally weighted Portfolio II offers the best risk reduction and the largest downside risk reduction for the conventional index, the sustainability index, the DJIM index, and the consumer services, consumer goods, health care and industrials sectors. For the remaining sectors, Portfolio IV provides the best performance in terms of risk reduction, VaR reduction, semivariance and regret reduction. It is worth noting that among the ten Islamic sectors, the best risk reductions are detected for the technology sector, followed by the energy and industrials sectors. The highest risk reduction is for the basic material-gold pairs followed by the energy-gold and the consumer services-gold pairs. These results indicate that including gold in the Islamic stock portfolios helps investors to manage effectively their portfolios.

For the oil market (see Table 8), we find that for all cases the optimally weighted Portfolio II offers the best risk reduction and the largest downside risk reduction than the other competing portfolios (i.e., Portfolio II and Portfolio III). Among the Islamic stock sectors, we show that the basic materials, energy and technology sectors present higher risk reductions and VaR reductions than the remaining sectors. This result shows that investors attain the highest risk reductions by including oil in the stock portfolio.

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[7] The reader can refer to Mensi et al. (2015a) for further information on the specification of these risk measures.

6. Conclusion

International stock markets and major commodities (particularly gold and oil) have recently become more integrated due to the financialization of commodity markets and the globalization of financial markets. These developments in the world's asset markets have made hedging more difficult, and thereby reducing the benefits from diversification. On the other hand, understanding the risk spillovers between the major commodities (oil and gold) and the behaviors of new business models such the Islamic stock markets (particularly at the sectoral level) is of great importance for the purpose of asset allocation and portfolio risk management.

The aim of this paper is to examine the risk spillovers between the two major commodity markets, crude oil and gold, and both the aggregate Dow Jones Islamic index and its associated ten stock sectors, using the dynamic equicorrelation models and the Diebold and Yilmaz (2012) index. For comparison purposes, we consider the aggregate conventional and sustainability indexes. The current study is conducted on daily data over the period November 9, 1998 through March 5, 2015, which includes several major financial crises that are accused of increasing correlations and causing portfolio re-balances. It also evaluates four portfolios in terms of risk reduction and downside risk reduction, using the risk measures VaR, semivariance and regret tools.

Our results find a dynamic equicorrelation between all the commodity and Islamic markets under consideration, as explained earlier. Moreover, the equicorrelation increases during the 2008–2009 global financial crisis period. Using the DY index, we show time-varying risk spillovers between oil, gold and the aggregate and the associated ten Islamic sectoral stock indexes. More interestingly, the results reveal that both the oil and gold markets and the Islamic energy, financial, technology and telecommunications sectors are net receivers of risk spillovers, while the DJIM index, consumer goods, consumer services, health care, industrials, utilities sectors are net contributors to risk spillovers. The basic materials sector is found to be risk spillover-neutral.

Using the downside risk reduction measures the VaR, semivariance and regret measures, we find that the optimally weighted portfolio offers the best risk reduction and the largest downside risk reduction for all the oil-Islamic sector pairs, the gold-DJIM, the gold-Islamic consumer services, the gold-Islamic consumer goods, the gold-Islamic health care and the gold-Islamic industrials sector. For the rest of the gold-Islamic sector pairs, the hedged portfolio provides the best risk reduction.

Additionally, gold offers better diversification benefits, risk reductions and downside risk reductions than oil. Based on this result, we can recommend gold as a better diversifier and risk reducer than oil. This finding may fit well with many of the wealthy faithful individual and institutional investors who reside in major oil-producing countries whose economies and stock markets are propelled by oil revenues. Further, the Islamic equity sectors show an increasing financial integration with the commodity markets. But these sectors do not respond in similar ways to the commodity price shocks. The basic materials, energy, financial, telecommunications and technology sectors are net risk receivers of spillovers, while the rest of sectors are net risk contributors.

Investors can consider commodity prices to predict the sector equity prices. More interestingly, investors can have the opportunity to allocate their portfolio in distinctive sectors. Finally, gold is helpful for investors dealing with the technology sector, while those investing in the basic materials sector can include oil in their portfolios for risk-reduction purposes. Finally, the Sharia-compliance rules are not restrictive enough to make the Islamic stock equity market very different from the conventional indexes and immune against commodity price shocks.

The results have important implications for investors and portfolio managers. Investing in gold as a complement to Islamic sector investments offers a better way for the investor to diversify their portfolio when their expectations are heterogeneous in terms of risk tolerance and time preference.

It will be intriguing in the future to extend this work by considering the impact of oil and gold on Islamic sector stock market liquidity in the short, medium and long runs.

Appendix A. Supplementary data

Supplementary data to this article can be found online at http://dx.doi.org/10.1016/j.eneco.2017.08.031.

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