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Delay-Aware Control Designs of Wide-Area Power Networks ⋆

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Abstract: A co-design of the implementation platform and control strategies for wide-area power networks is addressed. Limited and shared resources among control and non-control applications introduce delays in transmitted messages. The design is based on a delay-aware architecture and cloud computing has been proposed for damping wide-area oscillations. We accommodate possibly large delays in the network and take into account their values in the designs. Moreover, we design output feedbacks for the cases that some state variables are not accessible. The designs are verified through a simulation on 50-bus Australian model.

Keywords: Power systems stability, Wide-area measurement systems, Optimal operation and control of power systems, Cyber-physical Systems, Systems with time-delays

1. INTRODUCTION

The wide-area measurement systems (WAMS) technology using Phasor Measurement Units (PMUs) has been regarded as the key to guaranteeing stability, reliability, state estimation, control, and protection of next-generation power systems (Chakrabortty, 2012; Chakrabortty and Khargonekar, 2013; Phadke et al., 1983). However, with the exponentially increasing number of PMUs deployed in the North American grid, and the resulting explosion in data volume, the design and deployment of an efficient wide-area communication and computing infrastructure is evolving as one of the greatest challenges to the power system and IT communities. For example, according to UCAlug Open Smart Grid (OpenSG) ope, every PMU requires 600 to 1500 kbps bandwidth, 20 ms to 200 ms latency, almost 100% reliability, and a 24-hour backup. With several thousands of networked PMUs being scheduled to be installed in the United States by 2020, WAMS will require a significant Gigabit per second bandwidth. The challenge is even more aggravated by the gradual transition of the computational architecture of wide-area monitoring and control from centralized to distributed for facilitating the speed of data processing (Nabavi et al., 2015)

One of the greatest challenges for implementing wide-area control is the issue of communication delay. If a US-wide communication network capable of transporting gigabit volumes of PMU data indeed needs to be implemented then power system operators must have a clear sense of how the various forms of delays that are bound to arise in such networks, affect the stability of these control loops. One important question is - how can wide-area controllers be co-designed in sync with these communication delays in order to make the closed-loop system resilient and *delay-aware*, rather than just *delay-tolerant*? Since utilities are unlikely to establish highly expensive, dedicated communication links for these types of system-wide controls, the communication infrastructure must be implemented on top of their existing subnetworks. As a result, PMU data used for control will have to be transported over a *shared* resource, sharing bandwidth with other ongoing applications, giving rise to not only transport delays, but also significant delays due to queuing and routing. Currently, there is very little insight on how the different protocols for PMU data transport may lead to a variety of such delay patterns, and how controlling these delays can potentially help wide-area control designs The existing PMU standards, IEEE C37.118 and IEC 61850, only specify the sensory data format and communication requirements. They do not indicate any *dynamic* performance standard of the closed-loop system. In recent literature, several researchers have looked into delay mitigation in wide-area control loops (Chaudhuri et al., 2004; Wu et al., 2002; Zhang and Vittal, 2013). Especially relevant is the recent work in Zhang and Vittal (2013) where $\mathcal{H}_\infty$ controllers were designed for redundancy and delay insensitivity. All of these designs are, however, based on worst-case delays,

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which make the controller unnecessarily restrictive, and may degrade closed-loop performance.

Motivated by these concerns in our recent papers (Soudbakhsh et al., 2017), we presented a cyber-physical architecture for wide-area control using Arbitrated Network Control Systems (ANCS) for mitigating the destabilizing effects of network delays on small-signal models of power systems. The ANCS framework facilitates one in co-designing the wide-area controllers in sync with the knowledge about the delays arising from shared resources among control and non-control applications. The design in Soudbakhsh et al. (2017) investigates the case when all the delays are smaller than the sampling period h of the Synchrophaors, and also assumes full state availability. In reality, however, both of these assumptions may not hold. Therefore, in this paper we expand our results to a more practical case when (1) some delays in the network are larger than h, and (ii) the controller is implemented via output feedback instead of state feedback. We illustrate the effectiveness of proposed designs on a 50-bus Australian power system network consisting of 14 generators across four coherent areas.

The rest of the paper is organized as follows. In Section 2, the problem statement is described. Section 3 devotes to the extension of the ANCS design for accommodating delays larger than the sampling period, while Section 4 derives the output feedback control. Simulation results are shown in Section 5. Finally, Section 6 concludes the paper. Due to space limitations, the proofs are left out of this version, but are available in Dibaji et al. (2017).

2. PROBLEM STATEMENT

2.1 *Problem Description*

We consider a power system with a total of n generators distributed among p areas, with a_j generators each, $j = 1, \ldots, p$. Each area j has its own virtual machine (VM) which is responsible for computing the control inputs of the a_j generators (see Fig. 1). Assuming that each generator i has n_i states, $i = 1, \ldots, n$, which include rotor phase angle and frequency, excitation voltage, d-axis sub-transient flux, exciter states, power system stabilizer states, turbine/governor states, active-and reactive load modulation states, and states of Static Var Compensators, and FACTS devices, and has a scalar input which corresponds to the field excitation voltage, stacking all n_i states together, the network model can be compactly written as

$$\dot{x}(t) = A_c x(t) + B_c u(t), \tag{1}$$

where $B_c = [B_c^1, \ldots, B_c^n] \in \mathbb{R}^{N \times n}$, $\sum_{i=1}^{n} n_i = N$, and $A_c \in \mathbb{R}^{N \times N}$. The wide-area damping control problem is to design a global state-feedback controller $u(t)$ in (1), using discrete measurements sampled every h secon+++ds, such that the overall closed-loop is asymptotically stable with the closed-loop poles placed at desired locations which correspond to the requisite damping. The main challenge is that even though these N states are measured at area j, the measured information arrives at other areas $i \neq j$ with a delay, as they go through a network of VMs. In this paper, we assume that these delays are $\tau_s, \tau_m, \tau_\ell$, which correspond to the local-delay, intra-area delay, and inter-area delay, respectively, with $0 < \tau_s < \tau_m < \tau_\ell$. We

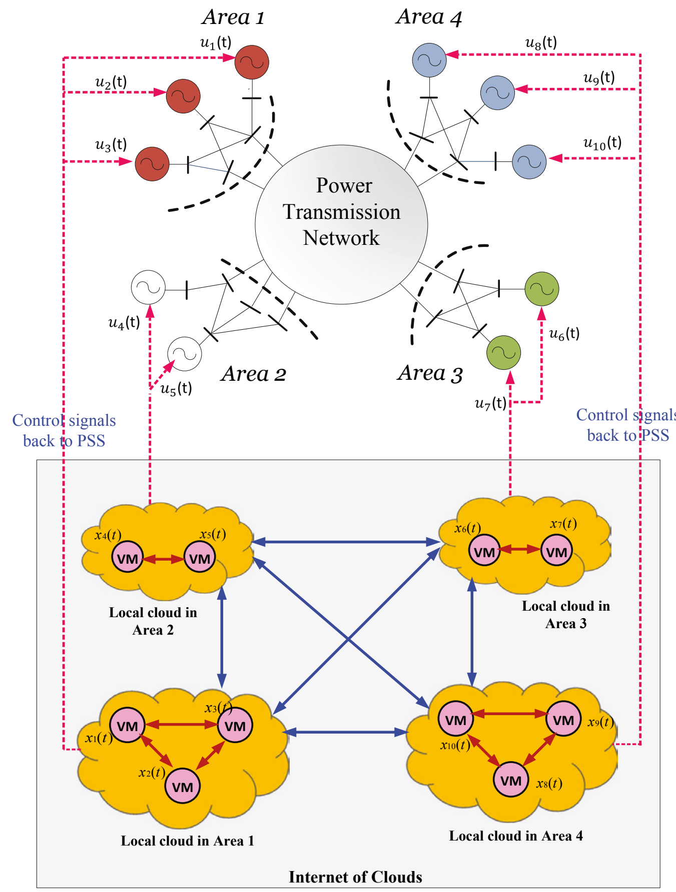

Fig. 1. Control of WAMS via Internet of Clouds

assume in this paper that $\tau_m < h$, while τ_ℓ is significantly large, and such that $4h < \tau_\ell < 5h$, and that all three delays are constants. Such assumptions are based on the typical values of these delays that can be expected to be encountered in a wide area network.

To have a better understanding of the problem, we provide an example:

Example 1. Assume that generators 1, 2, and 3 are in area 1 and generator 4 is in area 2 ($p = 2$). The control input of generator 1, for example, is obtained in the interval $[kh+\tau_m, kh+\tau_\ell)$ using $x_1[k]$, $x_2[k]$, $x_3[k]$ and $\hat{x}_4[k]$, where $x_i[k] \in \mathbb{R}^{n_i}$ is the vector of all state variables of generator i and $\hat{x}_i[k] \in \mathbb{R}^{n_i}$ is an estimation of the state measurements of generator i. The control inputs $u_{1j}[k]$, $j = 1, 2, 3$, are applied after each time new measurements arrive at the VM of area 1. Fig. 2 exhibits the architecture of designs for generator 1, where $\tau_\ell > 4h$.

2.2 *A Sampled-Data Plant Model with Delayed Inputs*

Given that the goal is the control of (1) using input at discrete instants, we convert (1) into a zero-order sampled-data model as follows.

$$x[k+1] = Ax[k] + Bu[k], \tag{2}$$

where

$$A = e^{A_c h} \quad \text{and} \quad B = \int_0^h e^{A_c s} B_c ds. \tag{3}$$

With the assumptions on the three delays $\tau_s, \tau_m, \tau_\ell$, we address the problem for three different cases : (i) $\tau_m < \tau_\ell -$

$4h$, (ii) $\tau_s < \tau_\ell - 4h < \tau_m$, and (iii) $0 < \tau_\ell - 4h < \tau_s$, with Figure 2 illustrating case (i). As shown in Fig 2 for Example 1, it follows that over every interval $[kh,(k+1)h)$, new information arrives from various state measurements at an area j at three different intervals. As will be seen later, this information is judiciously used in the control input. Let $D^i = \{\tau_{ij}\}$ denote the set of delays that corresponds to the computation of the control input of i-th generator. Also denote $D^i_{sm} = \{\tau_{ij} \mid \tau_{ij} < h, \tau_{ij} \in D^i\}$, $D^i_{4h} = \{\tau_{ij} - 4h \mid 4h < \tau_{ij} < 5h, \tau_{ij} \in D^i\}$, and $\Gamma^i = D^i_{sm} \cup D^i_{4h}$. If $|\Gamma^i| = g(i)$, γ^i can be defined as the finite sequence constructed from putting members of Γ^i in an ascending order, where $\sum_{i=1}^n g(i) = \mathcal{G}$. Then the input of generator i is given by

$$U_i(t) = \begin{cases} u_{ij}[k] & \text{if } t - kh \in [\tau'_{ij}, \tau'_{i(j+1)}),\ j < g(i), \\ u_{ig(i)}[k] & \text{if } t - kh \in [\tau'_{ig(i)}, h),\ j = g(i), \\ u_{ig(i)}[k-1] & \text{if } t - kh \in [0, \tau'_{i1}), \end{cases} \tag{4}$$

where τ'_{ij} is j-th member of the sequence γ^i. That is, $u_{ij}[k] \in \mathbb{R}$ is the control of i-th generator adjusted by the measurements of j-th arrival of new information at time k (see Fig. 2). Note that $|D^i| \geq g(i)$ which means that some inter-area information, with a delay greater than $4h$, may arrive simultaneously with those from the same area which is subjected to a much smaller delay. With the piecewise

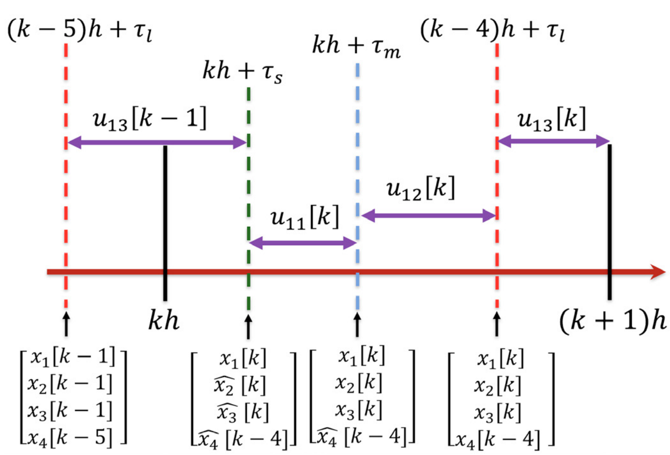

Fig. 2. A schematic on ANCS designs when $\tau_\ell > 4h$

constant control in (4), we integrate (1) by partitioning $[kh,(k+1)h)$ into $(g(i)+1)$ sub-intervals. Therefore, the following dynamic model is obtained

$$x[k+1] = Ax[k] + B_1 U[k] + B_2 U[k-1], \tag{5}$$

where

$$B^i_{j1} = \begin{cases} \displaystyle\int_0^{h-\tau'_{ij}} e^{A_c s} ds B^i_c & \text{if } j = g(i), \quad (6a) \\ \displaystyle\int_{h-\tau'_{i(j+1)}}^{h-\tau'_{ij}} e^{A_c s} ds B^i_c, & \text{if } j \neq g(i), \quad (6b) \end{cases}$$

$$B^i_{i2} = \int_{h-\tau'_{i1}}^{h} e^{A_c s} ds B^i_c, \tag{7}$$

and $B^i_{j1} \in \mathbb{R}^{N\times 1}$ is the coefficient of $u_{ij}[k]$ and $B^i_{i2} \in \mathbb{R}^{N\times 1}$ is the coefficient of $u_{ig(i)}[k-1]$ in (4). In an ideal case, where all measurements arrive with negligible delay, $B_2 = 0$ and $U[k]$ coincides with a sampled value $u[k]$ of dimension n. In contrast, with measurements arriving at different instances, n has expanded to $\mathcal{G}$ with

$$U[k] = [u_{11} \ \cdots \ u_{1g(1)} \ u_{21} \ \cdots \ u_{i(g(i))} \ \cdots \ u_{ng(n)}]^T, \tag{8}$$

Likewise, the matrices B_1 and B_2 can be written in the following forms:

$$B_1 = [B^1_{11} \ \cdots \ B^1_{g(1)1} \ \cdots \ B^i_{11} \ \cdots \ B^i_{g(i)1} \ \cdots \ B^n_{g(n)1}] \tag{9}$$

and

$$B_2 = [0 \ \cdots \ B^1_{g(1)2} \ \cdots \ 0 \ \cdots \ B^i_{g(i)2} \ \cdots \ 0 \cdots \ B^n_{g(n)2}]. \tag{10}$$

We note that the equations (5)-(7) are applicable for case (i) through case (iii) mentioned above. A few comments are worth making regarding the sampled-data plant model in Eq. (5). The plant model has been constructed using a delay-aware controller. That is, each control input $u_{ij}[k]$ in $U[k]$ acts on new information that becomes available during an interval $[kh,(k+1)h)$, with the new information arriving due to delays in measurements. These delays are grouped into three categories, with the first two delays τ_s and τ_m assumed to be smaller than the sampling period, as they are from generators that are local or those that are within the same area. The third category is due to measurements coming from other areas and hence correspond to delays that are much larger, and are therefore assumed to belong to the interval $[4h, 5h)$.

3. A DELAY-AWARE CONTROL DESIGN

3.1 Control Design

The starting point for the control design is the delay-aware plant model in (5) with the goal of designing $U[k] = \left[U_1[k]^T \ U_2[k]^T \ \cdots \ U_n[k]^T\right]^T$ so that the states $X[k]$ tend to zero for any initial conditions. Since our assumption is that all states are measurable, we proceed with a state-feedback based control design:

$$U[k] = \sum_{i=0}^{4} (K_i x[k-i] + G_i U[k-i-1]). \tag{11}$$

However, since the measured values are arriving at their intended recipients' location sporadically, at several distinct instances in any sampling interval $[kh,(k+1)h)$, we propose the following control input rather than a standard state feedback:

$$U_j[k] = \sum_{i=0}^{4} (K_{i,j}\overline{x}_{k,j}[k-i] + G_{i,j}\overline{U}_{k,j}[k-i-1]), \tag{12}$$

where $K_{i,j} \in \mathbb{R}^{g(i)\times N}$, $j = 1,\ldots,n$ is the j-th block of the gains in K_i corresponding to the j-th generator, i.e. $K_i = \left[K^T_{i,1} \ K^T_{i,2} \ \cdots \ K^T_{i,n}\right]^T$ and $G_{i,j} \in \mathbb{R}^{g(i)\times \mathcal{G}}$, where $G_i = \left[G^T_{i,1} \ G^T_{i,2} \ \cdots \ G^T_{i,n}\right]^T$. Also, the vectors $\overline{x}_{k,j}[t] = \left[\overline{x}^1_{k,j}[t]^T \ \overline{x}^2_{k,j}[t]^T \ \ldots \ \overline{x}^n_{k,j}[t]^T\right]^T$ and $\overline{U}_{k,j}[t] = \left[\overline{U}^1_{k,j}[t]^T \ \overline{U}^2_{k,j}[t]^T \ \ldots \ \overline{U}^n_{k,j}[t]^T\right]^T$ are defined as below, respectively:

$$\overline{x}^i_{k,j}[t] = \begin{cases} x_i[t] & \text{if } x_i[t] \text{ has arrived at VM of generator } j \text{ at time } k, \\ \hat{x}_i[t] & \text{if } x_i[t] \text{ has not arrived at VM of generator } j \text{ at time } k, \end{cases} \tag{13}$$

$$\overline{U}^i_{k,j}[t] = \begin{cases} U_i[t] & \text{if } U_i[t] \text{ is available at} \\ & \text{VM of generator } j \text{ at time } k, \\ \hat{U}_i[t] & \text{if } U_i[t] \text{ has not arrived at} \\ & \text{VM of generator } j \text{ at time } k, \end{cases} \quad (14)$$

where $x_i[t]$ is the state measurements of generator i at time t and $U_i[t]$ is the control input of generator i at time t. We note that $\overline{x}_{k,j}[t] \in \mathbb{R}^N$ is, indeed, how the VM of generator j sees the state variables of the network and is constructed of those entries of $x[t]$ and of $\hat{x}[t]$, available or estimated, respectively at time t for computation of control input of j-th generator. Each VM sends out its own $u_i[k]$ and $x_i[k]$ to other VMs as soon as it is computed or measured.

For applying the rule (12), they need to know exact value or estimated value of the following variables:

$$U_j[k-i-1] \quad \text{and} \quad x_j[k-i], \quad i = 0,1,2,3,4. \quad (15)$$

Having $B_i = \begin{bmatrix} B_{i1}^T & B_{i2}^T & \cdots B_{in}^T \end{bmatrix}^T$, where $B_{ij} \in \mathbb{R}^{n_j \times \mathcal{G}}$, for $j = 1,\ldots,n$ and $i = 1,2$. Also $A = \begin{bmatrix} A_1^T & \cdots & A_n^T \end{bmatrix}^T$, where $A_j^T \in \mathbb{R}^{n_j \times N}$.

The estimates $\hat{x}_i[t]$ in (13) are determined using the plant model in (5) as

$$\hat{U}_j[k-\ell] = \sum_{s=0}^{4} (K_{\ell,j}\overline{x}_{k,j}[k-s-\ell] + G_{i,j}\overline{U}[k-s-\ell-1]), \quad (16)$$

and

$$\hat{x}_j[k-\ell+1] = A_j x[k-\ell] + B_{1j}\overline{U}[k-\ell] + B_{2j}\overline{U}[k-\ell-1], \quad (17)$$

where $\ell = 1,2,3,4$. With this, Algorithm 1 must be implemented by VM $i, i = 1,\ldots,n$, for estimating the inter-area variables:

Algorithm 1 Inter-Area Estimations

```
procedure INTERSTAINP–ESTIMATES
    x̂_j[k−4] = A_j x[k−5] + B_1j U[k−5] + B_2j U[k−6]
    set ℓ = 4.
    for each j = 1,...,n, and 1 ≤ ℓ ≤ 4 do
        Compute Û_j[k−ℓ] through (16)
        Compute x̂_j[k−ℓ+1] through (17)
        ℓ = ℓ − 1
    end for
end procedure
```

Remark 1. As it is seen each VM i must have a memory with the capacity to store the communicated variables $(x_j[k], U_j[k])$ for all $j = 1,\ldots,n$ up to $(x_j[k-8], U_j[k-9])$ which comes from $s = \ell = 4$.

Remark 2. In this design, each VM, for estimations, needs to know the control gains of all other generators. This privacy shortage is left as a future research.

3.2 Stability Analysis

Noting that some of the elements of $x[k]$ arrive after four sampling intervals, an extended state-space description that includes $x[k], x[k-1],\ldots,x[k-5]$ is needed. This extended form is given by

$$W_{4h}[k+1] = A_{4h}W_{4h}[k] + B_{4h}U[k], \quad (18)$$

where

$$W_{4h}[k] = \begin{bmatrix} x[k] \\ U[k-1] \\ x[k-1] \\ U[k-2] \\ \vdots \\ x[k-5] \\ U[k-6] \end{bmatrix}, \quad B_{4h} = \begin{bmatrix} B_1 \\ I_{\mathcal{G}} \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix},$$

and $A_{4h} \in \mathbb{R}^{6(N+\mathcal{G})\times 6(N+\mathcal{G})}$ is

$$A_{4h} = \begin{bmatrix} A & B_2 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ I_N & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & I_{\mathcal{G}} & 0 & \cdots & 0 & 0 & 0 \\ & & & \vdots & & & \\ 0 & 0 & 0 & \cdots & I_{\mathcal{G}} & 0 & 0 \end{bmatrix}. \quad (19)$$

Note that the extended form (18) can be also written in the general form $mh < \tau_\ell < (m+1)h, m \geq 0$. In particular, the case $m = 0$ is subject of study in Soudbakhsh et al. (2017), where $A_h = \begin{bmatrix} A & B_2 \\ 0 & 0 \end{bmatrix}$ and $B_h = \begin{bmatrix} B_1 \\ I_{\mathcal{G}} \end{bmatrix}$. We first show that when the original plant-model is stable, then A_{4h} is Schur-stable. For this purpose, the following Lemma is needed:

Lemma 2. (Yuz and Goodwin, 2014) The matrix A in (3) is non-singular and all of its eigenvalues, provided that A_c in (1) is Hurwitz, are inside the unit circle.

The following proposition addresses the Schur-stability of A_{4h} in (19):

Proposition 3. If the system (1) is stable, A_{4h} is Schur-stable with N eigenvalues coinciding with all eigenvalues of A and the remaining ones at zero.

We now address the controllability of the extended state-space model in (19). The following lemma is useful to connect the controllability of (A_h, B_h) to (A_{4h}, B_{4h}). The proof follows from Theorem 1 in Liu and Fong (2012) and is omitted.

Lemma 4. The time-delay system

$$x[k+1] = \sum_{i=0}^{d_x} A_i x[k-i] + \sum_{i=0}^{d_u} B_i u[k-i],$$

is completely controllable if and only if

$$Y = [\sum_{i=0}^{d_x} \lambda^{d_x - i} A_i - \lambda^{d_x+1} I \quad \sum_{i=0}^{d_u} \lambda^{d_u - i} B_i]$$

has full rank at all roots of

$$\left| \sum_{i=0}^{d_x} \lambda^{d_x - i} A_i - \lambda^{d_x+1} I \right| = 0.$$

Proposition 5. If (A_h, B_h) is controllable, (A_{4h}, B_{4h}) is stabilizable.

Knowing that (A_{4h}, B_{4h}) is stabilizable and A_c is stable, for damping the oscillations, we can see that the state feedback control using the extended state equations (18)

$$U[k] = K_{4h}W_{4h}[k], \quad (20)$$

where

$$K_{4h} = [K_0 \ G_0 \ K_1 \ G_1 \ K_2 \ G_2 \ K_3 \ G_3 \ K_4 \ G_4 \ K_5 \ G_5]$$

can be applied for stabilizability. Using the estimations (16)-(17), the extended state feedback control (20) can be rewritten in the form of (12), where G_5 and K_5 are zero matrices due to the last $N+\mathcal{G}$ columns of A_{4h} which are zero. An optimal K_{4h} is achieved through the minimization of the quadratic cost function (Anderson and Moore, 2007)

$$J_{4h} = \sum_{0}^{\infty} \big(W_{4h}[k]^T Q_{4h} W_{4h}[k] + U[k]^T R U[k]\big),$$

where the wight matrices Q_{4h} and R are determined via the procedures described in Soudbakhsh et al. (2017).

The following remark shows the generality of the designs presented in this section for arbitrarily large delays.

Remark 3. All equations and results described above can be extended to the case that $mh < \tau_\ell < (m+1)h$, where $0 \le m$ is an arbitrary integer. To this aim, the equations (12), (18), W_{4h}, and all related variables can be rewritten based on W_{mh}, for a general m. However, the performance of the closed-loop system drops as m increases. If the inter-area delays are less than h, i.e. $m = 0$, D^i_{4h} is empty and τ'_{ij}s are equal to τ_{ij}s. Hence, (5), (7), and the control design (12) can be viewed as a generalization of the designs in Soudbakhsh et al. (2017).

4. EMPLOYING OUTPUT FEEDBACK

In this section, we remove the assumption that all system states are accessible. For example, we can consider the case where only the rotor phase angles and frequencies are measured, which is a realistic case. For the sake of simplicity, we also assume that $\tau_\ell < h$ and s state variables from N to be measurable. Also, to avoid repetition, communication protocols among VMs and estimation procedure is left out by putting $\overline{U}_{k,j}[t] = U[k]$. Therefore, by using an appropriate output matrix $C \in \mathbb{R}^{s\times N}$, we determine the output vector $y \in \mathbb{R}^s$ as

$$y[k] = Cx[k]. \tag{21}$$

The control problem is the regulation of the plant (5) using the available output signals given in (21).

4.1 Output Feedback Controller Design

Consider the observer dynamics is given as

$$x_o[k+1] = Ax_o[k] + B_2U[k-1] + B_1U[k] + L(y[k] - Cx_o[k]), \tag{22}$$

where $x_o \in \mathbb{R}^N$ is the observer state vector and $L \in \mathbb{R}^{N\times s}$ is a constant observer gain matrix. Subtracting (22) from (5), the observer error dynamics is obtained as

$$\tilde{x}[k+1] = (A - LC)\tilde{x}[k], \tag{23}$$

where $\tilde{x} = x - x_o$ is the observer error vector. The observer gain L can be obtained using pole placement or an LQR design where the following objective function is minimized

$$J_o = \sum_{0}^{\infty} \big(x_o[k]^T Q_o x_o[k] + U_o[k]^T R_o U_o[k]\big).$$

An alternative to the full order observer is the *reduced order observer* which is explained below.

Let's assume that $x_a \in \mathbb{R}^s$ refers to the measurable states vector and $x_b \in \mathbb{R}^{N-s}$ refers to the vector of states that need to be observed. Then, the plant dynamics of the power system can be partitioned as

$$\begin{aligned}
\begin{bmatrix} x_a[k+1] \\ x_b[k+1] \end{bmatrix} &= \begin{bmatrix} A_{aa} & A_{ab} \\ A_{ba} & A_{bb} \end{bmatrix} \begin{bmatrix} x_a[k] \\ x_b[k] \end{bmatrix} + \begin{bmatrix} B_2^a \\ B_2^b \end{bmatrix} U[k-1] \\
&\quad + \begin{bmatrix} B_1^a \\ B_1^b \end{bmatrix} U[k] \qquad (24) \\
y[k] &= \begin{bmatrix} I_{s\times s} & 0_{s\times(N-s)} \end{bmatrix} \begin{bmatrix} x_a[k] \\ x_b[k] \end{bmatrix}.
\end{aligned}$$

We can obtain the dynamics of the unmeasured states from (24) as

$$x_b[k+1] = A_{bb}x_b[k] + A_{ba}x_a[k] + B_2^bU[k-1] + B_1^bU[k]. \tag{25}$$

The dynamics of the reduced order observer is given as

$$\begin{aligned}
x_{bo}[k+1] &= A_{bb}x_{bo}[k] + A_{ba}x_a[k] + B_2^bU[k-1] \\
&\quad + B_1^bU[k] + L_r\Big(x_a[k+1] - A_{aa}x_a[k] \\
&\quad - B_2^aU[k-1] - B_1^aU[k] - A_{ab}x_{bo}[k]\Big), \qquad (26)
\end{aligned}$$

where x_{bo} is the observed value of the unmeasured state vector x_b and $L_r \in \mathbb{R}^{(N-s)\times s}$ is a constant reduced order observer gain matrix.

Subtracting (26) from (25) it is obtained that

$$\tilde{x}_{bo}[k+1] = \big(A_{bb} - L_rA_{ab}\big)\tilde{x}_{bo}[k], \tag{27}$$

where $\tilde{x}_{bo} = x_b - x_{bo}$ is the reduced order observer error vector. Similar to the full order observer case, the observer gain L_r can be obtained using pole placement or an LQR design, where the following objective function is minimized

$$J_{bo} = \sum_{0}^{\infty} \big(x_{bo}[k]^T Q_{bo} x_{bo}[k] + U_{bo}[k]^T R_{bo}$$

4.2 Stability Analysis

For the state accessible case, the control input (11), for $i = 0$, can be re-written as

$$U[k] = K_0^{(1)}x[k] + K_0^{(2)}\hat{x}[k] + G_0U[k-1], \tag{28}$$

where $K_0^{(1)}$ is a constant control gain matrix built by putting zero instead of the gains on K_0 whenever the corresponding state variables are not available. Likewise $K_0^{(2)} = K_0 - K_0^{(1)}$ which has non-zero elements to be effective for those state variables whose their estimates are used (Soudbakhsh et al., 2017). When the states are not accessible, the control input is modified as

$$U[k] = K_0^{(1)}x_o[k] + K_0^{(2)}\hat{x}_o[k] + G_0U[k-1], \tag{29}$$

where the *estimated observer state vector* $\hat{x}_o$ is calculated using (22) as

$$\begin{aligned}
\hat{x}_o[k] &= Ax_o[k-1] + B_2U[k-2] + B_1U[k-1] \\
&\quad + L(y[k-1] - Cx_o[k-1]). \qquad (30)
\end{aligned}$$

Theorem 6. The closed-loop output feedback control system consisting of (5), (29) and (30) is stable.

Remark 4. It is noted that for the case of $\tau_\ell > h$, the observer design follows the same procedure as explained above with an extended state-space description given in (18).

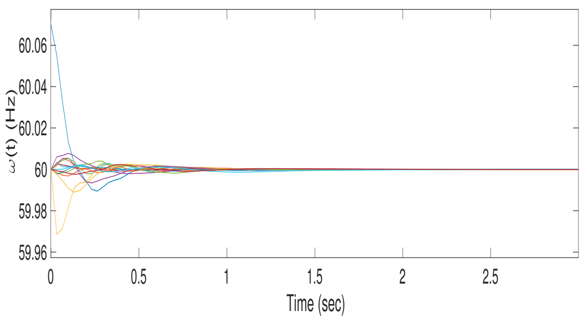

Fig. 3. Frequency of all generators: Large delay case

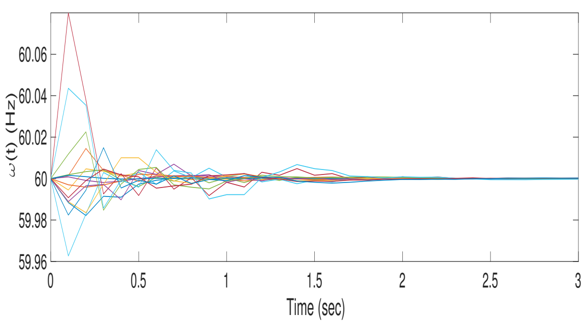

Fig. 4. Frequency of all generators: Output feedback controllers

5. SIMULATION RESULTS

We carry out our simulations on 14 generators with overall 200 state variables ($N = 200$) from Australian 50-bus network (Gibbard and Vowles, 2010). Like the example given in Soudbakhsh et al. (2017), generators 1, 2, 3, 4, and 5 belong to area 1, generators 6 and 7 belongs to area 2, generators 8, 9, 10, and 11 to area 3, and generators 12, 13, and 14 to area 4 ($p = 4$). We assumed that an impulse disturbance is present which simulated as a change in the initial condition, with $\omega_1(0) = 60.07Hz$.

5.1 Large Delay Case

First, we examine the large delay design. The delays and sampling time are assigned as following:

$$\tau_s = 10ms, \quad \tau_m = 30ms, \quad \tau_\ell = 155ms, \quad h = 33ms.$$

Fig. 3 illustrates the frequency of the generators under the designed controllers (12). As it is seen, they are all damped.

5.2 Output Feedback Controllers

Finally, we conduct simulations on the same network for the output feedback case, where the delays and sampling period are set to

$$\tau_s = 10ms, \quad \tau_m = 30ms, \quad \tau_\ell = 90ms, \quad h = 100ms.$$

The observer states' initial conditions are assigned to zero. In Fig. 4, frequency of all generators controlled by the designed control signals are presented.

6. CONCLUSIONS

We extended the ANCS control design of wide area power networks in two respects: To accommodate larger inter-area delays and to employ output feedbacks. We analyzed stability of both cases, with the assumption that the original open-loop system is stable. Simulation results on a 50-bus Australian network verified the proposed approaches. In future work, we intend to investigate the effect of cyber-physical attacks on such designs.

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