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Stability Analysis of a Human-in-the-Loop Telerobotics System with Two Independent Time-Delays

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Abstract: In this paper, stability of a human-in-the-loop telerobotics system with force feedback and communication delays is investigated. A general linear time-invariant time-delayed mathematical model of the human operator is incorporated into the system dynamics based on the interaction of the human operator with the rest of the telerobotic system. The resulting closed loop dynamics contains two independent time-delays mainly due to back and forth communication delay and human reaction time delay. Stability of this dynamics is characterized next on the plane of the two delays by rigorous mathematical investigation using Cluster Treatment of Characteristic Roots (CTCR). An illustrative numerical example is further provided in the results section along with interpretations.

Keywords: Telerobotics; Human-in-the-Loop Systems; Stability; Time-Delay System

1. INTRODUCTION

Teleoperation is a platform that enables a human to interact with a distant robot in order to accomplish a given task. Teleoperation systems have many applications in various fields including but not limited to space investigations, underwater operations, telediagnosis and telesurgery, and education; see, for example, (Ferre et al., 2007). In this technology, to improve the performance of the human operator and to give her/him a feel of the remote operating environment, certain perception signals are fed back to the master (human) side from the slave (robot) side. The transmitted signals can be categorized in three types: visual, auditory, and haptic; see, for example, (Hokayem and Spong, 2006; Sheridan, 1995). The purpose of this paper is to consider the signals of the third type, that is, haptic or force feedback, as it is this feature that is more relevant to human-in-the-loop applications, but also compromises stability more than the other types (Abidi et al., 2016).

One key issue in incorporating humans in the control loop is the latency (τ_c) in the teleoperation infrastructure. The authors in (Kaber et al., 2012), for example, investigated the effect of system latency on the human performance in a telesurgery task using a virtual reality simulator. They incorporated Fitt's law (Fitts, 1954) and one of its modified versions (Ware and Balakrishnan, 1994) to obtain quantitative measures of human motion time and task difficulty. They experimentally concluded that time lag could result in user performance degradation in terms of motion time and task difficulty.

Model-mediated teleoperation has also gained attention in the literature (Mitra and Niemeyer, 2008; Weber et al., 2009). In this approach, either a model of the master side is reflected on the slave side, or a model of the slave side is reflected on the master side in order to compensate the time delays and disturbances. Based on this idea, the authors in (Feth et al., 2010) used a Kalman filter for signal fusion on both slave and master sides assuming an upper bound for time-delays. They also considered a linear model for the human operator without considering human reaction time delays (τ_h) with the idea that in telerehabilitation applications, a therapist does not need to perform a sudden motion.

There are a number of methods in the literature to predict the motion of human operators. One well-known is using the minimum jerk model, which is based on the observation on intact primates that they generate a motion of a limb from an equilibrium point to another one (point-to-point) in a given time interval in the smoothest way possible by minimizing the mean-square jerk (Hogan, 1984). With the idea that much of human actions are more predictive (feedforward) than feedback-based (Berthoz, 2000), and also by using the Smith Predictor, the authors in (Smith and Christensen, 2009) introduced a control strategy to handle the command and measurement communication delays. They used the minimum jerk model to predict the human operator's future inputs. Based on the experiments, they concluded that the system with minimum-jerk human input predictor has better performance compared to that of the system with a standard Smith Predictor.

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To the best knowledge of the authors, many studies assumed that time-delays are homogeneous in teleoperation applications, with few exceptions; see, for example, (Cheong and Niculescu, 2008) and (Liacu et al., 2013). Moreover, rigorous mathematical analysis of human-in-the-loop telerobotic systems from a stability point of view is challenging especially in the presence of heterogeneous time-delays (Sipahi et al., 2011). Since human reaction time delay can also result in significant stability limitations (Acosta et al., 2015); in this paper, we consider the human operator as an element of the overall closed loop system dynamics. Specifically, we theoretically show how the human operator and the rest of the relerobotics system interact with each other and how the human reaction time delay and telecommunication delays affect the stability of the closed-loop system. To this end, we will use well-established human models with reaction delays (τ_h) interacting with a teleoperation model that has inherent communication delays (τ_c), and utilize stability analysis tools established for multiple delay systems, specifically CTCR in (Sipahi, 2005) and the references therein, to understand how human dynamics affects the closed-loop system.

The paper is organized as follows. In Section 2, we provide the problem formulation. In Section 3, we investigate the stability of the closed loop human-in-the-loop telerobotics system. In Section 4, we present a numerical illustration of the theoretical analysis, and we provide discussions and conclusions in Section 5.

2. PROBLEM FORMULATION

Consider the human-in-the-loop telerobotics system with the block diagram depicted in Fig. 1. Specifically, here we focus on a human model with reaction time delay expressed by the general linear time-invariant time-delayed model (Yucelen et al., 2017)

$$\dot{x}_h(t) = A_h x_h(t) + B_h \theta_1(t-\tau_h), \quad x_h(0) = 0, \tag{1}$$
$$F_h(t) = C_h x_h(t) + D_h \theta_1(t-\tau_h), \tag{2}$$

where $x_h(t) \in \mathbb{R}^{n_h}$ is the human state vector, $\tau_h \in \mathbb{R}^+$ is the human reaction time delay, $A_h \in \mathbb{R}^{n_h \times n_h}$, $B_h \in \mathbb{R}^{n_h \times n_{\theta_1}}$, $C_h \in \mathbb{R}^{n_{F_h} \times n_h}$, and $D_h \in \mathbb{R}^{n_{F_h} \times n_{\theta_1}}$ are "human operator system" matrices, and $F_h(t) \in \mathbb{R}^{n_{F_h}}$ is the human operator's force command. The input to the human dynamics is given by

$$\theta_1 \triangleq r(t) - y_m(t), \tag{3}$$

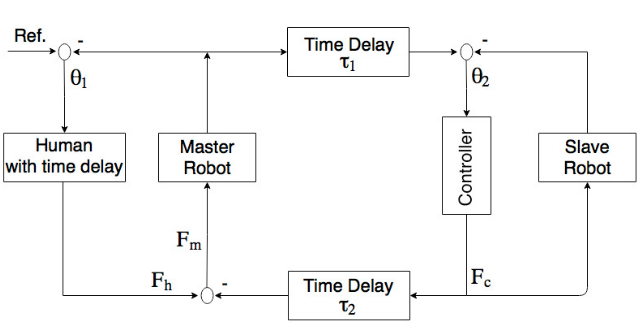

Fig. 1. Block diagram of the overall human-in-the-loop telerobotics system.

where $\theta_1(t) \in \mathbb{R}^{n_r}$ is the error vector with $r(t) \in \mathbb{R}^{n_r}$ defined as the reference input, and $y_m(t)$ as the master robot output.

Master robot is considered to be a system with the following dynamics

$$\dot{x}_m(t) = A_m x_m(t) + B_m F_m(t), \quad x_m(0) = 0, \tag{4}$$
$$y_m(t) = C_m x_m(t) + D_m F_m(t), \tag{5}$$

where $x_m(t) \in \mathbb{R}^{n_m}$ is the master robot state vector, $y_m(t) \in \mathbb{R}^{n_{y_m}}$ is the master robot output, and $A_m \in \mathbb{R}^{n_m \times n_m}$, $B_m \in \mathbb{R}^{n_m \times n_{F_m}}$, $C_m \in \mathbb{R}^{n_{y_m} \times n_m}$, and $D_m \in \mathbb{R}^{n_{y_m} \times n_{F_m}}$ are the master robot system matrices. Matrix $F_m(t) \in \mathbb{R}^{n_{F_h}}$ denotes the force input applied to the master robot. Here, it is given by

$$F_m(t) = F_h(t) - F_c(t-\tau_2), \tag{6}$$

where $F_c(t)$ is the slave-side controller output.

The slave robot dynamics is given by

$$\dot{x}_s(t) = A_s x_s(t) + B_s F_c(t), \quad x_s(0) = 0, \tag{7}$$
$$y_s(t) = C_s x_s(t) + D_s F_c(t), \tag{8}$$

where $x_s(t) \in \mathbb{R}^{n_s}$ is the slave robot state vector, $y_s(t) \in \mathbb{R}^{n_{y_s}}$ is the slave robot output, and $A_s \in \mathbb{R}^{n_s \times n_s}$, $B_s \in \mathbb{R}^{n_s \times n_{F_c}}$, $C_s \in \mathbb{R}^{n_{y_s} \times n_s}$, and $D_s \in \mathbb{R}^{n_{y_s} \times n_{F_c}}$ are the slave robot system matrices, and $\tau_2 \in \mathbb{R}^+$ is the feedback communication delay.

The controller dynamics is given as

$$\dot{x}_c(t) = A_c x_c(t) + B_c \theta_2(t), \quad x_c(0) = 0 \tag{9}$$
$$F_c(t) = C_c x_h(t) + D_c \theta_2(t), \tag{10}$$

where $x_c(t) \in \mathbb{R}^{n_c}$ is the controller state vector, and $A_c \in \mathbb{R}^{n_c \times n_c}$, $B_c \in \mathbb{R}^{n_c \times n_{\theta_2}}$, $C_c \in \mathbb{R}^{n_{F_c} \times n_c}$, and $D_s \in \mathbb{R}^{n_{F_c} \times n_{\theta_2}}$ are the controller system matrices. The error dynamics on the slave side controller reads

$$\theta_2(t) \triangleq y_m(t-\tau_1) - y_s(t), \tag{11}$$

where $\theta_2(t) \in \mathbb{R}^{n_{y_m}}$, and $\tau_1 \in \mathbb{R}^+$ is the feedforward communication delay.

With this given setup, the stability of the overall system subject to human and independent time-delays is investigated next.

3. STABILITY IN THE PRESENCE OF TWO INDEPENDENT DELAYS

Using (3) and (5), one can write

$$\theta_1(t) \triangleq r(t) - C_m x_m(t), \tag{12}$$

and using (5), (8), and (11), one obtains

$$\theta_2(t) \triangleq G_0 C_m x_m(t-\tau_1) - G_0 C_s x_s - G_0 D_s C_c x_c, \tag{13}$$

where the existence of $G_0 = (I + D_s D_c)^{-1}$ is assumed implicitly. Now, considering (2), (6), and (10), we can write

$$F_m(t) = C_h x_h(t) + D_h \theta_1(t-\tau_h) - C_c x_c(t-\tau_2) - D_c \theta_2(t-\tau_2). \tag{14}$$

Finally, by letting $\phi(t) \triangleq [x_h^T(t), x_m^T(t), x_c^T(t), x_s^T(t)]$, and using (4), (7), (9), (12), (13) and (14), we obtain the augmented state space representation of the dynamics in Fig. 1,

$$\dot{\phi}(t) = \mathcal{A}_0 \phi(t) + \mathcal{A}_{\tau_h} \phi(t-\tau_h) + \mathcal{A}_{\tau_1} \phi(t-\tau_1) + \mathcal{A}_{\tau_2} \phi(t-\tau_2) + \mathcal{A}_{\tau_1 \tau_2} \phi(t-\tau_1-\tau_2) + \mathcal{B}_{\tau_h} r(t-\tau_h), \tag{15}$$

where

$$\mathcal{A}_0 \triangleq \begin{bmatrix} A_h & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ B_m C_h & A_m & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & A_c - BcG_0C_s & \mathbf{0} & -B_c G_0 C_s \\ \mathbf{0} & \mathbf{0} & B_s C_c & A_s - B_s D_c G_0 C_s \end{bmatrix}, \tag{16}$$

$$\mathcal{A}_{\tau_h} \triangleq \begin{bmatrix} \mathbf{0} & -B_h C_m & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -B_m D_h C_m & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \tag{17}$$

$$\mathcal{A}_{\tau_1} \triangleq \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & B_c G_0 C_m & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & B_s D_c G_0 C_m & \mathbf{0} & \mathbf{0} \end{bmatrix}, \tag{18}$$

$$\mathcal{A}_{\tau_2} \triangleq \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -B_m C_c & B_m D_c G_0 C_s \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \tag{19}$$

$$\mathcal{A}_{\tau_1\tau_2} \triangleq \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -B_m D_c G_0 C_m & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \tag{20}$$

$$\mathcal{B}_{\tau_h} \triangleq \begin{bmatrix} B_h \\ B_m D_h \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}. \tag{21}$$

To simplify the calculations, we assume that communication delays in both directions are identical, $\tau_1 = \tau_2 = \tau_c$, however this delay is in general different from human reaction time delay (τ_h). This then leads to the following state space representation

$$\begin{aligned} \dot{\phi}(t) = \mathcal{A}_0\phi(t) + \mathcal{A}_{\tau_h}\phi(t-\tau_h) + \mathcal{A}_{\tau_c}\phi(t-\tau_c) + \\ \mathcal{A}_{\tau_c\tau_c}\phi(t-2\tau_c) + \mathcal{B}_{\tau_h} r(t-\tau_h), \end{aligned} \tag{22}$$

where

$$\mathcal{A}_{\tau_c} \triangleq \mathcal{A}_{\tau_1} + \mathcal{A}_{\tau_2}, \tag{23}$$

$$\mathcal{A}_{\tau_c\tau_c} \triangleq \mathcal{A}_{\tau_1\tau_2}. \tag{24}$$

The dynamics (22) is a linear time-invariant multiple-delay system, and in the following developments, its stability characteristics on the plane of $\tau_c - \tau_h$ will be investigated using CTCR, see (Sipahi and Olgac, 2005; Sipahi, 2005).

The characteristic equation of (22) is given as

$$CE = \det(sI - \mathcal{A}_0 - \mathcal{A}_{\tau_h}e^{-\tau_h s} - \mathcal{A}_{\tau_c}e^{-\tau_c s} - \mathcal{A}_{\tau_c\tau_c}e^{-2\tau_c s}) = 0. \tag{25}$$

Through some manipulations, one can find the general form of the characteristic equation as

$$CE = \sum_{k=0}^{n}\sum_{j=0}^{n-k}\sum_{l=0}^{n-k-j} a_{kjl}(s)e^{-(k\tau_h + (j+2l)\tau_c)s}, \tag{26}$$

where $a_{kjl}(s)$ are polynomials in "s". We next utilize the *Rekasius substitution* [1]

$$e^{-\tau_j s} = \frac{1 - T_j s}{1 + T_j s},\ T_j \in \mathbb{R},\ j = h, c, \tag{27}$$

which is an exact substitution for $s = j\omega_c$ roots of the characteristic equation. Then, we obtain a polynomial in T_j, which is given as

$$CE = \sum_{k=0}^{n}\sum_{j=0}^{n-k}\sum_{l=0}^{n-k-j} a_{kjl}(s)\left(\frac{1 - T_h s}{1 + T_h s}\right)^k \left(\frac{1 - T_c s}{1 + T_c s}\right)^{j+2l}. \tag{28}$$

Furthermore, (28) can be simplified by expanding it by $(1+T_h s)^n(1+T_c s)^{n-k}$, which does not bring any artificial $s = j\omega_c$ roots, since T_c and T_h are both real. Next, it can be shown that using the phase condition in (27), the following mapping between T_j and τ_j values holds:

$$\tau_j = \frac{2}{\omega_c}\left[\tan^{-1}(\omega_c T_j + k\pi)\right],\ k = 0, 1, \ldots;\ j = h, c. \tag{29}$$

It is important to note that $s = j\omega_c$ roots of (25) and (28) one to one match (Sipahi, 2005) (Sipahi and Olgac, 2005). Since we have the transformed characteristic equation in the polynomial form in (28), which is simpler than (25), we first calculate all the imaginary axis crossings $s = j\omega_c$ in terms of $T_c \in \mathbb{R}$ and $T_h \in \mathbb{R}$ from (28), for example, using Routh's array. Using these T_c and T_h values obtained from Routh's array, we can then use (29) to calculate the delays τ_j for which (25) has crossings at the same crossing $s = j\omega_c$.

Note that there are infinitely many delays corresponding to each pair (T_c, ω_c) and (T_h, ω_c) due to the counter k. The smallest positive of the delays and the corresponding imaginary axis crossings $\omega_c \in \Omega$ construct the so-called *"kernel curves"*, and the remaining positive delays construct the so-called *"offspring curves"*. In this problem, offspring curves follow the corresponding kernel curve in terms of stabilizing or destabilizing behavior of the root $s = j\omega_c$, which is associated with the property called *"Root Tendency (RT) invariance"*. RT for the specific problem at hand is calculated for $s = j\omega_c$ using

$$RT|_{s=\omega_c i}^{\tau_c} = sgn\{Im\,[H(s,\tau_h)]\}, \tag{30}$$

where

$$H(s,\tau_h) = \frac{\sum_{k=0}^{n}\sum_{j=0}^{n-k}\sum_{l=0}^{n-k-j}\left(\left(\frac{da_{kjl}(s)}{ds}\right)_{kjl} - (2l+j)\tau_c a_{kjl}\right)}{\sum_{k=0}^{n}\sum_{j=0}^{n-k}\sum_{l=0}^{n-k-j}(-a_{kjl}k)}. \tag{31}$$

In order to check the stability of a region on the plane of delays, one keeps τ_c fixed and uses the invariance property of RT with respect to time-delay τ_h to determine the number of unstable roots of the system on $\tau_c - \tau_h$, see details in the above-cited references.

4. RESULTS AND DISCUSSIONS

For the force reflecting telerobotics system considered in this numerical example, we employ a PI controller at the slave robot side, which makes the slave robot velocity follow the master robot velocity. The controller output is also fed back to the master robot side.

The Neal-Schmidt Model (Schmidt and Bacon, 1983) is deployed as the human operator's model, whose dynamics is given by

$$G_h = k_p \frac{T_z s + 1}{T_p s + 1} e^{-\tau_h s}, \tag{32}$$

[1] Note that this substitution for single delay systems was proposed in (Rekasius, 1980), its extensions to multiple delays as well as developments in the single delay case can be found in (Sipahi, 2005).

where $k_p \in \mathbb{R}^+$ is the human operator's gain, $T_z \in \mathbb{R}^+$ and $T_p \in \mathbb{R}^+$ are time constants, and $\tau_h \in \mathbb{R}^+$ is the human operator's reaction time delay.

The master and slave robot dynamics are given as

$$m_m \dot{v}_m(t) = F_m, \quad v_m(0) = 0, \tag{33}$$

$$m_s \dot{v}_s(t) = F_c, \quad v_s(0) = 0, \tag{34}$$

where $v_m(t) = \dot{x}_m(t) \in \mathbb{R}$ and $v_s(t) = \dot{x}_s(t) \in \mathbb{R}$. Therefore, master and slave robot transfer functions are given by

$$G_m = \frac{1}{m_m s}, \tag{35}$$

$$G_s = \frac{1}{m_s s}. \tag{36}$$

Next, the PI controller is formulated as

$$F_c(t) = B_c(\dot{x}_m(t-\tau_c) - \dot{x}_s(t)) + K_c \int_{t=t_0}^{t} (\dot{x}_m(\zeta - \tau_c) - \dot{x}_s(\zeta))d\zeta, \tag{37}$$

where $B_c \in \mathbb{R}^+$ and $K_c \in \mathbb{R}^+$ are the controller constants, and ζ is a dummy variable. Considering the velocity difference between the delayed master robot output and the slave robot output, $\delta_v = \dot{x}_m(t-\tau_c) - \dot{x}_s(t)$, as the input to the controller, and taking Laplace transform of (37), we write

$$\mathrm{F_c}(s) = B_c \Delta_v(s) + K_c \frac{\Delta_v(s)}{s}, \tag{38}$$

where $\mathrm{F_c(s)}$ and $\Delta_v(s)$ are the Laplace transforms of $F_c(t)$ and $\delta_v(t)$, respectively. Therefore, the controller transfer function is given by

$$G_c = \frac{\mathrm{F_c}(s)}{\Delta_v(s)} = \frac{B_c s + K_c}{s}. \tag{39}$$

Numerical values of the parameters in this case study are provided in Table 1.

Human model gain (k_p)	1
Human time constant (T_z)	10
Human time constant (T_p)	1
Controller proportional gain (B_c)	5
Controller integral gain (K_c)	10
Master robot mass (m_m)	1
Slave robot mass (m_s)	1

Table 1. Numerical data

For the closed loop system, using (25) the characteristic equation of the non-delayed system is given as

$$CE_{\tau_j=0} = \det(sI - \mathcal{A}_0 - \mathcal{A}_{\tau_h} - \mathcal{A}_{\tau_c} - \mathcal{A}_{\tau_c\tau_c}) = 0, \quad j = c, h. \tag{40}$$

Given the numerical values in Table 1, the poles of the non-delayed system are calculated as the roots of (40), which are -17.2731, -0.0920, and $-1.8175 \pm 1.7282j$. Since all the poles have negative real parts, the non-delayed system is stable.

Next, (22) is used to obtain (26), and then (28), which is then expanded by $(1+T_h)^n(1+T_c)^{n-k}$ and implemented in a Routh's array, parametric with respect to T_h and T_c. Fig. 2 depicts the exhaustive T_j values which render $s = j\omega_c$ roots from Routh's array analysis. Using $\omega_c \in \Omega$ and T_c, T_h in (29) then yields τ_c and τ_h. Fig. 3 depicts these imaginary axis crossings (ω_c) for delay values on the *kernel curves*.

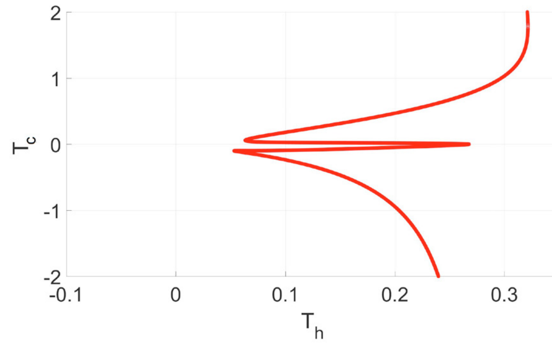

Fig. 2. All T_j combinations satisfying (28). These curves are called "core curves" (Sipahi and Olgac, 2005).

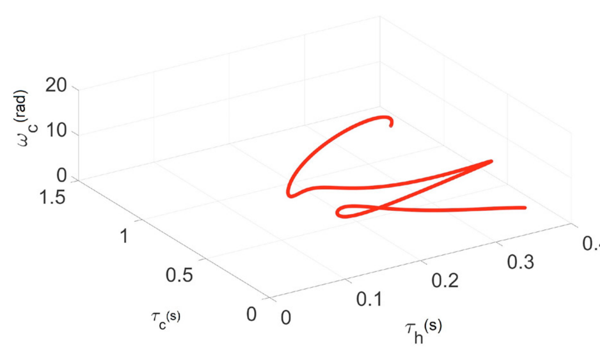

Fig. 3. Variation of imaginary axis crossing $\omega_c \in \Omega$ with respect to various values of τ_c and τ_h.

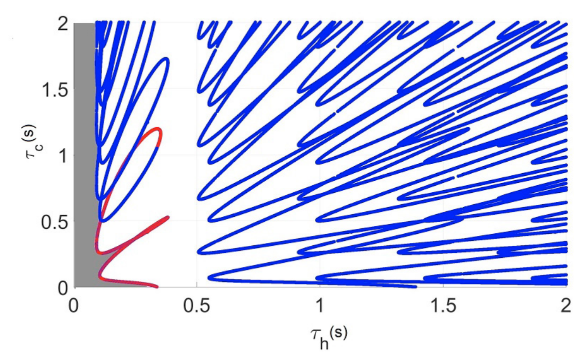

Fig. 4. Stability characterization of the human-in-the-loop telerobotics system in terms of communication τ_c and human reaction delays τ_h. Red line is the *kernel curve*, blue lines are the *offspring curves*. Shaded region shows the stable areas.

Finally, Fig. 4 provides the complete stability picture of the system for a range of time delays by assembling kernel curves (red) and offspring curves (blue) together, and identifying stable and unstable regions. Note that the gray area marks the stable region, which is attached to the origin of the delay plane, since the non-delayed system is also stable.

Now that stability is established with respect to τ_c and τ_h, we next perform simulations to validate the results. Fig. 5 shows the destabilizing effect of increased human reaction delay, for a given communication delay value, where the output of the master robot becomes unstable. It is noted

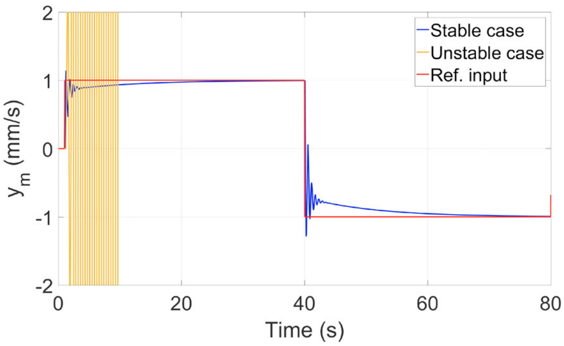

Fig. 5. Master system output for two cases: stable ($\tau_h = 0.05$s, $\tau = 0.1$s), and unstable ($\tau_h = 0.2$s, $\tau_c = 0.1$s).

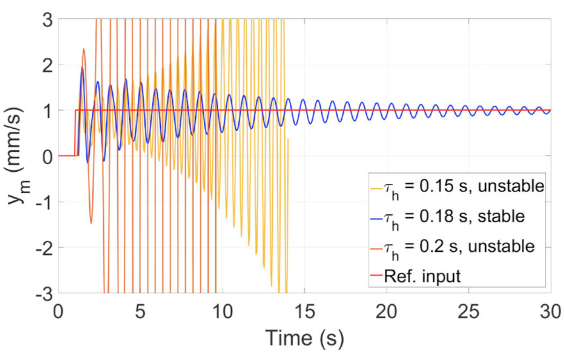

Fig. 6. Master system output for the case of $\tau_c = 0.25$s and three different τ_h values for which the unstable-stable-unstable transition is observed.

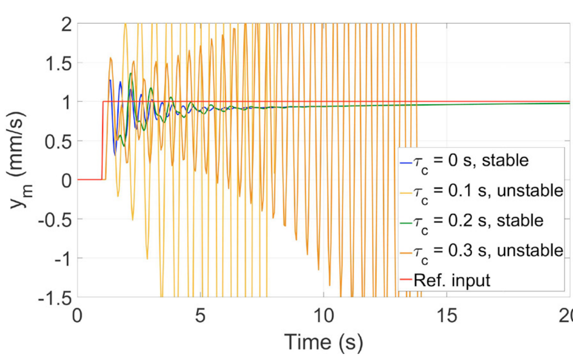

Fig. 7. Master system output for the case of $\tau_h = 0.15$s and four different τ_c values for which the stable-unstable-stable-unstable transition is observed.

that this is not always the case. In Fig. 6, it is shown that for the communication delay value of 0.25 seconds, increased values of the human reaction delay can cause a transition from unstable to stable and back to unstable behavior (see Fig. 4). However, τ_h is still comparably smaller than realistic human reaction time delays; see, for example, (Schmidt and Bacon, 1983). Moreover, a similar stability recovery phenomenon is observed for the case of a fixed human reaction time delay of 0.15 seconds, where increased values of the communication delay values cause stable to unstable transitions, consistent with Fig. 4, see Fig. 7 for time simulations.

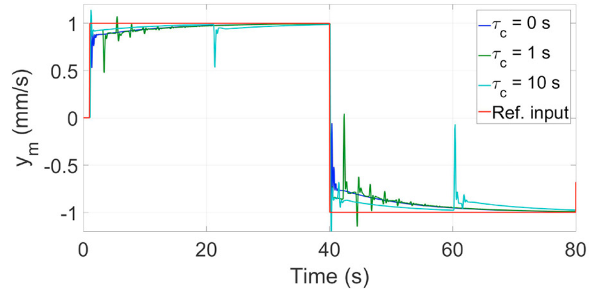

Fig. 8. Master system output for the case of $\tau_h = 0.05$s with various τ_c values.

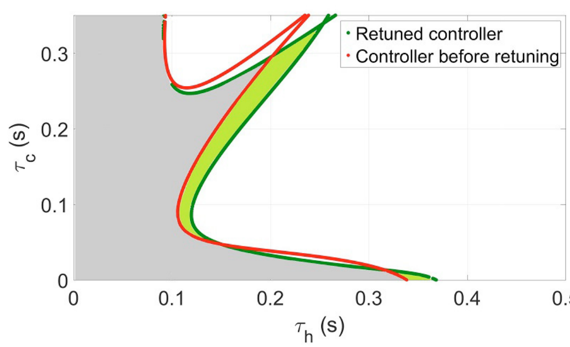

Fig. 9. Stability characterization of the human-in-the-loop telerobotics system in terms of communication delays τ_c and human reaction time delays τ_h, with retuned controller gains. Shaded grey region marks stable region before controller retuning. Shaded green region is the stable region achieved by controller retuning and depicts shift of stable region towards higher human reaction time delays τ_h for a range of communication delays τ_c.

It is noted that in Fig. 4, the area where human reaction delay is less than 0.1 seconds shows a stable region regardless of the communication delays. This observation points out the fact that human reaction time delay is indeed the main and strong limiting factor. Fig. 8 confirms this observation with the stable plots of the master robot output for various communication delay values when human reaction time delay is 0.05 seconds.

Next, we tune the controller gains with the intent to enlarge the stability regions. Fig. 9 shows an example where the closed-loop system can accommodate larger τ_h as gained by the marked green region. To obtain this plot, controller gains were tuned to $B_c = 4$ and $K_c = 18$.

Finally, in this paper we analyzed the stability of a class of telerobotic systems, namely bilateral force-reflecting telerobotic systems, where we explicitly consider human operator model as an element of the closed loop system, specifically focusing on the effects of human reaction time delay and communication delay. With the analytical framework of CTCR available, extensions of the approach to more complicated models with various architectures could be considered as the topic of future work.

5. CONCLUSION

A stability characterization of a human-in-the-loop telerobotic system was provided in this paper with respect to communication and human reaction time delays. The human operator model with time delay was used in the closed loop system dynamics, which was then analyzed with a mathematically rigorous stability analysis tool, namely, CTCR. It was shown that human reaction time delay can be the main limiting factor in achieving stability, and, interestingly, recovering stability with increased human reaction time delay could be possible. Moreover, with careful tuning of the controller, stable operating conditions of the closed-loop system could be enlarged on the plane of delays. Future research topics include exploring tools to effectively tune the controller gains to accommodate larger delays, and optimizing the closed-loop spectrum for improved transient performance.

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