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Markdown Budgets for Retail Buyers: Help or Hindrance?

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For many retailers, markdown decisions are taken by retail buyers whose compensation is based on sales revenue so their objective is to maximize it through the season. This implies that the buyers' objectives are not perfectly aligned with the overall profitability the firm. Many retailers set markdown budgets prior to the season to control margin erosion and increase profitability. Markdown budget constrains the buyers on the amount of discounts that they can apply on a given inventory of merchandise and sets a limit on the dollar value of markdowns for the season. While markdown budgets may be useful in preventing excessive discounts, they can have a detrimental effect on the buyers' ability to respond to poor market and remove distressed inventory. We investigate the effectiveness of this practice in aligning the incentives of buyers with that of the firm, and provide guidance on how these budgets should be established ahead of time. We consider a firm with a fixed inventory of a seasonable item, and a single chance to mark the price down. The retailer knows only the demand distribution at the beginning of the season, but the market information is revealed during the season to the buyer. We first characterize the buyer's markdown policy and understand the circumstances under which this can be different from the retailer's markdown policy. We use our model to determine the optimal markdown budget and quantify its effectiveness considering different factors such as the level of demand uncertainty, initial markup, and market's responsiveness to markdowns.

Key words: clearance sales; pricing policy; incentive alignment; fashion industry
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1. Introduction

1.1. Clearance Sales

For many retailers that sell seasonal goods such as apparel and accessories, long lead times prohibit replenishment of stocks during short seasons. For these companies, markdown is often the only lever to match supply and demand once they place their orders and set their initial price. According to some estimations, a typical retailer sells between 40% and 45% of its merchandise at a discount (The Wall Street Journal 2012). In U.S. department stores, this ratio may be as high as 76% for clothing (Daily News Record 2002). The majority of these discounts are permanent markdowns or clearance sales that occur later in the season to clear distressed inventory as most of the merchandise has little or no value at the end of the season. These markdowns are used to correct an initial mistake of ordering (overbuying) or poor pricing (high initial mark-ups). In other words, permanent markdowns are applied in response to an incorrect initial belief about demand and to clear space for new products, as discussed in one of the early descriptive studies by Lazear (1986).[1]

The major initiatives in industry to better match demand and supply can be categorized to accurate response (Fisher and Raman 1996) and quick response (Iyer and Bergen 1997). These initiatives are of critical importance in fast-fashion, which is characterized by affordable prices and frequent assortment changes (Caro and Gallien 2010). However, this change of focus from supply efficiency to response agility and accuracy has not been able to solve the retailers' problems completely. Zara, a leading fast fashion retailer, commits only 15–25% of production before the season begins, but still sells 15–20% of its items at less than full price (Fraiman et al. 2012).

While markdowns may not seem to be a very glamorous part of business, especially in comparison to other retail activities such as buying and advertising, they are often the place where the profits are made or lost. A carefully planned and executed markdown may very well salvage an otherwise unsuccessful season. Many retailers blame excessive markdowns for their recent financial troubles (The Wall Street Journal 2011). Seeing the impact on the bottom line, many retailers seek help to optimize their markdowns. The importance of markdown management is also evident from the abundance of software companies active in this area and the number of patents registered by this keyword. Many software vendors including IBM, Oracle, Predictix, Revionics, SAP, and SAS offer markdown optimization solutions to manage the timing and depth of markdowns, based on sales trends and inventory levels (Stores 2012).

1.2. Incentives: Retailer Vs. Buyer

Markdown decisions are usually delegated to retail fashion buyers (or merchandisers). Buyers have expertise in fashion trends, may have better demand information than the retailer, and are in fact responsible for most of the merchandising decisions such as what items to purchase and how much to order along with the pricing and markdown decisions for a specific product category. The incentives of these buyers may not be perfectly aligned with the profitability of the firm. While there may be different arguments in favor of delegation of markdown decisions to the buyers, our focus is on the profit losses caused by incentive misalignment and policies to limit those losses.

A buyer's performance evaluation and compensation is, at least partly, based on total sales of the category for which she is responsible (Clodfelter 2012). The retailer's overall profitability, however, requires a proper mix of total sales revenue and profit margin. This tension between achieving sales goals and maintaining profitable margins is a recurring theme in fashion retailing, see, for example, Women Wear Daily (1996). A factor that contributes to the incentive misalignment problem is the so-called "Retail Method of Accounting" which is practiced by an important number of retailers in many industries including fashion. In this method, the inventory and transactions are managed in retail dollars at aggregate levels assuming a constant mark-up percentage across different products. The cost of inventory sold is then calculated by dividing the revenue obtained to the mark-up percentage while in fact the accounting should be based on the actual cost paid for the merchandise. With this cost accounting method, the buyer's objective boils down to maximizing sales revenue (since net profit appears to be a fixed fraction of the sales revenue) and the buyer's focus shifts to getting rid of old merchandise with markdowns (that are earlier and deeper than what is optimal) and make way for new merchandise in the category she owns.

The above compensation system implies that the buyers objective is to maximize revenue, while the retailers objective is to maximize profit. Notice that this misalignment problem would not be relevant for the sales of the existing stock of the current product alone as the paid cost is sunk and both the retail and the buyer focus on increasing the revenue.

One tool that is heavily used in practice to control margin erosion is *markdown budget* which constrains the total dollar value of all markdowns a buyer can use. For instance, if an item with an initial price of $50 is marked down to a new price of $35, over a remaining stock of 1000 items, this decision would consume $15,000 of the markdown budget. The retailer establishes a markdown budget prior to the season and the total amount of discount applied on the inventory at time of the markdown cannot exceed this budget. Retailers put in place markdown budgets to limit the total value of merchandise discounts during the markdown period, in order to prevent excessive devaluation of the inventory.

While markdown budgets are common in industry, their role and whether they are useful is controversial. In an extensive review of manufacturing and retail operations in the apparel industry, Sen (2008) states that "markdowns are usually subject to the buyer's budget, limiting the responsiveness of these decisions to sales activity". IBM, the provider of the DemandTec Markdown Optimization solution, considers markdown budgets as one of the primary reasons for retailers not to implement a science-based markdown solution since staying within the budget leads to an illusion that the markdown decisions were effective (IBM 2015). Others including SAS Institute, SAP and Oracle seem to take the markdown budgets as input in their software solutions and generate markdown plans to stay within these budgets (Hassanzadeh et al. 2014, Sanli and Yao 2011, Veit et al. 2011). A recent review of markdown optimization solutions in industry notes that many retailers are now abandoning their markdown solutions and argue that competing performance motivations should be reconciled for the success of new software implementations (Nemett 2013).

Our review of industrial and research literature finds no evidence on whether markdown budgets are more help or hindrance in an environment where the buyers' incentives are not perfectly aligned with the retailer's overall profitability. We also found no guidance on the process through which these budgets should be determined prior to the season and the impact of various factors such as the uncertainty of demand (the fashion content of the products in question) or the market response to markdowns on these decisions. We attempt to fill this gap in the literature and investigate how one should manage conflicting supply chain interests, uncertain demand, and inflexible supply via markdown budgets.

1.3. Our Setting

We study markdown decisions of a retail firm which sells a fixed stock of a seasonable/perishable item over a finite selling season. At the beginning of the sales season, the retailer determines the initial price (or the initial mark-up). Initial prices are determined manually because of the judgment required to evaluate brands, quality, and design attractiveness (Talluri and van Ryzin 2004, p. 536). Prior to the season, the retailer sets the markdown budget.

We assume that the information regarding market size is fully revealed at the beginning of the sales season. While this assumption is mainly for simplicity and tractability, it may be justified partially based on what is observed in practice. For example, a consultant in fashion industry argues "a week after an item hits the floor, a merchant knows whether it's going to be a dog or a best-seller" (Chain Store Age 1999). Fisher (2009) also reports that forecasts based on early sales are significantly better (and are highly accurate) in comparison to forecasts made prior to season. A similar assumption is also made in Lago et al. (2016) in a different setting.

We assume that once the state of the market is learned, the demand throughout the season is deterministic and the retailer has a single chance of price change. These assumptions have been used in other papers as well, e.g., see Whang (2009) where a retailer chooses the timing of markdown as a function of the current time and his inventory level. As Smith (2015) argues "because the clearance period is relatively short and sales rates are declining, the early clearance markdowns tend to be the dominant decisions economically, thus reducing the importance of multi-stage optimization." The Wall Street Journal (2012) reports that "combination of two markdowns will never be as profitable as a single markdown. Arriving early enough to tempt customers, the first markdown gives the greatest boost to profits, and extra price cuts simply add profit-eroding labor costs."

There are other reasons as to why the first markdowns provide the lion's share of the profit compared to the next rounds of markdowns. First markdowns are usually placed throughout the store (mixed with regularly priced products), whereas further markdowns are placed in a certain reserved spot in the store and do not have the same exposure to foot traffic. Further markdowns also take smaller discounts compared to the first markdown. Finally, first markdown prices are still high enough for a good margin, but subsequent markdowns leave insufficient margin to cover labor costs.

Once the market reaction to the merchandise is known, the buyer has the opportunity to mark the price down once to a predetermined price level during the season. Thus, the decision made by the buyer is the timing of the markdown. In determining the timing of the markdown, the buyer's problem is to maximize the sales revenue subject to the markdown budget. If the inventory runs out (is cleared) before the season is over, then the shelf space will be used to sell other merchandise for the remainder of the season. Since the buyer's objective is to maximize sales revenue, the buyer does not consider the cost of goods sold for the new sales, while clearly the retailer's profits are net of the procurement costs. In other words, the source of incentive misalignment is the fact that the buyers compensation depends only on the sales, but not on the profit margin. Later, we discuss different possibilities for the use of the shelf space after the current product is depleted.

The buyer's markdown timing decision in this setting can be suboptimal with respect to the objective of the retailer in one of two ways. First, if the market for the merchandise is relatively weak, the buyer may mark the price down later than what is optimal for the retailer as she is constrained by a budget (since early in the season the markdown will be applied to more inventory). Second, if the market for the merchandise is relatively strong, the buyer may be inclined to mark the price down earlier than what is optimal for the retailer, clear the inventory and use the shelf space to generate more sales with a new merchandise as she is not considering the procurement costs. Our model allows us to investigate these two risks simultaneously. Using this model, we characterize the retailer's optimal markdown policy and the buyer's markdown decisions as a function of the market size. We provide a formal explanation for the possible inefficiencies of the buyer's decision, and explain why it can be different from the retailer's optimal markdown policy. Our model can also be used to determine the optimal markdown budget by the retailer and quantify the effectiveness of using markdown budgets for incentive alignment. Through a numerical study, we investigate various factors that moderate the size of optimal markdown budgets and resulting effectiveness. We find that there is no "one size fits all" for retailers when determining their markdown budgets, and one needs to consider the level of uncertainty, market size projections, profit margin and consumer's responsiveness to markdowns. Resulting effectiveness is also a function of these factors.

The rest of the study is organized as follows. In section 2, we review the research literature. In section 3, we introduce our model and characterize the optimal markdown policy for the retailer as well as the markdown policy that will be used by the buyer given her objective and the markdown budget. In sections 4 and 5, we report the results of our numerical study and explain the effect of different factors on the markdown budget set by the retailer. We provide a discussion of our results and conclude in section 6 along with future research directions.

2. Literature Review

2.1. Clearance Sales

An important strand of literature is focused on helping retailers through prescribing optimal clearance pricing and markdown policies of seasonal products with the inclusion of operational considerations. Smith and Achabal (1998) develop end-of-season inventory management policies that take into account the impact of time, inventory, and price on sales rates. Bitran et al. (1998) study clearance markdowns with a focus on coordinating between different stores of a retail chain, and find that their suggested method performs better than the current practice of the retail company. Mantrala and Rao (2001) provide a detailed discussion of a decision support system for markdown decisions which are applied to take the advantage of price differentiation and customers' nonstationary price sensitivity. In an empirical study, Heching et al. (2002) analyze an apparel retailer's markdown pricing by fitting a few common demand models to the sales data. Their analysis suggest that the retailer could improve revenue by making smaller markdowns earlier. Heching and Leung (2005) consider a retailer that determines possible markdowns at the beginning of each period subject to a markdown budget. They formulate an optimization problem to determine the initial price, markdown price, and markdown time; and report the difficulty in solving their optimization problem due to existing nonlinearities. Vakhutinsky et al. (2012) develop a markdown optimization model in which demand is a function of price, seasonality, and inventory; and then obtain a monotonically decreasing sequence of merchandise prices that maximizes the revenue. In Yao et al. (2015), the authors study the effect of multiple products/locations that share a common budget. Ramakrishnan (2012) reviews modern markdown management methods that are used in retail industry.

Markdown optimization can be considered as a special subclass of dynamic pricing with inventory considerations where price can only be adjusted downward. We refer to Elmaghraby and Keskinocak (2003) for an introduction and a review of earlier research in this area. New lines of research in dynamic pricing include considering strategic consumers (e.g., Aviv and Pazgal 2008), costly price adjustments (e.g., Celik et al. 2009) and learning (e.g., Besbes and Zeevi 2009).

Among the recent research that is more closely related to our work, Besbes and Maglaras (2012) study a problem where a set of financial milestone constraints on the revenues and sales need to be satisfied at different time points during the selling season. Osadchiy and Vulcano (2010) suggest a new markdown mechanism where price sensitive consumers can place binding reservations for the leftover inventory that the retailer will clear at the end of the selling season. Focusing on pre-season markdowns to speed up demand learning, Talebian et al. (2014) study a setting where the retailer determines assortment, but is uncertain about the market size of different products.

2.2. Incentives: Retailer Vs. Buyer

Despite the existing literature on markdown optimization, we are not aware of any scholarly work in regard to the incentive alignment between the retailers and buyers through markdown budgets. In other words, nearly all literature assumes that buyers' objective is the same as the retailer's objective of maximizing profit. The only exception is Goodwin (1992) who studies the effect of markdown budgets on sales and profit. The author reports that 77% of companies in the study pay bonuses to buying staff for the attainment of the sales budget that increases their earning between 5.5% and 14.5%. Interestingly, buyers who attained the budget were not necessarily more successful in bringing in sales and profit, compared to the ones who went over budget. He suggests that markdowns should be based on sales activity rather than budgeted prior to season.

Our model can be thought of as a two-stage Stackelberg game where retail is the leader and sets the markdown budget whereas buyer is the follower and decides the markdown timing. There are various applications of Stackelberg models in retailing and more generally supply chain management literature. However, most of these studies focus on inter-firm relations, e.g., price competition, category captainship, and joint contracts, while we focus on intra-firm interaction between a retail manager and a merchandise buyer. As an example of few models which study intra-firm interactions, Pekgun et al. (2008) study a firm for which pricing and lead time decisions are made by marketing and production departments respectively, and analyze the inefficiencies created by the decentralization of price and lead time decisions. They show that coordination can be achieved, using a transfer price contract with bonus payments.

In a broader scope, our work is related to the principal-agent models in economics literature, where the retailer (principal) cannot observe the true market size, and the buyer (agent) makes decisions on his behalf. In many mechanisms, the principal modifies the agent's objective function to align it with his goals. Our setting is unique as the retailer attempts to align the buyer's decision through constraints.

Within the literature on principal-agent models, our problem is particularly related to the literature on price delegation where research has focused on whether the sales force should be given the control over price (fully or partially) and if so what kind of compensation scheme should be used. Weinberg (1975) shows that if the sales personnel are commissioned based on gross margin (as opposed to sales), they will set prices so as to simultaneously maximize their own income and the firm's profits. Lal (1986) shows that price delegation is more profitable if the salesperson's information regarding the selling environment is superior to that of the firm's. Despite these and other theoretical results that are in favor of price delegation, in an empirical study, Stephenson et al. (1979) show that firms that give higher degree of pricing authority generate lower profits. Among the potential reasons for this discrepancy between the theory and practice, the authors question the control value compensation systems based on gross margins and argue that many sales representatives in the industry may use sales volume as a surrogate measure of their performance. The authors argue that, if these sales representatives are given price authority, they will use heavy discounting. Our study differs from the previous research on price delegation, as we consider a retail environment defined by perishable products, fixed inventory and valuable shelf space and investigate the effect of controlling pricing decisions through budgets. In this respect, the use of markdown budgets by fashion retailers to control the decisions of the buyers can be justified.

3. Model and Analysis

We consider a retail firm that sells an initial stock n of a fashion item over a sales period whose length is normalized to 1. The firm starts the season with an initial price normalized to 1, but has the option to reduce it to a predetermined markdown price $1 - \delta$ $(0 \leq \delta \leq 1)$ during the season [0, 1]. In retail terminology, δ represents *off-retail percentage* or *markdown depth* (Clodfelter 2012). We normalize the salvage value to 0.[2]

We represent the market size with full price with Θ, which is not known in advance and is a random variable. Its particular realization is denoted by θ. We denote λ to be the sales jump after the markdown so that the demand at the markdown price has a rate of $\theta(1 + \lambda)$. With regard to the markdown depth and sales jump after the markdown, we assume $(1 - \delta)(1 + \lambda) \geq 1$. This assumption ensures that the revenue rate increases with the markdown, for otherwise there is no incentive to reduce the price.

As discussed, the misalignment problem appears in the markdown process when it comes to valuing the empty shelf space. The buyer's value of the shelf space depends only on the generated revenue, while the retailer's value considers the profit margin. If the current item is depleted before the end of the season, the shelf space can be used to generate revenue for the remainder of the season. This can be done in one of the following ways. First, the retailer may extend the shelf space of an existing basic (e.g., basic shirts or jeans) item in its assortment. For most basic items, replenishment within the season is already planned and the retailer can increase the amount of replenishment within the season to respond to increased demand due to extended shelf space. Second, the retailer may extend the shelf space of another fashion item. For this, the retailer may have to work with a near-shore supplier with a shorter lead time. The use of near-shore suppliers for within season replenishment along with off-shore suppliers used for preseason orders (dual-sourcing) is an increasing trend even among the more traditional fashion companies (Just-style 2014, Just-style 2015). Finally, the retailer may introduce a new fashion item and allocate the shelf space to this new item. This option is viable for most fast-fashion retailers which are more vertically integrated and can reduce the lead times down to 2–6 weeks including the design (Caro and Martinez-de-Albeniz 2015). In fact, assortment changes within the season are planned for many categories and the timing of such changes are of crucial importance (Caro and Gallien 2007).

We use the revenue obtained from the sales of these new items to obtain the value of the shelf space per unit of time our analysis. This is similar to the revenue term used in Araman and Caldentey (2009) for a an inventory pricing problem with demand learning to capture the opportunity cost of the retailer's operation, except that the authors there assumed a one time terminal value, not a revenue per unit of time.

The retailer aims to maximize his profit. Since the procurement cost of the current item is already paid and sunk, the retailer aims to maximize the revenue of selling the current item. We can denote ζ to be the revenue that can be generated from the shelf space per unit of time. Then the value of the empty shelves per unit of time that is assigned by the retailer (i.e., "true value" of the shelf space) is $\gamma_r = m\zeta$, where $1 - m$ is the unit cost. The buyer's goal is to maximize her compensation which is the sum of her commission on the current seasonal item and her commission on the items that she uses to fill the empty shelf space once the initial item is cleared. Let c represent the ratio of the commission rate that the buyer collects for the replacement product (within season replenishment) to the commission that she collects for original product (preseason orders). Then the value of the empty shelf space per unit of time that is assigned by the buyer is $\gamma_b = c\zeta$. In cases where another item in the same category is used to fill the empty shelf space, it may be appropriate to set ζ to $\mathbb{E}[\Theta]$ where $\mathbb{E}[\Theta]$ is the expected demand per unit of time at full price for another item in the same category. This is the approach we take in our numerical experiments.

As explained before, the buyer, in contrast to the retailer, does not take into account the procurement cost in her decisions since she is not driven by profit, but by sales revenue. On the other hand, the commission percentage of basic products sold through empty shelf space can be lower than commission on original fashion products. This implies that shelf space has different values for the retailer and the buyer, and both cases of $\gamma_r \le \gamma_b$ and $\gamma_r > \gamma_b$ are possible. This difference explains the incentive issues between the retailer and the buyer that makes markdown decision on his behalf. The difference between the retailer's and buyer's decisions is demonstrated by different values they associate for the shelf space that becomes available for use once the current inventory is depleted. We do not need to have a special form of relationship between these two parameters for our analysis. We note here that when $\gamma_r = \gamma_b$, the incentives of the retailer and the buyer are aligned for the markdown decision we are studying here. However, the fact that the buyer's compensation depends only on the sales, but not on the profit margin may lead to other incentive alignment problems. For example, the buyer may be inclined to form an initial assortment that consists of higher price, but less margin products. Likewise, the buyer may want to pick a higher price, but less margin replacement item to fill the shelves once the current product is cleared. These product selection decisions are not in the scope of this study.

We can express the objective function, that is, expected revenue of the retailer ($\gamma = \gamma_r$) or the buyer ($\gamma = \gamma_b$) as follows:

$$\begin{aligned}\pi(\theta,t) = {} & \min\{t\theta, n\} + (1-\delta)\theta(1+\lambda) \\ & \min\left\{1-t, \frac{(n-t\theta)^+}{\theta(1+\lambda)}\right\} \\ & + \gamma\left(1 - t - \frac{(n-t\theta)^+}{\theta(1+\lambda)}\right)^+,\end{aligned} \tag{1}$$

where $(x)^+ = \max\{x, 0\}$. The first term in Equation (1) represents the expected revenue from sales at full price. The second term represents the expected revenue from sales at markdown price. The last term represents the expected value of the shelf space that will be utilized once the current inventory is depleted.

We denote

$$\pi^*(\theta) = \max_{t\in[01]}[\pi(\theta,t)], \tag{2}$$

to be the expected optimal revenue. The following theorem expresses the optimal markdown time in closed form.

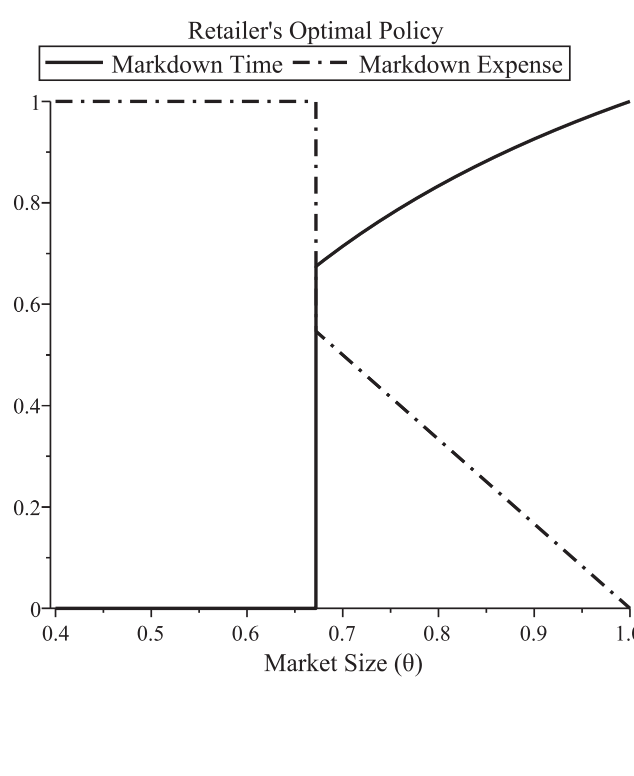
THEOREM 1. *The optimal markdown time is given by the following:*

$$t^*(\theta) = \begin{cases} \frac{n}{\theta}, & \text{if } n \le \theta, \\ \frac{\theta(1+\lambda)-n}{\theta\lambda}, & \text{if } \max\left\{\frac{n}{1+\lambda}, \frac{\lambda\gamma}{(1+\lambda)\delta}\right\} \le \theta \le n, \\ 0, & \text{if } \theta \le \max\left\{\frac{n}{1+\lambda}, \frac{\lambda\gamma}{(1+\lambda)\delta}\right\}. \end{cases} \tag{3}$$

Theorem 1 shows that the price should be marked down at the beginning of the season if the market size is less than the threshold $\max\{\frac{n}{1+\lambda}, \frac{\lambda\gamma}{(1+\lambda)\delta}\}$. In this case, the inventory will be depleted before the season is over and the empty shelves will be utilized for the remainder of the season. If the market size is larger than the threshold, a markdown time is chosen such that the inventory is depleted precisely at the end of the season. If $\theta \ge n$, it is optimal not to apply any markdown and have the inventory deplete at full price at time $\frac{n}{\theta}$ (which is taken as markdown time as convention).

An example is provided in Figure 1, where we plot the optimal markdown time and the amount spent on markdowns (markdown depth multiplied by the remaining inventory at the time of the markdown) as a fraction of maximum possible spending on markdowns (δn). Notice that δn corresponds to marking the price down for all inventory at the beginning of the season. When the market size θ is less than 0.672, it is optimal for the retailer to mark the price down at the beginning of the season. Otherwise, the

Figure 1 **Retailer's Optimal Markdown Policy** $n = 1$, $\delta = 0.5$, $\lambda = 1.5$, $\gamma = 0.56$



markdown time is such that the inventory is depleted precisely at the end of the season.

Since the buyer and the retailer assign different values for the shelf space, their objectives, given in Equation (1) are also different. The difference between objectives of the retailer and the buyer implies an incentive misalignment. Since the timing decision is actually made by the buyer, the actual markdown time is different from what is optimal for the retailer. Theorem 1 suggests that if $\gamma_r \le \gamma_b$, the threshold for the market size, before which it is optimal to mark the prices down right at the beginning of the season is larger for the buyer. That is, the buyer may be inclined to mark the prices down when it is still more profitable for the retailer to sell them at full price. On the other hand, if $\gamma_r > \gamma_b$, the threshold for the market size is smaller for the buyer, and the buyer may be inclined to mark the prices down later than optimal.

A common method in the retail industry to mitigate this conflict is using a markdown budget. Markdown budget is considered as a constraint on how much can be spent on markdowns. Based on whether buyer sets a higher value to empty shelves or a lower value, the inequality in the constraint has a different direction. Consider the case where $\gamma_r \le \gamma_b$, and let $\alpha \in [0, 1]$ denote the markdown budget represented as a fraction of maximum possible spending on markdowns. If the budget is set at α, then the retailer can mark the price down when there are at most αn units in inventory. Therefore any markdown should be delayed until at least $(1-\alpha)n$ units are sold. That is, the markdown time cannot be earlier than $\frac{(1-\alpha)n}{\theta}$, the time when the $[(1-\alpha)n]$th item is demanded under a demand function with rate θ. A similar argument can be made for the case $\gamma_r > \gamma_b$, where the buyer is less inclined to mark the price down than the retailer and the markdown budget enforces a minimum amount of markdown to be taken. Therefore, the buyer's optimal markdown time under a budget constraint can be written as:[3]

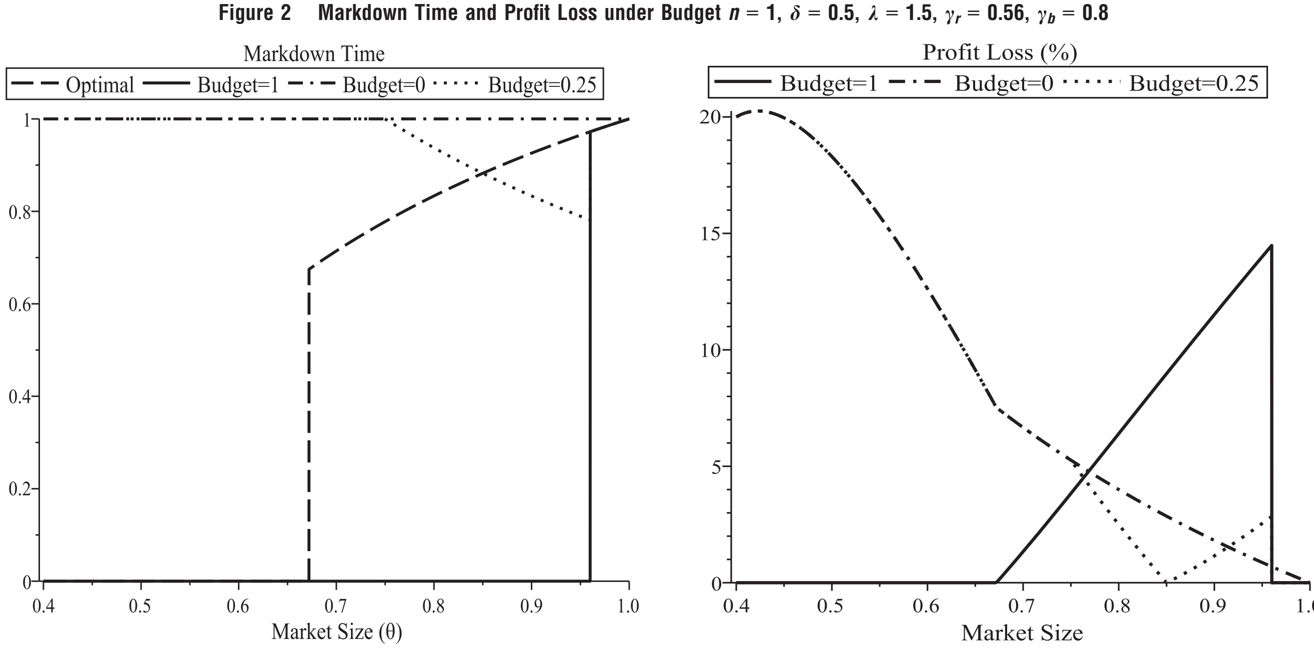
$$t_b^*(\theta|\alpha) = \begin{cases} \max\{t_b^*(\theta), \frac{n(1-\alpha)}{\theta}\}, & \text{if } \gamma_r \le \gamma_b, \\ \min\{t_b^*(\theta), \frac{n(1-\alpha)}{\theta}\}, & \text{if } \gamma_r > \gamma_b. \end{cases} \quad (4)$$

Note that the buyer's decision under a markdown budget constraint may be suboptimal. We measure the inefficiency of delegating the markdown timing decision to the buyer and controlling her through a markdown budget by comparing her resulting expected profit to what can be obtained if the retailer determines the markdown time on his own after the market size information is revealed. Particularly, we use $1 - \pi_r(\theta, t_b^*(\theta|\alpha))/\pi_r^*(\theta)$, which quantifies the profit loss as a fraction of the optimal revenue that would be obtained if the retailer determines the markdown time by himself.

Figure 2 illustrates how markdown time and profit loss change as a function of market size when $\gamma_r \le \gamma_b$. Notice that the case of "Budget = 1" correspond to the case where the budget constraint is completely relaxed, and the buyer can set markdown time as she wishes, and potentially different from what is optimal for the retailer.

It is interesting to note that markdown budget hurts the retailer when the markdown is needed the most, that is, when the market size is small. Notice that the

Figure 2 **Markdown Time and Profit Loss under Budget $n = 1$, $\delta = 0.5$, $\lambda = 1.5$, $\gamma_r = 0.56$, $\gamma_b = 0.8$**



buyer may not be able to give any markdown if market size is small because the budget constraint can never be satisfied. Also, notice that the buyer requires a better market than the retailer for not marking the price down right away. With the budget constraint, the buyer cannot mark down the product right away.

Figure 3 illustrates the change in markdown time and profit loss as θ changes for an example where $\gamma_r > \gamma_b$. Notice that in this case, the markdown budget is the minimum allowed discount.

When the budget is set to 1, the buyer has no option, but to mark the price down at the beginning at time 0. At the other extreme, when the budget is set to 0, there is no limit on how much discount the buyer can offer and the buyer can mark the price down whenever it is optimal for her to do so. This may lead her to mark the price down later than what is optimal for the retailer. This happens when the market is low. When the budget is set to an intermediate level (0.25), the buyer may be forced to marking the price down at a relatively good market despite the fact that this is not optimal for neither the buyer nor the retailer. Setting the budget to 1 may lead to substantial losses for the retailer for this example. Setting the budget to 0 dominates setting the budget to an intermediate level.

We assume a leader–follower framework for the markdown budget decision. The retailer is the leader and sets the budget first, and the buyer is the follower and determines the markdown time subject to the budget set by the retailer. When the retailer sets the budget, the season has not started and the information regarding the market size is not revealed yet. We represent the the retailer's belief about market size by random variable Θ. Therefore the retailer determines the markdown budget considering the uncertainty of the market size. We assume that the retailer is risk-neutral and determines the budget level α that maximizes its expected profit. More formally, the retailer's optimal markdown budget is given by

$$\alpha^* = \underset{0 \le \alpha \le 1}{\arg\max}\, \mathbb{E}_\Theta[\pi_r(\Theta, t_b^*(\Theta|\alpha))]. \quad (5)$$

We can measure the ex ante efficiency of an incentive alignment policy that uses the optimal markdown budget by profit loss percentage defined as:

$$\text{Profit Loss } (\%) = 1 - \frac{\mathbb{E}_\Theta[\pi_r(\Theta, t_b^*(\Theta|\alpha^*))]}{\mathbb{E}_\Theta[\pi_r^*(\Theta)]}, \quad (6)$$

which is the expected value of the profit loss as a fraction of the retailer's optimal expected revenue.

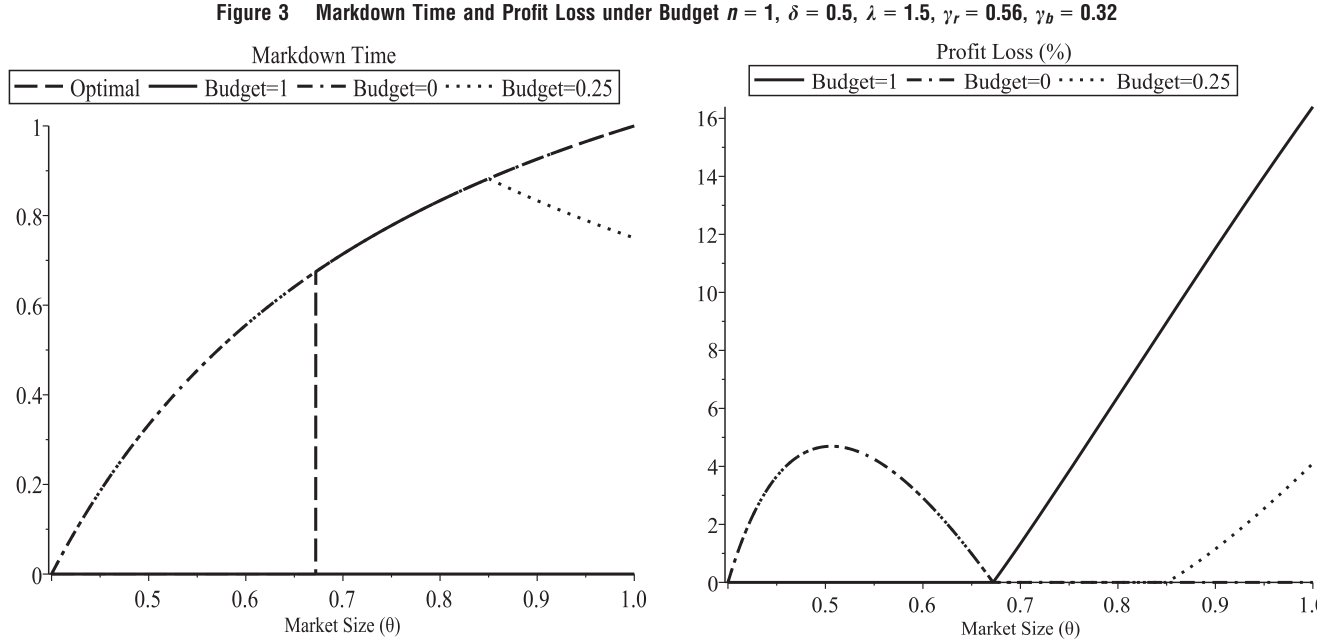
3.1. Extension: Stochastic Demand

Our results can be extended to a more realistic case where demand is stochastic. More precisely, we assume that the market size with full price follows a Poisson process with rate θ, so the expected revenue for a given markdown time $0 \le t \le 1$ is equal to:

$$\begin{aligned}\tilde{\pi}(\theta, \gamma, t) = \mathbb{E}\big[&\min\{N_\theta(t), n\} \\ &+ (1-\delta)\min\{N_{\theta(1+\lambda)}(1-t), \\ &(n - N_\theta(t))^+\} + \gamma(1 - A_n)^+\big]\end{aligned} \quad (7)$$

where $N_x(y)$ represents the number of arrivals over a period of length y in a homogeneous Poisson process with rate x, and A_n is the nth arrival time of a non-homogeneous Poisson process with a rate of θ in [0, t] and $\theta(1 + \lambda)$ in [t, 1]. Notice that event A_n

Figure 3 Markdown Time and Profit Loss under Budget $n = 1$, $\delta = 0.5$, $\lambda = 1.5$, $\gamma_r = 0.56$, $\gamma_b = 0.32$



may never happen before the end of the season, and therefore we look at $(1 - A_n)^+ = \max\{0, (1 - A_n)\}$.

The optimization problem corresponds to

$$\tilde{\pi}^*(\theta, \gamma) = \max_{\tau \in \mathcal{T}} \mathbb{E}[\tilde{\pi}(\theta, \gamma, \tau)], \qquad (8)$$

where $\mathcal{T}$ represents the set of stopping times τ satisfying two conditions: (1) $0 \leq \tau \leq 1$ and (2) $N_\theta(\tau) \leq n$ almost surely. This stochastic version of our problem is similar to Feng and Gallego (1995).

In order to capture the stochastic nature of demand, it is required to formulate models based on stochastic dynamic programming and solve them, usually through development of heuristics. Rather than attempting to solve the stochastic stopping problem and obtain $\tilde{\pi}^*(\theta, \gamma)$, we will propose a heuristic based on the deterministic version of the problem. The deterministic solution suggests a stopping-time heuristic for the stochastic problem. Denote $k(\theta, \gamma) = \theta t^*(\theta, \gamma)$. Let $t_{k(\theta, \gamma)} = k(\theta, \gamma)/\theta$ be the time it takes to sell $k(\theta, \gamma)$ units if the demand is deterministic with rate θ and let $A_{k(\theta, \gamma)}$ be the (random) time it takes to sell $k(\theta, \gamma)$ units if the demand follows a Poisson process with rate θ. The heuristic switches the price at $\min\{A_{k(\theta, \gamma)}, t_{k(\theta, \gamma)}\}$. The next theorem shows that this stopping-time heuristic is asymptotically optimal and suggests that retailers that deal with a large number of units may use the deterministic solution to manage their markdowns.

THEOREM 2. *The heuristic policy that switches the price at* $\min\{A_{k(\theta, \gamma)}, t_{k(\theta, \gamma)}\}$ *is asymptotically optimal (as n goes to infinity) in the stochastic problem. The expected revenue of these heuristic approaches* $\pi^*(\theta, \gamma)$.

Theorem 2 demonstrates that the stopping time heuristic is asymptotically optimal, which shows that marking down in an attempt to compensate for typical stochastic variations in demand captures only second order increases in revenue. These results are consistent with some evidences in retailing that the first round of markdowns has a substantially larger effect on revenues compared to other rounds of markdowns which have a secondary order impact (Talebian and van Ryzin 2014). There exist anecdotal evidences that some large department stores reduced the number of markdown rounds as it was shown that it was better to discard items earlier and create space for new items.

Theorem 2 means that for the stochastic problem given in Equation (8), the buyer's optimization is over $\mathcal{T}$ which now represents the set of stopping times τ such that $0 \leq \tau \leq 1$ and $(1 - \alpha)n \leq N_\theta(\tau) \leq n$ almost surely. In this case, we can also update the buyer's stopping-time heuristic based on the deterministic solution such that the buyer marks the price down at

$$\max\{A_{(1-\alpha)n}, \min\{A_{k(\theta,\gamma_b)}, t_{k(\theta,\gamma_b)}\}\}. \qquad (9)$$

Given that a typical retailer deals with a large volume of merchandise and based on the asymptotic optimality result provided in Theorem 2, we focus our attention to the deterministic problem from this point on.

4. Optimal Markdown Budgets

In the next two sections, we provide numerical examples to show that setting the markdown budgets may be a non-trivial task and investigate how different factors moderate the amount of markdown budget one should set prior to the season and resulting efficiency of the optimal markdown policy. In doing so, we did our best to calibrate our parameters with the industry. For example, according to US Census Bureau, gross margin was on the average 45% for clothing and clothing accessories stores in 2014 (US Census Bureau 2014). We use $m = 0.5$ in most of our examples, and also study the impact of m separately. Soysal and Krishnamurthi (2012) study a leading specialty apparel retailer, and report an average first markdown of 38% of the retail price. In their study of fast fashion, Cachon and Swinney (2011) choose markdown depths of 5%, 15%, and 25%. Caro and Gallien (2012) report that the average item at a Zara store collects 85% of its full price, while the usual range in the industry is 60–70%. We choose markdown depths of 15%, 20%, 30%, 40% and 45% in our numerical study.

We confirmed our choice of these two parameters also with the operations director of a major European apparel retail company. In addition, we discussed our assumptions and verified that most of our assumptions are valid for this company. Majority of the products are ordered only once prior to the season, and despite the retailer's efforts to order the right products prior to season, the company is also often faced with the necessity to liquidate its slow-moving inventory through markdowns. For this company, the percentage of items sold at a markdown are in the same range with leading fast-fashion companies including Zara and H&M. We learned that usually within a week after a product is put on shelves, the retailer has a very good understanding of the demand in the rest of the season. We also confirmed that most benefits are obtained in the first markdown, the retailer measures and monitors the value of the shelf space and there are incentive alignment problems between the buyer team and the retailer. Based on these observations, we believe that the assumptions of our model and the parameters of our numerical investigation are well calibrated with the company's operation.

In our analysis, we normalize the initial inventory n to 1. Remember that the demand rate at the full price is represented by θ, and the retailer's belief about market size is represented by random variable Θ. In order to reflect the difference between the value of shelf space that is assigned by the retailer and the buyer, we assume that the shelf space is used for another product in the same category. This new product's market size is identically distributed as and independent from the current product's market size. Therefore we set $\gamma_r = \mathbb{E}[\Theta]m$ and $\gamma_b = \mathbb{E}[\Theta]c$. Most of our analysis is for the case $\gamma_r < \gamma_b$ which is a more probable scenario.

We use the modified PERT distribution (Vose 2008) for the random variable Θ. The modified PERT distribution is a 4-parameter distribution and is frequently used to model expert opinion. Expert opinion is used to specify the minimum (a), maximum (b), most-likely (Mo) values and a fourth parameter (s) controls the shape. The probability density function of the modified PERT distribution is given as follows:

$$f(x) = \frac{(x-a)^{\eta_1 - 1}(b-x)^{\eta_2 - 1}}{B(\eta_1, \eta_2)(b-a)^{\eta_1+\eta_2-1}},$$

where $B(\eta_1, \eta_2)$ is the beta function, and

$$\eta_1 = 1 + s(\frac{Mo - a}{b - a}), \quad \eta_2 = 1 + s(\frac{b - Mo}{b - a}).$$

The mean (μ) and variance (σ^2) are

$$\mu = \frac{a + sMo + b}{s + 2}, \quad \sigma^2 = \frac{(\mu - a)(b - \mu)}{s + 3}.$$

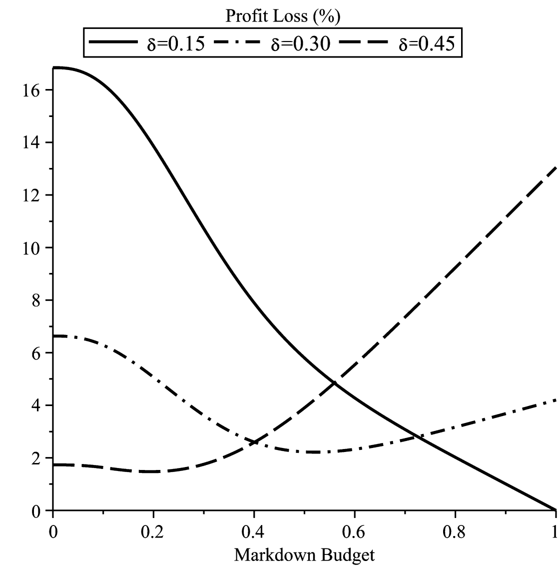
We use the modified PERT distribution since it has a bounded domain and we can easily control the shape and mode with two dedicated parameters.

Note that modified PERT distribution is a specific version of the four parameter Beta distribution. When $s = 0$, the distribution reduces to uniform distribution. Since PERT or Beta offer a lot of flexibility in modeling the shape of a probability distribution, they are commonly used in industry.

We assume that the maximum possible value for Θ is 1. This ensures that the retailer's initial stocking decision leads to 100% service level even in the case of best market outcome. In addition, we assume that the minimum possible value for Θ is $1/(1 + \lambda)$. This ensures that the retailer is able to finish his inventory by marking the price down at the beginning of the season even in the case of worst market outcome. Consistent with this discussion, we set $a = \frac{1}{1+\lambda}$ and $b = 1$.

We first note that setting a markdown budget is a non-trivial task when $\gamma_r < \gamma_b$. Figure 4 shows the percentage profit loss of the retailer as a function of the markdown budget for three different problems. The expected profit under a markdown budget policy for a particular setting is a complex function of the price,

Figure 4 Profit Loss (%), m = 0.5, c = 1, λ = 1, $\theta \sim$ PERT(0.5, 1, 0.9, 4)



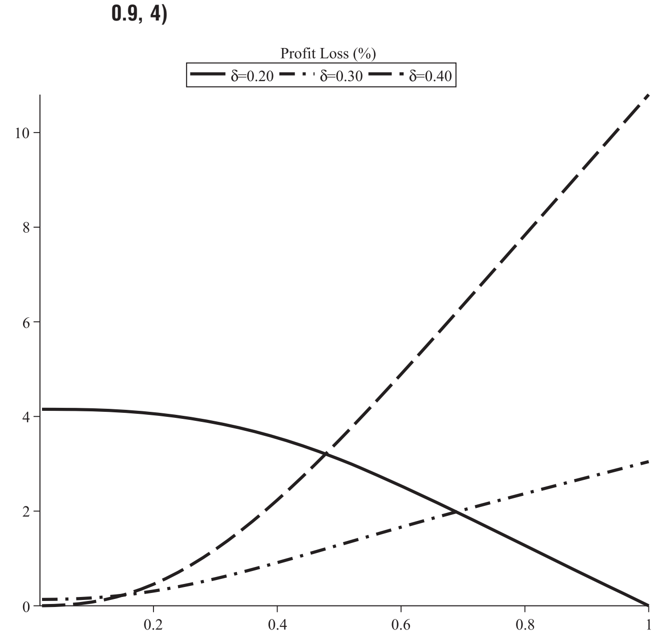
demand and uncertainty parameters as well as the amount of budget. A very small budget will limit the buyer's ability to respond to a low market and a very large budget will lead to the buyer marking the prices down when it is not really necessary. In a particular problem, a markdown budgeting policy's performance will depend on the probability of these two events taking place and the extent of the damages of these events on retailer profitability. In some settings, the risk of cutting the prices down prematurely outweighs the risk of not being able to respond to a bad market. In some of these problems, it may be optimal to set a markdown budget to close to zero and essentially stop the buyer from marking the price down altogether. An example is given for the markdown depth $\delta = 0.45$ where the losses due to an unnecessary price cut is substantial.

In some other settings, the risk of not being able to cut the price down when it is really necessary outweighs the risk of buyer making early markdowns. In some of these problems, it may be optimal to set the markdown budget to one and give the buyer the complete flexibility in marking the prices down. An example is given for the markdown depth $\delta = 0.15$. Since the depth of the markdown is small, the retailer should mark the price down immediately in most outcomes of the market. Therefore, the potential damage due to an earlier-than-necessary markdown is not high in comparison to delaying the markdown and having some leftover inventory at the end of the season.

Finally in some settings, the size of the markdown budget may have a more interesting effect on profitability. An example is given for the markdown depth $\delta = 0.30$. When the markdown budget is small, the buyer's ability to respond to a low market is very limited. When the markdown budget is at the intermediate range, it allows the buyer to adequately address the low market, hence the expected profit goes up (and the profit loss goes down) as the markdown budget approaches roughly the half of the maximum budget. However, once we go beyond this level, the risk of having an early markdown outweighs the benefit of responding to a bad market and the expected profit decreases again. Therefore, it is optimal to set the markdown budget to an intermediate level, that is, completely stopping or freeing the buyer is not an optimal policy.

Setting the optimal markdown budget is different in the case where $\gamma_r > \gamma_b$. We have done several numerical tests with different sets of parameters, and in all of them the profit loss is monotone as a function of markdown budget. It implies that optimal budget is either 0 or 1. Figure 5 shows the percentage profit loss, and as can be confirmed, the only candidates for the optimal budget is 0, that is, setting the buyer completely free, or 1, i.e., forcing the buyer to take markdowns right away. When the markdown depth δ is high, markdowns may lead to significant revenue losses if they are not taken at the right time. In this case, the retailer prefers to set the markdown budget to 0 and not force the buyer to take any markdowns. When the markdown depth δ is low, the revenue leak with the markdowns is not significant and the retailer prefers markdowns in most outcomes of the market. In this case, the retailer is better off by setting the budget to 1 and ensuring that the buyer marks the price down as soon as possible.

Figure 5 Profit Loss (%), $m = 0.5$, $c = 0.3$, $\lambda = 1$, $\theta \sim$ PERT(0.5, 1, 0.9, 4)



5. Comparative Statics

In this section, we report the results of our computational study that investigates the effect of various factors on the value of the optimal markdown budget and its efficiency. We start by focusing on the more complicated case where $\gamma_r < \gamma_b$. We first look at the effect of the most likely value of the demand distribution (*Mo*). Figure 6 reports the optimal markdown budget and the corresponding profit losses. Notice that the optimal budget is shown as a fraction of the maximum budget which is given by initial inventory Equation (1) multiplied by the depth of the markdown (δ) and is equal to δ.

Notice that when *Mo* is small, we expect the demand to be weak and anticipate marking the prices very early in the season. In this case, it is optimal for the retailer to set a markdown budget as high as possible and set the buyer almost free on her markdown decisions. Nevertheless, there is still some discrepancy between the motives of the buyer and the retailer. It is more profitable for the buyer to mark the prices earlier than what is optimal for the retailer leading to profit losses up to 2–3%. As *Mo* goes up, it is optimal to set the markdown budget to an intermediate level as the retailer wants to further limit the markdowns taken unnecessarily early by the buyer. The profit losses also increase. As *Mo* grows further, the retailer decreases the markdown budgets as it anticipates better market. When *Mo* is very large, the probability of earlier than optimal markdowns is reduced, leading to smaller profit losses.

In Figure 7, we investigate the effect of the shape parameter of the demand distribution (s). Notice that as s decreases the variance of the market size Θ increases. Clearly, as there is more uncertainty regarding the market, the retailer increases the markdown budget in order to allow the buyer respond to more probable low market. When the uncertainty is low, it is possible to finetune the markdown budget and force the buyer to make markdown decisions that are aligned with the profitability of the retailer. However, as uncertainty increases, this is no longer possible and the profit losses are larger.

In Figure 8, the effect of the sales jump λ, is investigated. Note that as λ changes, we also change the minimum value of the PERT distribution to $1/(1 + \lambda)$. As λ increases, the retailer finds the markdown to be an effective option since it can clear the inventory faster and use the shelf space for a new product for a longer period of time, and therefore increases the budget. Once λ reaches a certain high level, the retailer sets the budget to maximum and allow the buyer to be completely free in her markdown decisions to respond to market. The initial increase of λ however may lead the buyer to mark the price down earlier

Figure 6 Effect of *Mo*, δ = 0.3, *m* = 0.5, *c* = 1, λ = 1, $\theta \sim$ PERT(0.5, 1, Mo, s)

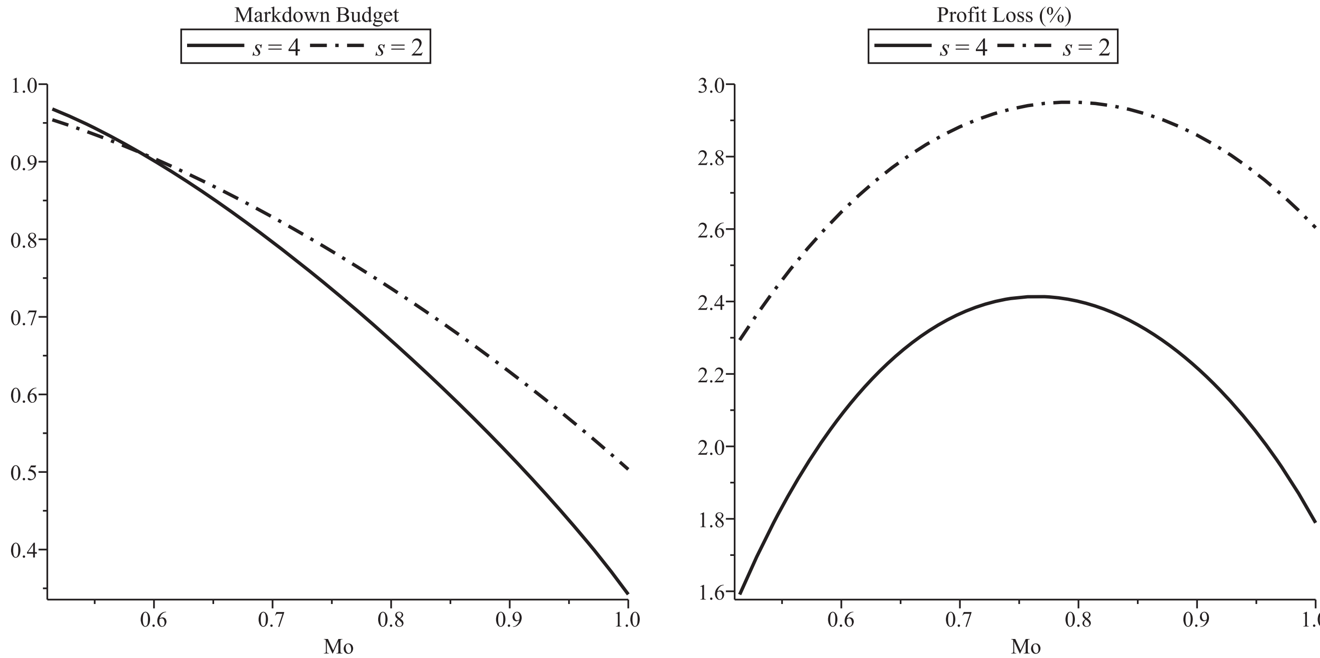
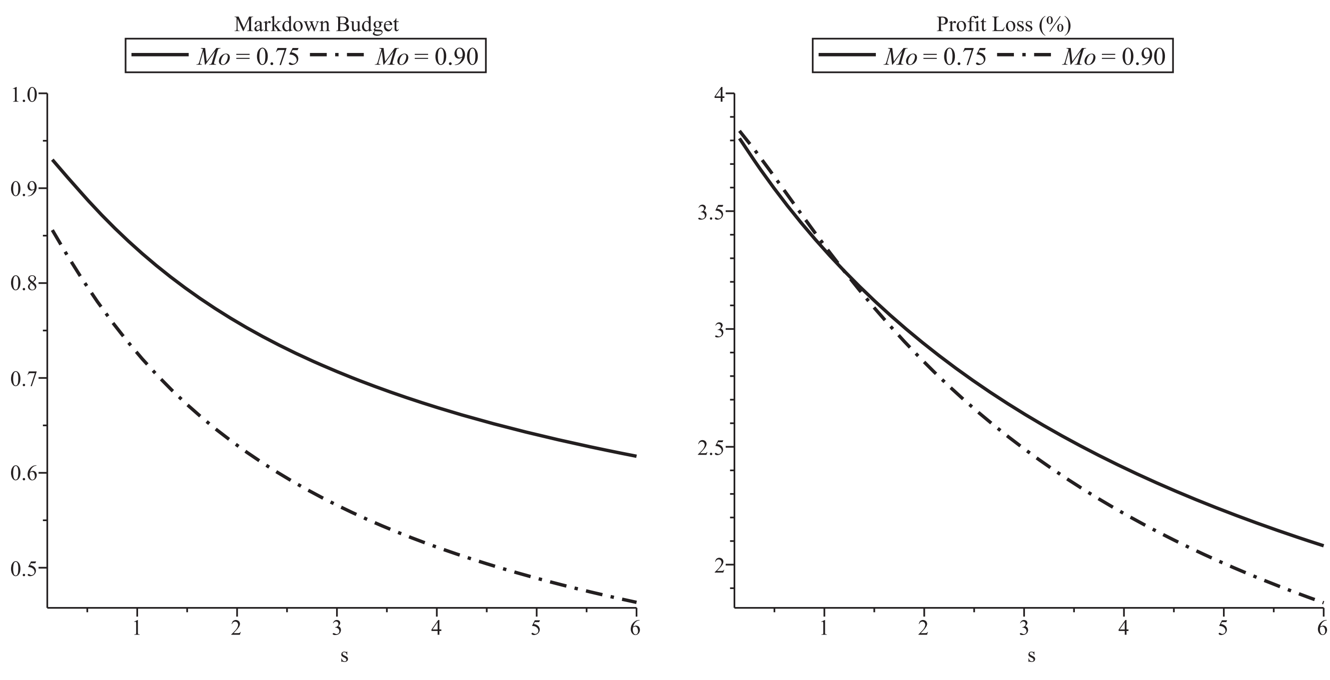


Figure 7 Effect of *s*, δ = 0.3, *m* = 0.5, *c* = 1, λ = 1, $\theta \sim$ PERT(0.5, 1, Mo, s)



than necessary yielding an increase in the profit loss. However, as λ increases further, the retailer's and buyer's incentives are better aligned and the gap goes down. It implies that as λ becomes smaller further, the actual budget goes up. The profit loss is smaller when λ is very low or very high, since these cases lead to a better alignment of the objectives of the retailer and the buyer. Conversely, the profit losses are higher when λ is at moderate levels.

In Figure 9, the effect of the profit margin m is investigated. Having a higher procurement cost hurts the retailer in case of an unnecessary markdown as this cost needs to be incurred when shelves are stocked with new products. Therefore, the retailer

Figure 8 Effect of λ, δ = 0.3, m = 0.5, c = 1, $\theta \sim$ PERT(1/(1 + λ), 1, 0.75, s)

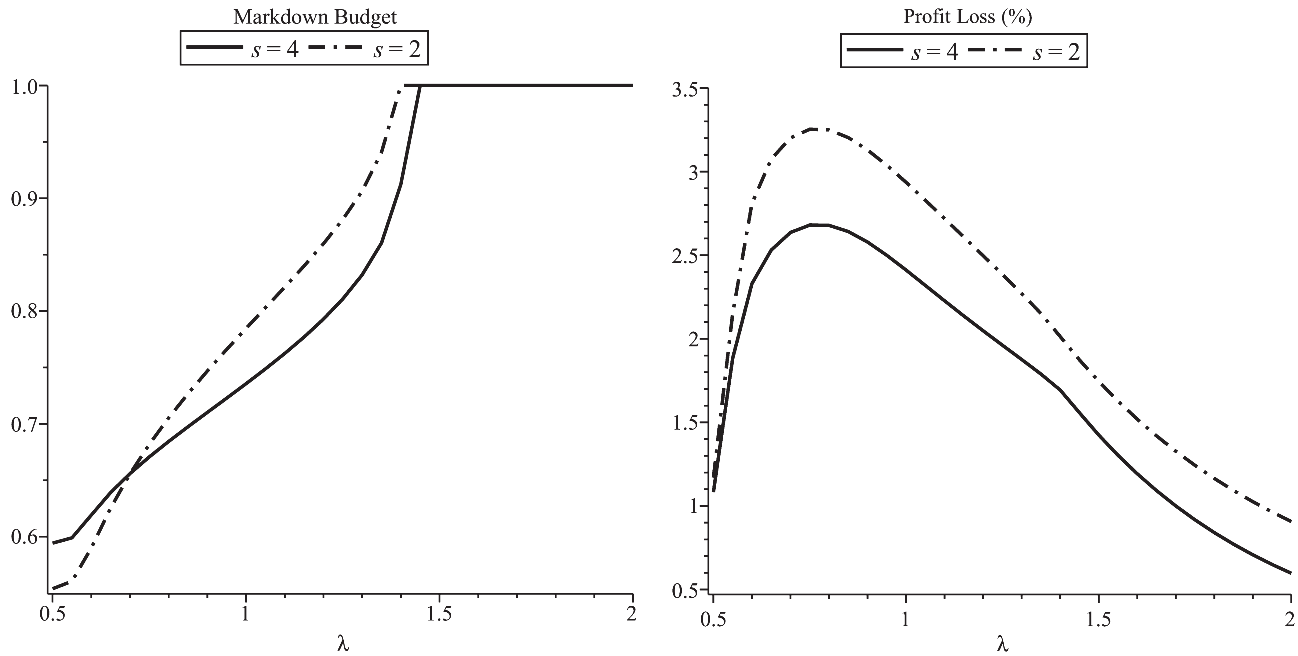
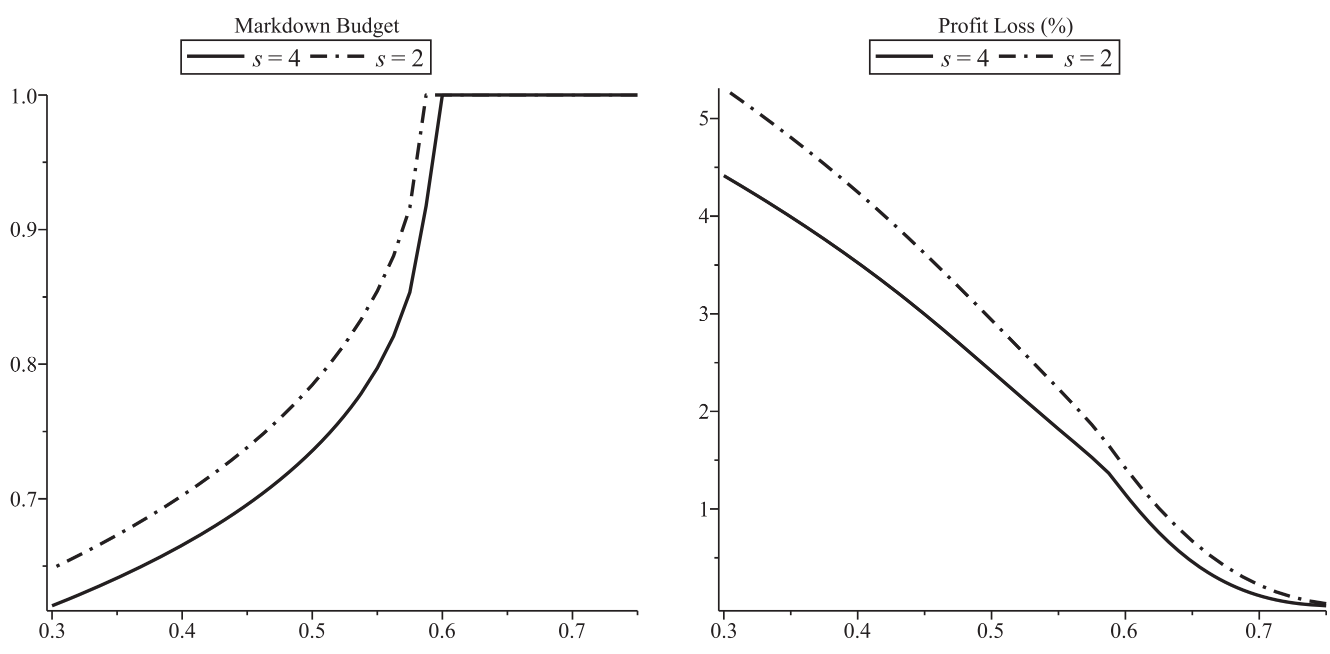


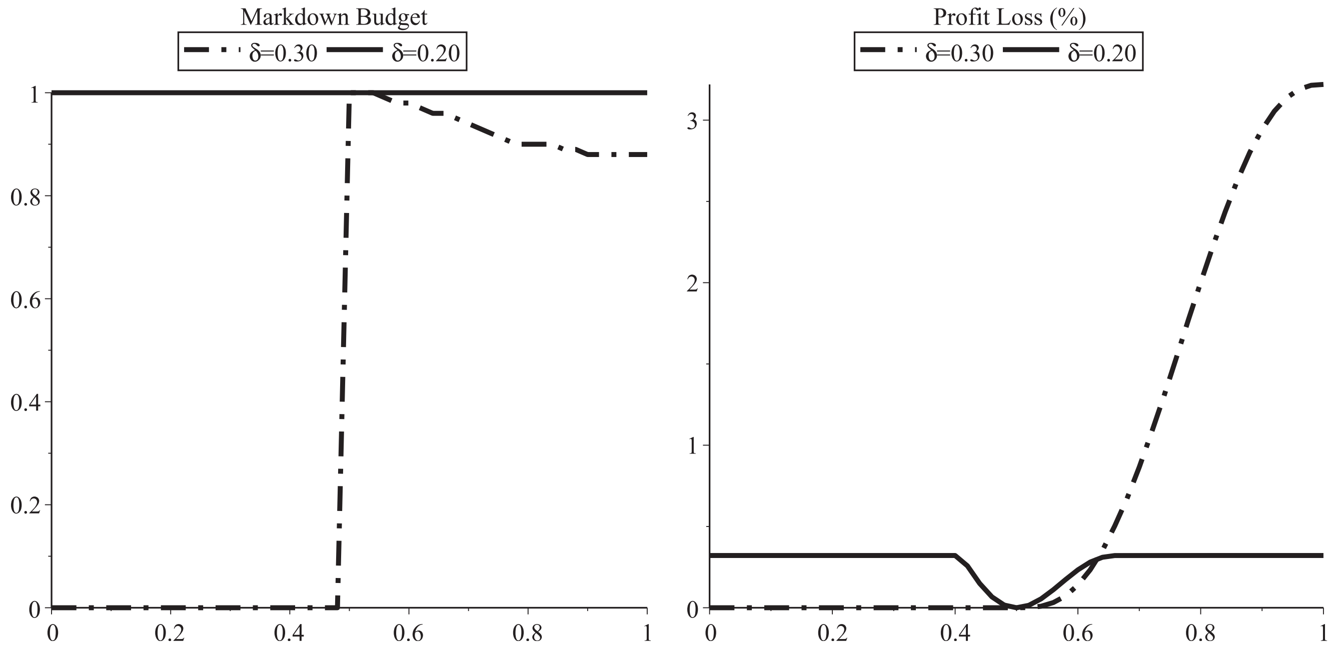
Figure 9 Effect of m, δ = 0.3, λ = 1, c = 1, $\theta \sim$ PERT(0.5, 1, 0.75, s)



reduces the budget as m goes down. As m goes down, we also see that the profit loss increases due to an increased misalignment of the buyer's and retailer's objectives. When m is very large, the procurement cost is very close to zero and the buyer's incentive is better aligned with the retailer's objective. Therefore, the retailer simply sets the budget to maximum and have the buyer make markdown decisions freely in this case.

In Figure 10, the effect of the buyer's commission parameter c is investigated. Note that when c is less (more) than m = 0.5, the empty shelf space is more (less) valuable for the retailer than the buyer. When c = m = 0.5, the shelf space valuations are the same

Figure 10 Effect of c, $\lambda = 1$, $m = 0.5$, $\theta \sim$ PERT(0.5, 1, 0.75, 4)



and there is no incentive misalignment. First note that when $\delta = 0.3$ and $c < m$, it is optimal for the retailer to set the budget to zero. In this case the profit losses are also zero. When $c = m$ it is optimal to set the budget to 1 as the incentives are aligned. As c increases beyond m, the markdown budget is lowered and the profit losses see a steady increase. When $\delta = 0.2$ it is optimal to set the budget 1 to for all values of c. When $c < m$, the retailer prefers the buyer to mark the price down at the beginning of the season. When $c > m$ the retailer sets the buyer free in markdown decisions. In both of these scenarios, because the revenue leak is very small, the profit losses are also very small. When $c = m$, the incentives are again aligned, the retailer sets the buyer free and the buyer makes a markdown decision that is also optimal for the retailer leading to zero profit loss.

6. Conclusions

In this paper, we study the markdown decisions of a retailer which sells a given inventory of fashionable product over a finite season. Different from previous research in this area, we consider the fact that the markdown decision is delegated to an agent (retail buyer), which is commonly observed in practice. The retail buyer may be better informed about the market, but her objective is to maximize sales, whereas, in contrast, the retailer's objective is to maximize profit. The retailer uses a markdown budget to control the buyer's decision and maintain a profitable margin. The buyer's markdown timing decision in this setting can be suboptimal with respect to the objective of the retailer in one of two ways. First, if the market for the merchandise is relatively weak, the buyer may mark the price down later than what is optimal for the retailer as she is constrained by a budget (since early in the season the markdown will be applied to more inventory). Second, if the market for the merchandise is relatively strong, the buyer may be inclined to mark the price down earlier than what is optimal for the retailer, clear the inventory and use the shelf space to generate more sales. Our model allows us to investigate these two risks simultaneously. Using this model, we characterize the retailer's optimal markdown policy and the buyer's markdown decisions and provide a formal explanation for the possible inefficiencies of the latter. Our model can also be used to determine the optimal markdown budget by the retailer and quantify the cost of the misalignment of the buyer's incentives with the objective of the retailer.

We find that a very small budget will limit the buyer's ability to respond to a low market and a very large budget will lead to the buyer marking the prices down when it is not really necessary. In a particular problem, a markdown budgeting policy's performance will depend on the probability of these two events taking place and the extent of the damages of these events on retailer profitability. Our numerical results show that using suboptimal budgets (either excessive or insufficient) may be destructive, leading to substantial profit losses. We also investigate the effects of various factors on the optimal budget and the resulting profit losses. Our results indicate that markdown budgets should be set higher when the market expectations are lower, the demand uncertainty is higher (i.e., the merchandise in question has higher fashion content), the market is more responsive to markdowns, and the initial markup is higher. The profit losses due to misalignment of the objectives are typically higher when the expected market is at moderate levels, the uncertainty is high, the initial markup is low, and the markdown depth is at moderate levels. Even when the markdown budgets are optimally determined, remaining losses can be as high as 4–5%.

We note that this study, to our knowledge, is to first to consider incentive issues in markdown pricing for retail firms. We study the effectiveness of using one particular (but commonly used) mechanism - markdown budget - to align the incentives of the buyers with the profitability of the firm. Future work can extend this study to many different directions. First, one can consider the use of other mechanisms and investigate whether these would lead to better incentive alignment in this setting. Second, we consider a single item and a single location; one can study a setting where markdowns need to be coordinated across different items and locations. Third, one can consider selling multiple versions of a product at a time and the possibility of product rollover strategies (see Liang et al. 2014). Fourth, one can investigate the effect of buyer's gradual learning about the market (as opposed to instantaneous learning in this study). Fifth, including competition is another possible avenue for extension; we refer to Sen (2016) for the effect of competition on markdown decisions.

We finally note that markdown budgets are in close relationship with another prevalent tool in retail industry: Open-To-Buy (OTB) system. The OTB system is used since the 1920s (Pasdermadjian 1954) and controls how much money a buyer can allocate for new purchases considering sales (regular and markdowns) and inventory. In this system, budgeted markdowns affect not only the markdown decisions of the buyer but also her purchasing decisions. Studying the effectiveness of this system when the demand is uncertain and the buyer's objective is in conflict with retailer's profitability may be an important avenue for future research.

Appendix A. Proofs

THEOREM 1. *Case 1: $n \geq t\theta$*

One of the following two cases can happen: i. $1 - t \leq \frac{(n-t\theta)}{\theta(1+\lambda)}$, *or* $\frac{\theta(\lambda+1)-n}{\theta\lambda} \leq t$:

$$\begin{aligned}\pi(\theta,\gamma,t) =& t\theta + (1-t)\theta(1-\delta)(1+\lambda)\\ =& \theta(1-\delta)(1+\lambda) + t\theta(\delta - \lambda + \delta\lambda)\\ \Rightarrow t^*(\theta,\gamma) =& \frac{\theta(\lambda+1)-n}{\theta\lambda}.\end{aligned}$$

ii. $\frac{(n-t\theta)}{\theta(1+\lambda)} \leq 1 - t$, *or* $t \leq \frac{\theta(\lambda+1)-n}{\theta\lambda}$:

$$\begin{aligned}\pi(\theta,\gamma,t) =& t\theta + (1-\delta)(n-t\theta) + \gamma\left(1 - t - \frac{n-t\theta}{\theta(1+\lambda)}\right)\\ =& n(1-\delta) + \gamma - \frac{n\gamma}{\theta(1+\lambda)} + t\left(\theta\delta - \frac{\gamma\lambda}{1+\lambda}\right)\\ \Rightarrow t^*(\theta,\gamma) =& \begin{cases}\max\left\{\frac{\lambda\gamma}{\delta(\lambda+1)}, \frac{n}{1+\lambda}\right\} \leq \theta : \frac{\theta(\lambda+1)-n}{\theta\lambda},\\ \theta \leq \max\left\{\frac{\lambda\gamma}{\delta(\lambda+1)}, \frac{n}{1+\lambda}\right\} : 0.\end{cases}\end{aligned}$$

Case 2: $n \leq t\theta$.

$$\begin{aligned}\pi(\theta,\gamma,t) =& n + (1-t)\gamma,\\ \Rightarrow t^*(\theta,\gamma) =& \frac{n}{\theta}.\end{aligned}$$

□

THEOREM 2. *The proof follows the same idea in Gallego and van Ryzin (1994), Theorem 5. Let $\pi^h(\theta, \gamma)$ be the expected revenue of the stopping-time heuristic. The hypothetical revenue of a deterministic problem can be defined as $\pi^*(\theta, \gamma) = max_t\pi(\theta, \gamma, t) = \pi(\theta, \gamma, t^*(\theta, \gamma))$. It is easy to show that $\tilde{\pi}^*(\theta, \gamma) \leq \pi^*(\theta, \gamma)$.*

We define a wasteful heuristic such that representing the profit of this wasteful heuristic by $\pi^w(\theta, \gamma)$, we have $\pi^w(\theta, \gamma) \leq \pi^h(\theta, \gamma)$.

Since $\frac{\pi^h(\theta,\gamma)}{\tilde{\pi}^*(\theta,\gamma)} \geq \frac{\pi^w(\theta,\gamma)}{\pi^*(\theta,\gamma)}$, it is enough to show that $\frac{\pi^w(\theta,\gamma)}{\pi^*(\theta,\gamma)}$ approaches to 1.

Case 1: $n \geq \theta \geq \max\left\{\frac{n}{1+\lambda}, \frac{\lambda\gamma}{(1+\lambda)\delta}\right\}$

This case correspond to markdown at middle of the season. In this case $t^*(\theta,\gamma) = \frac{n\theta(1+\lambda)-\gamma}{\theta\lambda}$ leading to

$$\begin{aligned}\pi^*(\theta,\gamma) =& \pi\left(\theta,\gamma,\frac{n\theta(1+\lambda)-\gamma}{\theta\lambda}\right)\\ =& \frac{\theta(1+\lambda)-n}{\lambda} + \left(\frac{n-\theta}{\lambda}\right)(1-\delta)(1+\lambda).\end{aligned}$$

We define the following wasteful heuristic. We reserve $\frac{\theta(1+\lambda)-n}{\lambda}$ units to be sold at full price, and the rest to be sold at the markdown price. We do not use possible empty shelves at end of the period. Since the wasteful reserves $\frac{\theta(1+\lambda)-n}{\lambda}$ units to be sold at the full price and $n - \frac{\theta(1+\lambda)-n}{\lambda} = \frac{(1+\lambda)(n-\theta)}{\lambda}$ units to be sold at the markdown price

$$\pi^w(\theta,\gamma) = \mathbb{E}\left[\min\left\{N_\theta(t), \frac{\theta(1+\lambda)-n}{\lambda}\right\}\right] + (1-\delta)\mathbb{E}\left[\min\left\{N_{\theta(1+\lambda)}(1-t), \frac{(1+\lambda)(n-\theta)}{\lambda}\right\}\right].$$

Based on Gallego (1992) and using the Cauchy–Schwartz inequality, one can show that:

$$\begin{aligned}\pi^w(\theta,\gamma) \geq& \frac{\theta(1+\lambda)-n}{\lambda} - \frac{1}{2\sqrt{\frac{\theta(1+\lambda)-n}{\lambda}}} \\ &+(1-\delta)\left(\frac{(1+\lambda)(n-\theta)}{\lambda} - \frac{1}{2\sqrt{\frac{(1+\lambda)(n-\theta)}{\lambda}}}\right) \\ &= \pi^*(\theta,\gamma) - \frac{1}{2}\left(\frac{1}{\sqrt{\frac{\theta(1+\lambda)-n}{\lambda}}} -(1-\delta)\left(\frac{1}{\sqrt{\frac{(1+\lambda)(n-\theta)}{\lambda}}}\right)\right).\end{aligned}$$

The last term approaches to zero as n and θ approach infinity, and therefore $\frac{\pi^w(\theta,\gamma)}{\pi^*(\theta,\gamma)}$ approaches 1.

Case 2: $n \leq \theta$.

This case corresponds to no markdown. In this case $t^*(\theta, \gamma) = \frac{n}{\theta}$ leading to

$$\begin{aligned}\pi^*(\theta,\gamma) =& \pi\left(\theta,\gamma,\frac{n}{\theta}\right) \\ =& n + \gamma\left(1-\frac{n}{\theta}\right).\end{aligned}$$

We define the following wasteful heuristic. We reserve all n units to be sold at full price

$$\pi^w(\theta,\gamma) = \mathbb{E}[\min\{N_\theta(t), n\}] + \gamma\left(1-\frac{n}{\theta}\right).$$

Based on Gallego (1992) and using the Cauchy-Schwartz inequality, one can show that:

$$\begin{aligned}\pi^w(\theta,\gamma) \geq& \\ =& \pi^*(\theta,\gamma) - \frac{1}{2\sqrt{n}}.\end{aligned}$$

The last term approaches to zero as n and θ approach infinity, and therefore $\frac{\pi^w(\theta,\gamma)}{\pi^*(\theta,\gamma)}$ approaches 1.

Case 3: $\theta \leq \max\left\{\frac{n}{1+\lambda}, \frac{\lambda\gamma}{(1+\lambda)\delta}\right\}$

This case corresponds to an immediate markdown. In this case $t^*(\theta, \gamma) = 0$ leading to

$$\begin{aligned}\pi^*(\theta,\gamma) =& \pi(\theta,\gamma,0) \\ =& n(1-\delta)(1+\lambda) + \gamma\left(1-\frac{n}{\theta(1+\lambda)}\right)^+.\end{aligned}$$

We define the following wasteful heuristic. We reserve all n units to be sold at full price

$$\pi^w(\theta,\gamma) = (1-\delta)\mathbb{E}\left[\min\{N_{\theta(1+\lambda)}(t), n\}\right] + \gamma\left(1-\frac{n}{\theta(1+\lambda)}\right)^+.$$

Based on Gallego (1992) and using the Cauchy–Schwartz inequality, one can show that:

$$\begin{aligned}\pi^w(\theta,\gamma) \geq& (1-\gamma)n(1+\lambda) - \frac{1}{2\sqrt{(1-\gamma)n(1+\lambda)}} \\ &+ \gamma\left(1-\frac{n}{\theta(1+\lambda)}\right)^+. \\ =& \pi^*(\theta,\gamma) - \left(1-\frac{n}{\theta(1+\lambda)}\right)^+.\end{aligned}$$

The last term approaches to zero as n and θ approach infinity, and therefore $\frac{\pi^w(\theta,\gamma)}{\pi^*(\theta,\gamma)}$ approaches 1. □

Notes

[1]In addition to overbuying and poor pricing, there can be other reasons for markdowns, e.g., price discrimination, selling errors, price competition, sales policies, and excitement creation (Wingate et al. 1972); these are out of this paper's scope.

[2]Representing initial price by p_1, markdown price by p_2, and salvage value by s, our normalization setting initial price to 1, and salvage value to 0 implies that $\delta = 1 - (p_2 - s)/(p_1 - s)$.

[3]Any markdown time later than 1 implies that there is no markdown.

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