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ORIGINAL PAPER

Inverse S-shaped probability weighting functions in first-price sealed-bid auctions

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Abstract It is often observed in first-price sealed-bid auction experiments that subjects tend to bid above the risk neutral Nash equilibrium predictions. One possible explanation for this overbidding phenomenon is that bidders subjectively weight their winning probabilities. In the relevant literature, the probability weighting functions (PWFs) suggested to explain overbidding imply the underweighting of all probabilities. However, such functions are not in accordance with the PWFs commonly used in the literature (i.e., inverse S-shaped functions). In this paper we introduce inverse S-shaped PWFs into first-price sealed-bid auctions and investigate the extent to which such weighting functions explain overbidding. Our results indicate that bidders with low valuations underbid, whereas those with high valuations overbid. We accordingly conclude that inverse S-shaped PWFs provide a partial explanation for overbidding.

Keywords First-price auctions · Overbidding · Subjective probability weighting · Inverse S-shaped functions

JEL Classification C72 · D44 · D81

1 Introduction

It is often observed in first-price sealed-bid auction experiments that subjects tend to bid above the *risk neutral Nash equilibrium* (RNNE) predictions (see Cox et al. 1988; Kagel 1995, among others). This *overbidding* phenomenon has often been explained

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using models with risk averse bidders. However, for such an explanation to be valid, bidders should be excessively risk averse. Accordingly, it is argued that risk aversion cannot be the only factor and may well not be the most important factor behind overbidding (see Kagel and Roth 1992). Along this line, several alternative explanations have been provided: ambiguity aversion (Salo and Weber 1995), regret theory (Filiz-Ozbay and Ozbay 2007), level-*k* thinking (Crawford and Iriberri 2007), and loss aversion (Lange and Ratan 2010).[1]

In addition to the above studies, a number of papers suggest subjective probability weighting as an alternative explanation for overbidding. To the best of our knowledge, Cox et al. (1985) are the first to present the idea of using subjective probability weighting in first price auctions. They propose that a power probability weighting function (PWF) is observationally equivalent to a model with risk aversion. Afterwards, Goeree et al. (2002) employ this idea utilizing a functional form which is originally suggested by Prelec (1998). They estimate that the PWF should be essentially convex over the whole range if it were to explain their experimental observations. Finally, Armantier and Treich (2009b) experimentally show that bidders tend to overbid as they underestimate their winning probabilities, whereas Armantier and Treich (2009a) analytically show that a star-shaped PWF[2] can explain overbidding in first-price auctions.

The above-mentioned PWFs imply the underweighting of all probabilities. Hence they are not in accordance with the PWFs commonly used in the literature (i.e., inverse S-shaped functions)(see Tversky and Kahneman 1992; Camerer and Ho 1994; Wu and Gonzalez 1996; Prelec 1998, among others).[3] In this paper we introduce inverse S-shaped PWFs into first-price sealed-bid auctions and investigate the extent to which such weighting functions explain overbidding.

Our results indicate that bidders with low valuations underbid if all bidders use the same inverse S-shaped PWF. We also show that (i) there exist cases under which all bidders always underbid and (ii) if the number of participants is sufficiently low, there exists a threshold valuation such that any bidder with a valuation higher than this threshold will overbid.[4] Therefore, we conclude that inverse S-shaped PWFs provide a *partial* explanation for overbidding. It is worth noting that these findings are somewhat consistent with the aforementioned experimental studies since overbidding is mostly observed for bidders with high valuations, whereas the submitted bids of subjects with

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[1] Salo and Weber (1995) show that greater aversion for ambiguity leads to higher bid amounts. Filiz-Ozbay and Ozbay (2007) introduce the concepts of winner and loser regret, and they explain overbidding by claiming that loser regret is more dominant. Crawford and Iriberri (2007) propose level-*k* thinking as a cause of overbidding. Finally, Lange and Ratan (2010) analyze overbidding in auctions using a multi-dimensional reference-dependent model.

[2] A function $F : [0, 1] \to [0, 1]$ with $F(0) = 0$ and $F(1) = 1$ is star-shaped if $F(x)/x$ is increasing in x.

[3] As a matter of fact, subjective probability weighting is suggested earlier by Karmarkar (1978) and by Kahneman and Tversky (1979). It is worth noting here that the PWF described by Kahneman and Tversky (1979) is essentially similar to an inverse S-shaped function; a function that overweights low probabilities and underweights moderate to high probabilities. Later, hints about inverse S-shaped PWFs are also given by Quiggin (1982).

[4] To obtain this overbidding result, we assume that the valuations are distributed according to the uniform distribution and utilize a specific PWF originally suggested by Prelec (1998).

low valuations are close to the RNNE predictions (see Filiz-Ozbay and Ozbay 2007; Armantier and Treich 2009b, among others).[5]

This paper is structured as follows: In Sect. 2, we present the related aspects of subjective probability weighting, we introduce inverse S-shaped PWFs into first-price sealed-bid auctions, and we investigate the unique symmetric Nash equilibrium. Section 3 concludes.

2 The model

2.1 On subjective probability weighting

Subjective probability weighting is supported by numerous individual decision-making experiments (see Camerer 1995, for a detailed review). It constitutes one of the key aspects of prospect theory (Kahneman and Tversky 1979) and cumulative prospect theory (Tversky and Kahneman 1992). Moreover, it is the main aspect of rank-dependent expected utility theory (Quiggin 1982) and dual theory (Yaari 1987). The bulk of relevant literature argues that a PWF appears to be inverse S-shaped (see Tversky and Kahneman 1992; Camerer and Ho 1994; Wu and Gonzalez 1996; Prelec 1998, among others). An increasing function $w : [0, 1] \to [0, 1]$ is inverse S-shaped if

(i) $w(0) = 0$ and $w(1) = 1$; and
(ii) there exists a unique $\bar{p} \in (0, 1)$ for which
- $w(\bar{p}) = \bar{p}$;
- $w(p) > p$ for every $p \in (0, \bar{p})$; and
- $w(p) < p$ for every $p \in (\bar{p}, 1)$.

Following this line of research, we study inverse S-shaped PWFs in this paper. In particular, we assume that all bidders employ an increasing, differentiable, and inverse S-shaped PWF when making their bidding decisions.

2.2 The auction framework

There is a single object to be sold, and there are n bidders in the player set N. Each bidder $i \in N$ assigns a monetary value to the auctioned object. The valuation v_i represents the maximum amount bidder i is willing to pay for the object and is only known to bidder i. In addition, each bidder knows that the valuations of other bidders are identically and independently distributed according to a cumulative distribution function F over $[0, 1]$. In this first-price sealed-bid auction framework, bidders simultaneously submit their bids. The bidder with the highest bid wins the auction and gets the object. For the case in which there are multiple bidders with the highest bid, the winner is determined randomly and with equal probabilities. The winner pays an amount equal to his/her bid, whereas the remaining bidders do not make any payment.

[5] We thank an anonymous reviewer for bringing this to our attention.

Consider a bidder $i \in N$ with valuation v_i and assume that each bidder $j \in N \backslash \{i\}$ follows[6] some increasing bid function $\beta_j : [0, 1] \to [0, \infty)$. Then if bidder i bids some $b \in [0, \infty)$, he/she wins the auction with probability $\prod_{j \neq i} F(\beta_j^{-1}(b))$, because b turns out to be greater than the bid of some $j \in N \backslash \{i\}$ with probability $F(\beta_j^{-1}(b))$. Consequently, the bidder faces the following lottery

$$L^{v_i}(b, (\beta_j)_{j \in N \backslash \{i\}}) = \left(\prod_{j \neq i} F(\beta_j^{-1}(b)),\ v_i - b;\ 1 - \prod_{j \neq i} F(\beta_j^{-1}(b)),\ 0 \right)$$

which describes a situation in which the bidder either wins the auction and receives a payoff of $v_i - b$ or does not win the auction and receives a payoff of zero. In this context, given the bid functions of other bidders, a best response of bidder i is the bid amount b^* that induces the lottery with the highest expected utility.

In this paper, we assume that all bidders subjectively weight probabilities with an inverse S-shaped PWF when evaluating these lotteries.[7] To fully concentrate on the effect of such weighting functions on bid amounts, we employ a standard linear utility function. Hence our model is in line with Yaari (1987)'s dual theory. Furthermore, it is assumed to be common knowledge that bidders have the same utility function and the same PWF.

2.3 The equilibrium analysis

We analyze symmetric equilibrium throughout the paper. The probability of winning the auction can then be represented by a function $G \equiv F^{n-1}$. Accordingly, when subjective probability weighting steps in, bidders have $w(G(\cdot))$ as their weighted probability of winning.

For the equilibrium analysis, take any bidder $i \in N$ with valuation v_i. His/Her expected utility from bidding $b \in [0, v_i]$ while all other bidders $j \in N \backslash \{i\}$ follow the same increasing, differentiable bid function $\beta : [0, 1] \to [0, \infty]$ is[8]

$$w\left(G(\beta^{-1}(b))\right)(v_i - b).$$

The analysis yields the following equilibrium bid function.

[6] A bidder is said to *follow* β if he/she bids $\beta(v)$ when his/her valuation is v.

[7] A natural question that arises is whether first-degree stochastic dominance relationships are preserved when we apply subjective probability weighting directly to the winning probabilities. The answer is provided by Goeree et al. (2002). Noting that the preferred solution would be to apply the weights to the cumulative distribution function, it is emphasized that the lotteries have only two outcomes in a first-price auction. Then, since the weighted probability of losing will be multiplied by 0 (which is the earning from losing), it is argued that applying the weights to the cumulative distribution function is equivalent to directly weighting the winning probabilities.

[8] It is straightforward that bidding any amount higher than own valuation is dominated by bidding 0. Accordingly, we do not consider those values of b in our analysis although they are in the bidder's strategy set.

Proposition 1 *In a first-price sealed-bid auction with subjective probability weighting, the unique symmetric equilibrium is given by*

$$\beta^*(v_i) = v_i - \frac{\int_0^{v_i} w(G(y))dy}{w(G(v_i))} \tag{1}$$

if all bidders subjectively weight probabilities with the same inverse S-shaped PWF, w.

Proof See "Appendix 2". □

It is worth noting that the above equilibrium bid function reduces to the *risk neutral Nash equilibrium* (RNNE) if the PWF is the identity function. Also note that the fraction in (1) is bidder i's net earning when he/she wins the auction; and it will be the only relevant part of the bid function when comparing β^* with the RNNE (denoted by β^*_{RN}).

2.4 On overbidding in first-price auctions

Considering the results of the earlier studies on subjective probability weighting in first-price auctions, one can conjecture that inverse S-shaped functions cannot *completely* explain overbidding. That said, our first objective is to check whether there exist valuations for which bidders underbid.

Proposition 2 *Consider a first-price sealed-bid auction with subjective probability weighting in which all bidders use the same inverse S-shaped PWF. Then there exists a valuation $\hat{v}$ such that $\beta^*(\hat{v}) < \beta^*_{RN}(\hat{v})$.*

Proof See "Appendix 2". □

The above proposition indicates that the above-mentioned conjecture is true. However, to what extent inverse S-shaped PWFs explain overbidding remains unanswered. We answer this question by checking the existence of valuations for which bidders overbid. At this point, we make two additional assumptions. First, we employ a functional form which is originally suggested by Prelec (1998) and is defined as

$$w(p) = \exp\left\{-(-\ln p)^{\alpha}\right\} \tag{2}$$

where $\alpha \in (0, 1)$. Second, we assume that F is the uniform distribution.[9]

Letting β^U and β^U_{RN} denote the corresponding unique symmetric equilibria under the uniform distribution, we first show that underbidding is possible for all values of valuations.

[9] The results of the following analysis depend on the distribution. In what follows, we prefer to adopt the uniform distribution, because overbidding is observed under the uniform distribution in the aforementioned experimental studies.

Proposition 3 *Consider a first-price sealed-bid auction with subjective probability weighting in which valuations are distributed according to the uniform distribution. If all bidders weight probabilities with the inverse S-shaped PWF given by* (2)*, then there exists a case under which all bidders always underbid.*

Proof Assume that $n = 10$ and $\alpha = 0.67$. Then for every $v \in (0, 1]$,

$$\beta^U(v) < \frac{9v}{10} = \beta^U_{RN}(v).$$

□

The next proposition states that if a certain regularity condition is satisfied, overbidding is *partially* explained under WAC.

Proposition 4 *Consider a first-price sealed-bid auction with subjective probability weighting in which valuations are distributed according to the uniform distribution. Assume that all bidders weight probabilities with the inverse S-shaped PWF given by* (2)*. If*

$$\int_0^1 \exp\left\{-(n-1)^\alpha (-\ln y)^\alpha\right\} dy \leq \frac{1}{n},$$

then there exists a unique critical valuation $v^* \in (0, 1]$ *such that* $\beta^U(v^*) = \beta^U_{RN}(v^*)$ *and any bidder with valuation* v *underbids if* $v < v^*$ *whereas he/she overbids if* $v > v^*$.

Proof See "Appendix 2". □

In addition, we have the following equation that uniquely characterizes the critical valuation v^*:

$$\frac{\int_0^{v^*} \exp\{-(n-1)^\alpha(-\ln y)^\alpha\} dy}{\exp\{-(n-1)^\alpha(-\ln v^*)^\alpha\}} = \frac{v^*}{n}.$$

We know by Propositions 2 and 4 that all overweighters underbid if all bidders weight probabilities with the inverse S-shaped PWF given by (2). In other words, subjective probability weighting causes bidders with low valuations to overestimate their chances of winning the auction, to which they respond by lowering their bids. Thus we relate our underbidding results with the overweighting interval of the PWF, which is given by $(0, \bar{p})$. On the other hand, the underweighting interval of the function gives bidders an incentive to increase their bid amounts. However, since the bids of overweighters are already below the RNNE predictions, there are some underbidding underweighters.[10] The effect of the underweighting interval becomes dominant for bidders with sufficiently high valuations, and there occurs overbidding.

[10] This follows by the continuity of equilibrium bid functions.

3 Concluding remarks

In this paper, we have introduced inverse S-shaped PWFs into a first-price sealed-bid auction framework. We have shown that inverse S-shaped PWFs cannot *completely* explain overbidding as bidders with low valuations underbid and that such weighting functions can *partially* explain overbidding as bidders with sufficiently high valuations overbid. It appears that the reason behind underbidding is the overweighting interval of inverse S-shaped PWFs.

This study is the first to use inverse S-shaped PWFs in first-price auctions. It can be considered a first step towards analyzing the reason(s) behind the discordance between the PWFs suggested to explain overbidding and inverse S-shaped PWFs commonly used in the literature. Our findings indicate that the level of discordance is greater for bidders with low valuations and if the number of bidders is high. Hence these issues may require special emphasis if one aims to unravel the reason(s) behind this discordance.

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Appendix 1

In first-price sealed-bid auctions, a participant wins the auction if every other bidder submits a bid less than that of the participant. Hence his/her winning probability is calculated by compounding the probabilities of other bidders' submitting such bids. Naturally, the timing of subjective probability weighting may lead to different theoretical predictions. In this paper, we assume that bidders directly weight their wining probabilities. This is in line with the standard method employed in earlier studies on subjective probability weighting in first-price auctions. In this "Appendix 1", we propose an alternative method: *weighting before compounding* (WBC). The following example demonstrates the difference between the standard method and WBC.

Example 1 Consider a lottery in which a fair coin is flipped twice. The lottery yields \$1 if both outcomes are heads and yields nothing otherwise. Obviously, the probability of winning the lottery is $1/4$. For this lottery, two possible weighting methods are as follows: (i) weighting the winning probability, which yields $w(1/4)$; or (ii) weighting the probabilities of each independent event separately and then compounding the weighted probabilities, which yields $w(1/2)^2$.

Clearly, (i) corresponds to the standard method. It stipulates that the probabilities of independent events are first compounded, and then this compounded probability will be distorted. On the other hand, (ii) corresponds to WBC. The idea behind this method resembles the one behind *prospective reference theory* introduced by Viscusi (1989).[11] According to this theory, a compound lottery is treated differently than the

[11] Prospective reference theory is able to predict several phenomena such as premiums for certain eliminations of a risk, the representativeness heuristic, the isolation effect, and the Allais paradox and related violations of the substitution axiom.

corresponding reduced lottery.[12] Under this method, the probability of each independent event is first distorted, and then these weighted probabilities will be compounded.

The equilibrium analysis: First, at a symmetric equilibrium, the weighted probability of winning is $w(F(\cdot))^{n-1}$ rather than $w(G(\cdot)) = w(F(\cdot)^{n-1})$. The equilibrium analysis follows similarly: Take an arbitrary bidder $i \in N$ with valuation v_i. His/Her expected utility from bidding $b \in [0, v_i]$ while all other bidders $j \in N\backslash\{i\}$ follow the same increasing, differentiable bid function $\beta : [0, 1] \to [0, \infty]$ is

$$w\left(F(\beta^{-1}(b)\right)^{n-1}(v_i - b).$$

The analysis yields the following equilibrium bid function.

Proposition 5 *In a first-price sealed-bid auction with subjective probability weighting, the unique symmetric equilibrium is given by*

$$\beta_B^*(v_i) = v_i - \frac{\int_0^{v_i} w(F(y))^{n-1}dy}{w(F(v_i))^{n-1}} \tag{3}$$

if all bidders subjectively weight probabilities before compounding with the same inverse S-shaped PWF, w.

Proof The first order condition with respect to b is

$$\frac{\partial w(F(\beta^{-1}(b)))^{n-1}}{\partial \beta^{-1}(b)} \frac{\partial \beta^{-1}(b)}{\partial b}(v_i - b) - w(F(\beta^{-1}(b)))^{n-1} = 0.$$

It then follows that

$$\beta_B^*(v_i) = v_i - \frac{\int_0^{v_i} w(F(y))^{n-1}dy}{w(F(v_i))^{n-1}}.$$

This bidding function is increasing in v_i, and $\beta_B^*(v_i)$ is not greater than v_i for any $v_i \in [0, 1]$. Hence β_B^* is the only candidate for a symmetric equilibrium. For verification, one can show that a bidder with valuation v_i bids $\beta_B^*(v_i)$ given that other bidders follow β_B^*. Thus β_B^* is the unique symmetric equilibrium. □

The results on overbidding: First, we prove the existence of an underbidder.

Proposition 6 *Consider a first-price sealed-bid auction with subjective probability weighting in which all bidders use the same inverse S-shaped PWF. Then there exist a valuation $\hat{v}_B$ such that $\beta_B^*(\hat{v}_B) < \beta_{RN}^*(\hat{v}_B)$.*

Proof Assume that bidders weight probabilities before compounding. Since w is inverse S-shaped, there exists a unique $\hat{v}_B \in (0, 1)$ such that $w(F(\hat{v}_B)) = F(\hat{v}_B)$. Consider a bidder with valuation $\hat{v}_B$. His/Her weighted probability of winning equals

[12] Note that the lottery in Example 1 can also be described as a compound lottery.

to his/her winning probability. Also, since every probability lower than $F(\hat{v}_B)$ is being overweighted,

$$\int_0^{\hat{v}_B} w(F(y))^{n-1}dy > \int_0^{\hat{v}_B} F(y)^{n-1}dy$$

for every $n \in \mathbb{N}$. It then follows that $\beta_B^*(\hat{v}_B) < \beta_{RN}^*(\hat{v}_B)$. □

For the following proposition, let β_B^U denote the equilibrium bid function under the assumption that F is the uniform distribution.

Proposition 7 *Consider a first-price sealed-bid auction with subjective probability weighting in which valuations are distributed according to the uniform distribution. Assume that all bidders weight probabilities before compounding with the inverse S-shaped PWF given by (2). Then there exists a unique critical valuation* $v_B^* \in (0, 1]$ *such that* $\beta_B^U(v_B^*) = \beta_{RN}^U(v_B^*)$ *and any bidder with valuation* v *underbids if* $v < v_B^*$ *whereas he/she overbids if* $v > v_B^*$.

Proof To show the existence of v_B^*, we first prove that a bidder with valuation 1 overbids. To do this, we first take the derivative of $\beta_B^U(1)$ with respect to α:

$$(1-n)\int_0^1 \exp\{-(n-2)(-\ln y)^\alpha\}\frac{\partial \exp\{-(-\ln y)^\alpha\}}{\partial\alpha}dy.$$

This expression turns out to be negative which implies that $\beta_B^U(1)$ is decreasing in α. Noting that $\beta_B^U(1) = \beta_{RN}^U(1)$ when $\alpha = 1$, we have $\beta_B^U(1) > \beta_{RN}^U(1)$ when $\alpha \in (0, 1)$. Recall that a bidder with valuation $\hat{v}_B$ underbids; i.e., $\beta_B^U(\hat{v}_B) < \beta_{RN}^U(\hat{v}_B)$. Since β_B^U and β_{RN}^U are both continuous, there exists v_B^* such that $\beta_B^U(v_B^*) = \beta_{RN}^U(v_B^*)$.

As for uniqueness, one needs to show that there exists a unique $v \in (0, 1]$ satisfying $\beta_B^U(v) - \beta_{RN}^U(v) = 0$. This expression is zero when $v = 0$, negative when $v = 1/e$, and positive when $v = 1$. Furthermore, it has a single extremum at some point in the interval $(1/e, 1]$. These jointly imply our claim that the critical valuation v_B^* is unique. In addition to this, any bidder with valuation v underbids if $v < v_B^*$ whereas he/she overbids if $v > v_B^*$. □

We have the following equation that uniquely characterizes the critical valuation v_B^*:

$$\frac{\int_0^{v_B^*}\exp\{-(n-1)(-\ln y)^\alpha\}dy}{\exp\left\{-(n-1)(-\ln v_B^*)^\alpha\right\}} = \frac{v_B^*}{n}.$$

Given the uniform distribution, notice that a regularity condition is no longer necessary in order for inverse S-shaped PWFs to *partially* explain overbidding. This is because a bidder with valuation 1 always overbids under WBC. Accordingly, we can conclude that overbidding is explained for a wider range of valuations under WBC in comparison to the standard method.

Appendix 2

Proof of Proposition 1. The first order condition with respect to b is

$$\frac{\partial w(G(\beta^{-1}(b)))}{\partial \beta^{-1}(b)} \frac{\partial \beta^{-1}(b)}{\partial b}(v_i - b) - w(G(\beta^{-1}(b))) = 0.$$

As we search for symmetric equilibrium, $b = \beta(v_i)$ should be the maximizer of the objective function; that is, $b = \beta(v_i)$ should solve the equation above. Thus,

$$\frac{\partial w(G(v_i))}{\partial v_i} \frac{1}{\beta'(v_i)}(v_i - \beta(v_i)) = w(G(v_i)).$$

After arranging terms, we obtain

$$\frac{\partial}{\partial v_i}(w(G(v_i))\beta(v_i)) = v_i \frac{\partial w(G(v_i))}{\partial v_i},$$

which implies

$$\beta^*(v_i) = \frac{1}{w(G(v_i))} \int_0^{v_i} y \frac{\partial w(G(y))}{\partial y} dy = v_i - \frac{\int_0^{v_i} w(G(y))dy}{w(G(v_i))}.$$

By differentiating, we see that β^* is increasing in v_i. Moreover, it is straightforward that $\beta^*(v_i)$ is not greater than v_i for any $v_i \in [0, 1]$. Thus β^* is the only candidate for a symmetric equilibrium.

To verify that β^* is an equilibrium, we first assume that every $j \in N \backslash \{i\}$ follows β^*. Note that bidding above $\beta^*(1)$ is dominated for bidder i. Suppose that bidder i acts as if his/her valuation is $z \in [0, 1]$ rather than v_i. Then it turns out that $z = v_i$ is a best response. Consequently, β^* is the unique symmetric equilibrium. □

Proof of Proposition 2. Take any $n \in \mathbb{N}$. Since w is inverse S-shaped, there exists a unique $\hat{v} \in (0, 1)$ such that $w(F(\hat{v})^{n-1}) = F(\hat{v})^{n-1}$. Consider a bidder with valuation $\hat{v}$. First note that his/her weighted probability of winning equals to his/her winning probability. Also, since every probability lower than $F(\hat{v})^{n-1}$ is being overweighted,

$$\int_0^{\hat{v}} w(F(y)^{n-1})dy > \int_0^{\hat{v}} F(y)^{n-1}dy.$$

It then follows that $\beta^*(\hat{v}) < \beta_{RN}(\hat{v})$ for every $n \in \mathbb{N}$. □

Proof of Proposition 3. Given the regularity condition, a bidder with valuation 1 overbids. Recall that a bidder with valuation $\hat{v}$ underbids; i.e., $\beta^U(\hat{v}) < \beta^U_{RN}(\hat{v})$. Since β^U and β^U_{RN} are both continuous, there exists v^* such that $\beta^U(v^*) = \beta^U_{RN}(v^*)$. The proof of uniqueness follows as in Proposition 7 (see "Appendix 1"). □

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