An integrated approach for airline scheduling, aircraft fleeting and routing with cruise speed control

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ABSTRACT

To place an emphasis on profound relations among airline schedule planning problems and to mitigate the effect of unexpected delays, we integrate schedule design, fleet assignment and aircraft routing problems within a daily planning horizon while passengers' connection service levels are ensured via chance constraints. We propose a nonlinear mixed integer programming model due to the nonlinear fuel consumption and CO_2 emission cost terms in the objective function, which is handled by second order conic reformulation. The key contribution of this study is to take into account the cruise time control for the first time in an integrated model of these three stages of airline operations. Changing cruise times of flights in an integrated model enables to construct a schedule to increase utilization of fuel efficient aircraft and even to decrease total number of aircraft needed while satisfying the same service level and maintenance requirements for aircraft fleeting and routing. There is a critical tradeoff between the number of aircraft needed to fulfill the required flights and overall operational expenses. We also propose two heuristic methods to solve larger size problems. Finally, computational results using real data obtained from a major U.S. carrier are presented to demonstrate potential profitability in applying the proposed solution methods.

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1. Introduction

Managing an airline is unavoidably expensive. One of the most basic costs is the price of purchasing the aircraft which range from 70 to 400 mio USD per unit Boeing (2015). Moreover, among the operational expenses, fuel has been the largest single cost term for the global airlines. According to IATA (2010) analysis on airline financial data, fuel expenses accounted for $210 billion in 2013 for global airline industry. On the other hand, unexpected delays are endemic in airline operations and to demonstrate their impact, the estimated total cost to the U.S. economy because of flight delays was as much as $41 billion in 2007 Rebollo and Balakrishnan (2014). Naturally, the efficient utilization of such expensive resources, decreasing operational expenses and higher robustness are objectives of any profitable airline. However, unfortunately these terms are inversely correlated, in other words higher utilization might cause higher operational cost, less robustness and vice versa. In this direction, generating a robust schedule with high aircraft utilization and less operational expenses at the same time is quite crucial for any airline. Therefore, in this paper, we propose an integrated model for robust schedule design, aircraft

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routing and fleeting with cruise speed control that aims efficient aircraft utilization and robustness within the consideration of operational expenses such as fuel consumption, CO_2 emission and spill cost.

Airline schedule design problem decides where to fly and in which frequency in consideration of market demand, profitability, available resources and the competitors. Due to its broad scope, Barnhart et al. (2003) state that building flight schedules from scratch is performed manually with limited optimization in the typical airline practice. Following the construction or design of a flight schedule, fleet assignment problem tries to find the optimal assignment of aircraft types to flights by considering the number of aircraft in each fleet and coverage of all flights. After fleet assignment decomposes flight networks into subnetworks in terms of a particular fleet type, maintenance routing problem assigns individual aircraft to these flights in consideration of the maintenance requirements. For a general review on airline schedule planning problems, we refer the reader to Barnhart and Cohn (2004).

Since the different airline schedule planning problems are strongly related to each other, several integrated models are proposed that take into account combinations of these problems to improve suboptimal solutions for the entire system. Lohatepanont and Barnhart (2004) consider schedule design and fleet assignment in an integrated way in which a base schedule and two flight lists including mandatory and optional flights are given. Starting from the base schedule they consider deleting/adding flights from/to the base schedule with respect to given flight lists. In a similar fashion, Sherali et al. (2013a) propose a model that integrates the schedule design and fleet assignment processes while considering flexible flight times, schedule balance, and recapture issues, along with optional legs, path/itinerary-based demands, and multiple fare-classes. Differently, they consider the flow of passengers along itineraries over the network together with flight scheduling and fleeting decisions in order to maximize profits. Integrating three problems enables to improve local optimal solutions, however tractability worsens as much as the scope of integration expands. Therefore, these integrated problems are modeled and solved for a daily planning horizon.

In the literature, there are studies related to our work in some aspects such as daily planning horizon, passenger connection, cruise speed control or maintenance considerations. Duran et al. (2015) propose a robust airline scheduling model with controllable cruise times. In their study, the tradeoff between the costs of cruise time change and idle time insertion is considered while passengers' connection service levels are ensured by chance constraints. Speed control is quite a recent concept in solving airline scheduling problems. Aktürk et al. (2014) is the first study that makes use of speed control in the context of airline schedule recovery from disruptions. Sherali et al. (2013b) propose an approach in which they integrate the schedule design, fleet assignment, and aircraft-routing problems within the consideration of flight selection, departure timing and maintenance requirements. For maintenance requirements, they use a limit on total flight time of each aircraft that might be different for each fleet type. As a solution method, they use Benders' decomposition and enhance the model via valid inequalities. Haouari et al. (2013) propose a model for daily maintenance routing problem in which they ensure maintenance feasibility by counter constraints on flight hours, take offs and number of days since the last maintenance checks for each aircraft. They present a compact polynomial-sized representation for the general aircraft routing model and they linearize and lift that representation. Moreover, in the study of Aloulou et al. (2013), a MIP model is proposed for the robust aircraft routing problem without directly accommodating maintenance constraints however by considering that the flights start and end in the single hub where maintenance checks are achieved overnight. Aloulou et al. (2013) capture robustness by an objective function pertaining to aircraft and passenger connections.

What distinguishes our work from the studies above and makes challenging simultaneously is cruise speed control and integration. To the best of our knowledge, this is the first study in which cruise speed/time is controlled within the integrated robust schedule design, aircraft fleeting and routing problem. In our study, the fuel consumption and CO_2 emission cost functions are nonlinear functions in cruise time and involve binary variables. We have shown that these nonlinear functions with binary variables can be transformed into a set of second order conic inequalities. Moreover, even if it is a special case of our problem, Parmentier (2013) showed that aircraft routing problem by itself is an NP-complete problem. In addition to aircraft routing problem, we consider robust airline scheduling and fleet type assignment problems in an integrated fashion that involves a large number of decision variables. For that reason, when the number of flights and aircraft increases, the problem size increases drastically. We also consider passengers' connection service levels with chance constraints as well as departure timing, idle time insertion and cruise speed control different from the aircraft routing problem. Changing cruise time of flights in an integrated model enlarges the solution space and enables to construct a schedule with new flight sequences, which could not be considered previously due to fixed cruise speed/time restriction. For two flights to be connected consecutively by the same aircraft, there must be enough time gap between departure times of these flights. This time gap is the sum of cruise time, non-cruise time, turnaround time and idle time. In other studies, the lower bound for this gap is taken as fixed, however cruise speed/time change enables to control this lower bound on the gap between departure times. By this means, in our study more flight connection alternatives could be generated.

The first contribution brought by our study is that aircraft utilization could be increased and even total number of aircraft needed to cover a set of flights could be decreased while ensuring equivalent service level and maintenance requirements. Due to having more alternatives on flight connections and compression of cruise time of flights, it is possible to increase the number of flights to be performed by an efficient aircraft. While this increase in the utilization of fuel efficient aircraft could reduce the minimum number of required aircraft to perform a set of flights. There is a critical tradeoff between the number of aircraft needed to fulfill the required flights and the overall operational expenses, such as fuel consumption costs.

The second is the robustness issue. Since we have more alternatives on flight connections, it is possible to generate better flight sequences in terms of robustness. For example, on a route having a flight with a great delay probability would require

an intervention for the following flights to be performed on time while removing the problematic flight from that sequence could render that intervention unnecessary. Our study has more options to make this type of changes on routing decisions so changing routes could improve the robustness.

Finally, we have used the recent advances in second order cone programming to handle the nonlinear mixed integer programming formulation. Furthermore, in order to solve the large scale problems in a reasonable time, we propose two heuristic algorithms. The first one is discretized approximation and cruise speed control algorithm, and the second one is multi-stage triplet search algorithm.

In the remaining of the paper, in Section 2, the problem definition is presented. Section 3 is devoted to the problem formulation and conic reformulation. In Section 4, heuristic methods are proposed and in Section 5 results of a computational study are presented.

2. Problem definition

Our problem is to solve robust airline schedule design, aircraft fleeting and routing problems within a daily planning horizon for a given set of flights and a set of aircraft in an integrated manner while considering maintenance requirements and passengers' connection service levels. In the proposed model, we determine the departure time, cruise time, an inserted idle time, if necessary, and aircraft fleet type for each flight along with the routing decisions. For each aircraft, the model determines whether that aircraft is going to be used and, if it is used, the flight sequence to be flown for that particular aircraft. These decisions are made in consideration of the idle time insertion cost, fuel consumption and CO_2 emission costs, spill cost and aircraft usage cost simultaneously.

The feasible set of solutions satisfy maintenance requirements, passengers' connection service levels along with the flight connection constraints in cases where two flights are performed consecutively by the same aircraft. For maintenance requirements, we adopt two basic rules for each aircraft. The first one is that we limit the total cruise time for each aircraft on a day. A similar approach is used by Sherali et al. (2013b), they use a limit (λ_t) on total flight hours of each aircraft on a day. The second one is that the first and the last airport of an aircraft is predetermined on a day similar to the study of Aloulou et al. (2013). In case when an aircraft is used, in order that aircraft to follow its ordinary maintenance checks, we secure that aircraft takes off from/lands to the particular airports at the beginning/end of a day. For passengers' connection, we use service levels with chance constraints and ensure necessary time for passengers to make connection with a particular probability, denoted as service level, between each pair of flights which is feasible for passengers' connection. Furthermore, we provide minimum turnaround time between flights which are performed by the same aircraft.

Similar to the study of Duran et al. (2015), in our model we take the flight time as the sum of cruise time and non-cruise time. Non-cruise time consists of the taxi-in and taxi-out stages as well as climb and descend stages of a flight, which include uncertainty depending on the airport congestion or weather conditions while cruise time refers to the time which is controllable by with speeding up the aircraft. Consequently, we take cruise time as a decision variable and non-cruise time as a given parameter. On the other hand, there are different methods in the literature to investigate the role of fuel cost on aircraft operating costs as discussed by Ryerson and Hansen (2013). In this study, we have used the following function proposed by Aktürk et al. (2014) and EUROCONTROL (2012) to calculate fuel consumption in kg for a given cruise time of flight i operated by aircraft type t:

$$F_i^t(f_i^t) = c_1^{i,t} \cdot \frac{1}{f_i^t} + c_2^{i,t} \cdot \frac{1}{(f_i^t)^2} + c_3^{i,t} \cdot (f_i^t)^3 + c_4^{i,t} \cdot (f_i^t)^2 \tag{2.1}$$

where $c_1^{i,t}$, $c_2^{i,t}$, $c_3^{i,t}$ and $c_4^{i,t}$ correspond to aircraft properties and the coefficients related to physical conditions of a flight for each flight i and aircraft type t pair as discussed in Aktürk et al. (2014). These coefficients for 399 different aircraft types are available in EUROCONTROL (2012). Fuel cost has been the largest operational expense for the global airlines. The EU emissions trading system (EU ETS) covers airlines as well as power plants and industrial plants in 31 countries. By introducing CO_2 emission costs, these trading systems aim at cutting CO_2 emissions by airline companies. Consequently, the fuel consumption and CO_2 emission costs are calculated by multiplying fuel consumption with their corresponding cost coefficients ($c_{fuel} + c_{CO_2}$).

In this study, similar to the study of Duran et al. (2015), passengers' connections are taken into account to develop a robust schedule such that mis-connections of passengers are minimized when a disruption occurs. Between two flights i and j, if the origin airport of the flight j is same as the destination airport of the flight i and the departure time of flight j is later than the arrival time of the flight i, the time needed for the passengers' connection is TP_{ij}. It is ensured that the percentage of the passengers' connection satisfied between flights (i, j) is greater than the minimum desired service level for each connecting flight pair, denoted as γ_{ij}, through a set of probabilistic chance constraints as discussed later.

2.1. Numerical example

In order to elaborate our problem definition and the model mechanics, in this section, we provide a numerical example. First, we present a published schedule in Table 1, which shows the planned departure time and cruise time for each flight, and idle time after each flight, if necessary, to satisfy the passenger connections. In our proposed approach, a new schedule

Table 1
Published schedule.

Tail#	Flight #	From	To	Departure time	Cruise time	Idle time
N3ELAA	2057	ORD	SJU	08:30	270	35.6
	2078	SJU	ORD	14:25	315	–
N3DUAA	2099	ORD	LAX	07:00	250	34.9
	1972	LAX	ORD	12:40	225	24.4
	1972	ORD	RDU	17:45	95	–
N412AA	2345	ORD	DFW	17:15	135	2.5
	2374	DFW	ORD	20:40	110	–
N4XGAA	2079	ORD	SAN	08:45	250	13.5
	1438	SAN	ORD	14:00	230	58.9
	346	ORD	LGA	19:50	115	–

with a new aircraft fleeting and routing is generated by considering idle time cost, fuel consumption and CO_2 emission costs, spill cost and daily usage cost simultaneously. In accordance with the marketing requirements, the departure times could be changed within ±15 min. The proposed schedule, given in Table 2, is feasible in terms of maintenance requirements because all aircraft land to the same airports as in the published schedule, and we also limit the flight hours of each aircraft regarding maintenance requirements. Further, passengers' service levels are ensured by the chance constraints in the proposed schedule. For this numerical example same service level is adopted with the published schedule for each passengers' connection.

There are major changes between two schedules such as the number of aircrafts used, routings of aircraft, idle time insertion and cruise times. The total number of aircraft used in the published schedule was four, while the proposed schedule clearly illustrates that the same passenger connection service levels can be attained with three aircraft. This improvement can be achieved by compressing the cruise time of flight 1438 around 14%, from 230 min to 198.4 min in the proposed schedule. In addition, the departure time of flight 2057 is changed from 8:30 to 8:45 in the proposed schedule. Fig. 1 represents the time space networks of the schedules. The horizontal axis represents the time and the vertical axis represents the airports. For each flight, there is an angular arrow nearby and a flight number. The horizontal arrows represent the turn around and idle time of an aircraft in the corresponding airport. The route of each aircraft is drawn by a different line style. For example, the solid arrows represent the operations of tail number N4XGAA.

We compare the performance of the published schedule with the proposed schedule in terms of the improvement in total cost by using the following formula:

$$\text{Cost Improvement} = 100 \times \frac{\text{Cost of the Published Schedule} - \text{Cost of the Proposed Schedule}}{\text{Cost of the Published Schedule}}$$

The cost calculation of the published schedule is shown in Table 3. The fuel consumption and CO_2 emission costs calculations are explained in former sections. Idle time costs are calculated by the multiplication of unit idle time cost of each aircraft and the idle time amount of that aircraft after each flight. For a given demand realization, spill costs are calculated by multiplication of the number of spilled passengers and the cost incurred when a spill occurs. In the published schedule, we assume that there is no spill cost. However, when we assign an aircraft with a smaller number of seats than the number of already assigned passengers, a spill cost can incur as shown in Table 4. Daily usage costs are the costs incurred when the corresponding aircraft are used. Total cost of the published schedule is $419,821.3.

As mentioned before, the total number of aircraft used is decreased from 4 to 3. In the proposed model, the most expensive aircraft in terms of daily usage cost is N3ELAA which is not a fuel efficient aircraft in comparison to N412AA and N3DUAA. Fig. 2 illustrates the differences in efficiency by providing the fuel consumption and CO_2 emission costs that would occur if flight 1438 was performed by these aircraft. Our model generates a schedule in which N3ELAA is not used due to its high daily usage cost and low fuel efficiency while the other aircraft are utilized more. Although the fuel efficiency of aircraft

Table 2
Proposed schedule.

Tail#	Flight #	From	To	Departure time	Cruise time	Idle time
N3DUAA	2079	ORD	SAN	08:38	249.5	8.4
	1438	SAN	ORD	13:45	198.4	–
	1972	ORD	RDU	18:00	94.8	–
N412AA	2099	ORD	LAX	06:55	249.9	–
	1972	LAX	ORD	12:25	221.7	–
	2345	ORD	DFW	17:26	133	–
	2374	DFW	ORD	20:55	110	–
N4XGAA	2057	ORD	SJU	08:45	275.9	1.2
	2078	SJU	ORD	14:10	293.6	–
	346	ORD	LGA	20:05	115	–
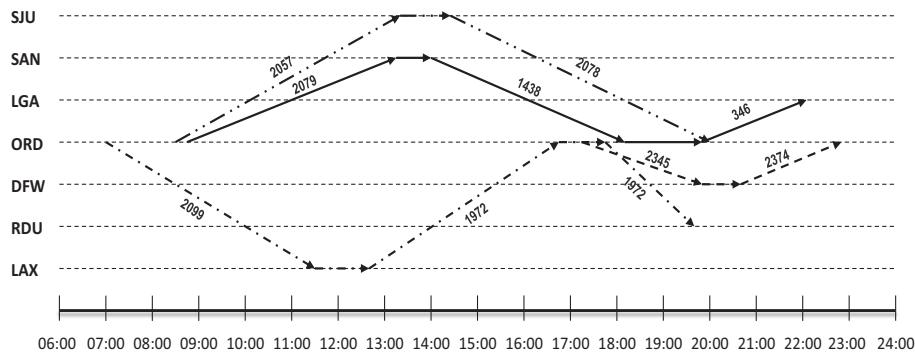
(a) Published schedule
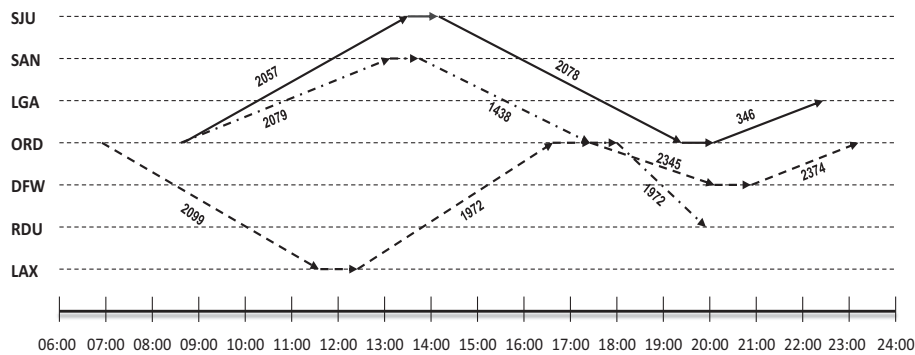
(b) Proposed schedule

Fig. 1. Time space networks.

Table 3
Cost calculation for the published schedule.

Tail#	Flight #	Fuel and CO_2	Idle time	Spill	Daily usage
N3ELAA	2057	7611	5334	–	90,000
	2078	8879.5	–	–	
N3DUAA	2099	5923.4	4955.8	–	85,200
	1972	5331	3461.9	–	
	1972	2250.9	–	–	
N412AA	2345	2319	347.2	–	84,000
	2374	1889.6	–	–	
N4XGAA	2079	6509.4	1944	–	86,400
	1438	5988.7	8481.6	–	
	346	2994.3	–	–	
Total		49696.8	24524.5	0	345,600

N4XGAA is worse than N3ELAA, the model prefers to use N4XGAA since its daily usage cost is smaller than N3ELAA. As a direct consequence of better aircraft utilization, the cost of daily usage is improved by 26%.

Another effect of higher efficient aircraft utilization is on the fuel consumption and CO_2 emission costs. Since utilizations of efficient aircraft are increased and better flight aircraft assignment is achieved, total cost of fuel consumption and CO_2 emission is improved around 10%. This improvement is achieved even if the cruise times of flights 2078, 1438 and 2345 are decreased by 7%, 14% and 2%, respectively. As an example of better flight aircraft assignment, in the published schedule flight 1438 is performed by aircraft N4XGAA with $5988.7 fuel and emission cost, while in the proposed schedule same flight is performed by aircraft N3DUAA with $5650 fuel and emission costs though the cruise time is compressed by 14% in albeit of additional fuel and emission costs. Since N3DUAA is more efficient than N4XGAA, the extra fuel and emission costs caused by that compression which is necessary to assign 1438 to N3DUAA can be compensated. In the proposed schedule, we increase the utilization of N412AA, which is the most fuel efficient aircraft. Furthermore, the necessity of idle time insertion after flights to satisfy the desired minimum service levels for passenger connections can be eliminated by changing routing decisions, cruise time and departure time.

While there is an improvement in sum of daily usage cost, idle time cost, fuel consumption and CO_2 emission costs, the spill cost is increased in the proposed schedule. The reason of this increase is that the fleet assignment in the published schedule is generated in consideration of aircraft capacity and passenger demand, however, in the proposed model the other cost terms are also considered. The improvement in the other cost terms overcomes the increase in the spill cost. While in the published schedule the spill cost is 0, the proposed model has a spill cost $2022.1. The spill of passengers are occurred in flights 2079, 1438, 2099 and 1972 in the proposed schedule.

When we compare the overall results, the total cost is improved 27% in the proposed schedule. The costs of both schedules are calculated at the planning stage. After the realizations of non-cruise times of each flight, the actual cost of the idle time and delay costs can be seen. For this purpose a simulation study can be done to compare the realized costs of the schedules. However, with this numerical example, we aim to show how the mechanics of the proposed model work.

3. Problem formulation

Our main aim in this paper is to integrate schedule design, fleeting and routing decisions along with aircraft cruise speed decisions so that the total cost is minimized while the passenger connection service level constraints are satisfied. In this section, we first present the mathematical formulation of the problem. Consequently, by the help of this model, we can easily demonstrate the interdependencies among these interrelated problems. Afterwards, we show the conic reformulation of the objective function and give the reformulation of the model.

Parameters

$Idle_t$: Unit cost of the idle time of aircraft t
Cap_t: Seat capacity of aircraft t
Dem_i: Passenger demand of flight i
$Daily_t$: The cost incurred when aircraft t is used
TA_{ij}^t: Turntime needed to prepare aircraft t between flights i and j
TP_{ij}: Turnaround time needed to connect passengers between flights i and j
$Cspl_i$: Opportunity cost of spilled passengers of flight i
λ_t: The total available cruise time of aircraft t on a day
γ_{ij}: minimum desired passengers' connection service level between flights i and j
c_{fuel}: Cost of fuel per kg of aircraft fuel consumption
c_{CO_2}: Cost of emission per kg of aircraft CO_2 emission
f_i^u, f_i^l: Upper and lower limit of the cruise time of flight i
d_i^u, d_i^l: Upper and lower limit of the departure time of flight i
e_b: Airport congestion coefficient for airport b
Or_i: Origin airport of flight i
Dn_i: Destination airport of flight i
NC_i: The random parameter denoting the non cruise time of flight i

Decision variables

x_{ij}^t: 1 if flights i and j are performed by the same aircraft t consecutively and is 0 ow.
y_i^t: 1 if flight i is the first flight performed by aircraft t and is 0 ow.
z_i^t: 1 if flight i is the last flight performed by aircraft t and is 0 ow.
d_i: Departure time of flight i
s_i^t: Idle time of aircraft t after flight i
f_i^t: Cruise time of flight i performed by aircraft t

In the proposed nonlinear mixed integer programming model, we utilize the following sets. T represents the set of aircraft that can be operated. F is the set of flight legs which have to be covered by using the aircraft in T. For each flight i in F, there are two sets which are U^i, upstream flights and D^i, downstream flights. U^i denotes the set of flights which can follow flight i in terms of departure time and origin, destination airports pair. It is easy to check origin destination pairs, since the destination airport of a flight in U^i has to be same with the origin airport of i. For departure time, there has to be a sufficient time for connection so that a flight in U^i and flight i might be performed by the same aircraft consecutively. If flight j is in U^i then, flight i is in D^j. A denotes the set of flights (i,j) such that flight $j \in D^i$, i.e. flight $i \in U^j$. If (i,j) is in A, it means that an aircraft can fly these flights consecutively. On the other hand, the connection (i,j) can also be avoided removing this pair from the set A, whenever there are some conditions on (i,j) restricting the chance of being performed by the same aircraft. In our numerical examples, we only consider the most necessary requirements which are the consistency between the arrival time/airport of the former flight i and the departure time/airport of the latter flight j. B denotes the set of airports to be considered in the model. F_e^t denotes the set of flights for an aircraft t in T such that the flights in F_e^t can be the last flight of aircraft t since they

terminate at the convenient airports in terms of maintenance feasibility. F_s^t denotes the set of flights for an aircraft t in T such that the origin airport of flights in F_s^t are the airport from which that aircraft starts the planning day. P_i is the set of flights such that the passengers in flight i might have connection.

For each $i \in F$ and $t \in T$, we redefine the fuel consumption as follows:

$$F_i^t(f_i^t) = \begin{cases} \left(c_1^{i,t} \frac{1}{f_i^t} + c_2^{i,t} \frac{1}{(f_i^t)^2} + c_3^{i,t} \left(f_i^t\right)^3 + c_4^{i,t} \left(f_i^t\right)^2 \right) & \text{if } \left(y_i^t + \sum_{j \in U^i} x_{ji}^t \right) = 1 \\ 0 & \text{if } \left(y_i^t + \sum_{j \in U^i} x_{ji}^t \right) = 0 \end{cases} \tag{3.1}$$

so that if aircraft t is not assigned to flight i, then $F_i^t(f_i^t) = 0$.

The proposed nonlinear mixed integer programming model is provided below:

$$\min \sum_{t \in T} \sum_{i \in F} \left(\sum_{j \in U^i} x_{ji}^t + y_i^t \right) \cdot Cspl_i \cdot max(0, Dem_i - Cap_t) + \sum_{i \in F} \sum_{t \in T} (c_{fuel} + c_{CO_2}) \cdot F_i^t(f_i^t) + \sum_{i \in F} \sum_{t \in T} y_i^t \cdot Daily_t + \sum_{i \in F} \sum_{t \in T} s_i^t \cdot Idle_t \tag{3.2}$$

$$\text{s.t. } \sum_{j \in U^i} x_{ji}^t + y_i^t - \sum_{j \in D^i} x_{ij}^t - z_i^t = 0 \quad \forall i \in F, t \in T \tag{3.3}$$

$$\sum_{i \in F} y_i^t \leqslant 1 \quad \forall t \in T \tag{3.4}$$

$$\sum_{t \in T} \left(y_i^t + \sum_{j \in U^i} x_{ji}^t \right) = 1 \quad \forall i \in F \tag{3.5}$$

$$\text{IF } \sum_{t \in T} x_{ij}^t = 1$$
$$\text{THEN, } d_j - d_i - TA_{ij}^t - \sum_{t \in T} f_i^t - E[NC_i] - \sum_{t \in T} s_i^t = 0 \quad \forall(i,j) \in A \tag{3.6}$$

$$Pr \left[NC_i \leqslant d_j - d_i - \sum_{t \in T} f_i^t - TP_{ij} \right] \geqslant \gamma_{ij} \quad \forall i \in F, j \in P_i \tag{3.7}$$

$$\sum_{i \in F} f_i^t \leqslant \lambda_t \quad \forall t \in T \tag{3.8}$$

$$\text{IF } \left(y_i^t + \sum_{j \in U^i} x_{ji}^t \right) = 1$$
$$\text{THEN, } f_i^l \leqslant f_i^t \leqslant f_i^u \quad \forall i \in F, t \in T \tag{3.9}$$
$$\text{ELSE}$$
$$f_i^t = 0$$
$$s_i^t = 0$$

$$y_i^t = 0 \quad \forall t \in T, i \in F \backslash F_s^t \tag{3.10}$$

$$z_i^t = 0 \quad \forall t \in T, i \in F \backslash F_e^t \tag{3.11}$$

$$d_i^l \leqslant d_i \leqslant d_i^u \quad \forall i \in F \tag{3.12}$$

$$s_i^t \geqslant 0 \quad \forall i \in F, t \in T \tag{3.13}$$

$$x_{ij}^t \in \{0,1\} \quad \forall(i,j) \in A, t \in T \tag{3.14}$$

$$y_i^t \in \{0,1\} \quad \forall i \in F, t \in T \tag{3.15}$$

The objective function (3.2) is the sum of the spill cost, fuel consumption and CO_2 emission cost, the daily aircraft usage cost and idle time cost, respectively. Constraint (3.3) is network balance equations. Constraints (3.4) and (3.5) are typical assignment constraints, such that each aircraft can be used for at most one flight sequence (string or path), and each flight can be performed by exactly one aircraft. Constraint (3.6) ensures that if two flights are performed by the same aircraft consecutively, time between the departures of these flights have to be greater than the sum of cruise time, non-cruise time and turnaround time. This time difference corresponds to idle time at the end of the first flight. Hence, when we add the idle time variable to (3.6), we obtain an equality constraint since the idle time is always nonnegative due to constraint (3.13). Constraint (3.7) is the chance constraint which ensures the service level of passengers' connection. This probabilistic constraint guarantees that the percentage of the passengers' connection satisfied between flights (i,j) is greater than the minimum desired service level for each connecting flight pair, denoted as γ_{ij}, such that the probability of the time between arrival

Table 4
Cost calculation for the proposed schedule.

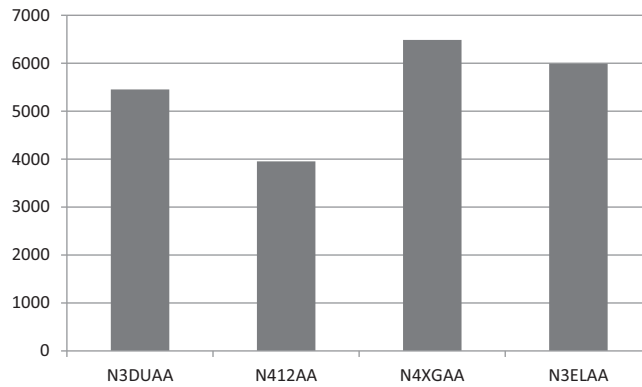| Tail#	Flight #	Fuel and CO_2	Idle time	Spill	Daily usage
N3DUAA	2079	5923.3	1192.8	453.1	85,200
	1438	5650	–	431.5	
	1972	2250.8	–	–	
N412AA	2099	4294.5	–	443.9	84,000
	1972	3866.4	–	693.6	
	2345	2320.4	–	–	
	2374	1889.6	–	–	
N4XGAA	2057	7035.2	172.8	–	86,400
	2078	8260.5	–	–	
	346	2994.3	–	–	
Total		44,485	1365.6	2022.1	255,600

Fig. 2. The fuel and CO_2 cost of flight 1438 with each aircraft type.

of flight i and departure of flight j being greater than the required connection time TP_{ij} is at least γ_{ij}. Due to constraint sets (3.6) and (3.7), there is a critical tradeoff between the passenger connection service levels defined in the constraint set and the decision variables s_i^t (the idle time of aircraft t after flight i, i.e. slack time), which are penalized in the objective function of the proposed model. When s_i^t values are increased, it is more likely to catch the connecting flight, i.e., higher passenger connection service levels. Constraint (3.8) ensures that total cruise time of an aircraft does not exceed a predetermined time limit in order to ensure maintenance feasibility. If a flight i is performed by aircraft t then constraint (3.9) limits cruise time change; cruise time of a flight cannot exceed the upper and lower bounds, else the corresponding variables f_i^t and s_i^t are set to zero. The aim of constraints (3.10) and (3.11) is to sustain maintenance policy intended in the published schedule, in this fashion first and last airport for each aircraft is determined. Constraint (3.12) sets the upper and lower bounds for departure time of each flight due to marketing requirements. As discussed above, it is guaranteed that the idle time is always nonnegative due to constraint (3.13). Constraints (3.14) and (3.15) guarantee that x_{ij}^t and y_i^t are binary variables and due to (3.3) all z_i^t are also binary variables.

3.1. Reformulation of the model

In the proposed model, we have a nonlinear cost terms in the objective function and a probabilistic chance constraint. Therefore, we conduct a conic reformulation of the model so that we can utilize the recent advances in second order cone programming to solve the proposed formulation using a commercial solver.

In the objective function, the cost functions involve nonlinearity due to controllable cruise time associated with the changing the cruise speed of the aircraft. To handle nonlinearity, nonlinear mixed integer optimization often requires too much computation time. On the other hand, it may not result in exact solutions. In order to shorten the solution time and obtain an optimal solution, in this section we show the conic quadratic reformulation of the fuel consumption function $F_i^t(f_i^t)$ cost as discussed in Aktürk et al. (2009) and Günlük and Linderoth (2010). To simplify the presentation, we drop the indices of the variables and parameters.

$$F(f) = \begin{cases} \left(c_1 \frac{1}{f} + c_2 \frac{1}{(f)^2} + c_3 (f)^3 + c_4 (f)^2 \right) & \text{if } w = 1 \\ 0 & \text{if } w = 0 \end{cases}$$

where $w = w_i^t = \left(y_i^t + \sum_{j \in U^i} x_{ji}^t \right)$, for simplicity.

$F(f)$ is discontinuous and therefore its epigraph $E_F = \left\{ (f, t) \in R^2 : F(f) \leqslant t \right\}$ is non-convex. In the next proposition, we describe how the convexity of E_F is obtained. A more detailed information can be found in Aktürk et al. (2009) and Günlük and Linderoth (2010).

Proposition 1. *The convex hull of E_F can be expressed as*

$$t \geqslant (c_1 \cdot q + c_2 \cdot \delta + c_3 \cdot \phi + c_4 \cdot v) \tag{3.16}$$

$$w^2 \leqslant q \cdot f \tag{3.17}$$

$$w^4 \leqslant f^2 \cdot \delta \cdot 1 \tag{3.18}$$

$$f^4 \leqslant w^2 \cdot \phi \cdot f \tag{3.19}$$

$$f^2 \leqslant v \cdot w \tag{3.20}$$

in the constraint set. Moreover, each inequalities (3.16)–(3.20) can be represented by conic inequalities.

Proof. Perspective of a convex function $F(f)$ is $wF(f/w)$ (Hiriart-Urruty and Lemaréchal, 2001). Since each of the nonlinear terms $\frac{1}{f}$, $\frac{1}{f^2}$, f^3 and f^2 is a convex function for $f \geqslant 0$, then epigraph of the perspective of each term can be stated as $\frac{w^2}{f} \leqslant q$, $\frac{w^4}{f^2} \leqslant \delta$, $\frac{f^3}{w^2} \leqslant \phi$, and $\frac{f^2}{w} \leqslant v$, respectively, where q, δ, ϕ and v are the variables required for defining epigraphs. Since w, $f \geqslant 0$, they can be written as stated in the proposition.

Finally, observe that (3.17) and (3.20) are hyperbolic inequalities, (3.18) can be restated as two hyperbolic inequalities

$$w^2 \leqslant u \cdot f \text{ and } u^2 \leqslant \delta \cdot 1$$

and (3.19) can be restated as

$$f^2 \leqslant u \cdot w \text{ and } u^2 \leqslant \phi \cdot f$$

which can be written as a conic quadratic inequality. $\square$

We now have a linear fuel cost in the objective function with a set of conic inequalities given in (3.22)–(3.25) as in Aktürk et al. (2014). Furthermore, we take the flight time as the sum of cruise and non-cruise times. In our integrated model, the cruise time of each flight is a decision variable, whereas the non-cruise time of each flight $i \in F$, NC_i, is a given parameter. Arikan et al. (2013) performed an extensive study on airline flight schedules across several U.S. airlines and showed that the log-Laplace distribution provides a good fit to the block time of a flight. Therefore, in our computational study, the non-cruise time of each flight, NC_i, is taken from a log-Laplace distribution. Consequently, probabilistic chance constraints could be represented in a closed form and written as second order conic inequalities in (3.26) as discussed in Duran et al. (2015). $F_X^{-1}(\gamma_{ij})$ is the value of quantile function for log-Laplace distribution at γ_{ij}. After the reformulation, we could solve the following mathematical model using a commercial solver such as IBM ILOG CPLEX.

$$\min \quad \sum_{t \in T} \sum_{i \in F} \left(\sum_{j \in U^i} x_{ji}^t + y_i^t \right) \cdot Cspl_i.max(0, Dem_i - Cap_t)$$

$$+ \sum_{i \in F} \sum_{t \in T} (c_{fuel} + c_{CO_2}) \cdot (c_1^{i,t} q_i^t + c_2^{i,t} \delta_i^t + c_3^{i,t} \phi_i^t + c_4^{i,t} v_i^t) \tag{3.21}$$

$$+ \sum_{i \in F} \sum_{t \in T} y_i^t \cdot Daily_t + \sum_{i \in F} \sum_{t \in T} s_i^t \cdot Idle_t$$

$$\text{s.to} \quad \left(y_i^t + \sum_{j \in U^i} x_{ji}^t \right)^2 \leqslant q_i^t \cdot f_i^t \qquad \forall i \in F,\ t \in T \tag{3.22}$$

$$\left(y_i^t + \sum_{j \in U^i} x_{ji}^t \right)^4 \leqslant (f_i^t)^2 \cdot \delta_i^t \cdot 1 \qquad \forall i \in F,\ t \in T \tag{3.23}$$

$$(f_i^t)^4 \leqslant \left(y_i^t + \sum_{j \in U^i} x_{ji}^t \right)^2 \cdot \phi_i^t \cdot f_i^t \qquad \forall i \in F,\ t \in T \tag{3.24}$$

$$(f_i^t)^2 \leqslant v_i^t \cdot \left(y_i^t + \sum_{j \in U^i} x_{ji}^t \right)^2 \qquad \forall i \in F,\ t \in T \tag{3.25}$$

$$d_j - d_i - \sum_{t \in T} f_i^t - TP_{ij} \geqslant F_X^{-1}(\gamma_{ij}) \qquad \forall i \in F,\ j \in P_i \tag{3.26}$$

$$(3.3)-(3.6),\ (3.8)-(3.15)$$

4. Heuristic methods

Although we could solve the small instances using the proposed integrated robust airline scheduling, aircraft fleeting and routing model, we could face some numerical stability problems due to the large variability of the problem parameters. Therefore, two heuristic methods are proposed to handle large size instances.

4.1. Discretized approximation and cruise speed control algorithm

To solve our problem faster than the integrated model, we propose discretized approximation model (hereafter DAM) which excludes nonlinearity caused by the cost function of fuel consumption and CO_2 emission. In this algorithm, the controllable cruise times, which are defined by a continuous function in the integrated model, are approximated through a set of discrete cruise time settings.

DAM is a mixed integer programming model which solves robust airline scheduling, aircraft fleeting and routing problems simultaneously. As distinct from the integrated model, in DAM, cruise times can only take a predetermined value from an interval rather than any value from that interval. For example, while cruise time of a flight in the integrated model can take any value between 85 and 115 min, in DAM the cruise time can be in increments of five minutes such as 85, 90, 95, 100, 105, 110 and 115. In this way, instead of a nonlinear cost function, we calculate the price of each cruise time option and use linear terms to denote the fuel consumption and CO_2 emission costs in the objective function. Consequently, DAM can be solved faster than the integrated model.

In order to adapt these changes, we introduce the following additional parameters and a set of binary variables for each flight and aircraft:

---

crs_{ik}^t: The kth cruise time option of flight i when it is performed by aircraft t
$cost_{ik}^t$: The cost of crs_{ik}^t which is equal to $(c_{fuel} + c_{CO_2}) \cdot F(crs_{ik}^t)$
σ_{ik}^t: 1 if cruise time of flight i takes the kth setting for aircraft t and is 0 ow.

The formulation of DAM is as follows:

$$\min \quad \sum_{t \in T} \sum_{i \in F} \left(\sum_{j \in U^i} x_{ji}^t + y_i^t \right) \cdot Cspl_i \cdot max(0, Dem_i - Cap_t) +$$

$$\sum_{i \in F} \sum_{t \in T} \sum_{k=1}^{p} \sigma_{ik}^t \cdot cost_{ik}^t + \sum_{i \in F} \sum_{t \in T} y_i^t \cdot Daily_t + \sum_{i \in F} \sum_{t \in T} s_i^t \cdot Idle_t \tag{4.1}$$

$$\text{s. to} \quad \sum_{k=1}^{p} \sigma_{ik}^t = \left(y_i^t + \sum_{j \in U^i} x_{ji}^t \right) \qquad \forall i \in F, \ t \in T \tag{4.2}$$

$$\sum_{k=1}^{p} \sigma_{ik}^t \cdot crs_{ik}^t = f_i^t \qquad \forall i \in F, \ t \in T \tag{4.3}$$

$$(3.3)-(3.6), \ (3.8)-(3.15), \ (3.26)$$

In DAM, the objective function is the sum of spill cost, the costs of fuel consumption and CO_2 emission, daily usage cost and idle time cost. As different from the integrated model, the cost of fuel consumption and CO_2 emission is represented by the linear term $\sum_{i \in F} \sum_{t \in T} \sum_{k=1}^{p} \sigma_{ik}^t \cdot cost_{ik}^t$. Constraints (4.2) and (4.3) ensure that when flight i is performed by aircraft t, the cruise time of that flight takes one of the discrete cruise time settings. The remaining constraints are same as with the integrated model.

We could solve the robust airline scheduling, aircraft fleeting and routing problem with DAM in a reasonable computation time. However, since we have discretized cruise time in DAM, there is still a chance to improve that solution by considering a continuous value of cruise time. Therefore, after fixing fleeting and routing with DAM, in order to decide continuous values of cruise time, we solve the cruise speed control model (hereafter CSCM), which is suggested by Duran et al. (2015). CSCM is a nonlinear second order cone programming model which solves robust airline scheduling problem by considering departure timing, cruise time control and idle time insertion. Since the fleeting and routing decisions are made by DAM, CSCM only deals with continuous decision variables. Therefore, even if it is a nonlinear model it can still be solved faster than the integrated model.

In order to use the fleeting and routing generated by DAM as parameters of CSCM, we introduce the following additional notation:

$\bar{w}_i^t$: A binary parameter that is 1 if flight i is performed by aircraft t in the solution of DAM
$\bar{A}$: Set of flights (i,j) such that (i,j) are two consecutive flights performed by the same aircraft in DAM

The formulation of CSCM is as follows:

$$\min \sum_{i \in F} \sum_{t \in T} \bar{w}_i^t \cdot \left((c_{fuel} + c_{CO_2})F(f_i^t) + s_i^t \cdot Idle_t \right) \tag{4.4}$$

$$\text{s.to } x_j - x_i - TA_{ij} - \sum_{t \in T} \bar{w}_i^t \cdot f_i^t - E[A_i] - \sum_{t \in T} \bar{w}_i^t \cdot s_i^t = 0 \qquad \forall (i,j) \in \overline{A} \tag{4.5}$$

$$f_i^l \cdot \bar{w}_i^t \leqslant f_i^t \leqslant f_i^u \cdot \bar{w}_i^t \qquad \forall i \in F,\ t \in T \tag{4.6}$$

$$s_i^t \leqslant M.\bar{w}_i^t \qquad \forall i \in F,\ t \in T \tag{4.7}$$

$$(3.8),\ (3.12),\ (3.13),\ (3.26)$$

In the objective function of CSCM, the costs of idle time insertion, fuel consumption and CO_2 emission are minimized. Constraint (4.5) ensures the aircraft connection for the routing generated by DAM. If aircraft t performs flight i as a result of DAM, constraint (4.6) keeps the cruise time of each flight between the upper and lower limits, otherwise f_i^t value is set to zero. Similarly, if aircraft t performs flight i as a result of DAM, then s_i^t can be nonnegative, otherwise it is set to zero by constraint (4.7).

In discretized approximation and cruise speed control algorithm (hereafter Heuristic 1), we solve two models sequentially. In Fig. 3, we illustrate the flow chart of the algorithm. Initially we solve the DAM and fix the fleeting and routing decision, then we solve the CSCM in order to find the minimum sum of idle time, fuel consumption and CO_2 emission costs, e.g. corresponding cruise time, for a given fleeting and routing decision. In this way, instead of solving the integrated model, which is a nonlinear MIP model; as a heuristic method we propose to solve first a MIP model, e.g., DAM, and then a nonlinear model, CSCM, so that total solution time is quite smaller than the integrated model as discussed earlier.

4.2. Multi-stage triplet search algorithm

As another observation, when we remove the cost terms related to the continuous decision variables from the objective function of the integrated model, we observe that solution time decreases drastically. This is due to two reasons. First, by removing fuel consumption and CO_2 emission costs, we exclude nonlinearity and get a MIP model as in the case of DAM. Second, when we remove the cost term of the idle time insertion as well as the cost term related to fuel consumption, aircraft fleeting and routing decisions are given more easily. To elaborate, without idle time, fuel consumption and CO_2 emission costs, the model only considers the tradeoff between daily usage and spill cost so it neglects the costs of idle time insertion and cruise time change. Therefore, we introduce daily usage and spill costs model (hereafter DSCM) in which the idle time insertion, fuel consumption and CO_2 emission costs are removed. All constraints of the integrated model in Section 4.1 are valid in DSCM, however the cost terms $\sum_{i \in F} \sum_{t \in T} (c_{fuel} + c_{CO_2}) \cdot F_i^t(f_i^t)$ and $\sum_{i \in F} \sum_{t \in T} s_i^t \cdot Idle_t$ are extracted from the objective function.

Although we get a non-dominated solution regarding daily usage and spill costs by DSCM, the generated solutions might have high cost of idle time and fuel consumption since DSCM takes idle time and fuel as free. In order to enhance a solution generated by DSCM, the cost of fuel and idle time should be taken into account. Thus, we take the solution of DSCM and then by fixing fleeting and routing of DSCM, we solve CSCM which is presented in the previous section.

Even if solving CSCM after DSCM can improve the cost of idle time and fuel, this improvement is local since CSCM is solved within the fixed fleeting and routing of DSCM. By driving DSCM to generate fleeting and routing which are also favorable regarding the costs of idle time and fuel, the sequential solution of DSCM and CSCM can find better solutions. For this
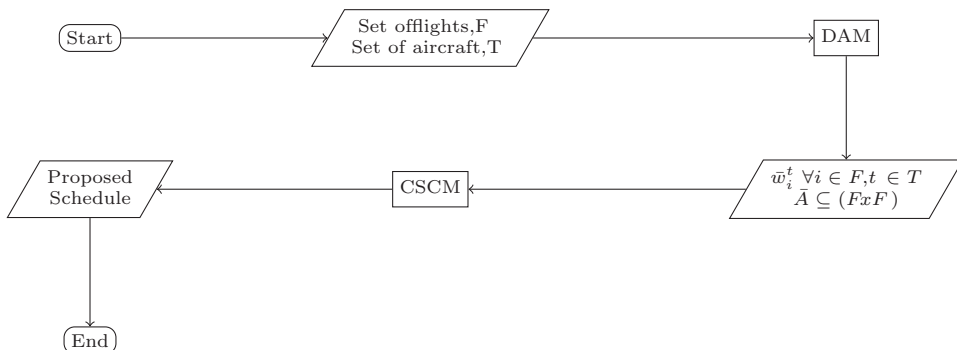

Fig. 3. Discretized approximation and cruise speed control algorithm.

purpose, initially we introduce triplet which is a constituent of a fleeting and routing in an airline schedule, then present a search algorithm over triplets to improve the solutions generated by the sequential solution of DSCM and CSCM.

Definition 1. A triplet, (i,j,t) is a collection of two consecutive flights i and j and the aircraft t which performs them. A triplet has the cruise times and idle time information regarding the flights in the triplet.

The cost of a triplet is calculated as follows: the fuel consumption and CO_2 emission costs are calculated for two flights and the idle time cost of the aircraft between these flights are summed and then the minimum required fuel consumption and CO_2 emission costs for these flights are extracted. The minimum required fuel consumption and CO_2 emission costs are calculated by assuming these flights are performed by the most efficient aircraft in the set of aircraft at optimum speed. Therefore, the cost of each triplet can be thought as the improvement capacity on the cost of that triplet.

In DSCM, each decision variable x_{ij}^t refers to a triplet so by fixing $x_{ij}^t = 0$ beforehand, we avoid DSCM to have triplet (i,j,t) in the generated solution. In this manner, we present multi-stage triplet search algorithm in which we employ a variable-fixing idea, and solve DSCM and CSCM models sequentially.

Algorithm 1: Two Stage Algorithm

    Input: Triplet (i,j,t), Set of Aircraft T, Set of Flights F
1 Fix corresponding $x_{ij}^t = 0$ for the given triplet (i,j,t);
2 Solve DSCM;
3 Set fleeting and routing;
4 Solve CSCM;
5 if *The problem is feasible* **then**
     | **Output**: The generated schedule
6 else
     | **Output**: null
7 end

Heuristic 2: Multi-Stage Triplet Search Algorithm

    Input: Set of Aircrafts T, Set of Flights F, Beam Size b, Depth Size d
1 Two stage algorithm (null, T, F);
2 Record the output solution of two stage algorithm to the set of solutions;
3 Generate the set of triplets from that solution, say S;
4 $l = 0$;
5 while a feasible solution exists or $l \leqslant d$ **do**
6 | Choose first b triplets with highest cost from S;
7 | Clear S;
8 | $k = 0$;
9 | **while** $k \leqslant b$ **do**
10 | Two stage algorithm (kth triplet, T, F);
11 | Record the generated solution to the set of solutions;
12 | Add generated triplets from two stage algorithm solution to S;
13 | $k = k + 1$;
14 | **end**
15 | $l = l + 1$;
16 end
    Output: The best solution from the set of solutions

The multi-stage triplet search algorithm, as illustrated in Heuristic 2, takes a set of aircraft and a set of flights with given beam size b and depth size d as an input. As an initialization step at the root node, we apply two stage algorithm, given in Algorithm 1, without a triplet. Afterwards, the set of triplets, S, is generated from the root node solution. After initialization step, a procedure which repeats itself for each depth starts. At the beginning of each depth, we have a set of triplets coming from the previous step, S, note that for the first depth, set of triplets comes from the root node solution. From S, we choose first b triplets with highest cost and then remove them from the solution set. For each chosen triplet, b times at total for a depth, we apply two stage algorithm to get a solution and add the generated triplets to S for the next depth. Since we apply this procedure d times, we get one solution from the root node and b solutions at each depth. In sum, we eventually report the best solution among $b \cdot d + 1$ feasible solutions generated by the multi-stage triplet search algorithm.

5. Computational study

In this section, we compare the performances of three solution methods which are the integrated approach and two heuristics, in terms of different airline cost components against the published schedule along with the required CPU times. In this computational study, there are four experimental factors, and their corresponding levels are given in Table 5.

The first factor is the fuel cost, which is the price of jet fuel per ton. The fuel prices are taken as \$1.8/gallon for the lower setting and \$3.6 gallon for the higher setting, or accordingly \$600/ton and \$1200/ton, respectively. Another important factor is the opportunity cost of spilled passengers of flight i, $Cspl_i$. We again consider two different levels of base spill cost, denoted as Factor B. The base spill cost is used as a multiplier to calculate the spill cost of a flight per passenger as discussed in a recent study conducted by Duran et al. (2015). One of the main contributions of the integrated approach is to improve the aircraft utilization by solving the aircraft scheduling, routing and fleeting problems simultaneously along with the cruise speed control decisions. On the other hand, there is a significant tradeoff between the aircraft utilization and overall operational expenses, such as fuel and carbon emission costs. Due to a possible increase in the utilization of a fuel efficient aircraft, total fuel consumption cost could be decreased for the entire flight network in albeit of additional spill costs. Consequently, we need to quantify the aircraft utilization along with the operational expenses in our objective function. We have used the parameter $Idle_t$ for each aircraft type t as a surrogate parameter to evaluate a possible tradeoff between the aforementioned aircraft utilization and operational expenses in our proposed mathematical model. Since it is difficult to accurately estimate these parameters, we have taken the daily usage cost of each aircraft type, $Daily_t$, as one of the experimental factors. The third experimental factor, v, is used for the calculation of the daily usage cost of an aircraft. In our study, we consider the daily usage cost of an aircraft which refers to the sum of the fixed operating costs and the lost opportunity cost when that aircraft is used as discussed in Section 2. We calculate daily usage cost related to the unit idle time cost since it is rational that a valuable aircraft has both high unit idle time cost and daily usage cost. For each aircraft type, we set its daily usage cost to low and high levels, respectively, to observe its impact on the overall results as follows.

$$Daily_t = v \cdot Idle_t \cdot 60 \tag{5.1}$$

The last factor is the minimum desired service level of the passengers' connections. As distinct from the first three factors, this factor has three levels. While the first three factors have linear effects on the objective function, the effect of service level on the objective function is not as straightforward as the other factors since it is not a direct cost term of the objective function. Therefore, in order to reveal its effect in the experimental design, we choose to adopt three levels on this factor.

In this study, we use the flight network, which is in the work of Aktürk et al. (2014). The published schedule of a major U.S. airline is provided in Table 6. The flight information were taken from the Bureau of Transportation Statistics (BTS) database (BTS, 2010). In the published schedule, each column represents the tail number, flight number, departure and arrival airport, departure time, flight block time and arrival time of each flight respectively. From the published schedule, we take the set of flights and set of aircraft as input in our model.

Among these 114 flights, we generate three different flight networks. The first one is 23 flight network; the second is 35 flight network and the last one is 114 flight network as given in Table 6. We conduct all computational study over these flight networks. Furthermore, we have taken 3 replications for each factor combination, e.g. out of $2 \cdot 2 \cdot 2 \cdot 3$ combinations in a full-factorial experimental design, resulting in 72 randomly generated runs for each flight network.

5.1. Analysis on the integrated model

We start the analysis on the results of our computational study by pointing out the significance of our integrated approach. For this purpose, we compare the cost of the published schedule, which is generated by the sequential approach, and the proposed schedule, which is the output of our proposed integrated robust airline scheduling, aircraft fleeting and routing model with cruise speed control.

We first analyze the 23 flight network in terms of operational efficiency. In particular, this small network consists of 23 flights that are performed by seven aircraft. The tail numbers of these aircraft are N3DUAA, N5DXAA, N412AA, N531AA, N4XGAA, N544AA and N3ELAA. The corresponding flights are available in Table 6. In this schedule, the average number

Table 5
Factor values.

Factors	Description	Levels		
		Low(0)	Medium(1)	High(2)
A	Fuel cost	\$600	–	\$1200
B	Base spill cost	\$15	–	\$60
C	v	1	–	10
D	Service level	75%	85%	95%

Table 6
Published schedule for 114 flight network.

Tail #	Flight #	From	To	Dep. time	Dur.	Tail #	Flight #	From	To	Dep. time	Dur.
N530AA	398	ORD	LGA	06:15	134	N3ETAA	1704	ORD	EWR	06:35	125
	319	LGA	ORD	09:25	170		1883	EWR	ORD	09:30	160
	2329	ORD	DFW	13:35	155		810	ORD	DCA	13:10	105
	2364	DFW	ORD	17:00	150		2013	DCA	ORD	15:45	135
N459AA	394	ORD	LGA	06:50	135	N3DYAA	2013	ORD	LAS	19:00	250
	321	LGA	ORD	10:00	170		1063	ORD	LAX	08:50	275
	366	ORD	LGA	13:55	140		874	LAX	ORD	14:30	255
	347	LGA	ORD	17:15	170		874	ORD	BOS	19:45	135
N531AA	2303	ORD	DFW	06:45	155	N3DRAA	1021	ORD	LAS	08:30	245
	2336	DFW	ORD	10:10	140		1544	LAS	ORD	13:25	215
	1053	ORD	AUS	13:25	170		1544	ORD	DCA	18:00	105
	336	AUS	ORD	17:00	165	N5DXAA	1048	ORD	MIA	07:35	190
	336	ORD	LGA	20:40	125		1763	MIA	ORD	11:55	200
N4XGAA	2079	ORD	SAN	08:45	270		1899	ORD	MIA	16:20	185
	1438	SAN	ORD	14:00	250	N454AA	2441	ORD	ATL	06:30	120
	346	ORD	LGA	19:50	135		1986	ATL	ORD	09:15	135
N598AA	1341	ORD	SFO	07:50	295		1872	ORD	MCO	12:25	160
	348	SFO	ORD	13:30	265		1131	MCO	ORD	15:50	185
	1521	ORD	TUS	19:15	235	N4YMAA	1137	ORD	MSY	08:20	145
N439AA	2455	ORD	PHX	07:10	240		1768	MSY	ORD	11:30	150
	358	PHX	ORD	11:55	210		1768	ORD	PHL	15:05	125
	358	ORD	LGA	16:25	145		1697	PHL	ORD	18:00	155
	371	LGA	ORD	20:00	155	N467AA	1823	ORD	PBI	09:20	175
N475AA	407	ORD	STL	06:20	70		2067	PBI	ORD	13:00	200
	755	STL	ORD	08:35	75		2067	ORD	STL	17:15	70
	755	ORD	SAT	10:45	180		1186	STL	ORD	19:10	80
	408	SAT	ORD	14:30	160	N536AA	2305	ORD	DFW	07:45	160
	408	ORD	PHL	18:05	125		2344	DFW	ORD	11:35	140
N3EEAA	876	ORD	BOS	06:35	130		1201	ORD	STL	14:50	65
	413	BOS	ORD	09:35	185		1815	STL	ORD	17:00	80
	413	ORD	SNA	13:45	275		1815	ORD	SLC	19:15	270
	1262	SNA	ORD	19:10	230	N420AA	1686	ORD	RDU	06:50	110
N4YDAA	451	ORD	SFO	09:45	295		2435	RDU	ORD	09:25	135
	554	SFO	ORD	15:45	265		2435	ORD	PHX	12:35	235
N3ERAA	496	ORD	DCA	06:45	100		1206	PHX	ORD	17:15	205
	1715	DCA	ORD	09:15	130	N546AA	1462	ORD	EWR	08:00	140
	1715	ORD	LAS	12:25	255		1387	EWR	ORD	11:25	160
	1708	LAS	ORD	17:20	220		1397	ORD	MCO	15:00	160
N5CLAA	1425	ORD	SNA	08:25	280		1221	MCO	ORD	18:25	175
	556	SNA	ORD	14:00	240	N4WPAA	2311	ORD	DFW	09:05	155
	1940	ORD	MIA	19:25	180		2348	DFW	ORD	12:35	140
N535AA	2460	ORD	RSW	06:45	165		1797	ORD	STL	15:50	70
	564	RSW	ORD	10:20	185		1982	STL	ORD	18:00	80
	1446	ORD	EWR	14:55	165		1339	ORD	SAN	20:15	270
	1411	EWR	ORD	18:45	165	N5EBAA	2375	ORD	EGE	08:10	175
N3DMAA	568	ORD	FLL	07:25	175		2378	EGE	ORD	12:25	165
	711	FLL	ORD	11:10	195		1677	ORD	SNA	18:40	270
	2021	ORD	SJU	15:25	275	N3DUAA	2099	ORD	LAX	07:00	270
N544AA	2463	ORD	MCI	06:25	90		1972	LAX	ORD	12:40	245
	754	MCI	ORD	08:40	90		1972	ORD	RDU	17:45	115
	2321	ORD	DFW	11:15	155	N3ELAA	2057	ORD	SJU	08:30	290
	2356	DFW	ORD	14:40	140		2078	SJU	ORD	14:25	335
	2487	ORD	DEN	17:50	165	N3DTAA	2363	ORD	HDN	09:50	170
N3EBAA	1565	ORD	MSP	06:40	90		2318	HDN	ORD	13:40	170
	779	MSP	ORD	09:00	85	N412AA	2345	ORD	DFW	17:15	155
	779	ORD	SAN	11:35	260		2374	DFW	ORD	20:40	130
	1358	SAN	ORD	16:45	235						
	1358	ORD	BOS	21:50	125						

of flights operated by a single aircraft is approximately 3. The average utilization of these aircraft is around 73%. Note that we consider the utilization of an aircraft as the ratio of the total time spent in the air, i.e., total cruise time to the total available cruise time for that aircraft denoted by λ_t in our problem formulation.

We next track the changes of the proposed model at these performance measures in order to interpret the impact of the integrated approach as well as the cruise time flexibility. Table 7 illustrates the values for these measures of the proposed schedule with respect to the different factor values. As the results over all instances row indicates that the integrated model yields higher aircraft utilization while decreasing the total number of aircraft required to perform these flights. Moreover, we provide the average cruise time change per flight because it is worth to note that when the daily usage cost factor is high (i.e, $C = 2$), the model achieves to decrease the total number of aircraft in all instances by controlling cruise time more aggressively.

It is clear that the integration and cruise time change increase the efficiency of the flight schedule in comparison to sequential approach however it is intricate to evaluate how valuable this increase because of the network effect and the non-trivial cost function of the fuel consumption. As a next step, we provide how these changes correspond to cost improvement. We calculate the cost improvement for total cost and other cost terms for each factor setting as follows:

$$\text{Improvement} = \frac{\text{Cost of Alternative} - \text{Cost of Our Model}}{\text{Cost of Alternative}} \qquad (5.2)$$

As it is shown in Table 8, our integrated model provides around 17% average improvement on the total cost of the published schedule over all factor levels. Moreover, from Table 8, the tradeoff between daily usage cost and fuel consumption and CO_2 emission costs can be deduced. When daily usage cost is high, the improvement over the fuel and CO_2 emission costs is limited around 4% at maximum and the cost improvement over daily usage cost is around 14% at minimum. That means in order to decrease total number of aircraft used, the model compresses the cruise time of flights in albeit of additional fuel consumption. Similarly, when the cost of fuel consumption and CO_2 emission is high, the average improvement on the daily usage cost decreases from 9% to 7% approximately. This is an important advantage of the proposed model, since we could now evaluate a possible tradeoff between the number of aircraft needed to cover all flights and the corresponding overall operational expenses.

5.1.1. Scenario analysis

The extensive computational study results intrigue several questions on model behavior and its performance. In this part, some insights into model dynamics are provided by realizing different scenarios.

Table 7
Efficiency of the proposed model.

		Avg. aircraft util. (%)			Avg. # of flights per route			Avg. cruise time change per flight (%)			# of aircraft used		
		Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.
A	0	72	79	84	3.29	3.61	3.83	0.82	1	1.30	6	6.42	7
	2	72	78	84	3.29	3.56	3.83	0.83	1	1.09	6	6.5	7
B	0	72	78	84	3.29	3.56	3.83	0.95	1.02	1.30	6	6.5	7
	2	72	79	84	3.29	3.61	3.83	0.82	0.98	1.09	6	6.42	7
C	0	72	73	84	3.29	3.33	3.83	0.82	0.94	1.06	6	6.92	7
	2	84	84	84	3.83	3.83	3.83	1.04	1.06	1.3	6	6	6
D	0	72	79	84	3.29	3.58	3.83	0.82	1.01	1.09	6	6.46	7
	1	72	79	84	3.29	3.58	3.83	0.82	1.01	1.30	6	6.46	7
	2	72	79	84	3.29	3.58	3.83	0.82	0.99	1.05	6	6.46	7
All ins.		72	79	84	3.29	3.58	3.83	0.82	1	1.30	6	6.46	7

Table 8
Cost improvement over the published schedule.

		Fuel and CO_2 (%)			Idle time (%)			Daily usage (%)			Total (%)		
		Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.
A	0	4.2	8.3	14.5	89.3	93.8	96	0	8.6	14.7	14.9	18	22.8
	2	4.2	9.8	14.5	89.3	93.5	95.7	0	7.4	14.7	13.4	16	19.8
B	0	4.2	9.3	14.5	89.3	93.4	95.7	0	7.4	14.7	14.5	17.9	22.8
	2	4.2	8.2	14.5	89.3	93.8	96	0	8.6	14.7	13.4	16	21.2
C	0	4.2	13.4	14.5	89.2	93.1	96	0	1.23	14.7	13.7	18.9	22.8
	2	4.2	4.2	4.2	91.8	94	95.7	14.7	14.7	14.7	13.4	15.1	16.6
D	0	4.2	8.8	14.5	92.2	94.6	96	0	8	14.7	13.5	17.1	22.8
	1	4.2	8.8	14.5	91.3	93.9	95.4	0	8	14.7	13.4	17	22.6
	2	4.2	8.8	14.5	89.3	92.3	93.8	0	8	14.7	13.4	16.9	22.3
All ins.		4.2	8.8	14.5	89.3	93.6	96	0	8	14.7	13.4	17	22.8

What if cruise time change is not allowed?

We continue the analysis on the results of our computational study by indicating what would happen without cruise time control. By this means, we present the analysis on the integrated model and compare its performance with other integrated models, which do not consider cruise speed control. For a fixed cruise time setting, we also consider two models. Both models are integrated ones such that airline scheduling, aircraft fleeting and routing decisions are made within the consideration of passengers' service level and maintenance requirements. The first model tries to minimize the sum of spill cost, fuel consumption and CO_2 emission cost, idle time cost and daily usage cost without cruise time control. The second one tries to minimize the sum of spill cost, idle time cost and daily usage cost but considers neither fuel consumption nor CO_2 emissions cost nor cruise time control. Hereafter, Alternative1 refers to the model, which considers fuel and CO_2 emission cost but not cruise time control, and Alternative2 refers to the model, which considers neither fuel nor CO_2 emissions cost nor cruise time control.

In Table 9, we present the total cost improvements provided by the cruise time control strategy over 23 flight network. When all the results are considered, it is seen that cruise time control can achieve 8% and 10% average cost improvement over Alternative1 and Alternative2, respectively. When we examine the effect of different factor levels, the cost improvement of cruise time control is remarkably increasing depending on factor C. That means, when higher utilizations of aircraft are more valuable, cruise time control is getting more crucial. The essence of the matter is that cruise time control can decrease the total cost around 9% and this average improvement can increase up to 11% when the daily usage cost is higher.

What if the same service levels with the published schedule are used?

Another critical question that might arise is whether the cost improvements over published schedule are achieved by the effect of service level changes in the proposed schedule or not. While comparing the proposed schedule and the published schedule previously, we take the service level as a factor of the experimental design. However, in this part, we use the same minimum service levels with the published schedule on each passenger connection in the proposed schedule.

In Table 10, it is seen that there is around 14% improvement on the average over the published schedule even though we use the same service levels on passenger connections in the proposed schedule. This improvement is achieved by considering cruise time change along with the fleeting and assignment decisions in the integration of these subproblems. Therefore, it can be deduced that although the same service levels of the published schedule are used, it is possible to improve the total cost by using this novel approach.

5.2. Analysis on heuristic methods

In this section, we analyze the proposed heuristic methods in terms of gap and improvement. Initially, the gap between the integrated model and the heuristic methods are examined with 23 flight network. We calculate the gap between the optimal solution of the integrated model and the solution of heuristic methods as follows:

Table 9
Total cost improvement of cruise speed control.

		Imp. over Alt1 (%)			Imp. over Alt2 (%)		
		Min.	Avg.	Max.	Min.	Avg.	Max.
A	0	3.9	8.8	12.4	4.9	9.9	13.0
	2	4.7	7.9	10.4	7.7	11.2	13.1
B	0	4.7	8.2	12.4	7.7	10.6	13.0
	2	3.9	8.5	12.1	4.9	9.2	12.3
C	0	3.9	5.7	7.7	4.9	8.1	10.8
	2	9.7	11.0	12.4	10.4	11.7	13.0
D	0	4.2	8.5	12.4	5.3	10.0	13.0
	1	4.1	8.4	12.4	5.1	10.0	13.0
	2	3.9	8.3	12.4	4.9	9.8	13.0
All ins.		3.9	8.4	12.4	4.9	9.9	13.0

Table 10
Cost improvement over the published schedule with same service levels.

		Fuel and CO_2 (%)			Idle time (%)			Daily usage (%)			Total (%)		
		Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.
A	0	4.1	5.8	14.5	52.7	60.6	62.2	0	12.3	14.7	10.2	14.2	16
	2	4.1	9.3	14.5	52.8	57.5	62.2	0	7.4	14.7	9.7	13.3	15.4
B	0	4.1	7.6	14.5	52.8	59.1	62.2	0	9.8	14.7	13.3	14.7	16
	2	4.1	7.5	14.5	52.7	59	62.2	0	9.8	14.7	9.7	12.8	14.5
C	0	4.2	11	14.5	52.7	55.9	62.2	0	4.9	14.7	9.7	13.8	16
	2	4.1	4.2	4.2	62.2	62.2	62.2	14.7	14.7	14.7	12.1	13.7	15
All ins.		4.1	7.6	14.5	52.7	59	62.2	0	9.8	14.7	9.7	13.7	16

Table 11
Gap of heuristic methods over 23 flight network.

		Heuristic 1 (%)			Heuristic 2 (%)		
		Min.	Avg.	Max.	Min.	Avg.	Max.
A	0	0.000	0.000	0.001	0.000	0.3	2.1
	2	0.000	0.000	0.002	0.000	0.4	3.7
B	0	0.000	0.000	0.002	0.000	0.5	3.7
	2	0.000	0.000	0.002	0.000	0.2	2.1
C	0	0.000	0.000	0.002	0.000	0.7	3.7
	2	0.000	0.000	0.000	0.000	0.000	0.001
D	0	0.000	0.000	0.001	0.000	0.4	3.7
	1	0.000	0.000	0.000	0.000	0.3	3.7
	2	0.000	0.000	0.002	0.000	0.2	2.1
All ins.		0.000	0.000	0.002	0.000	0.3	3.7

Table 12
Improvement of heuristic methods over 35 flight network.

		Heuristic 1 (%)			Heuristic 2 (%)		
		Min.	Avg.	Max.	Min.	Avg.	Max.
A	0	0.000	3.4	14.9	−0.041	3.3	14.9
	2	0.000	2.3	14.0	−0.291	2.1	13.6
B	0	0.000	2.6	14.9	−0.149	2.4	14.9
	2	0.000	3.0	13.9	−0.291	3	13.6
C	0	0.000	2.8	13.9	−0.291	2.5	13.6
	2	0.000	2.9	14.9	−0.149	2.9	14.9
D	0	0.000	3.1	14.9	−0.291	2.9	14.9
	1	0.000	3.1	13.9	−0.128	2.9	13.6
	2	0.000	2.4	13.2	−0.149	2.2	13.2
All ins.		0.000	2.9	14.9	−0.291	2.8	14.9

Table 13
Gap between the heuristic methods and best lower bound over 35 flight network.

		Heuristic 1 (%)			Heuristic 2 (%)		
		Min.	Avg.	Max.	Min.	Avg.	Max.
A	0	−5.783	−2.148	0.000	−6.002	−2.218	0.000
	2	−6.203	−2.655	0.000	−6.690	−2.859	0.000
B	0	−6.203	−2.601	0.000	−6.690	−2.828	0.000
	2	−5.707	−2.202	−0.002	−5.722	−2.248	−0.003
C	0	−6.203	−4.200	0.000	−6.690	−4.429	0.000
	2	−1.231	−0.603	−0.002	−1.406	−0.647	−0.003
D	0	−5.094	−2.091	0.000	−5.650	−2.244	0.000
	1	−5.059	−2.279	0.000	−5.346	−2.407	0.000
	2	−6.203	−2.835	−0.188	−6.690	−2.964	−0.188
All ins.		−6.203	−2.470	−0.002	−6.690	−2.611	−0.003

$$\text{Gap} = \frac{\text{Heuristic} - \text{Optimal}}{\text{Optimal}} \qquad (5.3)$$

As it can be seen from Table 11, the gaps of the heuristic methods are less than 1% on the average at all instances. It is worthwhile to point out that Heuristic 1 has 0.002% maximum gap over all factor levels and all instances. Although, Heuristic 2 has maximum gap 3.7% over all instances, when factors C and B are at their high levels, it has the maximum gap of 0.001% and 2.1%, respectively. This is due to the fact that Heuristic 2 fixes the routings and fleeting decisions by just considering daily usage and spill cost. When these cost terms dominate the other cost terms on routings and fleeting decisions, ignoring fuel and CO_2 emission and idle time costs at routing and fleeting gets less significant. Therefore, Heuristic 2 causes smaller gaps when factors B and C are at their high levels and factor A is at the low level.

Afterwards, we compare the performances of heuristic methods with the integrated model over 35 flight network to see a possible impact of a larger flight network. Due to computational requirements of the integrated model, we put a time limit of 5400 s for the exact approach and report the best incumbent solution generated by the integrated model in a given time limit. Consequently, we calculate the improvement of solutions of heuristic methods over the best solution as follows:

$$\text{Improvement} = \frac{\text{Cost of the Best Incumbent Solution} - \text{Cost of the Heuristic Method}}{\text{Cost of the Heuristic Method}} \tag{5.4}$$

As reported in Table 12, both heuristic methods now surpass the best incumbent solution of the integrated model on the average due to the time limit of 5400 s. While Heuristic 1 achieves 2.9%, Heuristic 2 achieves 2.8% average improvement over all instances. The negative improvement in Heuristic 2 means that the best solution generated in 5400 s by the integrated model has smaller objective value than Heuristic 2 for certain instances.

In Table 13, we compare the cost of the heuristic methods and the cost of the best upper bound obtained in 5400 s. However, the optimal value of the integrated model is between the best upper bound and the best lower bound. Therefore, we also provide a comparison of the performance of the proposed heuristic methods against the best lower bound obtained by the commercial solver in 5400 s. The gap between the cost of the heuristic methods and the best lower bound value is found by the following equation.

$$\text{Gap} = \frac{\text{Best Lower Bound} - \text{Cost of the Heuristic Method}}{\text{Cost of the Heuristic Method}} \tag{5.5}$$

It is worthwhile to note that the gap between the heuristic methods and the best lower bound is approximately 3% on the average for both heuristic methods. Together with the analysis over the best incumbent solution, this observation implies that the heuristic methods perform quite close to the integrated model over the 35 flight network as well as for the 23 flight network.

Finally, we analyze the heuristic methods over the 114 flight network. As discussed earlier, the integrated model cannot find a feasible solution in six hours so we compare the performance of the heuristic methods over the cost of published schedule in Table 14. It can be seen that both heuristic methods provide around 15% average improvement over all instances in comparison to the published schedule.

In the proposed model, for constraint (3.26), we take the minimum desired service levels for passenger connections, γ_{ij}, equal to the levels of factor D, e.g. 75%, 85% and 95%, respectively. On the other hand, we could also calculate the probability of successful passengers' connection for each connecting passenger, denoted as β_{ij}, by replacing Constraint (3.7) with the following constraints:

$$Pr\left[NC_i \leqslant d_j - d_i - \sum_{t \in T} f_i^t - TP_{ij} \right] \leqslant \beta_{ij} \qquad \forall i \in F, \; j \in P_i$$

$$\beta_{ij} \geqslant \gamma_{ij} \qquad \forall i \in F, \; j \in P_i$$

Table 14
Improvement of heuristic methods over the published schedule in 114 flight network.

		Heuristic 1 (%)			Heuristic 2 (%)		
		Min.	Avg.	Max.	Min.	Avg.	Max.
A	0	10.6	17.9	25.9	10.6	16.7	25.9
	2	9.8	14.6	19.8	9.2	12.3	15.9
B	0	11.3	17.1	25.9	9.8	14.9	23.5
	2	9.8	15.4	24.5	9.2	14.1	22.7
C	0	14.6	21.0	25.9	12.2	18.3	23.5
	2	9.8	11.5	12.4	9.2	10.7	11.9
D	0	9.9	16.4	25.9	9.4	14.6	23.5
	1	9.8	16.3	25.9	9.3	14.6	23.4
	2	9.8	16.0	25.3	9.2	14.2	22.6
All ins.		9.8	16.2	25.9	9.2	14.5	23.5

Table 15
CPU time analysis.

	Integrated model (in sec)			Heuristic 1 (in sec)			Heuristic 2 (in sec)		
	Min.	Avg.	Max.	Min.	Avg.	Max.	Min.	Avg.	Max.
23 FN	18	968	4048	0.8	1.1	1.8	10.5	12.4	15.3
35 FN	4243.4	5381.1	5400	3.2	5.7	7.8	31.6	40.2	54.7
114 FN	–	–	–	724.8	5173.4	5400	1702.6	3254.3	5400

Consequently, the achieved service levels of the proposed schedule are around 98%, 98% and 99%, respectively, on the average. However in the published schedule, each connection has a different service level and some of them are actually lower than even 75%, which is the lowest level of factor D in the computational study.

In summary, both heuristic methods have quite small optimality gaps over 23 flight network; while heuristic methods surpass the best solution of the integrated model in 5400 s over 35 flight networks. Moreover, heuristic methods work quite faster than the integrated model in terms of the CPU time. When the performance of the heuristic methods are compared, it can be said that Heuristic 1 surpasses the Heuristic 2 in terms of optimality gap and improvement over the best solution and published schedule over all flight networks. In Table 15, even for the CPU time, Heuristic 1 is more favorable than Heuristic 2 in 23 and 35 flight networks, however in 114 flight the CPU time of heuristic2 is 2000 s less than the CPU time of Heuristic 1 on the average, which could be attributed to the higher number of binary variables due to the discretization step in Heuristic 1. In other words, in 114 flight network even if the improvement of Heuristic 2 is 1.8% smaller than the improvement of Heuristic 1, Heuristic 2 works 2000 s faster on the average.

6. Conclusion

For an integration of robust airline scheduling, aircraft fleeting and routing problems, this novel consideration of cruise speed/time control enables us to make following contributions. First, aircraft utilization could be evaluated in terms of considering the entire flight network. We also show that total number of aircraft needed to cover a set of flights could be decreased while ensuring equivalent service level and maintenance requirements. Therefore, there is a significant tradeoff between aircraft utilization and overall operational expenses. Moreover, due to a possible increase in the utilization of a fuel efficient aircraft, total cost of fuel consumption could be decreased for the entire flight network. Second, the robustness can be achieved with smaller cost. To elaborate, on a route having a flight with a great delay probability would require idle time insertion for the following flights to be performed on time; while removing the problematic flight from that sequence could render that intervention unnecessary. Due to cruise speed control, our study has more options to make this type of changes on routing decisions.

In addition to the contributions due to cruise speed control, we put forward the following contributions regarding the solution of the problem. First, we propose a nonlinear mixed integer programming model and present its second order conic reformulation. Furthermore, to solve large scale problems in a reasonable computation time, we propose two heuristic methods, which are discrete approximation and cruise speed control algorithm, and multi-stage triplet search algorithm. Finally, we conduct a computational study in which we analyze these three proposed solution methods and compare their performances among each other and against the published schedule.

Since we deal with the integration of three airline schedule planning problems, one immediate future research direction can be integration of the last problem, crew assignment problem. Although there are different studies proposed to integrate the crew assignment problem to one of these three stages; as far as we know, there is not a study which integrates these four problems within the consideration of passengers' service levels and cruise speed control.

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