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Circular-Planned Diagrid Systems and an Interrelated Technique Using Planar Elements

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Abstract This paper presents the development of circular-planned diagrid systems in architecture and varying approaches in patents in relation to these developments, together with a technique introduced for producing diagrid implementations using planar elements. A circular plan creates a curved surface at its periphery that leads to the diagrid system on a curved surface, resulting in unique geometric configurations. Patents based on these geometric configurations and their details, registered between 1896 and 2016, are documented. A technique is devised by the authors as a reinterpretation of the diagrid system via planar components with its own parameters and principles. This technique may be utilized by design offices or by the students of architecture for prototyping or modelling a circular-planned diagrid structure in a precise, fast and economical manner by means of conventional CNC manufacturing techniques.

Keywords Diagrid systems · Algorithms · CAAD · CAD (grasshopper) · Geometry · Form making

An Introduction to Circular-Planned Diagrid Systems

The term "diagrid" is the combination of the terms "diagonal" and "grid", and a diagrid system is defined as a structure replacing vertical structural elements with diagonal ones, as a variation of the conventional orthogonal column-beam organization (Boake 2012: 131). In an orthogonal structure, columns and beams act under different sets of forces, like gravitational and lateral forces respectively. A diagrid system alters the directionality of vertical elements to diagonal. This enables

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the vertical elements to perform also under lateral forces. Horizontal elements may or may not be included in diagrid systems, depending on the structural conditions required by the design.

Diagrid systems can be utilised on both planar and curved surfaces. Although geometrically more complex than planar ones, the first implementations of diagrid systems were on circular plans, both for patents and as built examples. The first example of a circular-planned diagrid system was *Glashaus*, a pavilion at the *Cologne Deutscher Werkbund 1914* (Weston 2004: 40) (Fig. 1). It was designed by Bruno Taut as a quadrangulated concrete dome with a diameter of 11.06 m, and was raised on 14 columns (Nielsen 2015: 116). Although no slabs were present within the dome structure of the *Glashaus*, the diagonal wireframe formation of the dome created a circular-planned diagrid system. The diagrid formation was composed of linear concrete braces, forming the frames of 14 triangular and 98 quadrilateral coloured glass panels. Implementation of a diagrid system on the circular plan created intricate details because of the surface's curvilinear quality and the diagonal formations of the braces. There were numerous complex calculations made by Taut to form the overall structure by preserving the planarity of the quadrilateral glass panels (Nielsen 2015: 152), which is still a complicated task to realize today even via computational tools (Glymph et al. 2004).

Even before *Glashaus*, the first built example of a diagrid system in geometrical terms was the 37 m high *Shukhov Tower* in Novgorod, Russia, built in 1896 (Fig. 2). The tower was designed by Vladimir Shukhov using a special technique that Shukhov later patented (Beckh 2015: 66). Shukhov built more than 200 towers using the same technique, that range from 37 meters to 130 meters in height. In structural terms, Shukhov's towers are catenary lattice frames, but the geometric configurations of the

Fig. 1 Glashaus by Bruno Taut (image under Creative-Commons License 3.0)

Fig. 2 Shukhov Tower by Vladimir Shukhov—photograph taken by Sergei Arssenev. Licensed under Creative Commons Licence—attribution: https://creativecommons.org/licenses/by/3.0/. The image has not been modified

towers denote diagrid systems and they are considered the diagrid system's predecessors (Ritchie 2012). Shukhov had registered his technique through patents No. 1894, No. 1895 and No. 1896, all dated March 12th, 1899. They cover a variety of lattice shells for towers (Arssenev 2010). Among his patents, patent No. 1896 specifically defines lattice towers with linear elements, and it is important in denoting diagrid formations. Geometrically, vertical slanted linear elements around a circle in two opposing directions would create a rotational hyperboloid form (Pottmann et al. 2007: 297) and Shukhov has utilized this aspect in his structures.

With the *30 St. Mary Axe* building in London, UK, by Foster and Partners (Fig. 3), diagrid systems have become a primary structural system of the building (Foster et al. 2008: 272). Like conventional high-rise buildings, *30 St. Mary Axe* still has a central core. This core is accompanied by twelve pairs of peripheral diagonal columns resulting in a varying circular plan with column-free interiors. The cross-section of *30 St. Mary Axe* is formed from the combination of several concave arcs; the double-curved surface of the building differentiates diagrid nodes at each level together with connection angles and details (Boake 2014: 41). The structural diagrid organization of *30 St Mary Axe* has also been used in *Tornado Tower* and *Canton Tower*—the former by CICO Consulting Architects and Engineers, located in Doha, Qatar, and the latter by IBA, in Guangzhou, China—both built in 2008. Both towers' cross-sections are convex arcs, hence double-curved surfaces. They also have conventional structural core systems, but the final form of the towers is shaped by peripheral diagrid systems, enabling their architects to span larger column-free organizations in the interiors (Boake 2014: 33).

Fig. 3 30 St. Mary Axe Building by Foster and Partners—photograph taken by youflavio. Licensed under Creative Commons Licence—attribution: https://creativecommons.org/licenses/by/3.0/. The image has not been modified

There also exists another group of diagrid systems that govern planar surfaces. The first implementation of the diagrid system with planar organization in a high-rise orthogonal building dates to 1963, in the *IBM Building*, Pittsburgh, USA, designed by Curtis and Davis Associates (Fig. 4). The building has a rectangular plan, and the continuous aluminium diagrid system all around the building creates a tubular structure. Diagrid systems were initially utilized as cladding systems and as assisting structures on the perimeters of the buildings, functioning as exoskeletons. Similarly, in the *IBM Building*, the diagrid system acts both as a curtain wall and a structural system, enabling column-free interiors (Boake 2014: 25). In geometrical terms, the diagrid system of the *IBM Building* is composed of four planes with a single connection detail used throughout the system. However, it is still considered geometrically and structurally complex, as the angular connections of braces have created difficult joint details when compared to the system's orthogonal counter-parts, and the structural analysis of the system has led to numerous complex calculations (Robertson 2008: 67). The implementation of diagrid systems on circular plans makes the joint details even more complex as an outcome of surface curvilinearities. It is for this reason that, for a diagrid system, the main challenge becomes the manufacture of the joints and the mounting processes (Moon 2009: 403). The same challenge is valid even for prototypes.

This challenge is reflected in the patents that have sought to come up with innovative ways to build up diagrid systems. Patents on diagrid systems from 1899 to 2016 are listed in Appendix. In addition to patents related to circular-planned

Fig. 4 IBM Building by Curtis and Davis—photograph taken by Prof. Ivars Peterson, used by permission

diagrid systems, all patents dealing with diagrid configurations in geometrical terms have been included. In its broadest sense, the patents can be grouped into two: the first group dealing with the totality of diagrid systems and the second group focusing solely on the connection details. This grouping can be traced in a chronological pattern. While patents from 1899 to 2010 have mainly dealt with the totality of diagrid systems, from 2010 onwards patents only focused on joint details of the system, with specific materials and dimensions. The first group has also been categorized into two: patents dealing with planar surfaces and curved surfaces.

Patents dealing with planar surfaces were initiated by the patent of architect Alfred Butts, dated 1950. His patent proposes a diagonal construction system for walls and floors (Fig. 5). There are several more patents related to planar surfaces, which are differentiated by area of application, function, material, scale or connection details. Unlike patents on circular-planned systems, patents on planar surfaces have progressed before their architectural counterparts. However, all patents, whether circular-planned or planar, can also be traced through architectural examples previously built.

Besides the patents by Vladimir Shukhov, one of the first patents employing a circular-planned diagrid formation is by Ichiro Nozawa, in 1934. It is entitled "Arcuate truss" and describes a structural scheme for parabolic, semi-circular or dome-like forms utilising a network of standardized, diagonally positioned arched elements (Fig. 6). Nozawa's patent is differentiated from Shukhov's patents, as it defines various structures with specific final forms through discrete elements. There are other patents related to circular-planned diagrid configurations, but the architectural instances were predecessors when compared with patent registrations.

All patents proposing a configuration for the diagrid systems also deal with the joint details of the system in particular because of the joint detail's geometric complexity in a diagonal formation. After 2010, there is a common tendency among

Fig. 5 Alfred Butt's patent (US patent 2534852A)
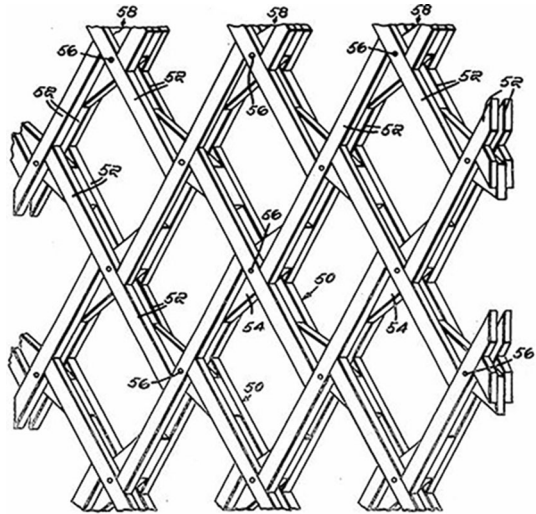
Fig. 6 Ichiro Nozawa's Arcuate Truss patent (US patent 1976188)
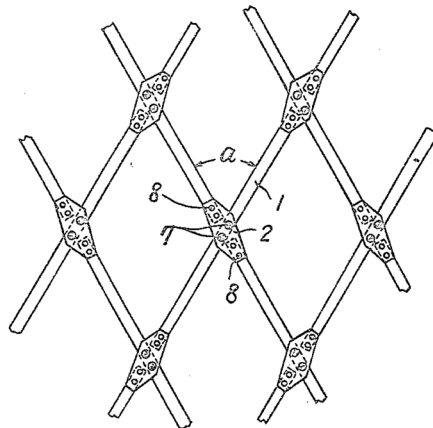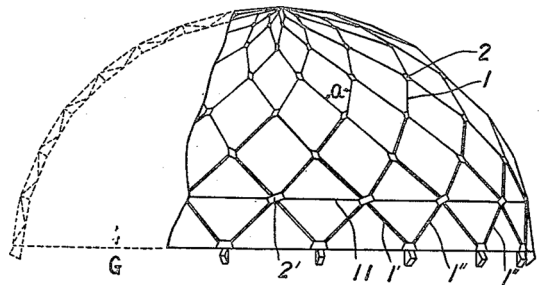
patents to replace the focus of the patent from the overall configurations of the diagrid system to joint details. This shift of interest displays the significance of joint details in diagrid systems. The patents that deal with joints mainly focus on the

elements' cross-sections, their materials or their connections. Such patents often propose a common single detail for every joint of the system. In a planar surface, a single detail can be suitable for implementing the whole system. However, if the patent is dealing with curved surfaces, the joints should be differentiated specifically for each connection through their angles or dimensions, and this would turn the proposed detail into a generic one for the system. The stated condition is a geometrical characteristic of diagrid systems and it is valid for all stages of design, even for prototyping or modelling.

The Proposed Technique

The proposed technique aims to present a geometrical variation of diagrid systems together with their joints. This study focuses on structures with circular plans due to their relatively complex geometry and a wide range of application possibilities in architecture (Steadman 2015). A circular-planned surface can be either single or double-curved, depending on the cross-section. While a vertical straight line as a cross-section for a circular plan would end up with a single-curved cylinder shape, a curve generates a double-curved surface. Besides the cylinder, any straight line or a jagged line with a circular plan would be a single-curved surface.

In all the analysed projects and patents, there is a common tendency to form the diagrid system through discrete elements from node to node, and this inevitably breaks the axial continuity of elements. Moon states that it is highly advantageous to maintain the axial continuity in diagrid systems (2005: 80). Within this framework, this study is an effort to obtain uniform brace angles in diagrid systems on circular plans. In parallel to the structural advantages specified by Moon, the proposed technique offers additional improvements, such as reduced costs, ease of assembly and a simple manufacturing process.

In a conventional circular-planned diagrid system, only three parameters are used: "cross-section curve", "number of braces" and "elevation difference of nodes." These parameters apply even to Taut's *Glashaus*. Taut used a concave arc as a cross-section, had 14 braces and set the elevation difference of nodes at approximately 170 cm. The increasing values for all three parameters would make the final diagrid structure finer. The conventional circular plan diagrid technique follows a relatively simple geometric formation. First, for creating the surface division patterns, the target surface (Fig. 7a) is intersected with successive horizontal planes from preferred node elevations (Fig. 7b). Subsequently, intersection curves are divided to half the number of braces on each level (Fig. 7c). To create the braces, points on every other level are translated in half the interval length (Fig. 7c) and lastly, every point is connected to its neighbouring six points, i.e. two upper and two lower, except the ones in the bottom and top levels, in which only two connections are made (Fig. 7d). Horizontal connections are kept, depending on the structural necessities or design intentions (Fig. 7e).

Grasshopper is used for developing the algorithm (www.grasshopper3d.com) because of the geometrical characteristics of the technique and the software's powerful parametric capabilities in generating variations through parameters
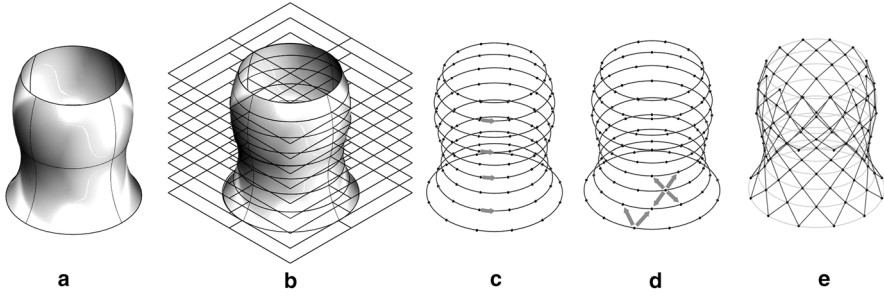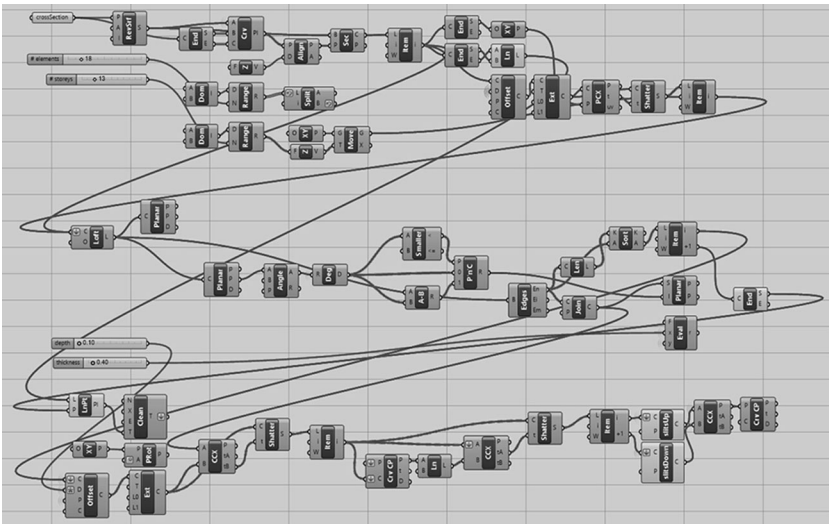
Fig. 7 Conventional diagrid formation

Fig. 8 Screenshot of Grasshopper algorithm

(Fig. 8). Initially, the algorithm computes the surface division pattern like a conventional diagrid technique. Then, instead of generating braces on each level separately, a holistic approach is taken and the sets of braces having the same direction are regarded as single ribs. To attain the ribs, a plane is created for each set of braces from the set's top, middle and bottom points. Circular-plan geometry of the surface guarantees the non-linear formation of points and creates a plane in all possible variations. The planes are then intersected with the target surface, generating a single intersection curve with every plane, and finally curves are offset with a specified distance to create the planar ribs. The ribs are further processed for the computation of the slits at every intersection (Fig. 9). The major differentiation of the technique from a conventional diagrid system is the ribs' linear to planar transformation.

The intersecting quality of the ribs makes an almost solid joint and, if the CNC machines allow the manufacture of the ribs as single pieces, the procedure is completed. If the dimensional limitations do not allow the production of pieces as
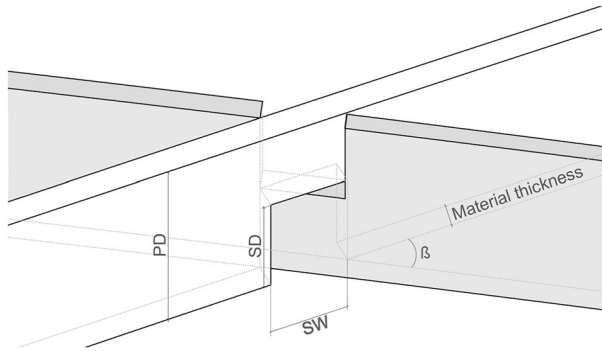
Fig. 9 Slit formation detail

wholes, according to the CNC machine's dimensions or the designer's intentions, ribs can be subdivided into smaller segments. As the ribs and subdivisions are planar, the segments can easily be connected via butt joints and filler plates. This also enables axial continuity of elements to be preserved in a torsion-free manner.

The slits are formed by a simple mathematical operation. Slit depths (*SD*) are equal to half of the piece depths (*PD*) to create the intersecting quality of pieces, and slit widths (SW) are calculated with the following equation, where ß is equal to the angle between elements:

$$SW = \text{Material thickness} \times (\operatorname{cosec} \beta + \cot \beta) \text{ where } SD = PD/2.$$

With the proposed technique, the connection points are shifted to mid-brace zones through slits and the joints are turned into generic and simple connections with their only variable being the slit widths. The slits and their connection method are introduced as an innovative solution to diagrid joints. By intersecting the pieces in a non-perpendicular and bidirectional fashion, complex nodes of the diagrid system are simplified. Additionally, the curvilinear quality of circular-planned diagrid systems creates varying directions of slits, resulting in an interlocking quality of the structure without any need for additional fixtures.

Implementation and Assessment

To demonstrate the potentials of the technique, an implementation is realized and three variations of the technique with different cross-sections are generated. For the implementation, a circular-planned diagrid system with a cross-section of a 1400 mm radius convex arc is manufactured and the number of ribs is 14 in both directions (Fig. 10). The diameter of the structure is 1150 mm, with a height of 1100 mm, and 4 mm thick medium-density fibreboard (MDF) is used. However, it should be noted that the technique is suitable for any planar material, and thickness, size or number of elements can also be varied, generating different implementations. The assembly consists of 140 pieces with 5 subdivisions on each rib. Butt joints with 15 mm × 60 mm × 1 mm sized steel filler plates are used for reconnecting
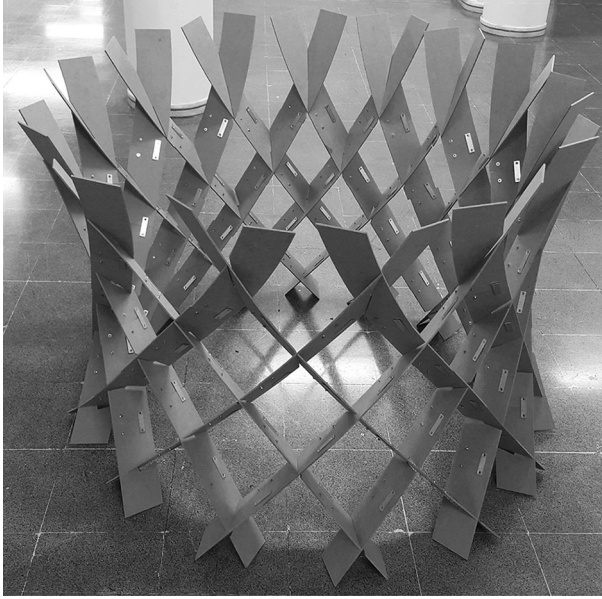
Fig. 10 Assembled structure

Fig. 11 Intersection of pieces and butt joint detail with filler plates

subdivisions (Fig. 11). Contrary to the excessive number of pieces of the structure, the assembly process is simple. There exist two types of pieces: inner and outer, and by connecting conjugate pieces to each other initial elements can be formed (Fig. 12). The fragmented nature of the ribs allows for the assembly process in a linear fashion, starting from the ground level elements and connecting every level of elements on the already assembled structure via filler plates.

The pieces can be subdivided from any point, and the number of subdivisions may vary. Although the CNC machine's dimensions enabled the manufacture of whole pieces, the pieces are subdivided in between slits to test the capacity of the
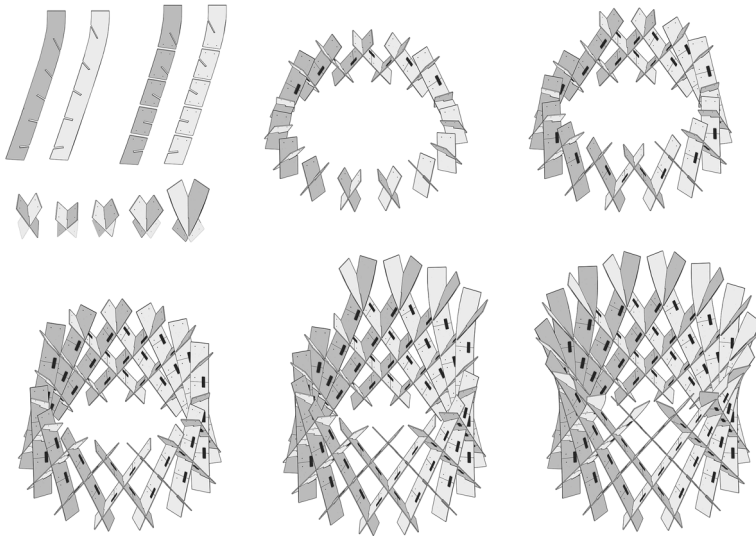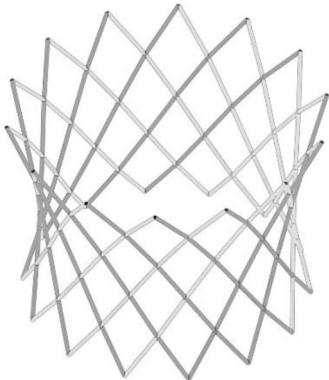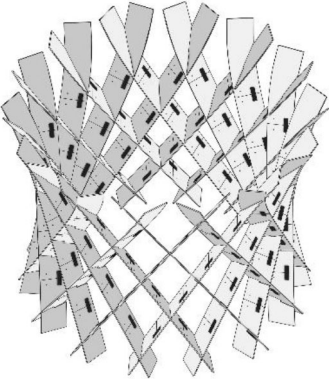
Fig. 12 Assembly sequence

butt connection technique. Butt joints performed effectively, as the dimensions of the digital model and physical structure are within only 4% differentiation. While the diameter of the digital model is 1150 mm, the diameter of the physical construct is measured as 1200 mm and the height has decreased from 1100 to 1075 mm. If a more precise production is aimed at, the structure may also be fixed from several points at the base.

In Table 1, a comparative assessment of the proposed technique and the conventional one can be found. For the comparison, the same cross-section, dimensions and parameters are used. Physical dimensions and material type are not applicable for the conventional system, as the implementation is not manufactured. The proposed technique is found to be advantageous in terms of inclination angles and total rib length, with the only disadvantage being varying intervals of node elevations. The constancy of inclination angles creates a straight and continuous bracing system and it is assumed that this would help to overcome torsional forces among braces. In terms of final surface characteristics, conventional circular-planned diagrid systems discretize surface continuity, due to straight braces. It can be improved by using curvilinear braces, but this would highly increase the complexity and costs of manufacturing (Boake 2014: 36). Instead, the proposed technique can produce a finer surface quality by using planar elements. In the proposed technique, each set of braces is oriented in a unidirectional fashion and the curvature of the outer surface can be preserved by manufacturing from sheet material with conventional CNC machines.

To show further possibilities of the technique, variations with three different cross-sections are presented in both diagrid systems (Fig. 13). The cross-sections include a straight vertical line, a jagged line and an S-shaped curve, and the cross-

Table 1 Dimensions and values of two alternatives

Technique used		
	Conventional diagrid system	Proposed technique
Cross-section	Convex arc with 1400 mm radius	
Dimensions (Digital model)	1150 mm (d)–1100 mm (h)	
Dimensions (Physical model)	NA	1200 mm (d)–1075 mm (h)
Number of ribs	28 (14 × 2)	
Number of pieces	140	
Inclination angles (from bottom to top)	54.37°	58.37°
	56.19°	
	57.34°	
	57.77°	
	57.53°	
	56.76°	
Node elevations (from bottom to top)	0 mm	
	220 mm	72 mm
	440 mm	252 mm
	660 mm	474 mm
	880 mm	717 mm
	1100 mm	924 mm
Total rib length	34.47 m (123 cm each rib)	31.73 m (113 cm each rib)

sections are shown beside the side views of each variation. The same number of ribs is generated in all variations. Exemplary pieces of each assembly are also presented with no subdivisions. It is shown that the technique can generate the system through cross-sections with different formal qualities. The generated parts of three variations show major differentiations as an outcome of changing curvatures of the surfaces.

As an outcome of circular plans and equidistant node elevations, the braces always have an S-shape. Therefore, the conventional diagrid system cannot preserve axial continuity even if the cross-section is a straight line. On the contrary, by
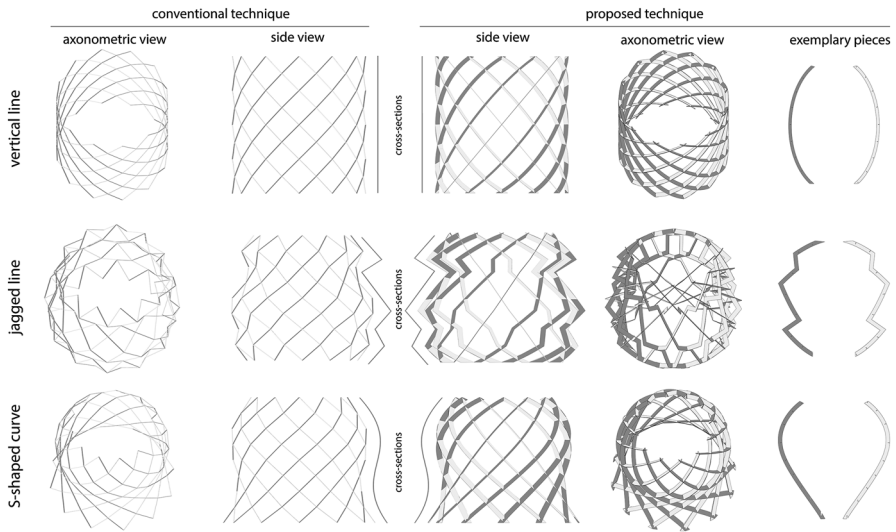
Fig. 13 Variations of both techniques with different cross-sections

having planarity in braces, the proposed technique conserves axiality throughout the system. There do not exist major differences between the two techniques in the first variation, because of the cross-sections' simple geometric qualities. In the second variation, it can be clearly seen that in the conventional diagrid system, the amount of detail is highly dependent on the number of braces. Although the jagged line has five corners, the system divides the cross-section into vertically equidistant points and creates the braces accordingly. In the second variation, the number of braces is not accurate enough to convey the geometry of the cross-section. The horizontal planes should coincide with the vertices to attain the desired form. On the other hand, the proposed technique can govern the geometry successfully, regardless of the characteristics of the cross-section or number of braces in the system. In the third variation with a curvilinear cross-section, conventional diagrid technique discretizes continuity of the braces and the surface curvature, whereas the proposed technique can generate the exact geometry following the surface curvature continuously, whether it is jagged or curved.

Discussion and Future Work

The proposed technique has several advantages. The most significant is the transformation of the linear elements of the diagrid system into planar ones, allowing the fabrication of pieces through sheet materials. The ability to construct a complex geometry from planar elements is highly beneficial in terms of material requirements and manufacturing ease (Dunn 2012:88). Additionally, discrete linear elements with varying directions in conventional diagrid systems are transformed in a continuous and unidirectional fashion

Because of the varying angles of linear elements, one of the most criticized features of the diagrid system is the complexity and high costs of the joint details (Moon 2005: 103). In the proposed technique, points of brace intersections have been shifted from the braces' end points to mid points, and this has simplified the joint detail of the diagrid system. The only variation among connections is the varying slit widths. This feature also allows a simple 2-axis manufacturing technology like routers, laser cutters or water jets, to be used for fabrication.

The 2-axis CNC manufacturing processes inevitably generate pieces with vertical sides on all edges. This results with a somehow imperfect connection detail for the pieces, due to their angular intersections. The outer surface of the structure also becomes discontinuous because of the perpendicular quality of the sides. Therefore, although the planar and intersecting quality of pieces creates a highly advantageous property, they may also be problematic depending on the required conditions of the end product. If the surface quality of the final structure is one of the main concerns of the manufacturing process, a 2-axis CNC machine will not be suitable; instead, a 5-axis CNC machine would be appropriate. This would enable varying edge orientations both on the inner and outer surfaces of the elements. This should result in a more intricate and solid connection. Even if the manufacturing process for this type of construction is more intensive and costly, the final surface quality and structural characteristics of the assembly would be much more efficient.

The technique can also be improved both formally and structurally by transforming the ribs into multi-layered formations (Fig. 14). Instead of having a rib with a single layer, multiple layers with minute differentiations that are dependent on the surface form would create a discretized three-dimensional form. Each constituent of the ribs can still be manufactured by a 2-axis CNC machine, but several two-dimensional layers would create a three-dimensional rib and it would be more akin to the actual shape. The use of multiple layers would also invalidate the need for filler plates, as the possible interlacing patterns of the layers would enable the connection of subdivisions through bolts. The multi-layered formation can be regarded as an intermediary step for the transition from a 2-axis to a 5-axis CNC machine.
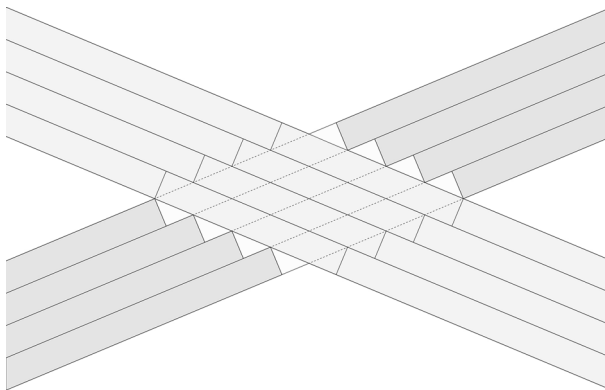
Fig. 14 Multi-layered formation

As the current study mainly focuses on the geometrical characteristics of the diagrid systems and has the same principles as conventional diagrid systems, it is limited with respect to the computational aspects of the technique. Structural characteristics of the technique should be studied via larger implementations and finite element analysis (FEA) methods to understand its potentials and limits. Because it shares very close geometrical characteristics with the conventional diagrid systems, the proposed technique is expected to perform effectively with appropriate detailing and dimensions.

Conclusion

This study is an effort to devise a generic technique for circular-planned diagrid system details. Although, within the scope of this study, the technique is implemented once, the algorithm can produce the pieces of any cross-section in any number, thickness or number of subdivisions, as shown in the variations. The generation of interlocking planar pieces regardless of the cross-section, makes the technique highly adaptable. The simple and generic connection detail of the ribs and the ease of assembly process are the other major advantages of the technique. It is expected that the presented algorithm and technique would be useful for design offices and students of architecture to create and prototype circular-planned diagrid systems.

Appendix

Author	Title	Patent no.	Date	Country	Surface type
Vladimir Grigoryevich Shukhov	Lattice towers in the form of hyperboloid of rotation with rectilinear forming lines, which connect ring bases with the reinforcement by intermediate rings	No. 1896	March 12, 1899	Russia	C
Ichiro Nozawa	Arcuate truss	US1976188 (A)	February 5, 1924	United States	C
Hartmann Louis Auguste	New type of wall and constructions including application	FR789499 (A)	October 9, 1934	France	C
Butts Alfred M	Structural units of grid-like construction providing supports for walls, floors, or the like	US2534852 (A)	October 29, 1935	United States	P
O'c Parker Brooks	Truss structure and supporting column	US2709975 (A)	December 19, 1950	United States	P

Author	Title	Patent no.	Date	Country	Surface type
Fentiman Arthur E	Wall construction	US2976968 (A)	July 6, 1955	United States	P
Kiewitt Gustel R	Roof structure	US2985984 (A)	March 28, 1961	United States	P
Schmidt Alexander, Uebelguenn Otto and Klasen Theobald	Structural latticework support	GB897899 (A)	May 30, 1961	Great Britain	P
Caldwell Alfred	Indoor-outdoor swimming pool and enclosure therefore	US3094708 (A)	May 30, 1962	United States	C
Lussky Frederic G	Large-diameter framed structure	US3603051 (A)	June 25, 1963	United States	C
Waters Terrance J	Hyperboloid buildings	US3618277 (A)	September 7, 1971	United States	C
Rosenblatt Joel H	Hyperbolic tower structure	US3922827 (A)	November 9, 1971	United States	C
Giovanni Simone	Modular reticular bearing structure for domed shelters	US4194327 (A)	December 2, 1975	United States	C
Hipkins Jim l	Method of making a reinforced preformed building wall	US4597813 (A)	July 1, 1986	United States	P
Iwata Mamoru; Nagai Eiichiro; Hayashi Kenichi	Structure with buckling constraint diagonal column as element	JPH09317001 (A)	December 9, 1997	Japan	P
Sun Gongmin, Meng Xiangyi, Ma Xiujuan	Composite diagrid for cantilever screen mesh	CN 2316047	April 28, 1999	China	P
Tripsianes Lazaros C	Dome structure	WO0063503 (A1)	October 26, 2000	United States	C
Takeshima Ichiro, Kamoshita Tsutomu	Building structural body	JP3811708 (B1)	August 23, 2006	Japan	P
Murazaki Shuji	Diagonal brace structure	JP2008267108 (A)	November 6, 2008	Japan	P
Zhou Xuhong, He Yongjun	Cylindrical-surface intersected three-dimensional truss system giant network structure with single-layer latticed intersected cylindrical shell substructure	CN 101709590	May 19, 2010	China	C

Author	Title	Patent no.	Date	Country	Surface type
U Young Kyu, Lee Jong Hyock, Kim Sang Dae	Diagrid structure	KR20100088373 (A)	August 9, 2010	South Korea	D
Lee Dong Kyu	Diagrid joining apparatus	KR20100130371 (A)	December 13, 2010	South Korea	D
Hatamoto Sai, Higuchi Satoshi	Diagonal column frame	JP2011026835 (A)	February 10, 2011	Japan	P
Yang Lichao	Diagrid sleeve structure for restricting connection of high strength concrete nodes	CN 102031829	April 27, 2011	China	D
Choi Sung Mo, Lee Seong Hui and Kim Young Ho	Segment of a diagrid system and a construction method of a concrete filled diagrid node using the same for enhancement of strength	KR20110126479 (A)	November 23, 2011	South Korea	D
Choi Sung Mo, Lee Seong Hui and Kim Young Ho	Joint structure of concrete filled steel pipe bracings in a diagrid system and a construction method thereof	KR20110126480 (A)	November 23, 2011	South Korea	D
Hou Xiaomeng, Zhou Wei, Zhang Jianxin, Cao Zhenggang, Hu Haibo, Zheng Wenzhong	Diagrid sleeve structure for restricting connection of high strength concrete nodes	CN 102031829 (B)	April 18, 2012	China	D
Kim Jong Ho, Kim Tae Jin, Kang Dae Eon and Cho Jeong Hyeok	Steel diagrid joint structure and a construction method thereof using a casting method	KR1020120069296 (A)	June 28, 2012	South Korea	D
Lee Dong Kyu, Kim Jin Ho	Diagrid structure of building	KR20120075283 (A)	July 6, 2012	South Korea	D
Lee Dong Kyu	Diagrid joining structure	KR20120122506 (A)	November 7, 2012	South Korea	D
Sitque David Lee	Diagrid joining structure	KR101206792 (B1)	November 30, 2012	South Korea	D

Author	Title	Patent no.	Date	Country	Surface type
Lee Dong Kyu	Tube for strengthening connection and diagrid joining structure using the same	KR20130005020 (A)	January 15, 2013	South Korea	D
Lee Dong Kyu, Kim Jin Ho	Diagrid joining structure, capable of effectively distributing a load which is applied to a brace unit	KR101274992 (B1)	June 17, 2013	South Korea	D
Lee Dong Kyu	Diagrid joining structure	KR20140087199 (A)	July 9, 2014	South Korea	D
Lee Joo Ho, Jang Sung Hoon, Jeon Hyun Soo, Kim Sung Yoon	The pipe diagrid joint structure and the compression test structure using the same	KR20160089924 (A)	June 21, 2016	South Korea	D

P planar surfaces, *C* curved surfaces, *D* details

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